

PANIC 2021, September 8<sup>th</sup>, 2021

# A model calculation of T-odd gluon TMD distributions at twist-2

**Francesco Giovanni Celiberto**

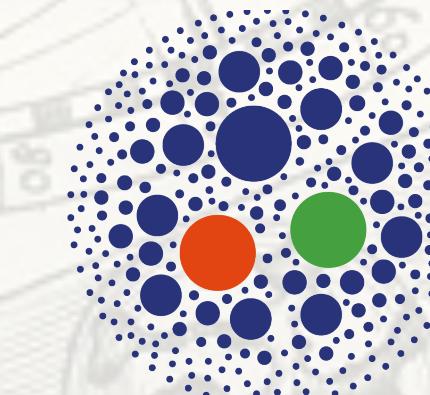
ECT\*/FBK Trento & INFN-TIFPA

**ECT\***

EUROPEAN CENTRE FOR THEORETICAL STUDIES  
IN NUCLEAR PHYSICS AND RELATED AREAS



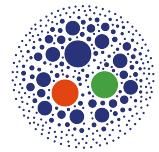
Trento Institute for  
Fundamental Physics  
and Applications



**HAS QCD**

HADRONIC STRUCTURE AND  
QUANTUM CHROMODYNAMICS

# Gluon TMDs: a largely unexplored territory



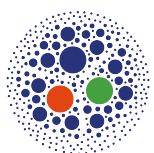
**Theory:** different **gauge-link** structures...

...more diversified kind of **modified universality!**



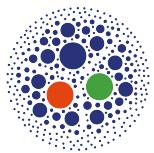
**Pheno:** golden channels for extraction  
of quark TMDs are subleading for gluon TMDs

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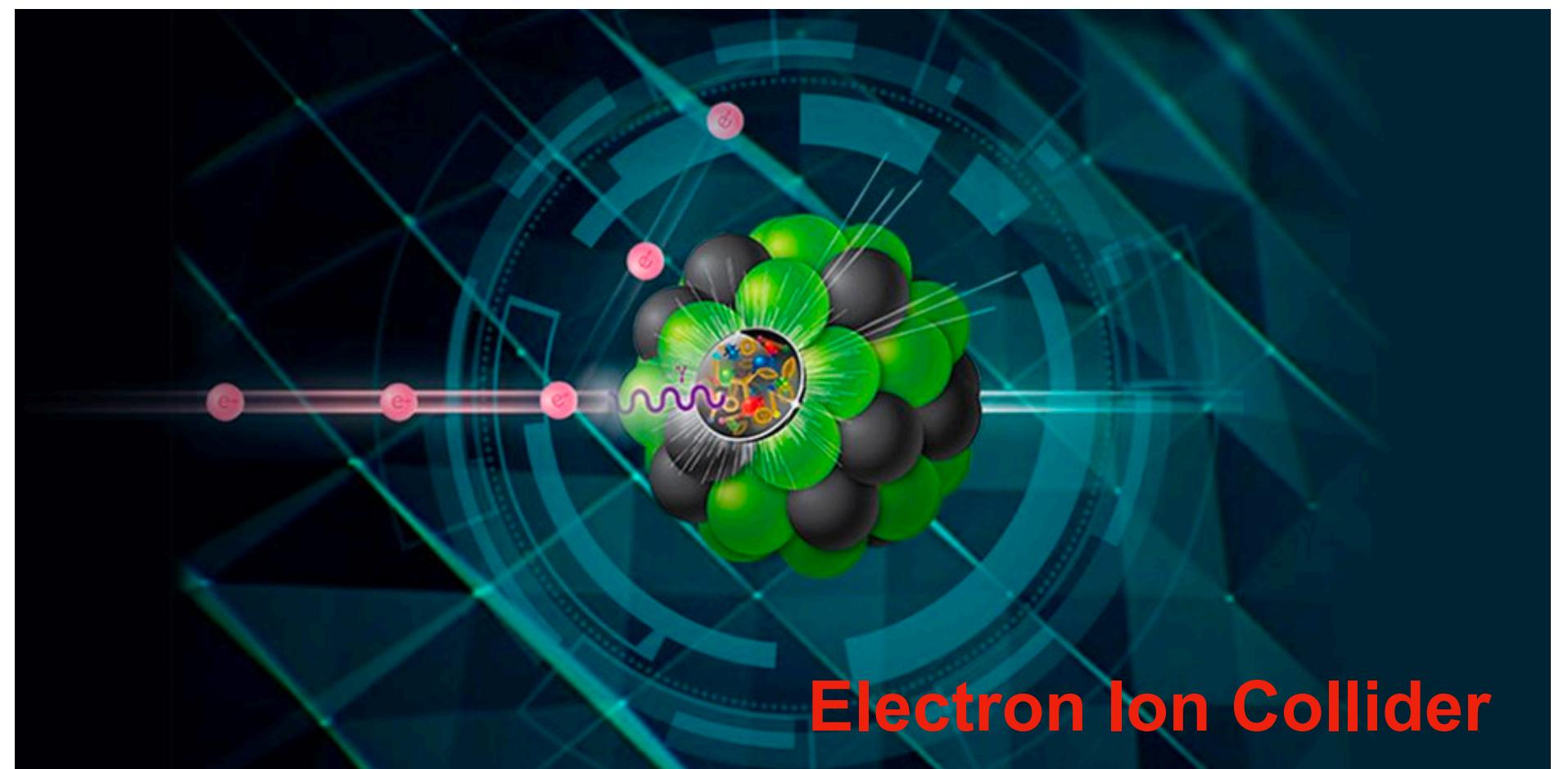


