Generalized parton distributions (GPDs) of sea quark in the proton from nonlocal chiral effective theory

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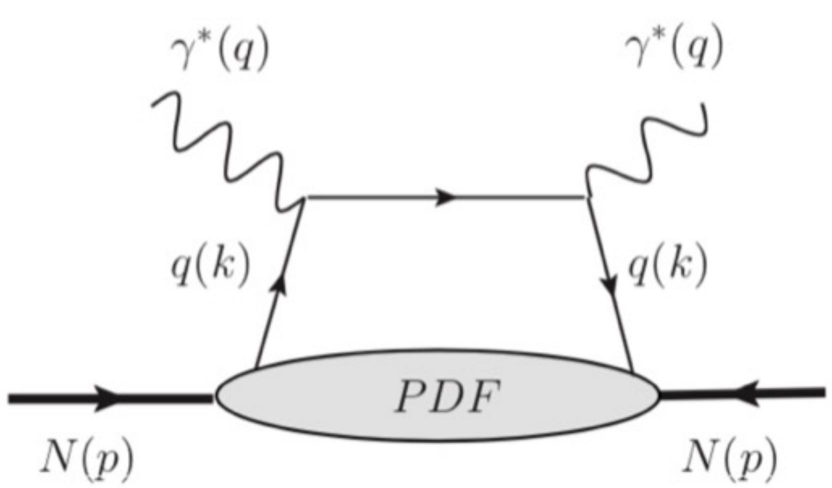
Outline

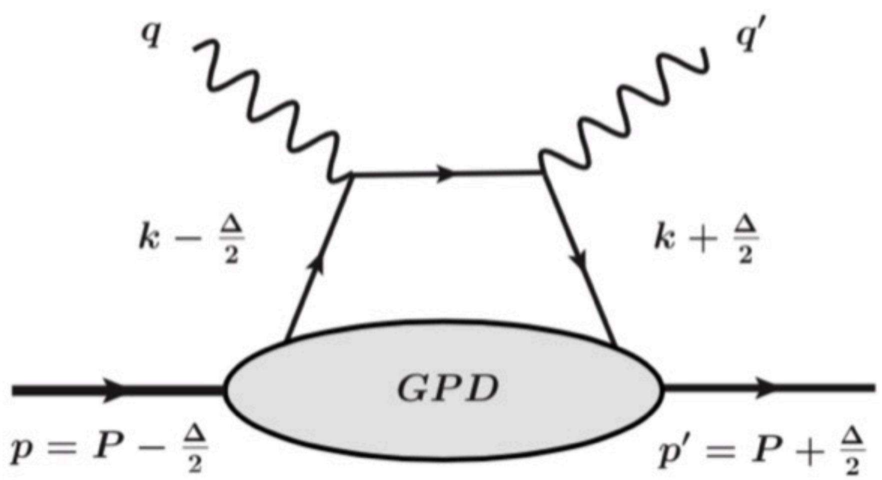
- Introduction to the unpolarized GPDs
- Nonlocal chiral effective theory
- Convolution formulas for GPDs of sea quark
- Numerical results
- Summary and outlook

Deeply virtual Compton scattering

✦ **Forward Compton scattering** → **Deeply virtual Compton scattering**

Ji, PRD 55(1997)





✦ **Parameters in GPDs:**

Skewness: $\xi = -\dfrac{\Delta \cdot n}{2P \cdot n} = -\dfrac{\Delta^+}{2P^+}$

The square of transfer momentum: $t = \Delta^2 = (p' - p)^2$

Average quark momentum fraction: $x = \dfrac{k^+}{P^+}$

Unpolarized quark GPDs of nucleon

- **Matrix element of the unpolarized quark GPDs in the proton:**

$$
\begin{aligned}
V^q &= \frac{1}{2}\int \frac{dz^-}{2\pi}\, e^{ix(P\cdot z)} \langle p'|\, \bar{q}(-\tfrac{1}{2}z)\, \tilde{\not n} q(\tfrac{1}{2}z)\, |p\rangle \Big|_{z=\lambda n} \\
&= \frac{1}{2P\cdot n}\left[H^q(x,\xi,t)\, \bar{u}(p')\tilde{\not n} u(p) + E^q(x,\xi,t)\, \bar{u}(p') \frac{i\sigma^{\beta\alpha}\Delta_\alpha \tilde{n}_\beta}{2m} u(p) \right],
\end{aligned}
$$

