

Analysis of Isospin Symmetry for Fragmentation Functions

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K.B. Chen, Z.T. Liang, Y.K. Song and S.Y. Wei, arXiv:2108.07740



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Motivation

➤ Motivation

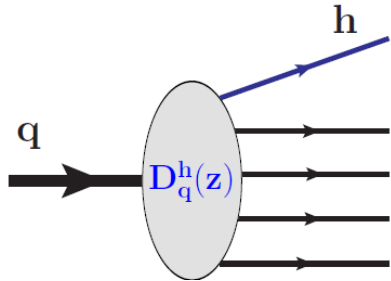
➤ Isospin symmetry of fragmentation functions

➤ Fit to Belle Λ polarization data

➤ Summary

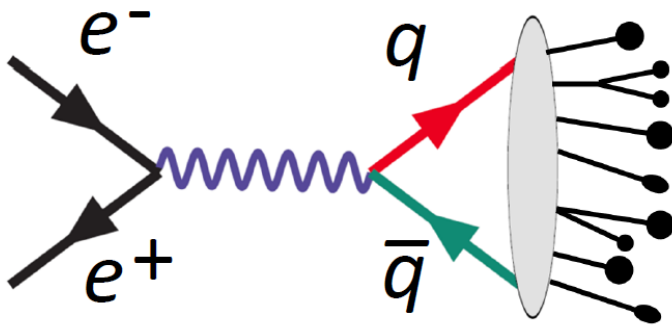
■ Fragmentation functions

Fragmentation functions (**FFs**)



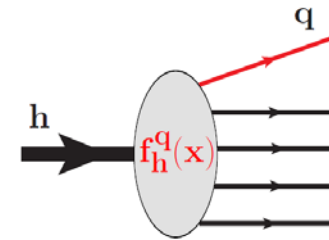
Hadronization mechanism

Hadron momentum distribution inside a parton jet



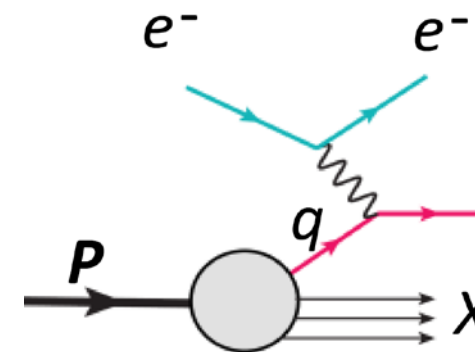
e^+e^- annihilation to hadrons

Parton distribution functions (**PDFs**)



Hadron structure

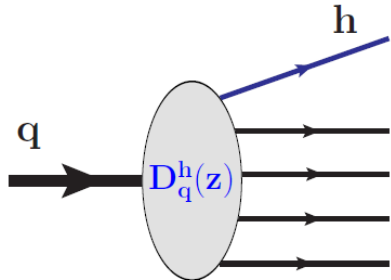
Parton momentum distribution inside a hadron



Lepton-nucleon deep inelastic scattering

■ Fragmentation functions

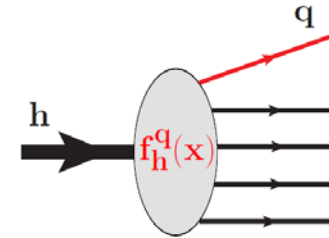
Fragmentation functions (**FFs**)



Hadronization mechanism

Hadron momentum distribution inside a parton jet

Parton distribution functions (**PDFs**)



Hadron structure

Parton momentum distribution inside a hadron

Why PDFs and FFs important?

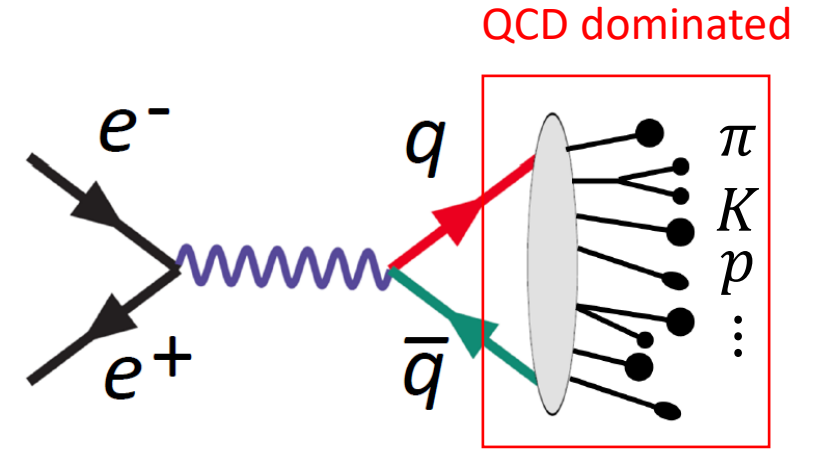
- Insights into the properties of strong interaction
- Inputs for describing high energy reactions
- Prerequisite for doing new physics researches
-