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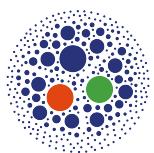


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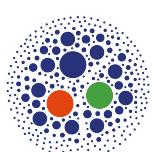


Electron Ion Collider

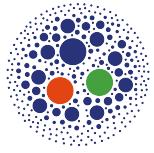
## Motivation



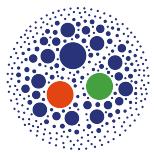
Gluon-TMD PDFs: *core* sector of **EIC** studies



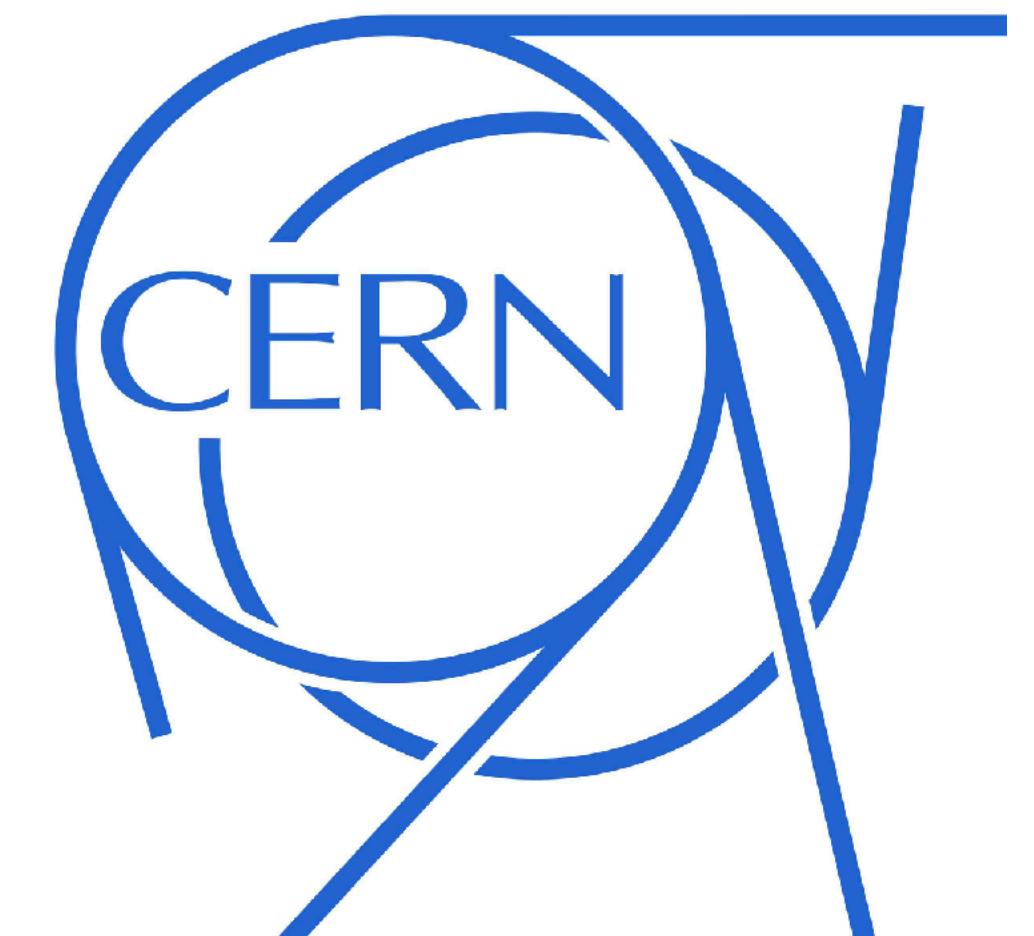
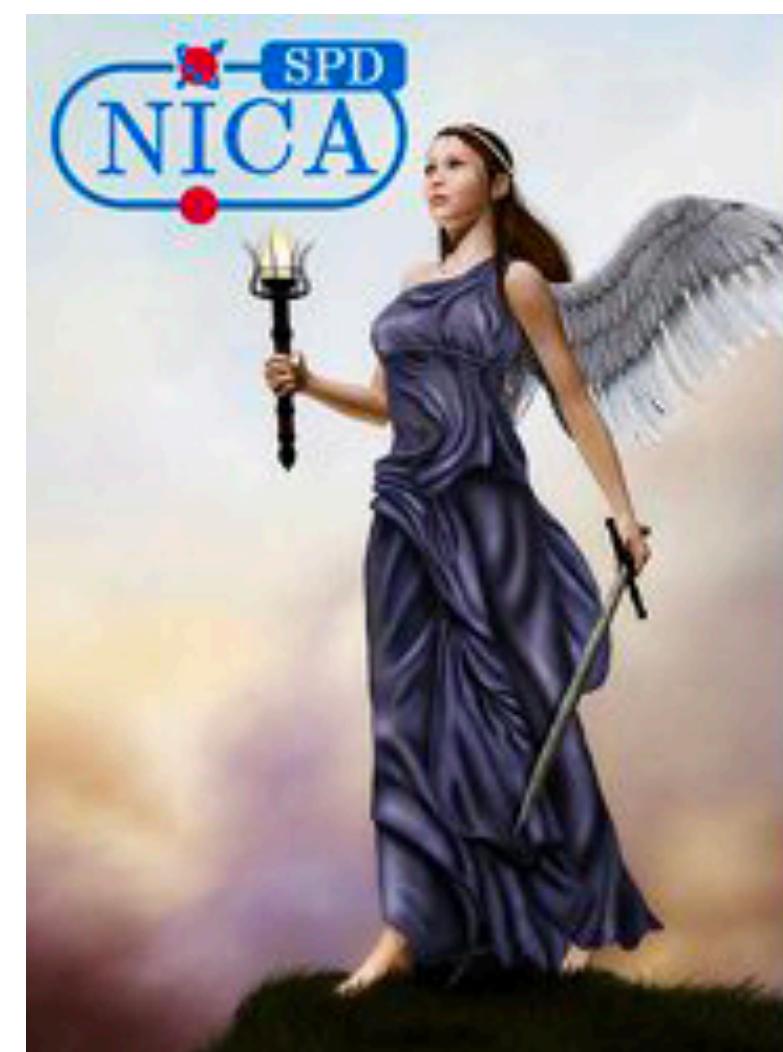
Need for a *flexible* model, suited to *pheno*