- **Parton distribution function($\xi = t = 0$):**

$$
H^q(x > 0,0,0) = q(x), \qquad H^q(x < 0,0,0) = -\bar{q}(-x)
$$

- **Dirac and Pauli form factors:**

$$
F_1^q(t) = \int_{-1}^{1} dx\, H^q(x,\xi,t), \qquad F_2^q(t) = \int_{-1}^{1} dx\, E^q(x,\xi,t)
$$

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Nonlocal chiral lagrangian

F. He and P. Wang, PRD(97), 2018
F. He and P. Wang, PRD(98), 2018

- **The local interaction between proton, Λ and Kaon:**

$$\mathcal{L}_K{}^{local} = -\int dx \frac{D+3F}{\sqrt{12}f}\bar{p}(x)\gamma^\mu\gamma_5\,\Lambda(x)(\partial_\mu + ie_s\,\mathscr{A}_\mu^s(x))K^+(x),$$

- **Corresponding nonlocal Lagrangian:**

$$\mathcal{L}_K{}^{nl} = -\int dx \int dy \frac{D+F}{\sqrt{12}f}\bar{p}(x)\gamma^\mu\gamma_5\Lambda(x)(\partial_\mu + i\,e_s\mathscr{A}_\mu^s(x))\Big(\exp[ie_s\int_x^y dz^\nu\,\mathscr{A}_\nu^s(z)]\,K^+(y)F(x-y)\Big),$$

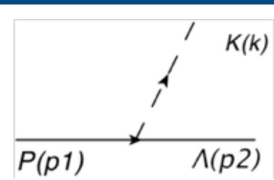
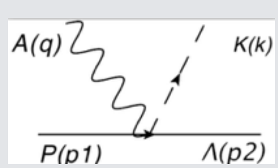
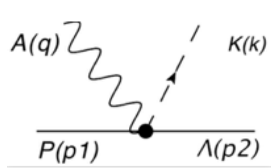
- **Nonlocal Lagrangian is invariant under the following gauge transformation:**

$$K^+(y) \to e^{-i\alpha(y)}K^+(y), \qquad \Lambda(x) \to e^{i\alpha(x)}\Lambda(x), \qquad \mathscr{A}_\mu(x) \to \mathscr{A}_\mu(x) - \frac{1}{e}\partial_\mu\alpha(x),$$

Comparison of local and nonlocal cases

- **Expand the nonlocal Lagrangian**

$$
\begin{aligned}
\mathcal{L}_K{}^{nl} = & -\int dx \int dy \frac{D+F}{\sqrt{12}f}\bar{p}(x)\gamma^\mu\gamma_5\Lambda(x)(\partial_\mu + i\, e_s \mathscr{A}^s_\mu(x))\Big(\exp[ie_s\int_x^y dz^\nu\, \mathscr{A}^s_\nu(z)]\, K^+(y)F(x-y)\Big), \\
= & -\int dx \int dy \frac{D+F}{\sqrt{12}f}\bar{p}(x)\gamma^\mu\gamma_5\Lambda(x)\partial_{\mu,x}\Big(K^+(y)F(x-y)\Big) \\
& -\int dx \int dy \frac{D+F}{\sqrt{12}f}\bar{p}(x)\gamma^\mu\gamma_5\Lambda(x)(i\, e_s \mathscr{A}^s_\mu(x))\Big(K^+(y)F(x-y)\Big) \\
& -\int dx \int dy \frac{D+F}{\sqrt{12}f}\bar{p}(x)\gamma^\mu\gamma_5\Lambda(x)\partial_{\mu,x}\Big(ie_s\int_x^y dz^\nu\, \mathscr{A}^s_\nu(z)\, K^+(y)F(x-y)\Big),
\end{aligned}
$$

