

# TMD cross-section factorization for dijet production at the EIC

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U N I V E R S I D A D  
**COMPLUTENSE**  
M A D R I D



# Outline

## **Dijet production**

- Kinematic region
- Cross-section factorization
- New TMD Soft Function

## **$\zeta$ -prescription & evolution**

## **Plots**

Based on the work published by  
Rafael F. del Castillo, Miguel G. Echevarría, Yiannis Makris & Ignazio Scimemi  
<https://arxiv.org/abs/2008.07531v4>

...and following work soon to be published

# Motivation

- Gluon transverse momentum dependent distributions (TMDs) are difficult to access due to the lack of clean processes where the factorization of the cross-section holds and incoming gluons constitute the dominant effect.

- We consider two processes which are presently attracting increasing attention

$$\ell + h \rightarrow \ell' + J_1 + J_2 + X$$

Dijet

$$\ell + h \rightarrow \ell' + H + \bar{H} + X$$

Heavy-meson

Dominguez, Xiao, Yuan, 2013

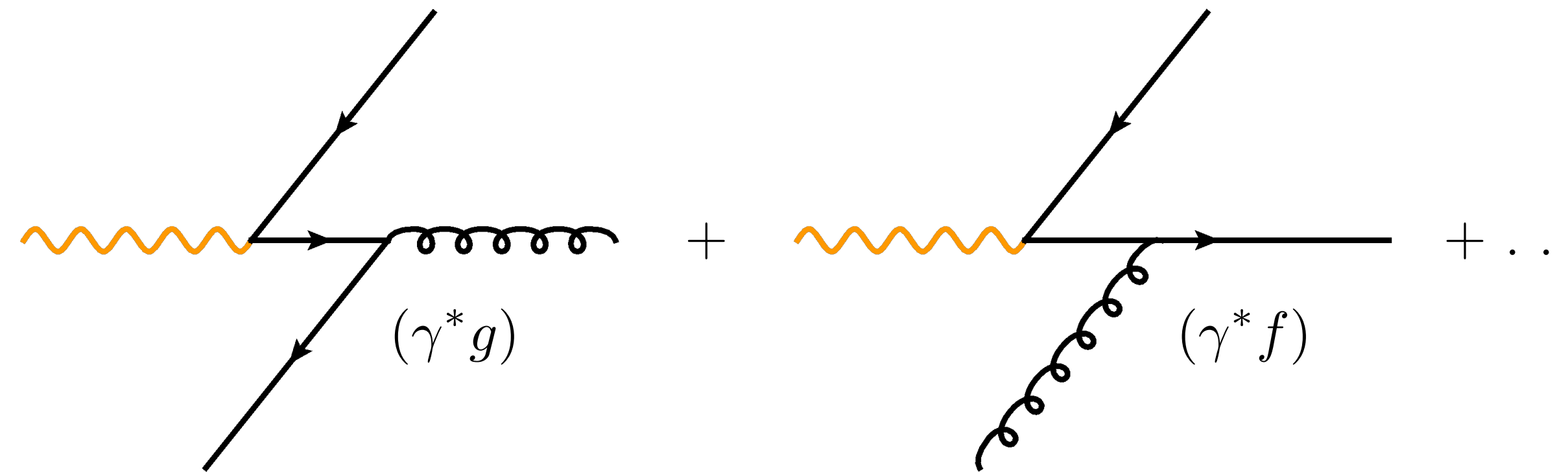
Boer, Brodsky, Mulders, Pisano, 2011

Zhang, 2017

# Dijet production

$$\ell + h \rightarrow \ell' + J_1 + J_2 + X$$

dijet LO process:



- Sensitive of polarized and unpolarized TMDPDFs
- Experimental observation should be possible in the future EIC Page, Chu, Aschenauer, 2020
- Jets here described have  $p_T \in [2, 40]$  GeV and are found in the central rapidity region
- Factorization within SCET