Fragmentation functions and isospin symmetry

- ✓ Fragmentation processes are dominated by **strong interactions**
- ✓ **Isospin symmetry** is a fundamental property of strong interactions



FFs respect isospin symmetry:
equal under the exchange of $u \leftrightarrow d$.



$$D_{1u}^{\pi^+(u\bar{d})}(z) \stackrel{u \leftrightarrow d}{=} D_{1d}^{\pi^-(d\bar{u})}(z),$$

$$D_{1u}^{K^+(u\bar{s})}(z) \stackrel{u \leftrightarrow d}{=} D_{1d}^{K^0(d\bar{s})}(z)$$

$$D_{1u}^{p(uud)}(z) \stackrel{u \leftrightarrow d}{=} D_{1d}^{n(udd)}(z),$$

$$D_{1u}^{\Lambda(uds)}(z) \stackrel{u \leftrightarrow d}{=} D_{1d}^{\Lambda(uds)}(z)$$

$$D_{1u}^{\Xi^0(u ss)}(z) \stackrel{u \leftrightarrow d}{=} D_{1d}^{\Xi^-(d ss)}(z),$$

$$D_{1u}^{\Omega^-(sss)}(z) \stackrel{u \leftrightarrow d}{=} D_{1d}^{\Omega^-(sss)}(z)$$

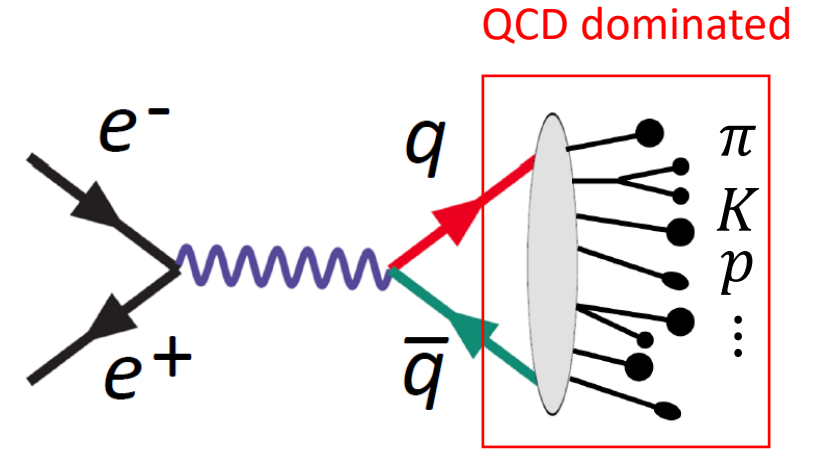
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■ Fragmentation functions and isospin symmetry

- ✓ Fragmentation processes are dominated by **strong interactions**
- ✓ **Isospin symmetry** is a fundamental property of strong interactions



FFs respect isospin symmetry:
equal under the exchange of
 $u \leftrightarrow d$.



But:

Hadron electroweak decays
violate isospin symmetry



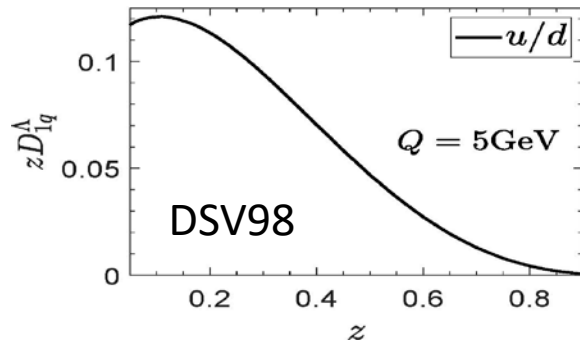
Isospin symmetry for FFs **can be violated!!**