Vertexes	Feynman rules in local case	Feynman rules in nonlocal case
	$\frac{D+3F}{\sqrt{12}f}k_\mu\gamma^\mu\gamma^5$	$\frac{D+3F}{\sqrt{12}f}k_\mu\gamma^\mu\gamma^5F(k)$
	$\frac{D+3F}{\sqrt{12}f}\gamma^\mu\gamma^5$	$\frac{D+3F}{\sqrt{12}f}\gamma^\mu\gamma^5F(k)$
	Nonexistence	$\frac{D+3F}{\sqrt{12}f}k_\nu\gamma^\nu\gamma^5\frac{[F(k+q)-F(k)](2k+q)^\mu}{2k\,.\,q+q^2}$ **(Additional vertex)**

$$F(k) = \frac{(\Lambda^2 - M_K^2)^2}{(\Lambda^2 - k^2)^2}$$

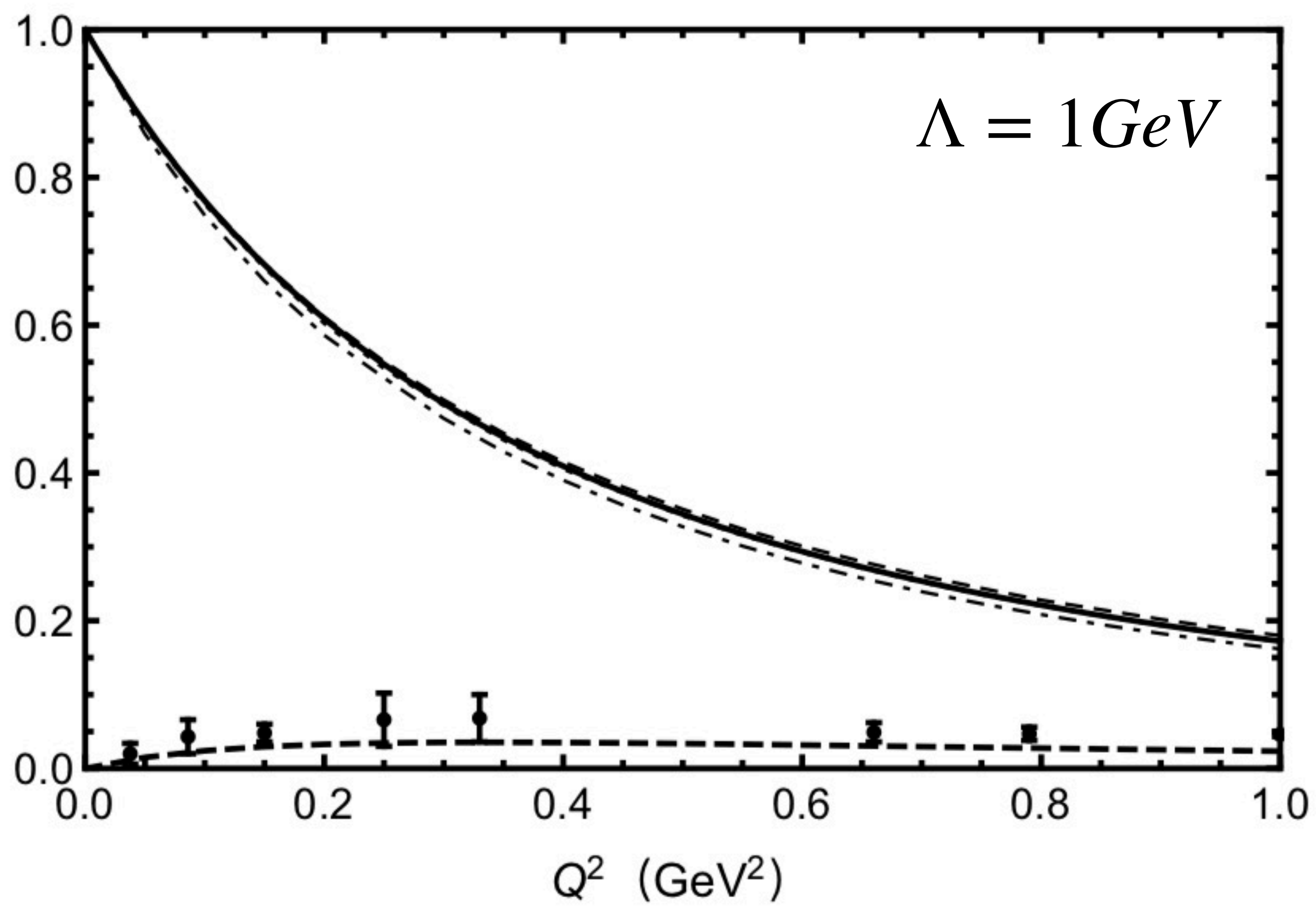
F(k) is the Fourier transformation of F(x-y)