**Unpolarized** and **polarized** gluon TMDs



*Consistent* framework for quark TMDs

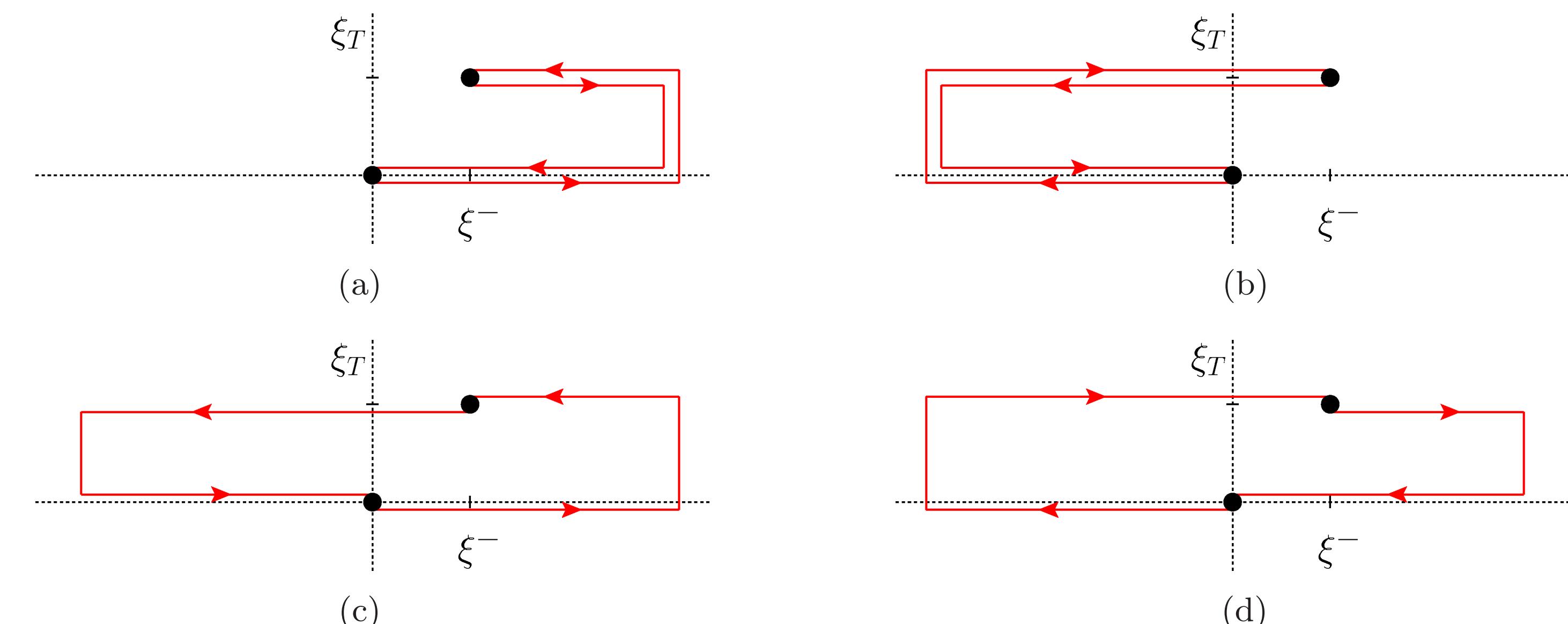


# Gluon TMDs: gauge links and modified universality

- \* **Single-spin asymmetries** → process dependence of TMDs via **gauge links**
- \* **Color flow** → integration paths of gauge links calculable