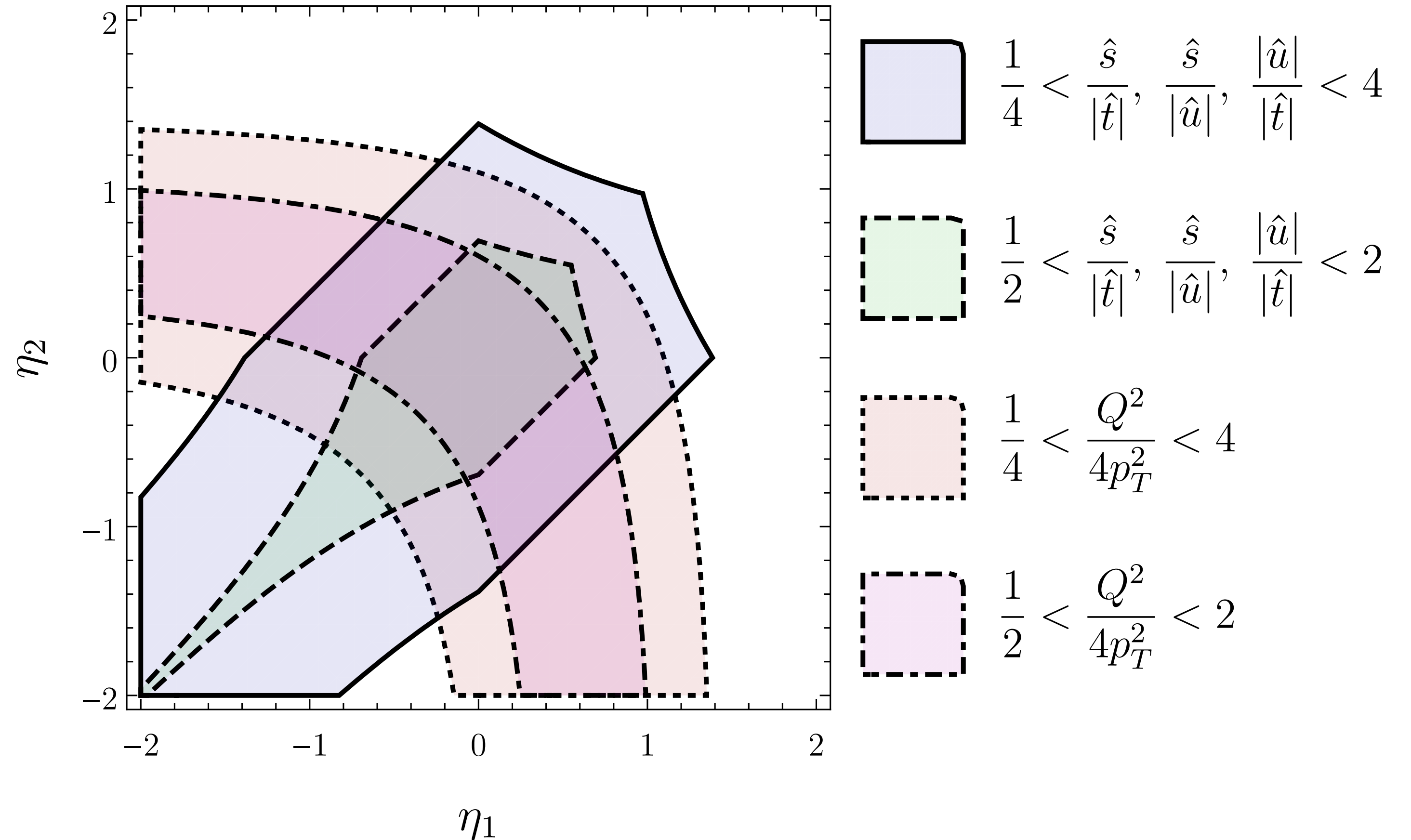
# Kinematic region

## Dijet production

$$\mathbf{r}_T = \mathbf{p}_{1T} + \mathbf{p}_{2T}$$

$$p_T = \frac{|\mathbf{p}_{1T}| + |\mathbf{p}_{2T}|}{2}$$

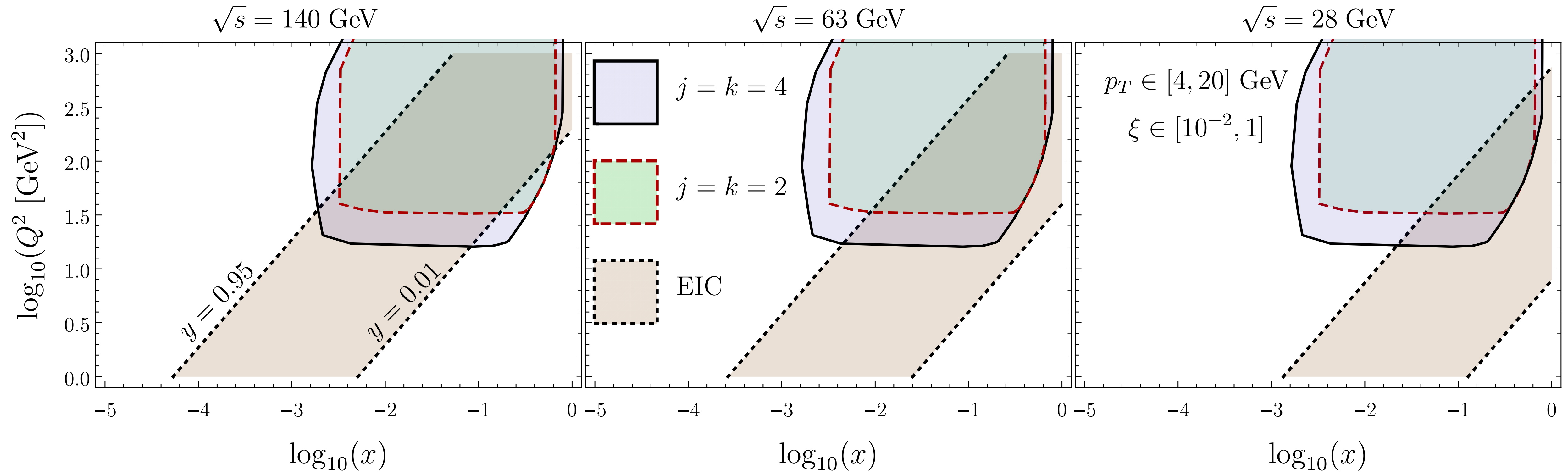
$$|\mathbf{r}_T| \ll p_T$$



Factorization holds for  $|\mathbf{r}_T| \ll p_T$  and for the central rapidity region

# Kinematic region vs EIC coverage

## Dijet production



$$\frac{1}{j} < \frac{\hat{s}}{|\hat{t}|}, \frac{\hat{s}}{|\hat{u}|}, \frac{|\hat{u}|}{|\hat{t}|} < j \quad \frac{1}{k} < \frac{Q^2}{4p_T^2} < k$$

Overlapping increases with higher beam energies

# Cross-section factorization

## Dijet production

$$\frac{d\sigma}{dx d\eta_1 d\eta_2 dp_T d\mathbf{r}_T}$$

- $x$  Bjorken variable
- $\eta_i$  jet pseudorapidity
- $p_T$  transverse momentum
- $\mathbf{r}_T$  transverse momentum imbalance

We measure over

$$(\gamma^* g) \quad \frac{d\sigma(\gamma^* g)}{dx d\eta_1 d\eta_2 dp_T d\mathbf{r}_T} = \sum_f H_{\gamma^* g \rightarrow f \bar{f}}^{\mu\nu}(\hat{s}, \hat{t}, \hat{u}, \mu) \int \frac{d^2\mathbf{b}}{(2\pi)^2} \exp(i\mathbf{b} \cdot \mathbf{r}_T) F_{g,\mu\nu}(\xi, \mathbf{b}, \mu, \zeta_1) \\ \times S_{\gamma g}(\mathbf{b}, \eta_1, \eta_2, \mu, \zeta_2) \left( C_f(\mathbf{b}, R, \mu) J_f(p_T, R, \mu) \right) \left( C_{\bar{f}}(\mathbf{b}, R, \mu) J_{\bar{f}}(p_T, R, \mu) \right)$$

$$(\gamma^* f) \quad \frac{d\sigma^U(\gamma^* f)}{dx d\eta_1 d\eta_2 dp_T d\mathbf{r}_T} = \sigma_0^{fU} \sum_f H_{\gamma^* f \rightarrow g \bar{f}}^U(\hat{s}, \hat{t}, \hat{u}, \mu) \int \frac{d^2\mathbf{b}}{(2\pi)^2} \exp(i\mathbf{b} \cdot \mathbf{r}_T) F_f(\xi, \mathbf{b}, \mu, \zeta_1) \\ \times S_{\gamma f}(\mathbf{b}, \zeta_2, \mu) \left( C_g(\mathbf{b}, R, \mu) J_g(p_T, R, \mu) \right) \left( C_{\bar{f}}(\mathbf{b}, R, \mu) J_{\bar{f}}(p_T, R, \mu) \right)$$

# Unpolarized & linearly polarized cross-section

## Dijet production

$$F_g^{\mu\nu}(\xi, \mathbf{b}) = f_1(\xi, \mathbf{b}) \frac{g_T^{\mu\nu}}{d-2} + h_1^\perp(\xi, \mathbf{b}) \left( \frac{g_T^{\mu\nu}}{d-2} + \frac{b^\mu b^\nu}{b^2} \right)$$

$$H_{\gamma^* g \rightarrow f \bar{f}}^{\mu\nu} = \sigma_0^{gU} H_{\gamma^* g \rightarrow f \bar{f}}^U \frac{g_T^{\mu\nu}}{d-2} + \sigma_0^{gL} H_{\gamma^* g \rightarrow f \bar{f}}^L \left( -\frac{g_T^{\mu\nu}}{d-2} + \frac{v_{1T}^\mu v_{2T}^\nu + v_{2T}^\mu v_{1T}^\nu}{2 v_{1T} \cdot v_{2T}} \right)$$

Unpolarized cross-section

$$\begin{aligned} \frac{d\sigma^U(\gamma^* g)}{dx d\eta_1 d\eta_2 dp_T d\mathbf{r}_T} &= \sigma_0^{gU} \sum_f H_{\gamma^* g \rightarrow f \bar{f}}^U(\hat{s}, \hat{t}, \hat{u}, \mu) \int \frac{d^2 \mathbf{b}}{(2\pi)^2} \exp(i\mathbf{b} \cdot \mathbf{r}_T) f_1(\xi, \mathbf{b}, \mu, \zeta_1) \\ &\quad \times S_{\gamma g}(\mathbf{b}, \zeta_2, \mu) \left( C_f(\mathbf{b}, R, \mu) J_f(p_T, R, \mu) \right) \left( C_{\bar{f}}(\mathbf{b}, R, \mu) J_{\bar{f}}(p_T, R, \mu) \right) \end{aligned}$$

Linearly polarized  
cross-section

$$\begin{aligned} \frac{d\sigma^L(\gamma^* g)}{dx d\eta_1 d\eta_2 dp_T d\mathbf{r}_T} &= \sigma_0^{gL} \sum_f H_{\gamma^* g \rightarrow f \bar{f}}^L(\hat{s}, \hat{t}, \hat{u}, \mu) \int \frac{d^2 \mathbf{b}}{(2\pi)^2} \exp(i\mathbf{b} \cdot \mathbf{r}_T) h_1^\perp(\xi, \mathbf{b}, \mu, \zeta_1) \\ &\quad \times \frac{s_b^2 - c_b^2}{2} S_{\gamma g}(\mathbf{b}, \zeta_2, \mu) \left( C_f(\mathbf{b}, R, \mu) J_f(p_T, R, \mu) \right) \left( C_{\bar{f}}(\mathbf{b}, R, \mu) J_{\bar{f}}(p_T, R, \mu) \right) \end{aligned}$$