Nucleon electromagnetic form factors

F. He and P. Wang, PRD(98), 2018

- **Numerical results of nucleon electromagnetic form factors with nonlocal chiral effective theory**

$$F(k) = \frac{(\Lambda^2 - M_K^2)^2}{(\Lambda^2 - k^2)^2}$$



Solid line: $1/(1+Q^2/0.71^2)^2$

Dashed line: G_P^E

Dash-Dotted line: G_P^M/μ_P

The best choice for Λ is about 1GeV !

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Convolution formula for GPDs

Matrix element for GPDs in quark level:

$$V_q = \frac{1}{2}\int \frac{dz^-}{2\pi} e^{ix(Pz)} < p'|O_q|p > |_{z=\lambda n}, \quad \text{where} \quad O_q = \bar{q}(-\frac{1}{2}z)\gamma^+ q(\frac{1}{2}z)$$

Match to the hadron level

$$V_q = \frac{1}{2}\sum_H \int_0^1 dy\, \theta(0 \le \frac{x}{y} \le 1) \times q_H^v(\frac{x}{y},0,t) \times \int \frac{dz^-}{2\pi} e^{iy(Pz)} < p'|O_H|p >$$

Input GPD　　Splitting function

Where O_H is twist-2 hadron operator, $q_H^v(\frac{x}{y},0,t)$ is quark valance GPD in hadron state H.

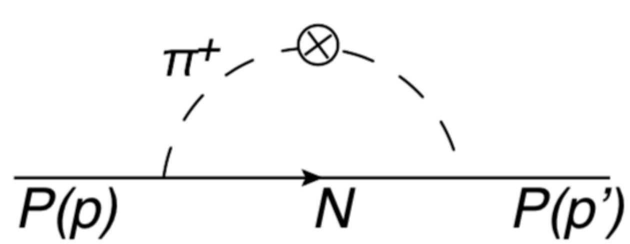
Sullivan process

① **Hadron operator for π^+ :** $\quad O_{\pi^+} = \pi^-(\frac{z}{2})\partial^+\pi^+(-\frac{z}{2}) - \partial^+\pi^-(\frac{z}{2})\pi^+(-\frac{z}{2})$

② **The hadron matrix element can be written as**

$$V_H = \int \frac{dz^-}{2\pi} e^{iy(Pz)} < p' | O_{\pi^+} | p > \; = \bar{u}(p') \left\{ \gamma^\mu f(y,0,t) + \frac{i\sigma^{\mu\nu} q_\nu}{2m_N} g(y,0,t) \right\} u(p),$$