- \* Gluon TMDs → more complicated structure with respect to quark **staple links**
- \* **Factorization-preserving** processes → two main kinds of **modified universality**
- \* Different classes of processes → distinct gluon TMDs, **not related** to each other

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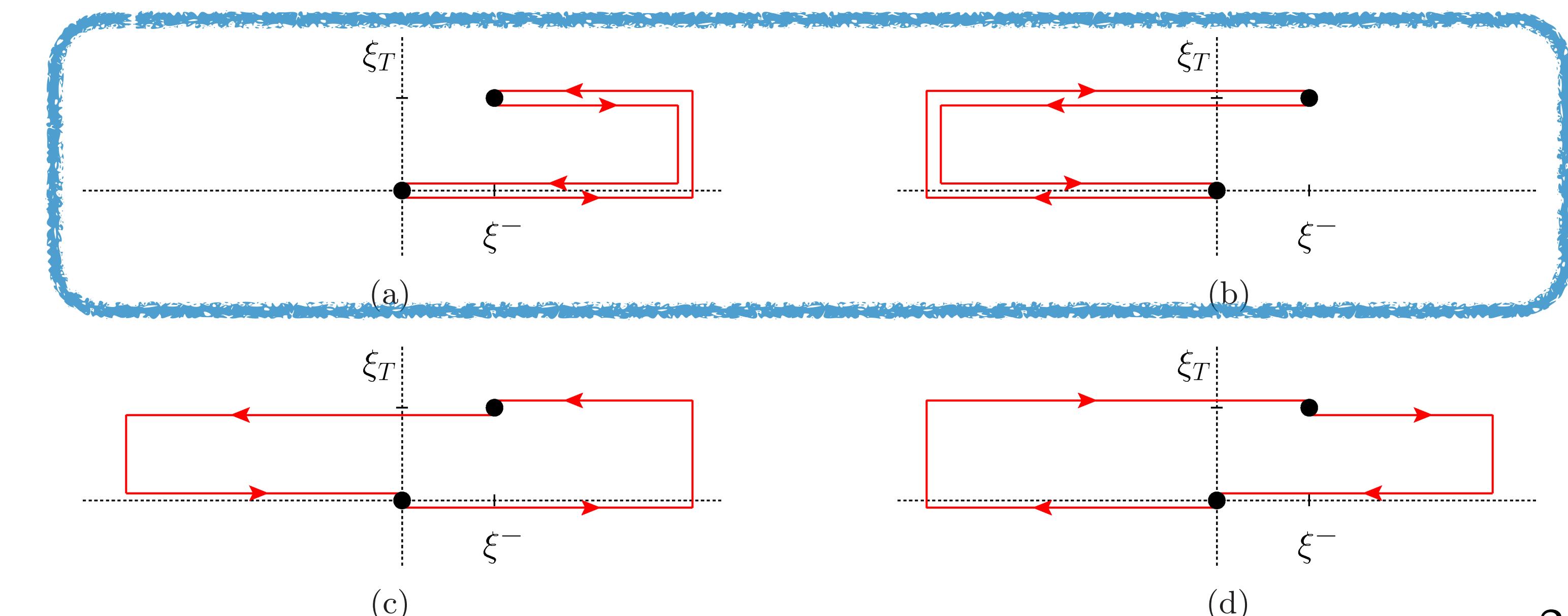