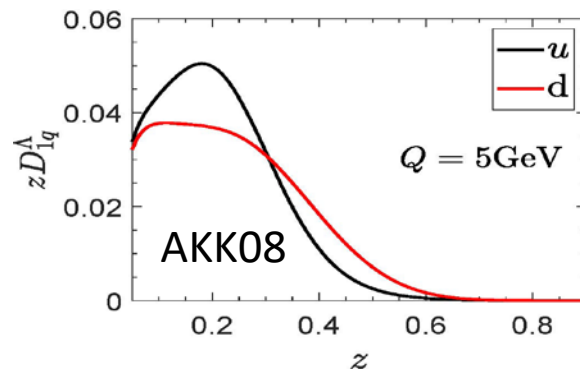
Motivation

Examples of Λ FFs

Unpolarized Λ FFs

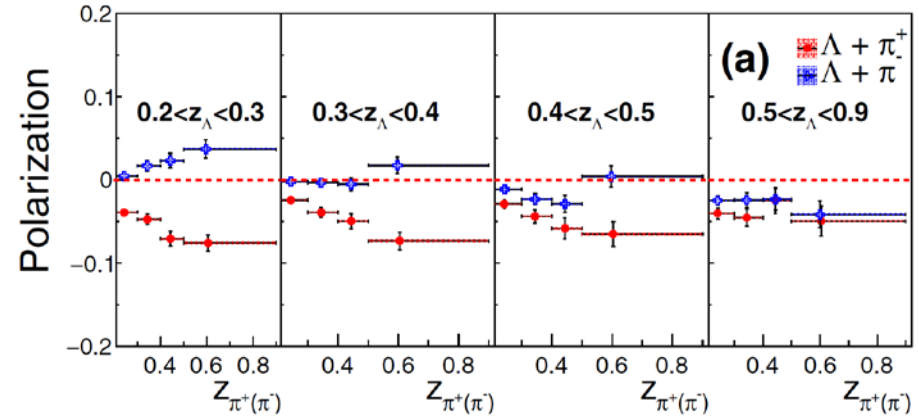


Isospin symmetric



Isospin violating

Belle data on Λ transverse polarization



Y. Guan et al. (Belle Collaboration), PRL 122, 042001 (2019)

$$e^+e^- \rightarrow \Lambda(uds)\pi^+(u\bar{d}) + X$$

$$\updownarrow u \leftrightarrow d$$

$$e^+e^- \rightarrow \Lambda(uds)\pi^-(d\bar{u}) + X$$

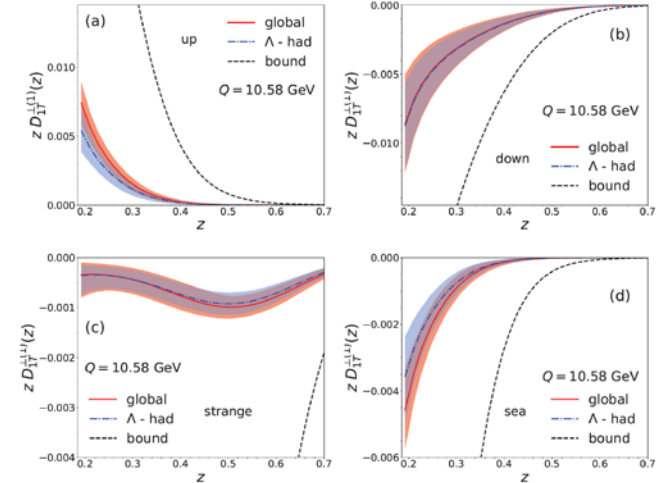
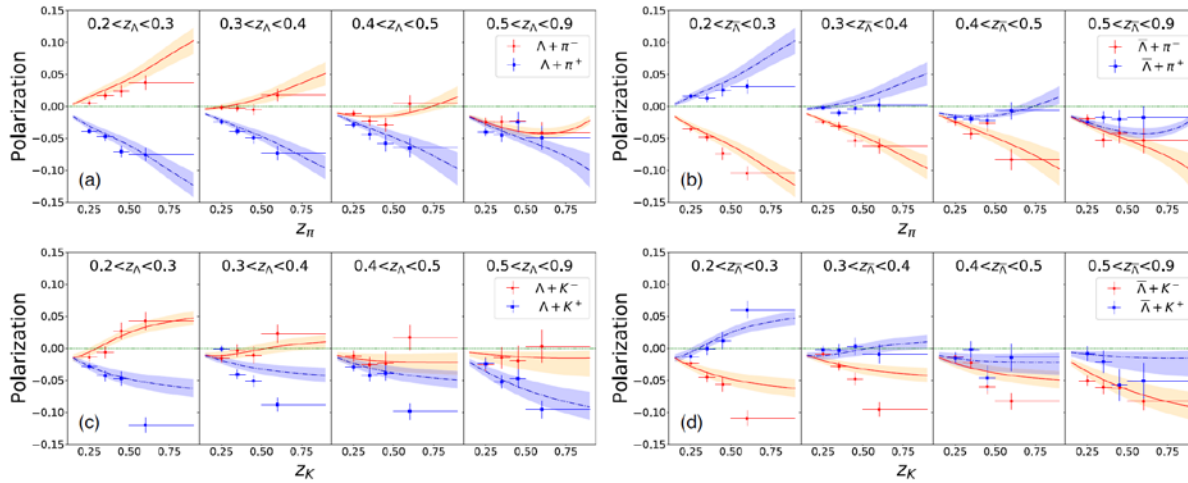
Isospin violation for polarized Λ FFs??