- **Rainbow diagram**



Splitting functions

③ **Sea quark GPDs in the proton can be obtained by the following convolution formalism:**

$$H^{\bar{d}}(x,0,t) = \int_x^1 \frac{dy}{y} H^{\bar{d}}_{\pi^+}(\frac{x}{y},0,t) f(y,0,t), \qquad E^{\bar{d}}(x,0,t) = \int_x^1 \frac{dy}{y} H^{\bar{d}}_{\pi^+}(\frac{x}{y},0,t) g(y,0,t)$$

$H^{\bar{d}}_{\pi^+}(\frac{x}{y},0,t)$ **is the valance quark GPD in π^+, we use the phenomenological expression** $H^{\bar{d}}_{\pi}(x,0,t) = \bar{d}_\pi(x) F_\pi(t)$

M. Aicher, A. Schafer and W. Vogelsang, PRL105(2018), 252003

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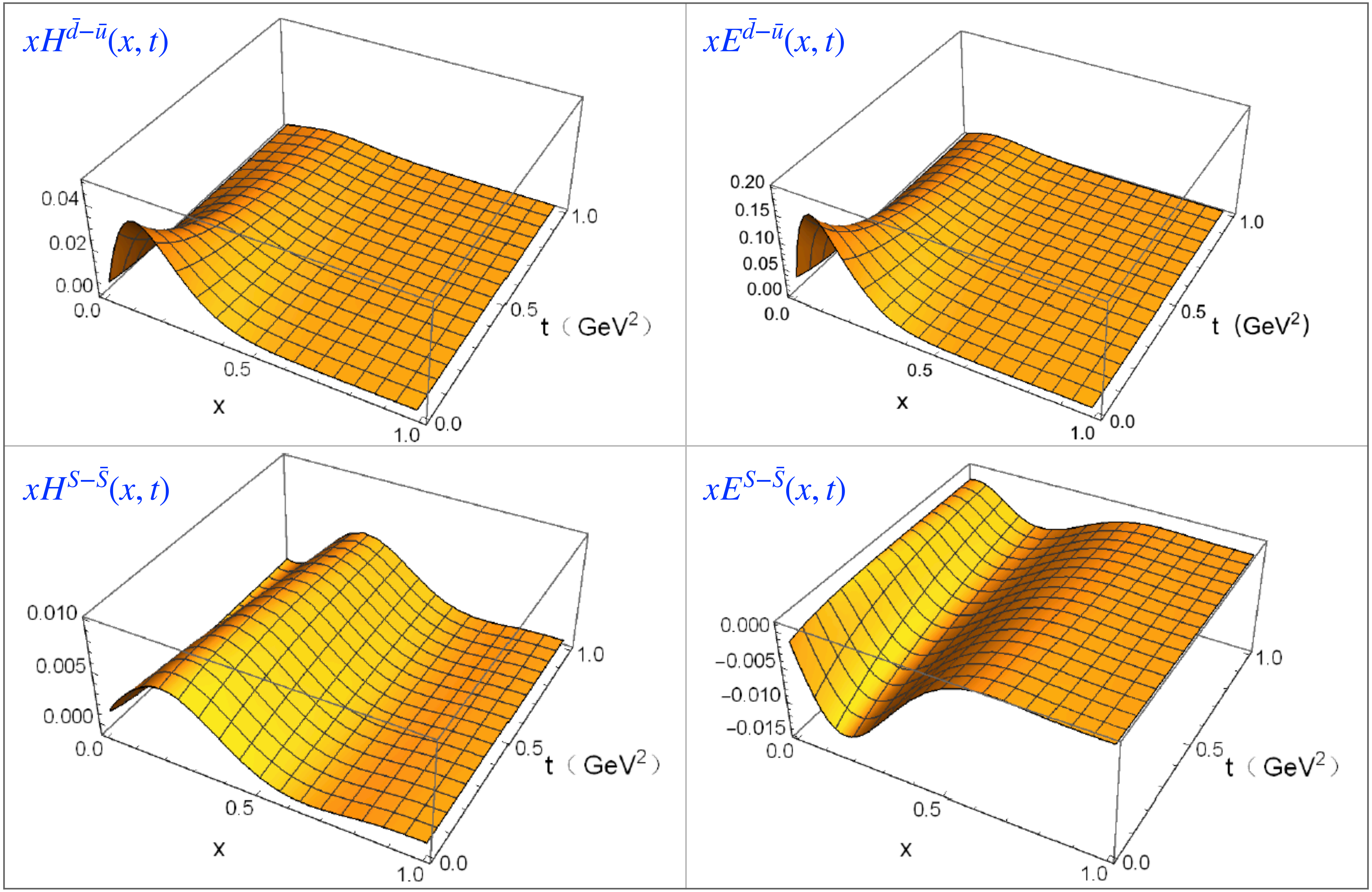
Numerical results ($\xi = 0$)