# New soft function

$n$  - incoming beam direction

$v_1$  - jet 1 direction

$v_2$  - jet 2 direction

Soft  
function

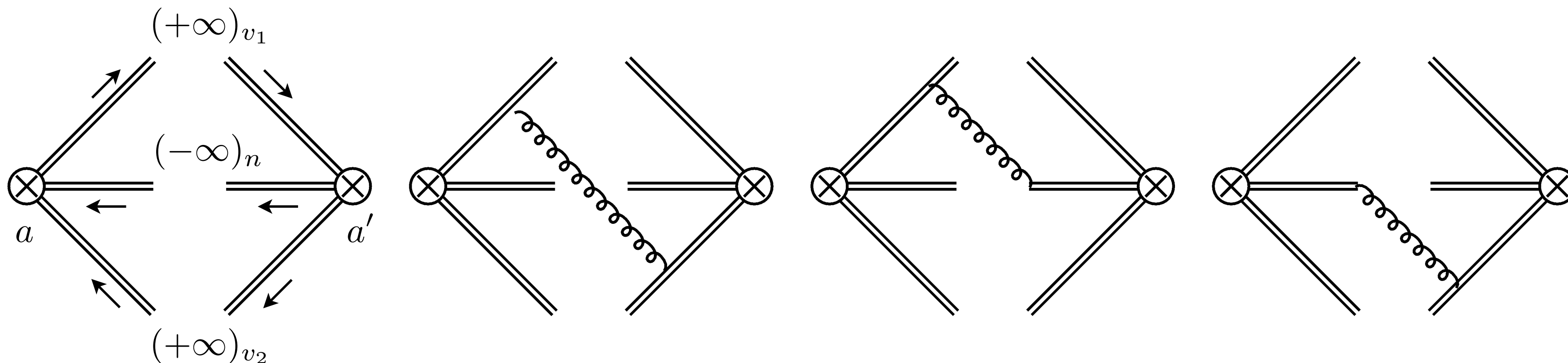
$$\hat{S}_{\gamma g}(\mathbf{b}) = \frac{1}{C_F C_A} \langle 0 | \mathcal{S}_n^\dagger(\mathbf{b}, -\infty)_{ca'} \text{Tr} \left[ S_{v_2}(+\infty, \mathbf{b}) T^{a'} S_{v_1}^\dagger(+\infty, \mathbf{b}) \right. \\ \left. \times S_{v_1}(+\infty, 0) T^a S_{v_2}^\dagger(+\infty, 0) \right] \mathcal{S}_n(0, -\infty)_{ac} | 0 \rangle$$

$$\hat{S}_{\gamma f} = \hat{S}_{\gamma g}(n \leftrightarrow v_2)$$

Wilson  
lines

$$S_v(+\infty, \xi) = P \exp \left[ -ig \int_0^{+\infty} d\lambda v \cdot A(\lambda v + \xi) \right] \quad S_v^\dagger(+\infty, \xi) = P \exp \left[ ig \int_0^{+\infty} d\lambda \bar{v} \cdot A(\lambda \bar{v} + \xi) \right]$$

$$S_n(+\infty, \xi) = \lim_{\delta^+ \rightarrow 0} P \exp \left[ -ig \int_0^{+\infty} d\lambda n \cdot A(\lambda n + \xi) e^{-\delta^+ \lambda} \right] \quad \delta - \text{regulator !!!}$$

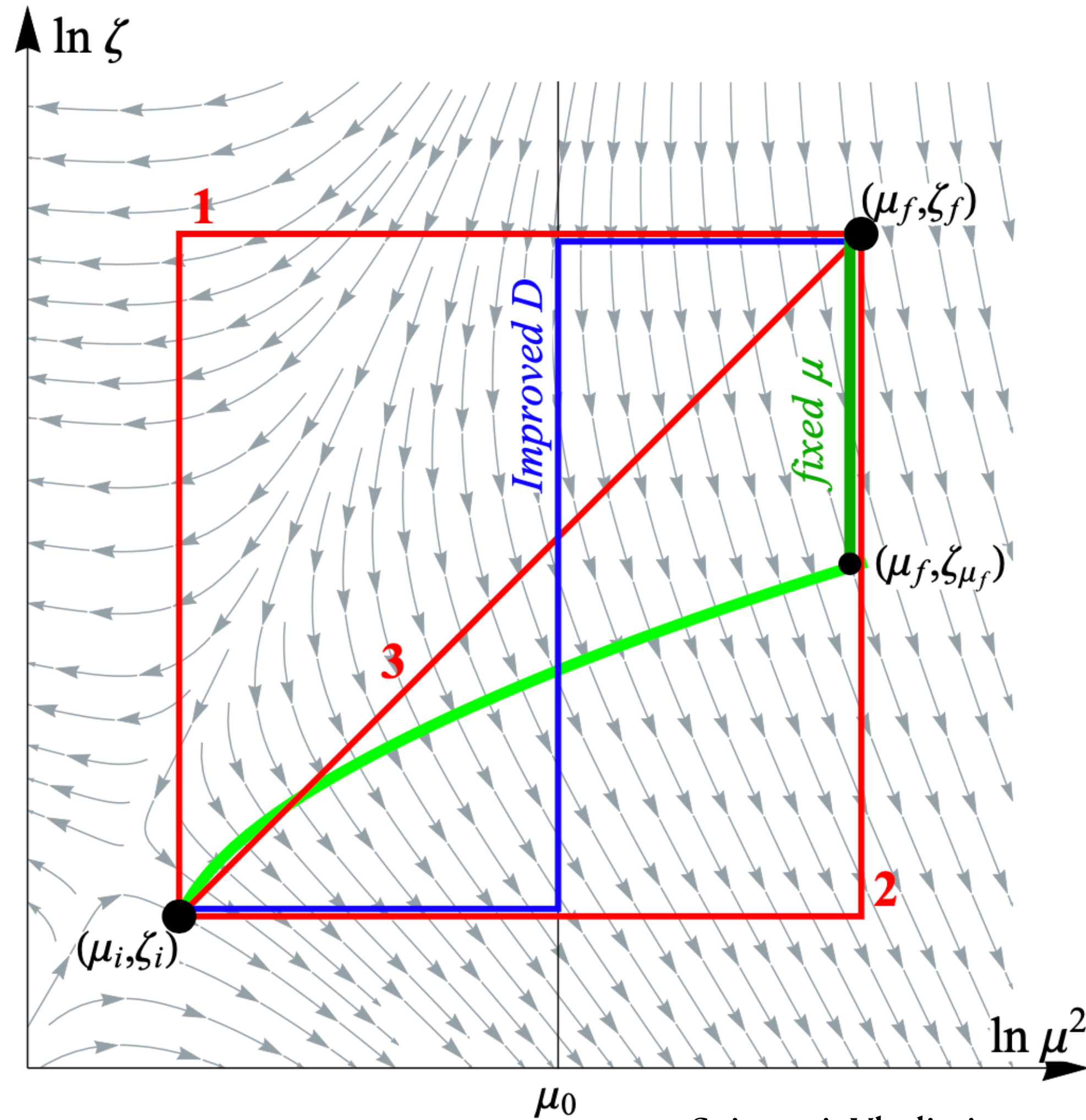


Echevarría, Scimemi, Vladimirov, 2016

+ virtual diagrams  
at one-loop order...