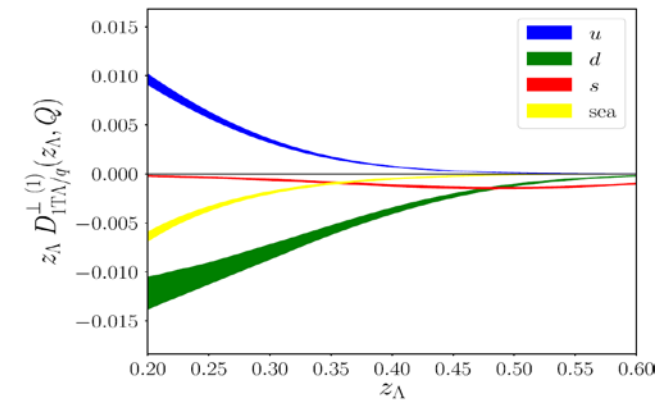
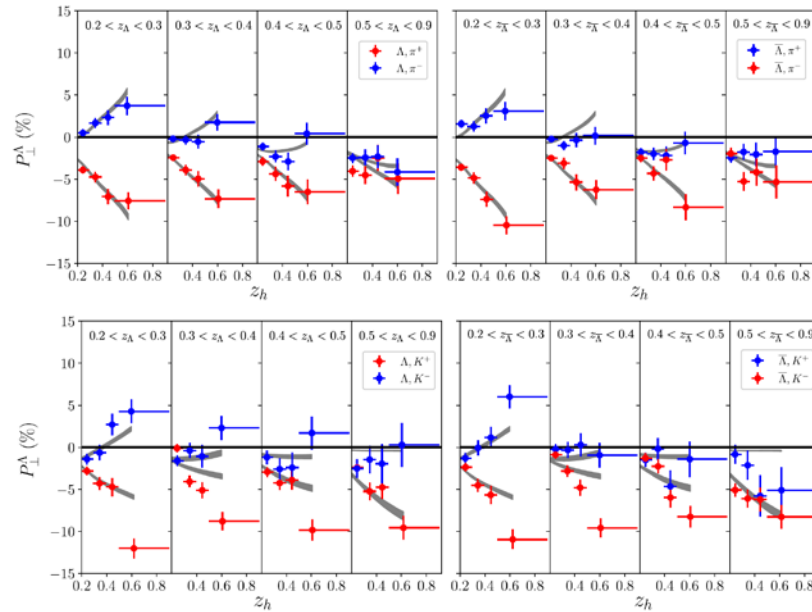
Motivation

Fittings

D'Alesio, Murgia, Zaccheddu,
PRD 102, 054001



Callos, Kang, Terry,
PRD 102, 096007



Both fittings have large isospin
symmetry violation for $D_{1Tq}^{\perp\Lambda}$



Motivation

- To what extent the isospin symmetry violation can be accommodated in FFs if it's held in QCD?
- Do the Belle data really mean that there is a large isospin symmetry violation of Λ FFs?



Isospin symmetry of fragmentation functions

➤ Motivation

➤ **Isospin symmetry of fragmentation functions**

➤ Fit to Belle Λ polarization data

➤ Summary



Isospin symmetry of fragmentation functions

■ The formalism

➤ **Unpolarized FFs:** $D_{1q}^h(z) = D_{1q}^{h,\text{dir}}(z) + D_{1q}^{h,\text{dec}}(z)$

$$= D_{1q}^{h,\text{dir}}(z) + \sum_{h_j} Br(h, h_j) \int dz' K_{h,h_j}(z, z') D_{1q}^{h_j}(z')$$

direct fragmentation decay contributions

✓ Direct FFs: $D_{1q}^{h,\text{dir}}(z)$ are isospin symmetric

✓ Decay contributions, consider: $J^P = 0^-$ (pseudo-scalar) and 1^- (vector) mesons,

$$J^P = \frac{1}{2}^+ \text{ (octet) and } \frac{3}{2}^+ \text{ (decuplet) baryons.}$$

✓ Higher excited resonance states (very small production rates, mostly strong decay) are included into the direct FFs effectively.

✓ Isospin violation may arise from EW-decays, determined by the branch ratio $Br(h, h_j)$.