F. He et al., in preparation

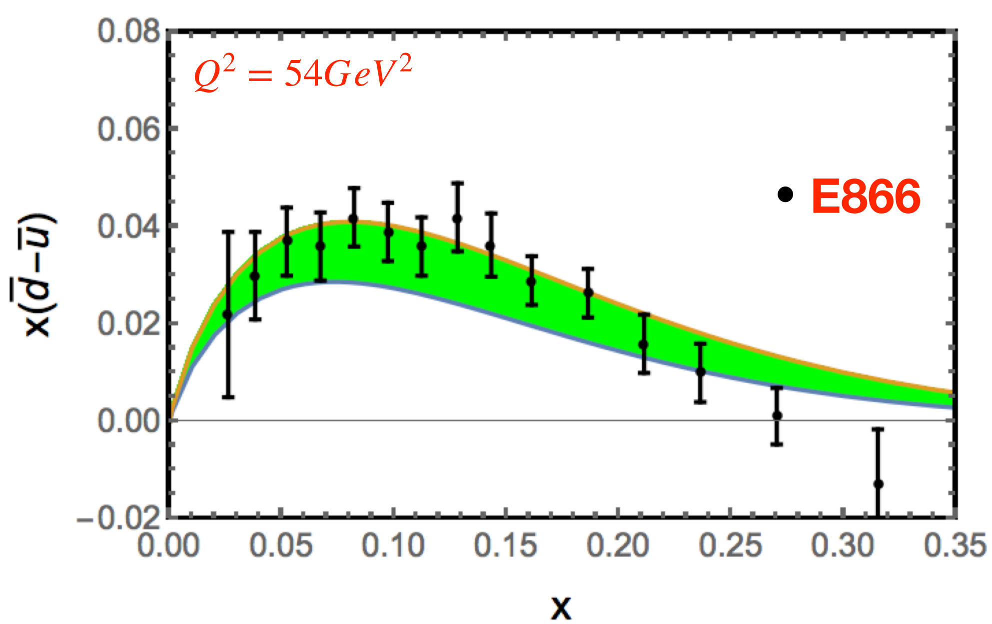


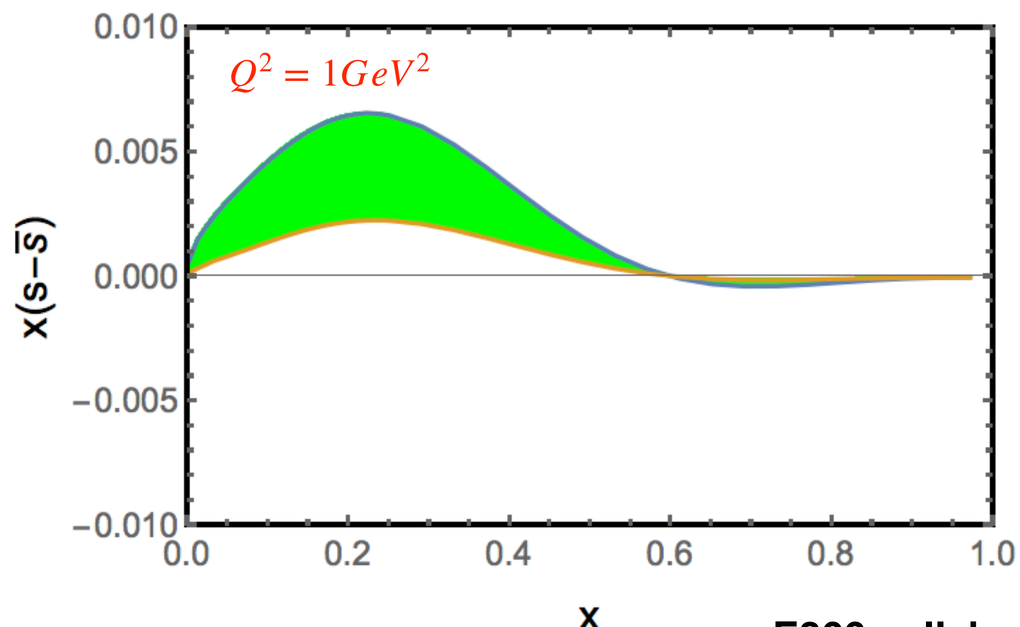
Numerical results (sea quark flavor asymmetry)

F. He et al., in preparation

$$F(k) = \frac{(\Lambda^2 - M_K^2)^2}{(\Lambda^2 - k^2)^2}$$

- sea quark asymmetry($\Lambda = 1GeV \pm 0.1GeV, Q_0^2 = 1GeV^2$)





E866 collaboration

- The lowest momentum:

$$< \bar{d} - \bar{u} > = \int_0^1 (\bar{d}(x) - \bar{u}(x))dx = 0.090 - 0.131, \qquad < \bar{d} - \bar{u} > = 0.118(12)$$

- The second momentum for strange quark:

