Explicit renormalization of the nucleon-nucleon interaction in chiral EFT with a finite cutoff.

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in collaboration with E. Epelbaum

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Chiral EFT

$$\mathcal{L}_{\text{eff}} = \mathcal{L}_{\pi}^{(2)} + \mathcal{L}_{\pi N}^{(1)} + \mathcal{L}_{NN}^{(0)} + \mathcal{L}_{NN}^{(2)} + \ldots$$

Consistent with symmetries

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$$\text{expansion parameter: } Q = \frac{q}{\Lambda_{\text{hard}}}$$

$$q \in \{|\vec{p}|\,, M_\pi\}\,, \quad \Lambda_{\text{hard}} \sim \Lambda_\chi \sim M_\rho$$

Weinberg power counting for NN-interaction

Weinberg, S., **NPB363**, 3 (1991)

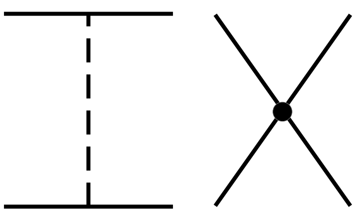
For potential (2N-irreducible) contributions:

$$D = 2L + \sum_{i=\text{vertices}} \left(d_i + \frac{n_i}{2} - 2 \right)$$

d_i – number of derivatives and quark masses

n_i – number of nucleon fields, L – number of loops

$\mathcal{O}(Q^0)$



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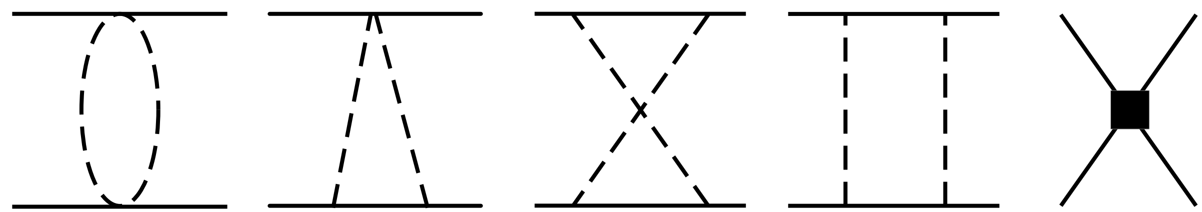

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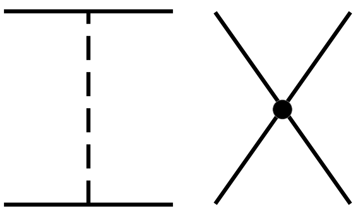

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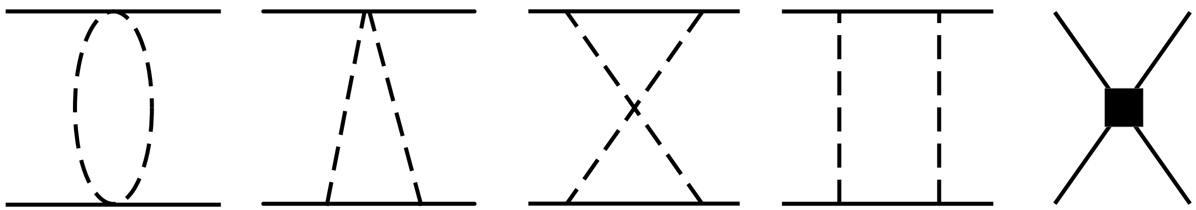

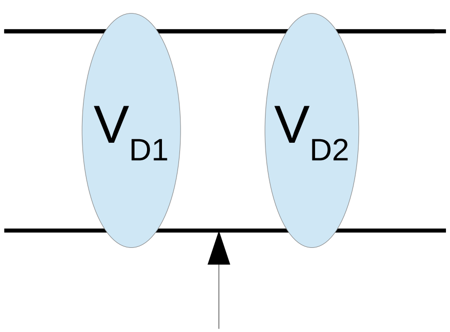

$$\sim \frac{m_N q}{\Lambda^2_{\text{hard}}} \sim 1$$

Enhancement due to the infrared singularity: V_0 must be iterated

$$T_0 = V_0 + V_0 G V_0 + V_0 G V_0 G V_0 + \ldots$$

$$T_2 = V_2 + V_2 G V_0 + V_0 G V_2 + V_2 G V_0 G V_0 + \ldots$$

Regularization

Divergent:

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G. P. Lepage, nucl-th/9706029
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Consistent with EFT (power counting)?