Isospin symmetry of fragmentation functions

- Ξ production:** $\delta D_{1q}^{\Xi}(z) \stackrel{\text{def}}{=} D_{1u}^{\Xi^0}(z) - D_{1d}^{\Xi^-}(z)$ $D_{1u}^{\Xi^0,\text{dir}}(z) = D_{1d}^{\Xi^-, \text{dir}}(z)$ $D_{1u}^{\Xi^0, \Xi^*}(z) = D_{1d}^{\Xi^-, \Xi^*}(z)$

$$D_{1q}^{\Xi^i}(z) = D_{1q}^{\Xi^i, \text{dir}}(z) + D_{1q}^{\Xi^i, \Xi^*}(z) + D_{1q}^{\Xi^i, \Omega^-}(z) \qquad D_{1u}^{\Omega^-}(z) = D_{1d}^{\Omega^-}(z)$$

Ω^- decays to Ξ^0 and Ξ^- unequally: $\begin{cases} Br(\Omega^- \rightarrow \Xi^0 \pi^-) = (23.6 \pm 0.7)\% \\ Br(\Omega^- \rightarrow \Xi^- \pi^0) = (8.6 \pm 0.4)\% \end{cases} \rightarrow \delta Br(\Xi, \Omega^-) = (15.0 \pm 1.1)\%$

$$\delta D_{1q}^{\Xi}(z) = \delta D_{1q}^{\Xi, \Omega^-} = \delta Br(\Xi, \Omega^-) \int dz' K_{\Xi, \Omega^-}(z, z') D_{1u}^{\Omega^-}(z') \leq \text{a few percent}$$

➔ *Tiny isospin violation for Ξ FFs, i.e., $D_{1u}^{\Xi^0}(z) \approx D_{1d}^{\Xi^-}(z)$*

- Λ production:** $\delta D_{1q}^{\Lambda}(z) \stackrel{\text{def}}{=} D_{1u}^{\Lambda}(z) - D_{1d}^{\Lambda}(z)$

$$D_{1q}^{\Lambda}(z) = D_{1q}^{\Lambda, \text{dir}}(z) + D_{1q}^{\Lambda, \Omega^-}(z) + D_{1q}^{\Lambda, \Sigma^*}(z) + D_{1q}^{\Lambda, \Xi}(z) + D_{1q}^{\Lambda, \Sigma^0}(z)$$

Notice: Ξ^0 and Ξ^- added together gives no isospin violation for Λ FFs via $\Xi \rightarrow \Lambda \pi$ (branch ratio 100%).

➔ *No isospin violation for Λ FFs, i.e., $D_{1u}^{\Lambda}(z) = D_{1d}^{\Lambda}(z)$*



Isospin symmetry of fragmentation functions

➤ **Polarized FFs:**
$$\Delta D_{1q}^{h,\text{dec}}(z) = \sum_{h_j} Br(h, h_j) \int dz' K_{h,h_j}(z, z') t_D^{h,h_j}(z) \Delta D_{1q}^{h_j}(z')$$

polarization transfer factor

The difference of t_D^{Λ, Ξ^0} and t_D^{Λ, Ξ^-} is very small through the weak decay of $\Xi \rightarrow \Lambda\pi$, leads to very tiny isospin violation for polarized Λ FFs.

★ Conclusion:

No isospin violation for unpolarized Λ FFs, only tiny violation for polarized. Isospin symmetry should be kept in the parameterizations of Λ FFs.

Can we describe the Belle data under the constraint of isospin symmetric Λ FFs?



Fit to Belle Λ polarization data

➤ Motivation

➤ Isospin symmetry of fragmentation functions

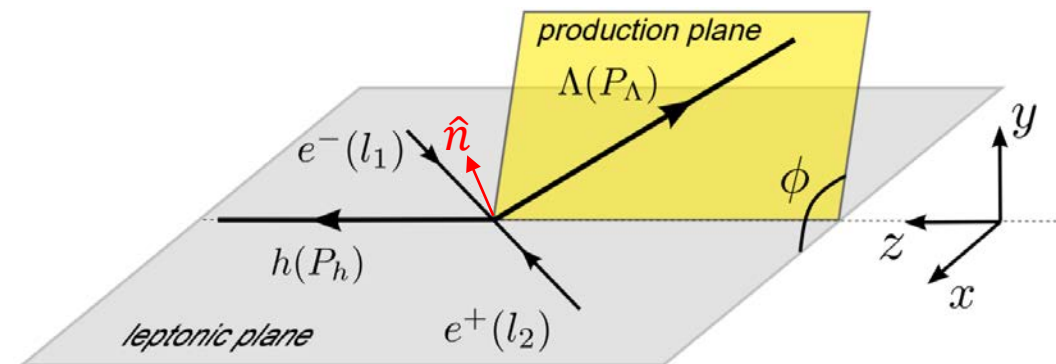
➤ **Fit to Belle Λ polarization data**

➤ Summary

Λ transverse polarization in $e^+e^- \rightarrow \Lambda h X$

w.r.t. the normal direction of the production plane (\hat{n})