$$< xs^- > = < x(s - \bar{s}) > = \int_0^1 x(s(x) - \bar{s}(x))dx = 0.0007 - 0.0021,$$

W. Bentz, et.al,PLB 693 (2010)

	$\langle xs^- \rangle$
Mason et al. [8]	0.00196 ± 0.00143
NNPDF [9]	0.0005 ± 0.0086
Alekhin et al. [30]	$0.0013 \pm 0.0009 \pm 0.0002$
MSTW [31]	$0.0016^{+0.0011}_{-0.0009}$
CTEQ [32]	$0.0018^{+0.0016}_{-0.0004}$
This work (Eq. (10))	0.0 ± 0.0020

NNPDF at Q = 1GeV : $< xs^- > = 0.0009 - 0.0053$ **(More recently)**

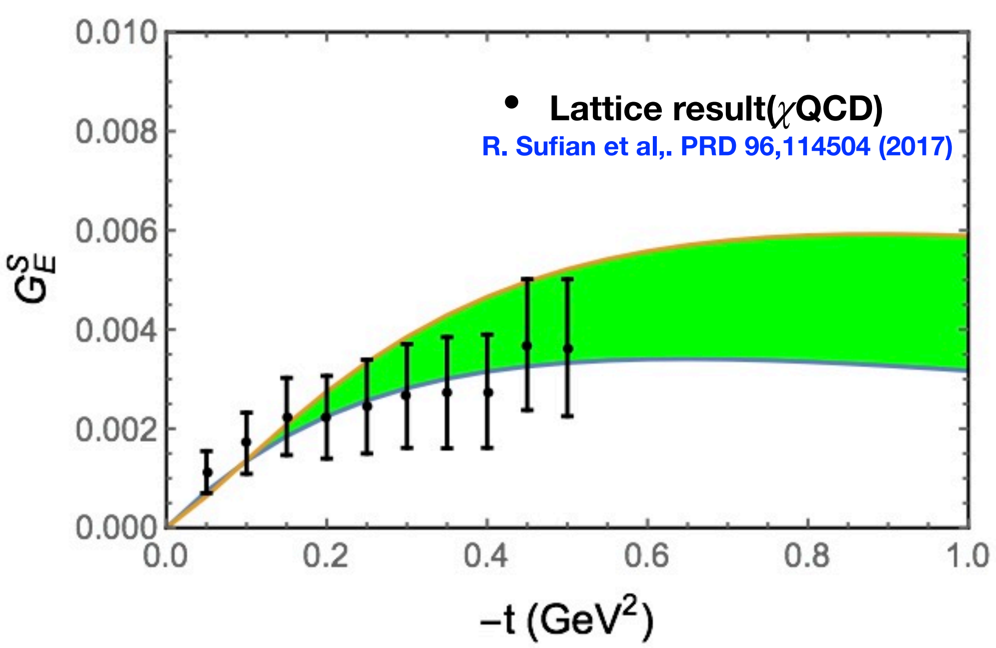
Numerical results (Strange form factors)