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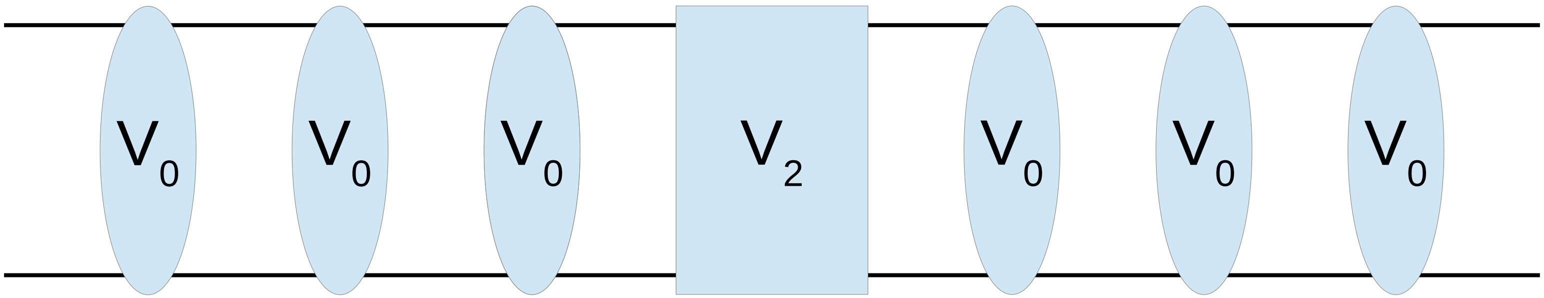
Non-perturbative effects?

BPHZ subtraction scheme

N. N. Bogoliubov, O. S. Parasiuk, **AM97**, 227 (1957); K. Hepp, **CMP2**, 301 (1966); W. Zimmermann, **CMP15**, 208 (1969)



Subtraction operation: $\mathbb{T}(X)(p', p, p_{\rm on}) = X(p'=0, p=0, p_{\rm on}=0)$

Forest formula: $\mathbb{R}(T_2^{[m,n]}) = T_2^{[m,n]} + \sum_{U_k \in \mathcal{F}^{m,n}} \prod_{(m_i,n_i)\in U_k} (-\mathbb{T}^{m_i,n_i}) T_2^{[m,n]}$

Forests:
nested sequences of subdiagrams
=nonoverlapping, not disjoined

$$U_k = \left((m_{k,1}, n_{k,1}), (m_{k,2}, n_{k,2}), \ldots\right),$$
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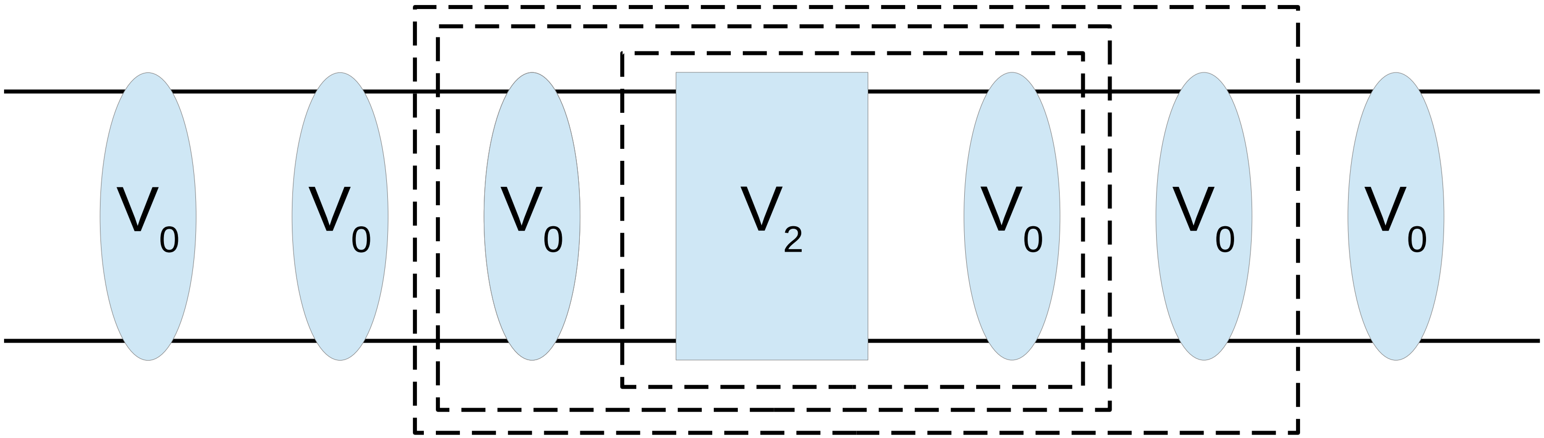