$$P_\Lambda(z_\Lambda, z_h, p_{\Lambda\perp}) = \frac{\mathcal{C}[wD_{1Tq}^{\perp\Lambda}D_{1\bar{q}}^h] + (q \leftrightarrow \bar{q})}{\mathcal{C}[D_{1q}^\Lambda D_{1\bar{q}}^h] + (q \leftrightarrow \bar{q})}$$



$$\mathcal{C}[wD_{1Tq}^{\perp\Lambda}D_{1\bar{q}}^h] \equiv \sum_q e_q^2 \int d^2p_T d^2p_{hT} \delta^2\left(\frac{z_\Lambda}{z_h}p_{hT} + p_T - p_{\Lambda\perp}\right) w D_{1Tq}^{\perp\Lambda}(z_\Lambda, p_T) D_{1\bar{q}}^h(z_h, p_{hT}), \quad w = \frac{\vec{p}_{\Lambda\perp} \cdot \vec{p}_T}{z_\Lambda M_\Lambda |\vec{p}_{\Lambda\perp}|}$$

Gaussian ansatz:

$$D_{1Tq}^{\perp\Lambda}(z_\Lambda, p_T) = D_{1Tq}^{\perp\Lambda}(z_\Lambda) \frac{1}{\pi\Delta^2} \exp(-p_T^2/\Delta^2)$$

$$D_{1Tq}^{\perp\Lambda}(z_\Lambda) = N_{Tq} \frac{(\alpha_q + \beta_q - 1)^{\alpha_q + \beta_q - 1}}{(\alpha_q - 1)^{\alpha_q - 1} \beta_q^{\beta_q}} \times z_\Lambda^{\alpha_q} (1 - z_\Lambda)^{\beta_q} D_{1q}^\Lambda(z_\Lambda)$$

$p_{\Lambda\perp}$ integrated Λ transverse polarization:

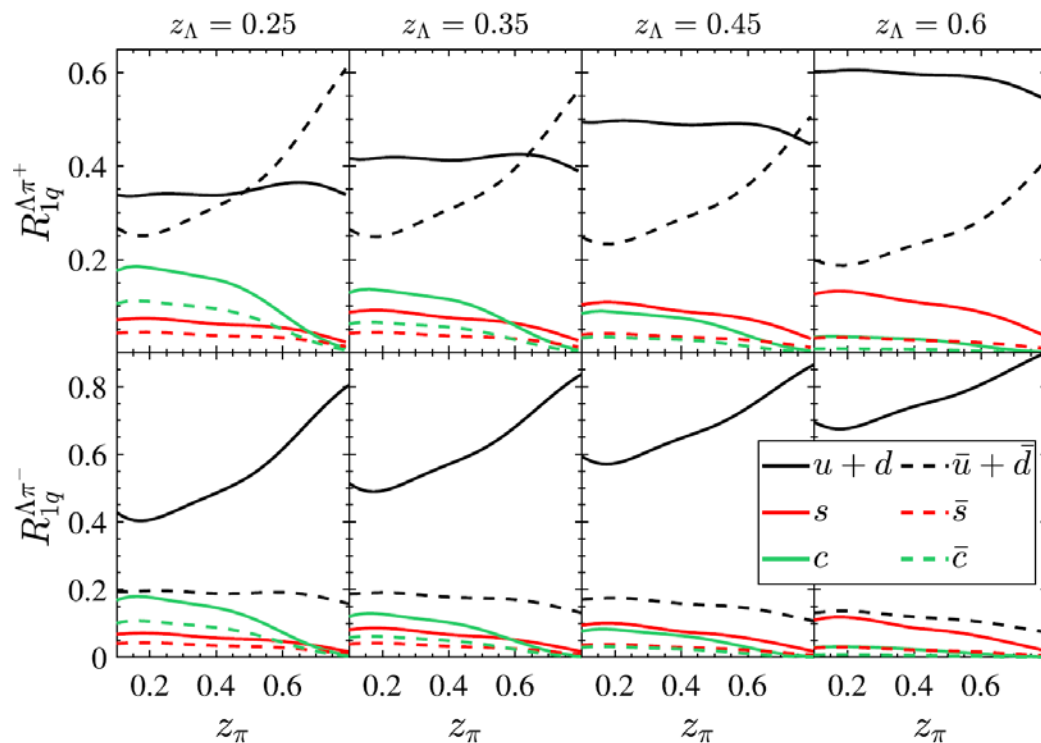
$$P_\Lambda(z_\Lambda, z_h) = \frac{\sqrt{\pi} z_h \Delta / 2 z_\Lambda M_\Lambda}{\sqrt{z_h^2 + z_\Lambda^2 \Delta_h^2 / \Delta^2}} \sum_q [R_{1q}^{\Lambda h}(z_\Lambda, z_h) \frac{D_{1Tq}^{\perp\Lambda}(z_\Lambda)}{D_{1q}^\Lambda(z_\Lambda)} + (q \leftrightarrow \bar{q})]$$