# Evolution, double-scale evolution

fixed  $\mu$  evolution



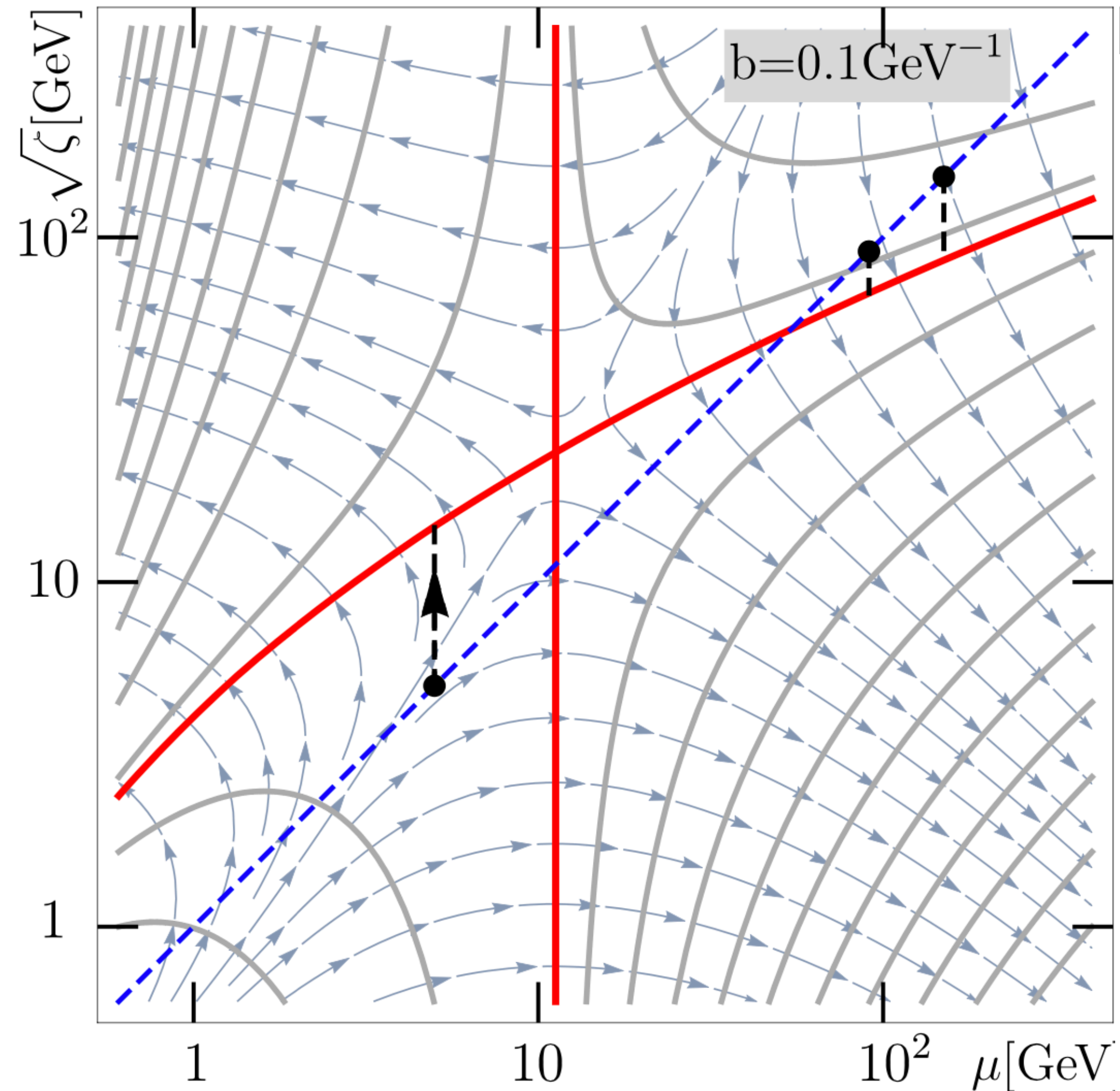
Describe evolution of functions depending on two scales



$\mu$ scale	Renormalization scale	$\varepsilon$ regulator (DR)
$\zeta$ scale	Rapidity scale	$\delta$ regulator

Scimemi, Vladimirov, 2018  
Scimemi, Vladimirov, 2020

# Evolution, $\zeta$ -prescription



Scimemi, Vladimirov, 2018  
Scimemi, Vladimirov, 2020

## fixed $\mu$ evolution

Evolution kernel is given by

$$S(\mathbf{b}; \mu_f, \zeta_{2,f}) = \exp \left[ \int_P (\gamma_S(\mu, \zeta_2) d \ln \mu - \mathcal{D}_S(\mu, b) d \ln \zeta_2) \right] S(\mathbf{b}; \mu_0, \zeta_{2,0})$$

$$\left. \begin{aligned} \frac{d}{d \ln \mu} S(\mathbf{b}; \mu, \zeta) &= \gamma_S(\mathbf{b}; \mu, \zeta) S(\mathbf{b}; \mu, \zeta) \\ \frac{d}{d \ln \zeta} S(\mathbf{b}; \mu, \zeta) &= -\mathcal{D}_S(\mathbf{b}, \mu) S(\mathbf{b}; \mu, \zeta) \end{aligned} \right\} \longrightarrow \boxed{\nabla F = \mathbf{E} F}$$

$$\mathbf{E} = (\gamma_S(\mathbf{b}, \mu, \zeta), -\mathcal{D}_S(\mathbf{b}, \mu))$$

Equipotential (null-evolution) line is given by  $\gamma_S = 2\mathcal{D}_S \frac{d \ln \zeta_\mu}{d \ln \mu^2}$

**gluon channel solution**  $\zeta_{2,\mu}^{\gamma^*g}(\mathbf{b}, \mu) = \left( \frac{\mu}{\mu_0} \right)^{\frac{2C_F}{C_A}} \zeta_{2,0} e^{v_S(\mathbf{b}, \mu)}$  ↗ perturbative

$$R_S((\mu_0, \zeta_{2,0}) \rightarrow (\mu_f, \zeta_f)) = \left( \frac{\zeta_f}{\zeta_{2,\mu}(\mathbf{b}, \mu_f)} \right)^{-D_S(\mathbf{b}, \mu_f)}$$

Saddle point

# Plots for phenomenological analysis

<https://teorica.fis.ucm.es/artemide/>  
<https://github.com/vladimirovalexey/artemide-public>

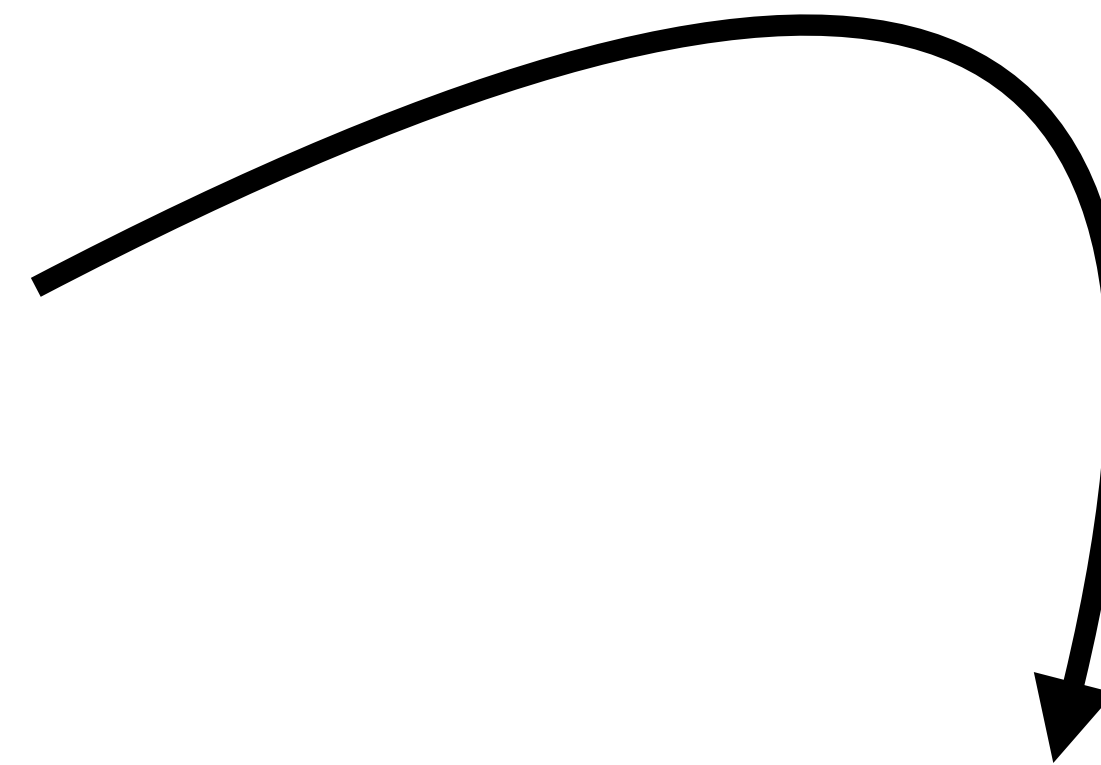
- We use **arTeMiDe** to obtain the plots
- TMDPDF and TMDFF structure and evolution is included arTeMiDe
- SF double-scale evolution and jet functions included as new modules