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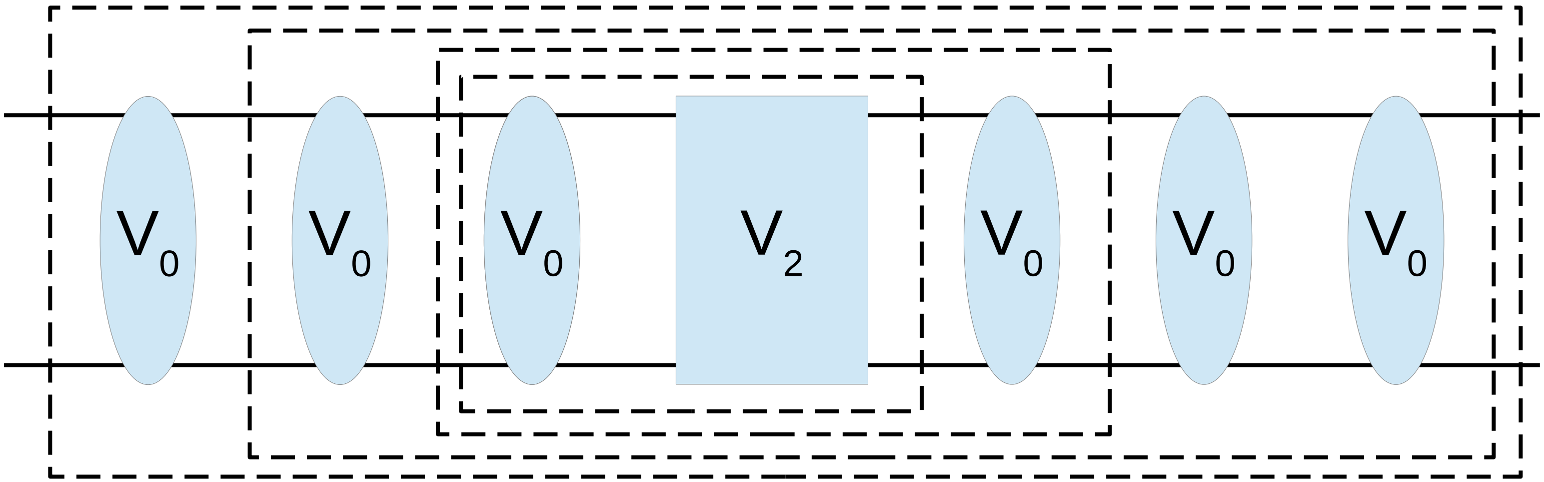

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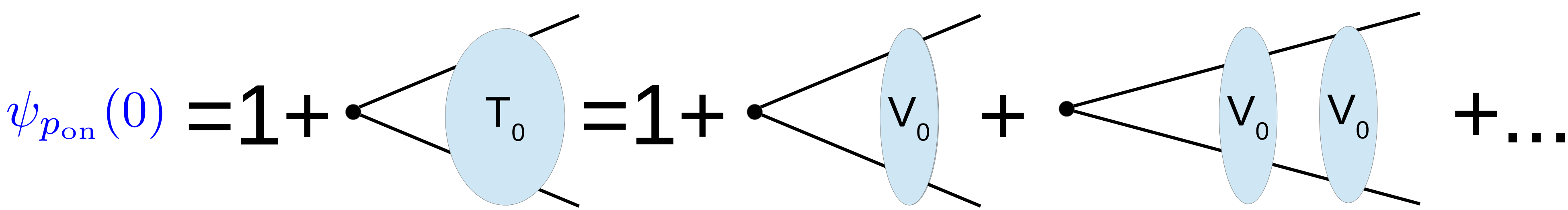
$$\left|\mathbb{R}(T_2^{[m,n]})(p_{\rm on})\right| \leq \frac{8\pi^2 \mathcal{M}_1}{m_N \Lambda_V} \left(\mathcal{M}_2 \frac{\Lambda}{\Lambda_V}\right)^{m+n} \frac{p_{\rm on}^2}{\Lambda_\chi^2} \log \Lambda / M_\pi$$