$$\text{relative weight: } R_{1q}^{\Lambda h}(z_\Lambda, z_h) = \frac{e_q^2 D_{1q}^\Lambda(z_\Lambda) D_{1\bar{q}}^h(z_h)}{\sum_f e_f^2 D_{1q}^\Lambda(z_\Lambda) D_{1\bar{q}}^h(z_h) + (f \leftrightarrow \bar{f})}$$



Fit to Belle Λ polarization data

$R_{1q}^{\Lambda\pi}$ calculated using DSV and DHESS FFs



➔ **parameterization schemes:**

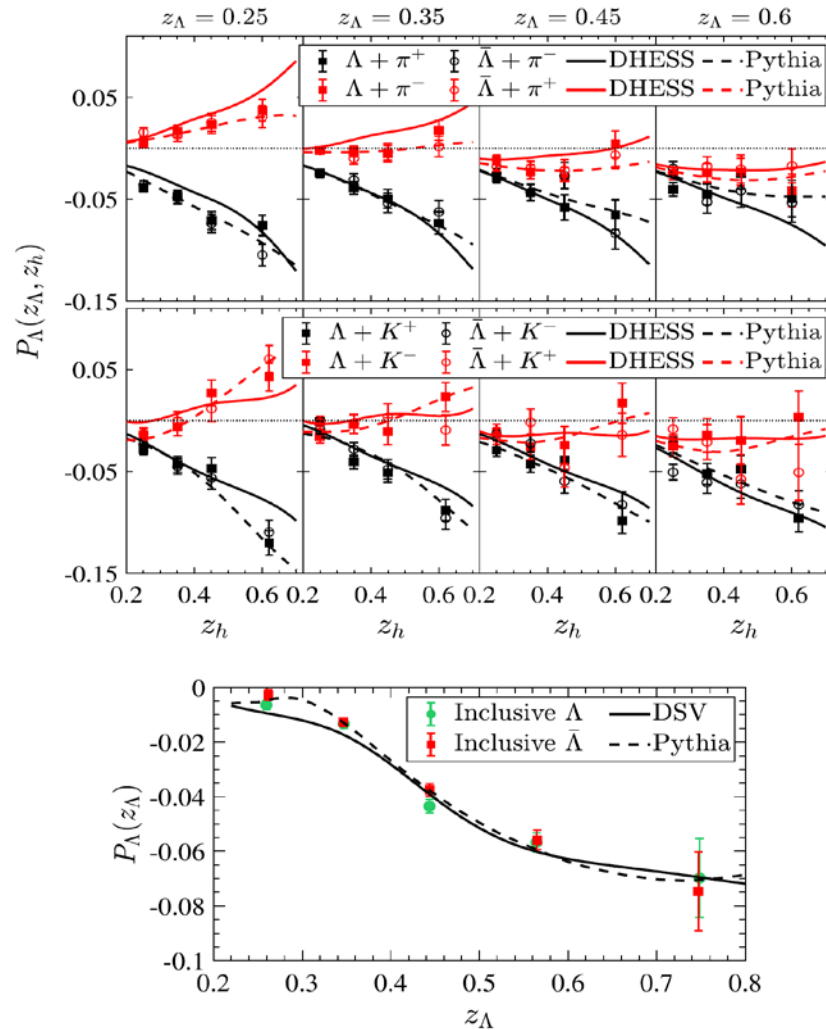
- Keep isospin symmetry, i.e., $u = d$ and $\bar{u} = \bar{d}$
- s contributes differently from u, d
- c, \bar{c} not negligible and different from light flavors

Six groups of parameters for $D_{1Tq}^{\perp\Lambda}(z_{\Lambda})$,
 $q = (u/d), s, c, (\bar{u}/\bar{d}), \bar{s}, \bar{c}$



Fit to Belle Λ polarization data

Fitting results



parameter	u, d	s	c	\bar{u}, \bar{d}	\bar{s}	\bar{c}
$\frac{\Delta}{M_\Lambda} N_{Tq}$	0.391 0.245	-0.391 -0.148	0.0278 0.108	-0.456 -0.231	-0.430 0.523	0.401 -0.324
α_q	1.38 2.41	6.91 1.54	1.43 5.14	1.00 1.86	2.64 1.74	11.6 1.02
β_q	3.98 7.69	0.646 0.551	14.3 15.0	0.0319 2.35	2.77 14.9	14.9 2.41

The Belle data can be well described using polarized Λ FFs with isospin symmetry.