$$p_T = 20 \text{ GeV} \quad (p_T \sim Q)$$

$$\sqrt{s} = 140 \text{ GeV}$$

Integrated over  $x$

Central rapidity region



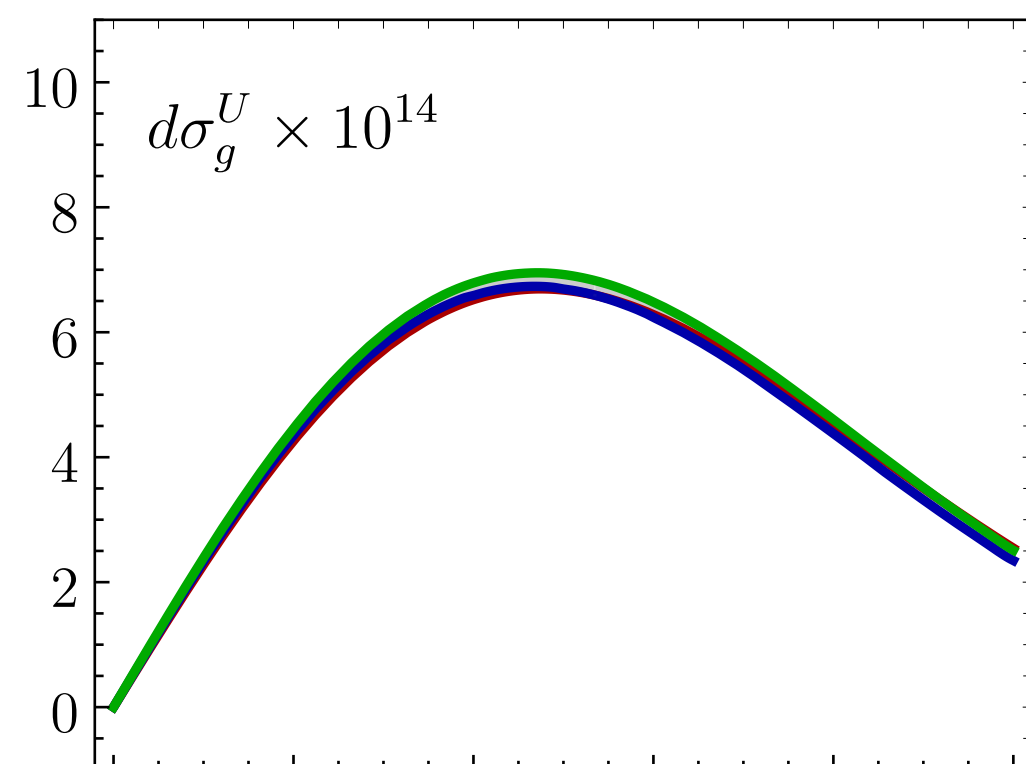
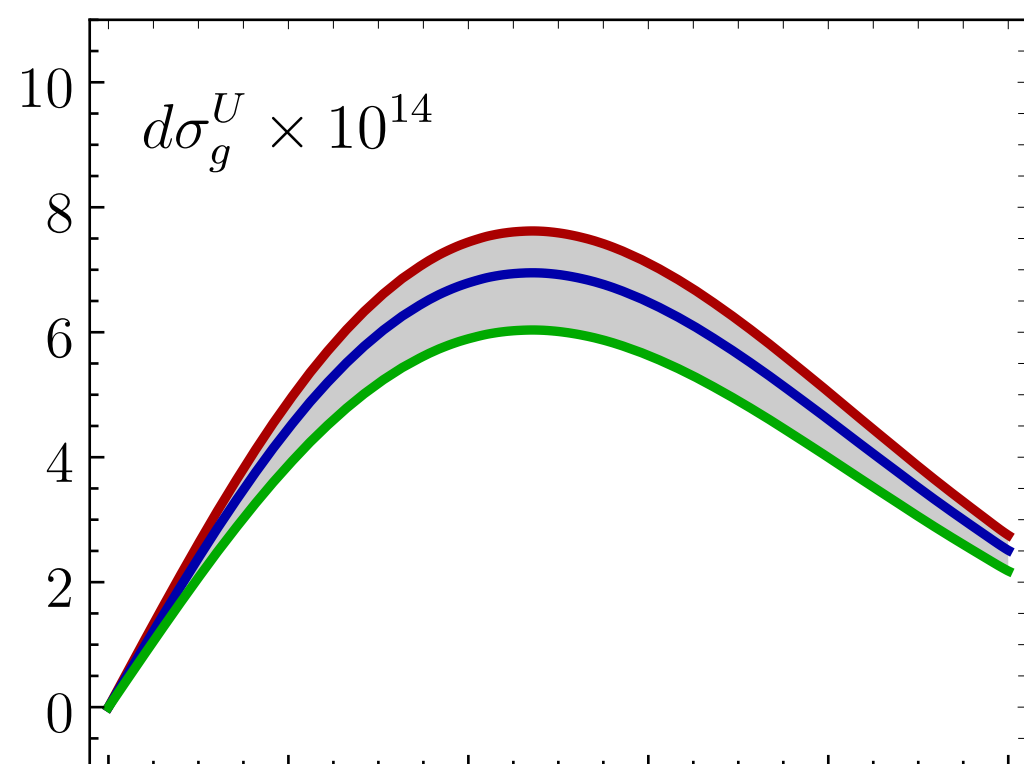
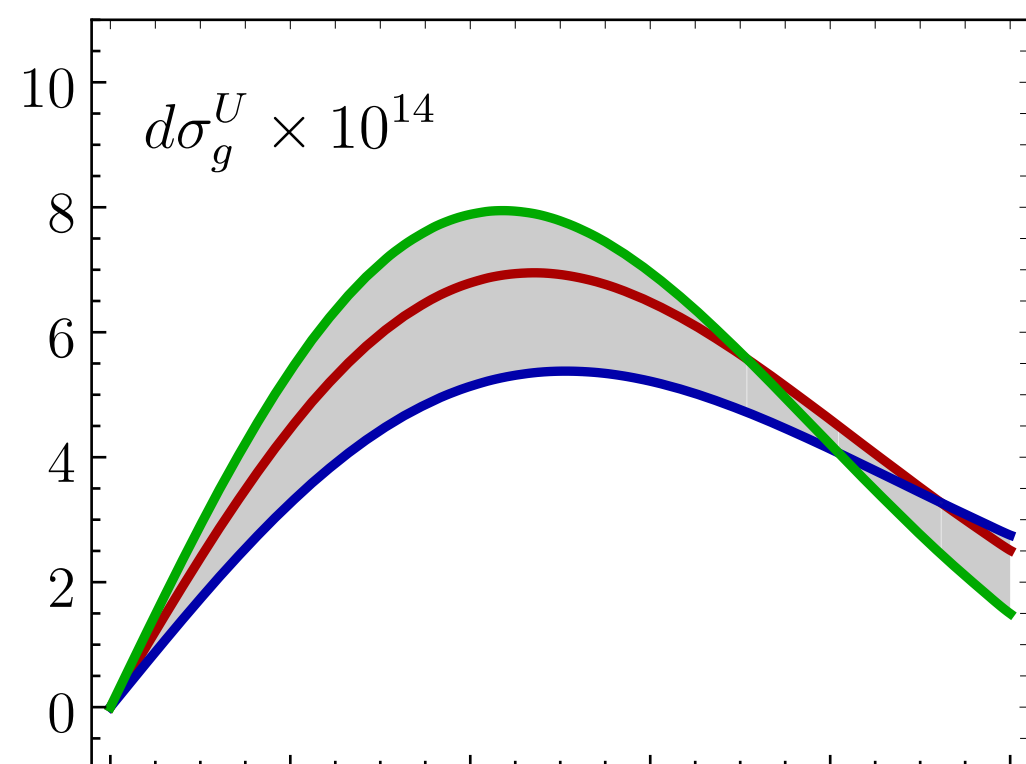
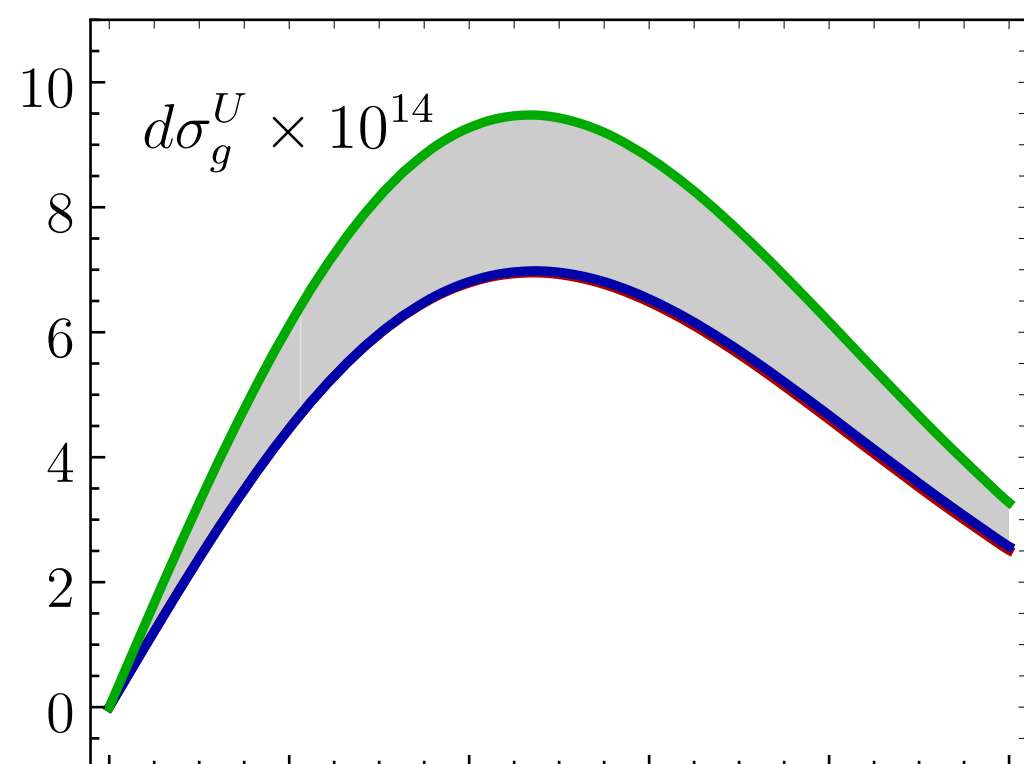
CSF