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## Weiszäcker-Williams (WW)

(a) [ + , + ] or (b) [ - , - ]

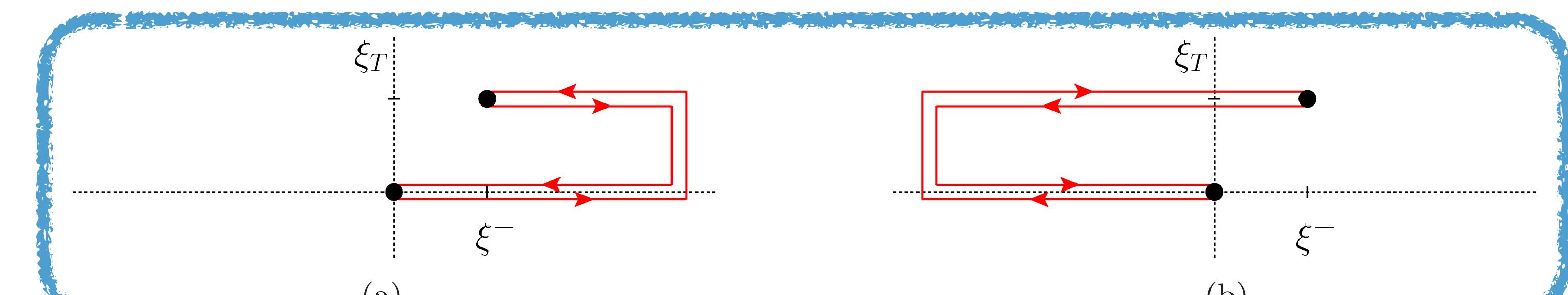


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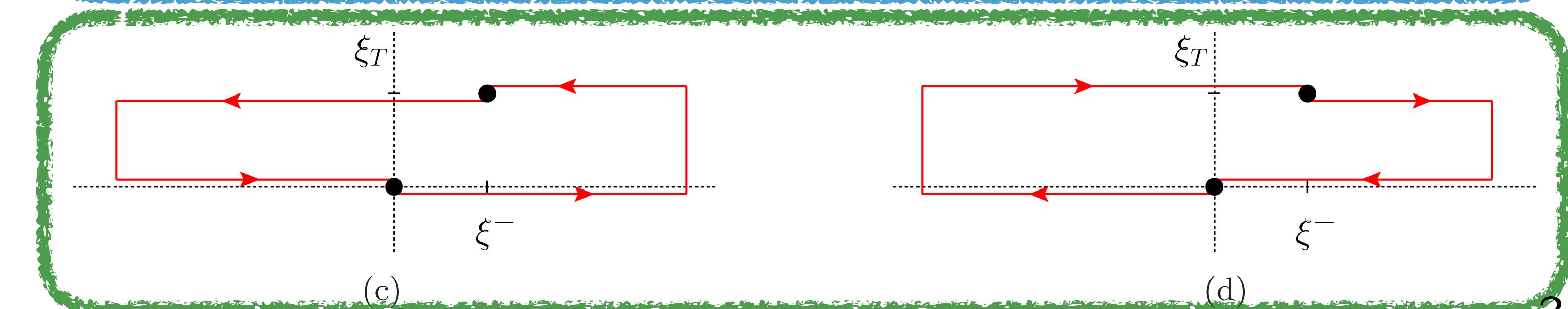
## Weiszäcker-Williams (WW)

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## Dipole (DP)

(c) [ + , - ] or (d) [ - , + ]



# **T-even and T-odd gluon TMD PDFs at twist-2**

gluon pol.