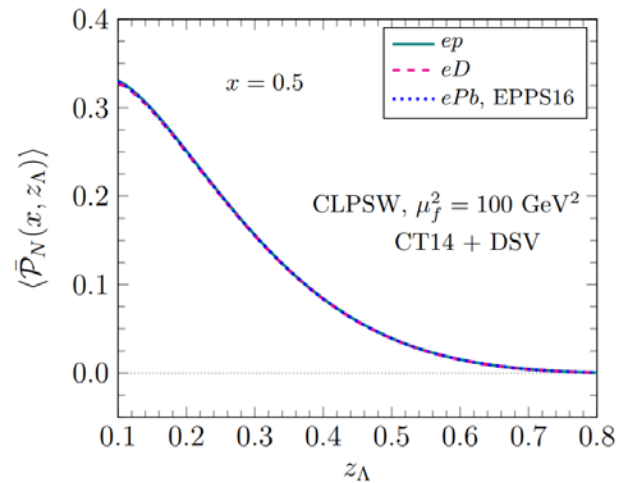
Current e^+e^- data cannot distinguish different scenarios of isospin symmetry or violation.



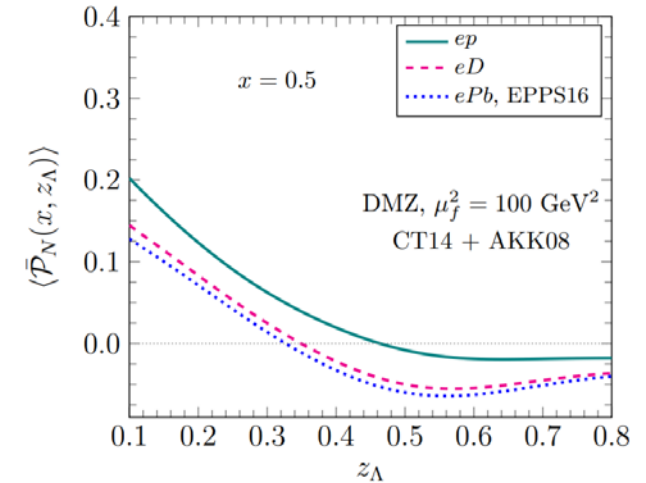
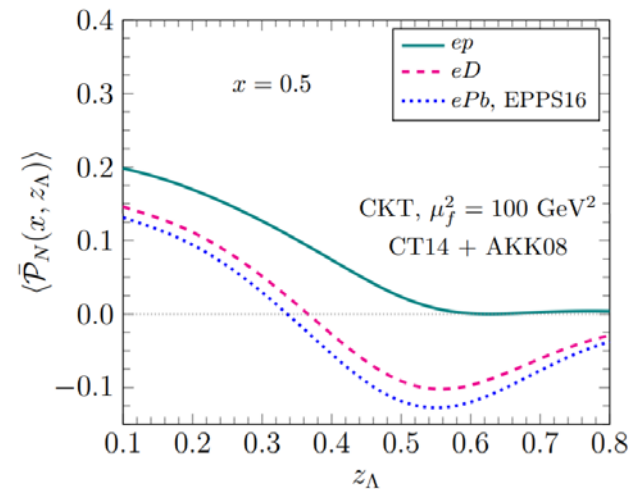
Fit to Belle Λ polarization data

Checking isospin symmetry in SIDIS off nucleus

Using isospin symmetric parameterizations for polarized Λ FFs



Using isospin violating parameterizations for polarized Λ FFs



Chen, Liang, Song and Wei, arXiv:2108.07740

A clean test!



Summary

➤ Motivation

➤ Isospin symmetry of fragmentation functions

➤ Fit to Belle Λ polarization data

➤ **Summary**



Summary

- We make a systematic analysis of isospin symmetry for FFs by considering hadron decay contributions.
- There is no isospin violation for unpolarized Λ FFs, tiny violation for polarized.
- The Belle Λ polarization data can be well described under isospin symmetric Λ polarized FFs. There is no need to introduce large isospin violation at current stage.
- The isospin symmetry can be clearly checked at EIC with SIDIS off different nucleus.

Thank you for your attention!