Hard

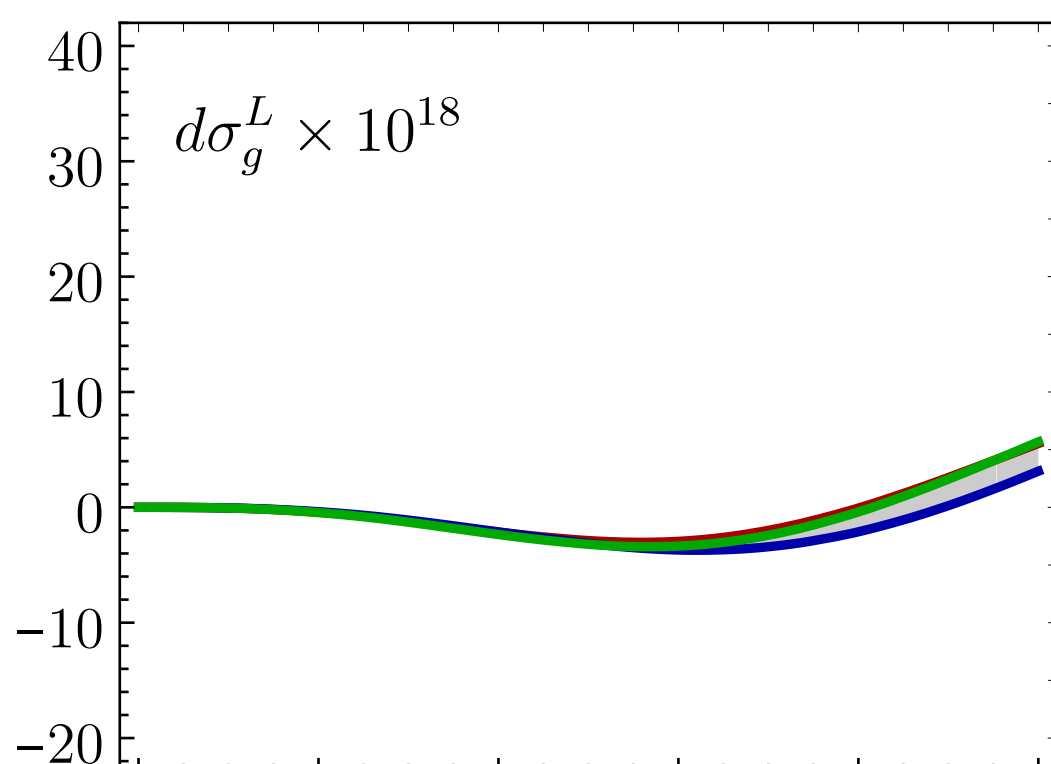
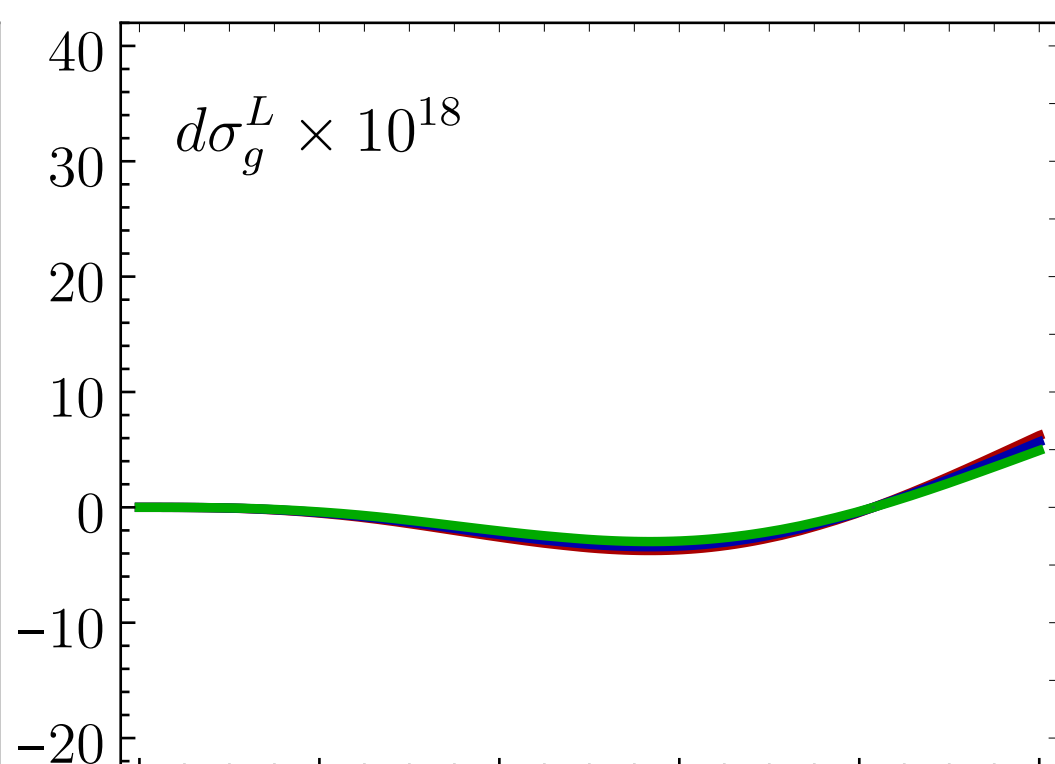
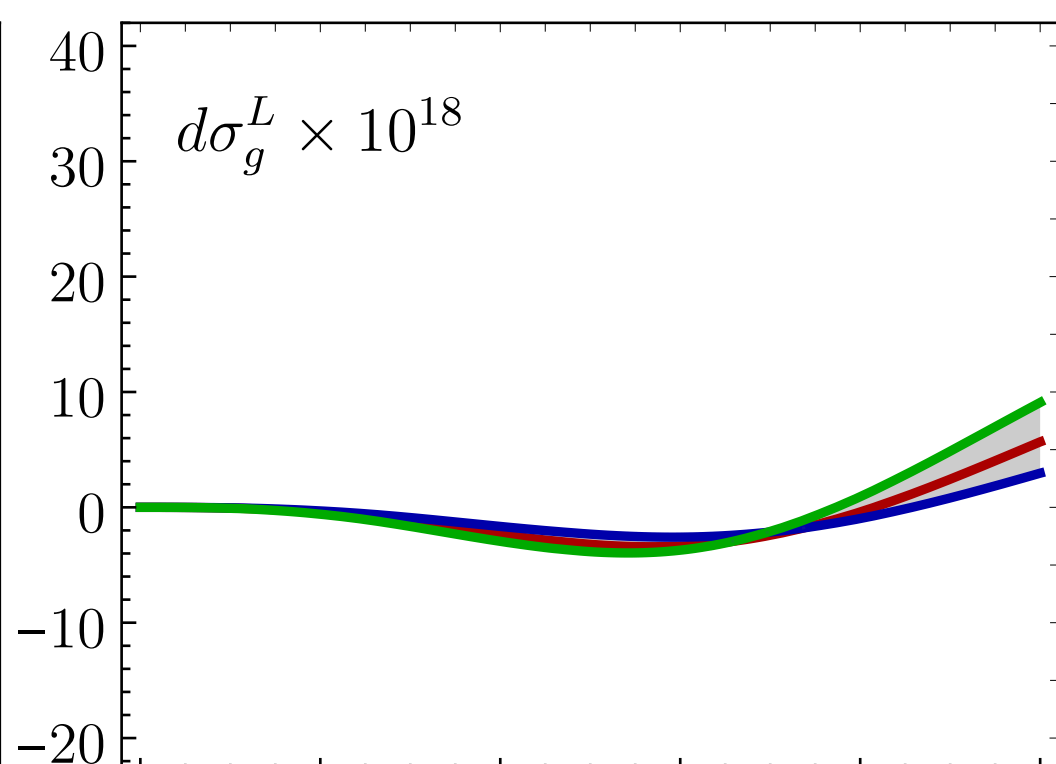
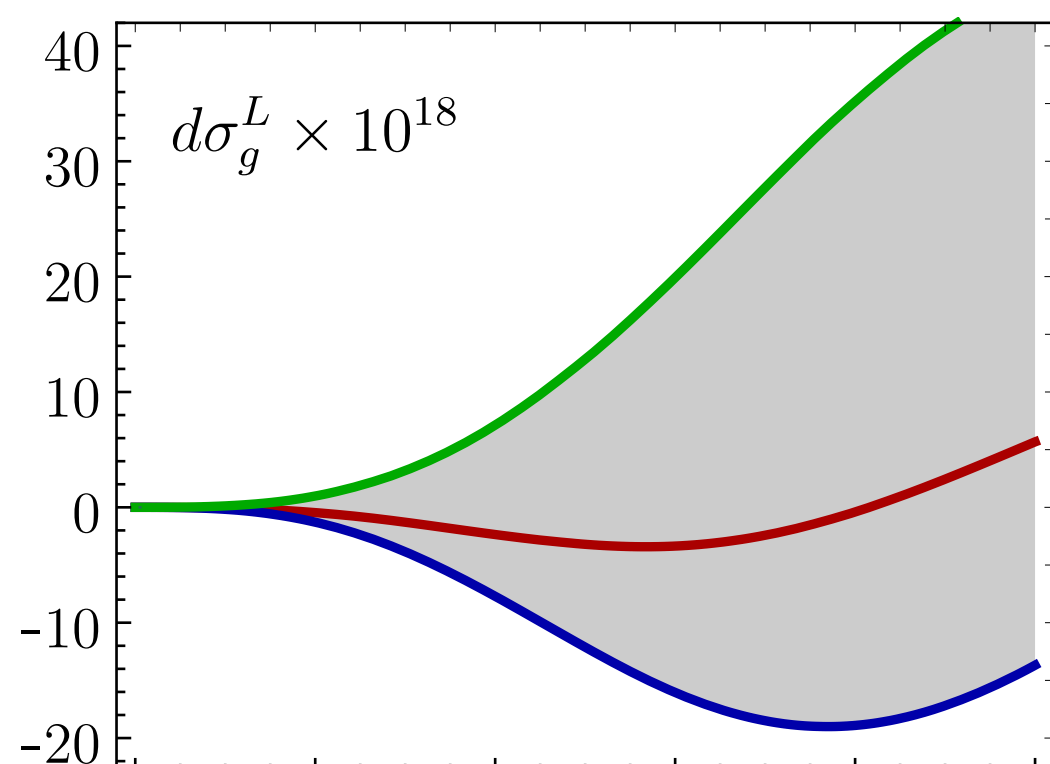
Jet