$$\mathcal{M}_1 \, , \mathcal{M}_2 \sim 1$$

S-waves. Subtractions. Non-perturbative LO

The series for $R(T_2^{[m,n]})$ can be summed explicitly

$$\mathbb{R}\left(T_2\right)\left(p_{\rm on}\right) = \sum_{m,n=0}^{\infty} \mathbb{R}\left(T_2^{[m,n]}\right)\left(p_{\rm on}\right) = T_2(p_{\rm on}) - T_2(p_{\rm on} = 0)\left[\frac{\psi_{p_{\rm on}}(0)}{\psi_{p_{\rm on}=0}(0)}\right]^2$$



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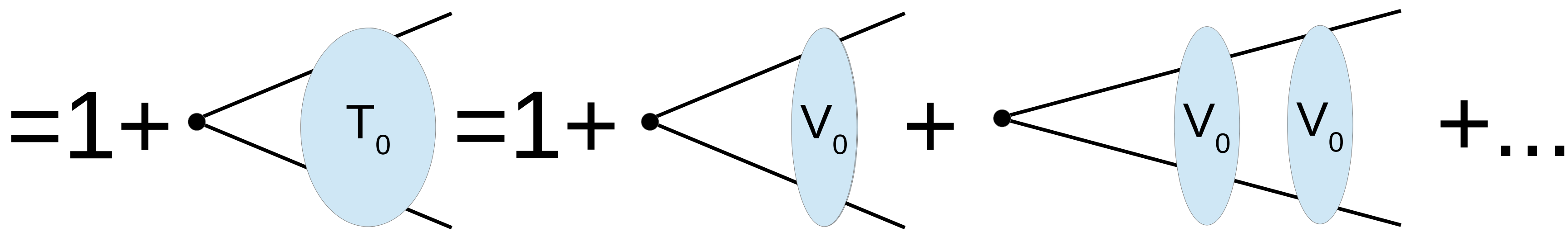
$\psi_{p_{\rm on}}(0)$ 

$$\mathbb{R}\left(T_2\right)\left(p_{\rm on}=0\right) = 0$$

Non-perturbative NLO. Fredholm formula.

$$R = \frac{1}{\mathbb{1} - GV_0} = \frac{N}{D}\,, \quad \bar{R} = \frac{1}{\mathbb{1} - V_0 G} = \frac{\bar{N}}{D}$$

Convergent series in V_0:

$$N = \sum_{i=0}^{\infty} N_i\,, \quad D = \sum_{i=0}^{\infty} D_i$$

Fredholm determinant contains the non-perturbative dynamics:

$$D = D(p_{\rm on})$$

(Quasi-) bound state:

$$D(p_{\rm on}) \sim \frac{p_{\rm on}}{M_\pi}$$

$$T_0(p_{\rm on}) = \frac{N_0(p_{\rm on})}{D(p_{\rm on})}\,, \quad T_2(p_{\rm on}) = \frac{N_2(p_{\rm on})}{D(p_{\rm on})^2}$$

Non-perturbative NLO. Renormalizability condition

$$\mathbb{R}\left(T_2\right)\left(p_{\rm on}\right) = \frac{1}{D(p_{\rm on})^2}\left[N_2(p_{\rm on}) - N_2(0)\frac{N_\psi(p_{\rm on})}{N_\psi(0)}\right]$$

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Inside the convergence radius

More constraints at higher orders!

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  NLO interaction is treated perturbatively.
  LO interaction is summed (perturbatively) up to an arbitrary order
  or treated non-perturbatively

- ✔ Power-counting breaking contributions at NLO can be absorbed
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Outlook

- ➢ General case: N^2LO, N^3LO,…

- ➢ Other systems: currents, few body,…