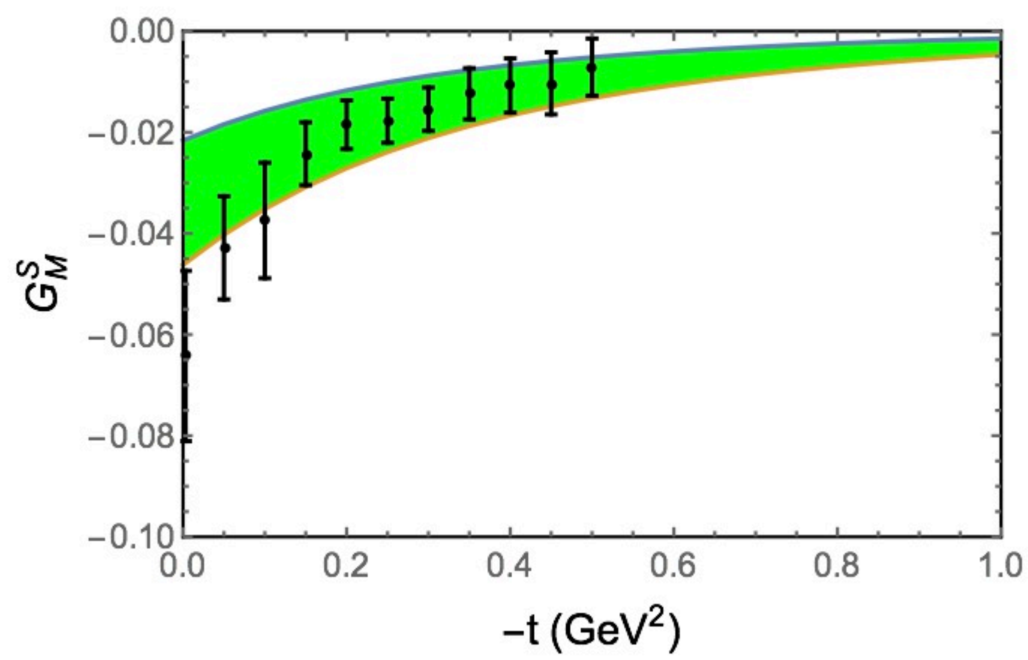
F. He et al., in preparation

- **Strange Dirac and Pauli Form factor:**

$$F_1^S = \int_0^1 \{H^S(x,0,t) - H^{\bar{S}}(x,0,t)\}dx \qquad F_2^S = \int_0^1 \{E^S(x,0,t) - E^{\bar{S}}(x,0,t)\}dx$$

- **Strange Electromagnetic Form factor:**





Summary and Outlook

- **In summary**

  - We proposed a nonlocal and gauge invariant chiral Lagrangian, the cut-off is naturally introduced in this method.
  - We calculate the sea quark zero-skewness GPDs, the asymmetry of sea quark PDF and strange electromagnetic Form factor can be got from GPDs, which are consistent with experimental or Lattice results.

- **In outlook**

  - We can extend our method to calculate the nonzero skewness GPDs, polarized GPDs…
  - This method also can be used to calculate the transverse momentum dependent PDFs.

Thank you for your attention!