OPE

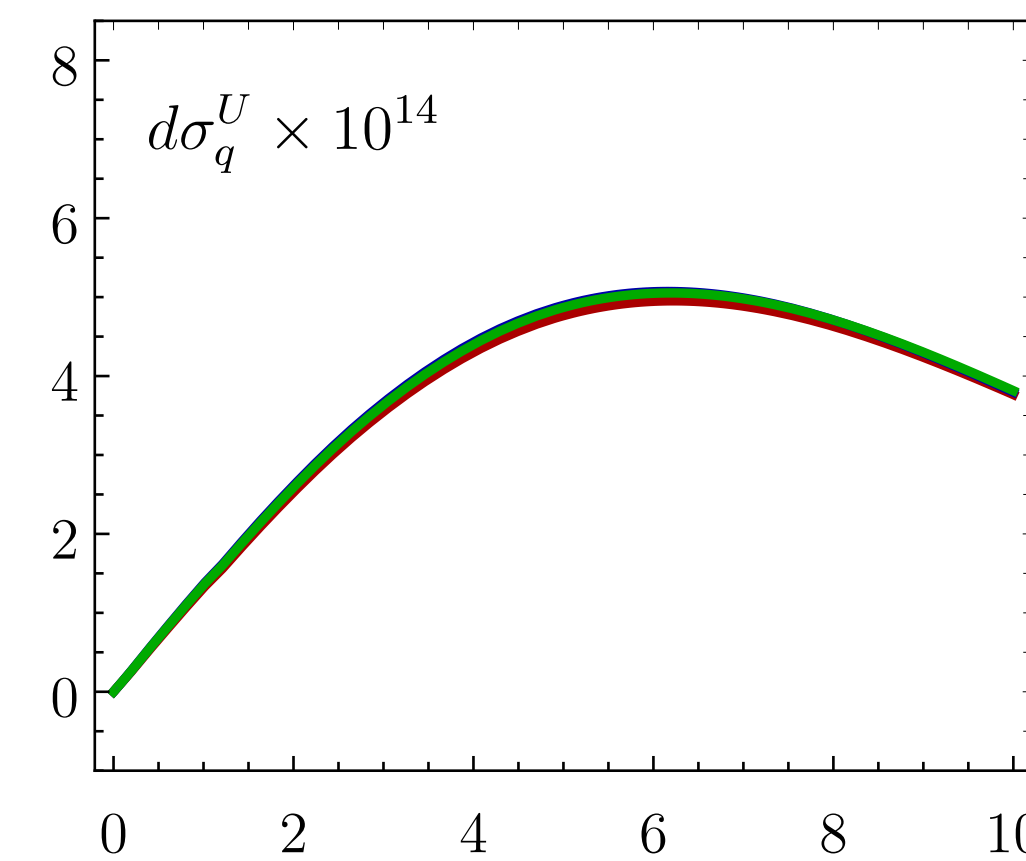
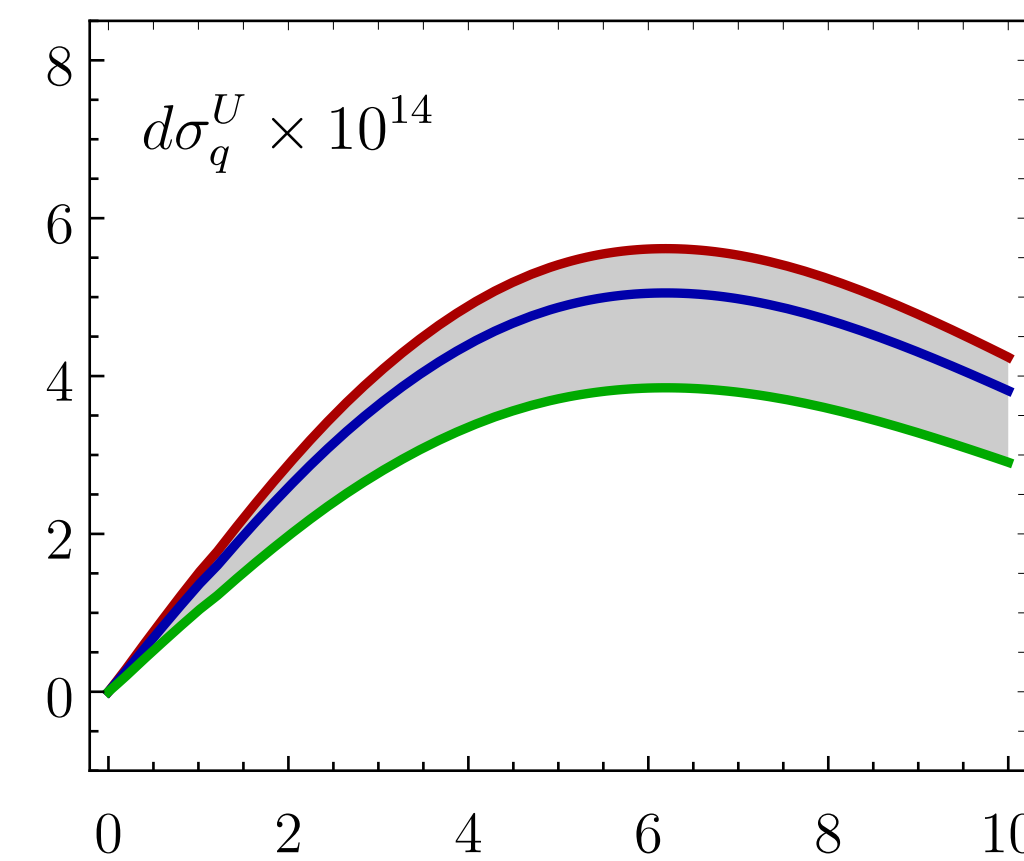
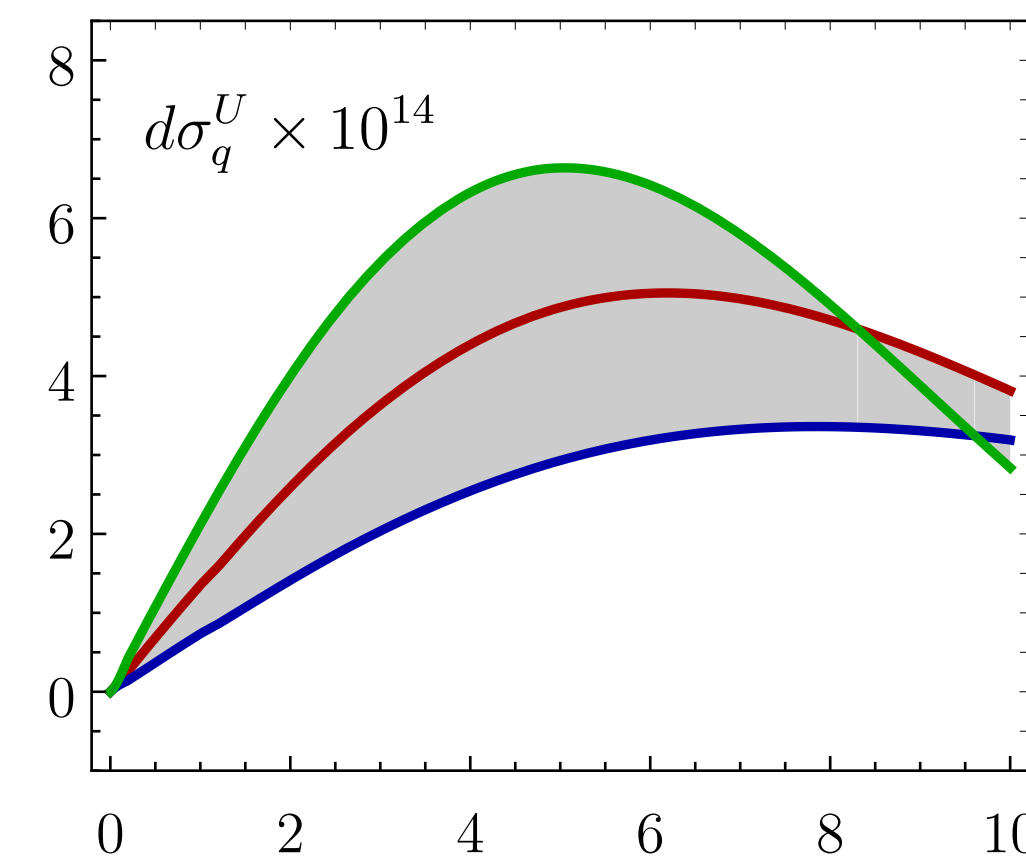
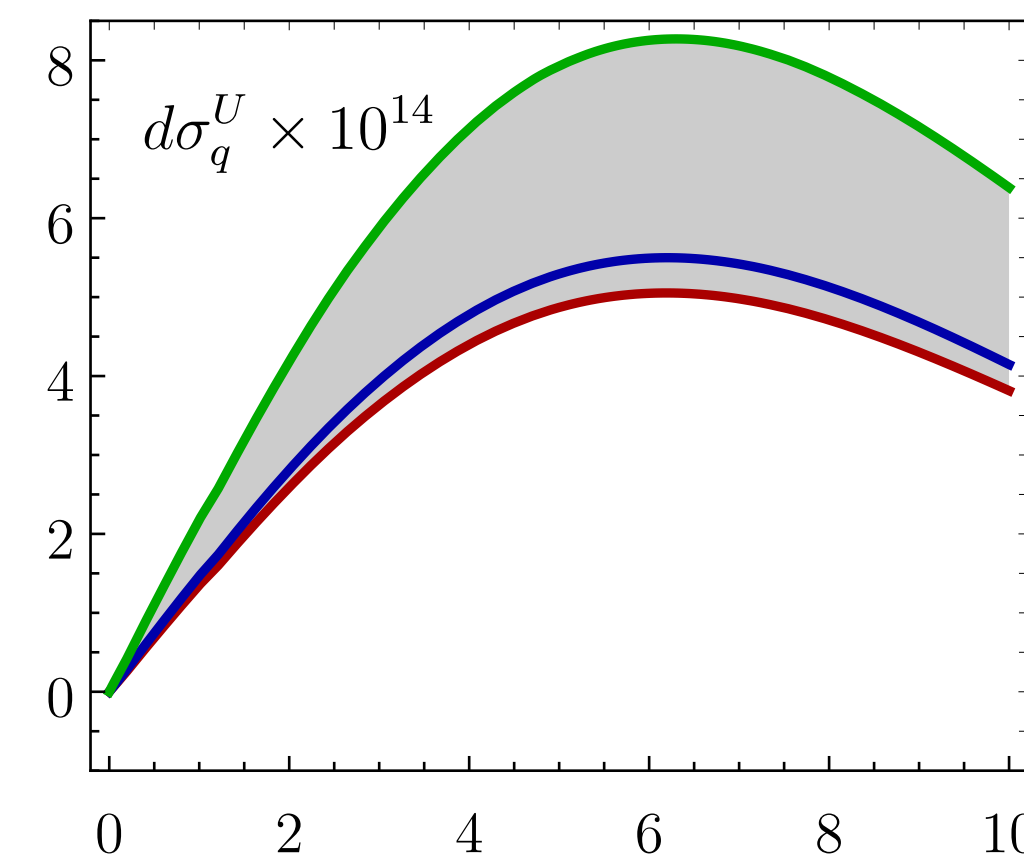
Unpolarized  
gluon channel



Linearly  
polarized  
gluon channel



Unpolarized  
quark channel

 $q_T$  [GeV]

# Conclusion

- We have established factorization for dijet production
- Can be potentially observed in the future EIC
- We have been able to compute the new TMD Soft Function up to NLO and its anomalous dimension up to three-loops
- Rapidity structure of this new SF allows us to use the  $\zeta$ -prescription
- The presence of the new SF makes the gluon TMDPDF extraction non-trivial
- Analysis of the numerical result for the cross-section shows the effect of linearly polarized gluon TMDs can be neglected compared to unpolarized gluon TMDs

**Thank you for listening!**