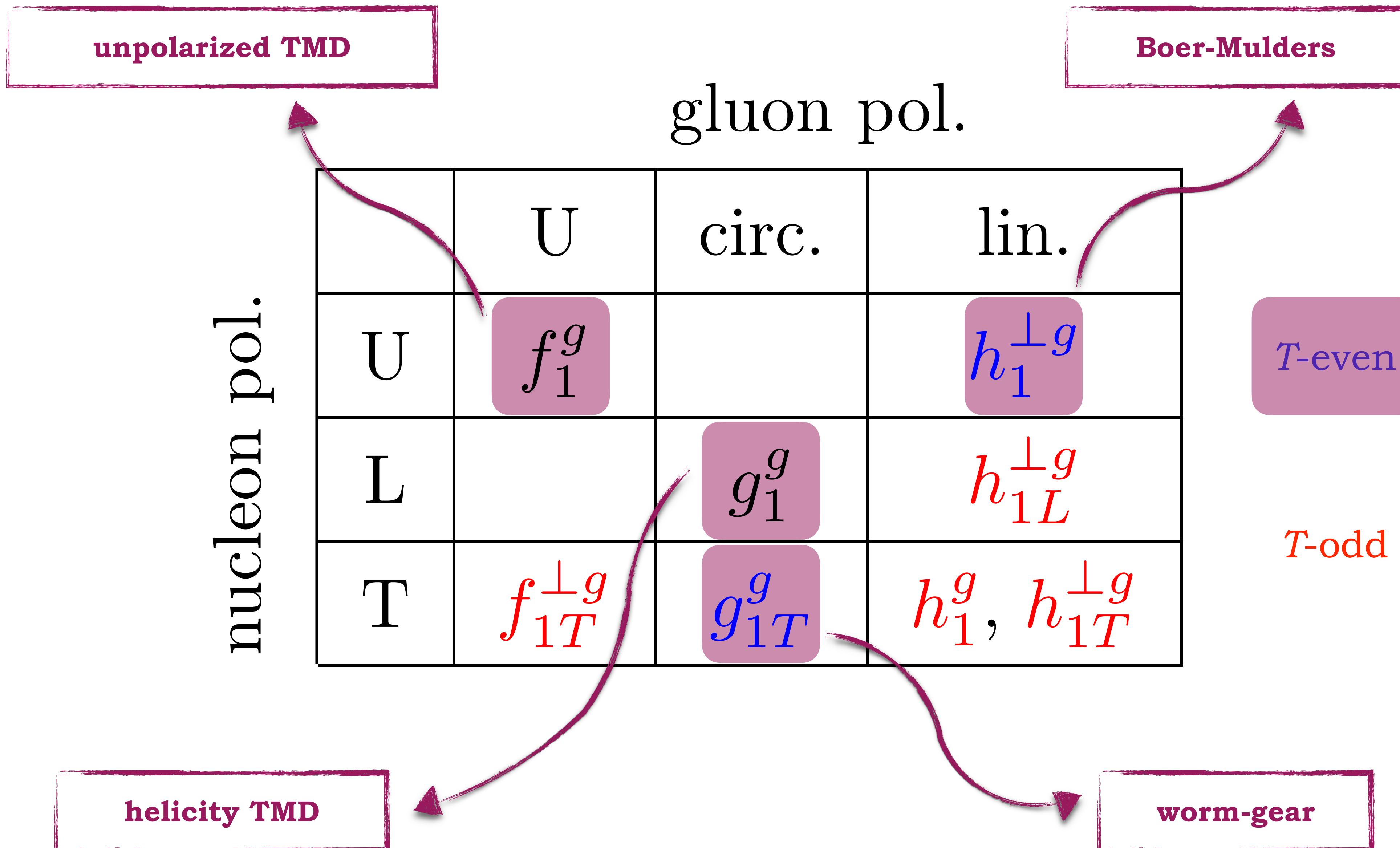
	U	circ.	lin.
U	$f_1^g$		$h_1^{\perp g}$
L		$g_1^g$	$h_{1L}^{\perp g}$
T	$f_{1T}^{\perp g}$	$g_{1T}^g$	$h_1^g, h_{1T}^{\perp g}$

*T-even*

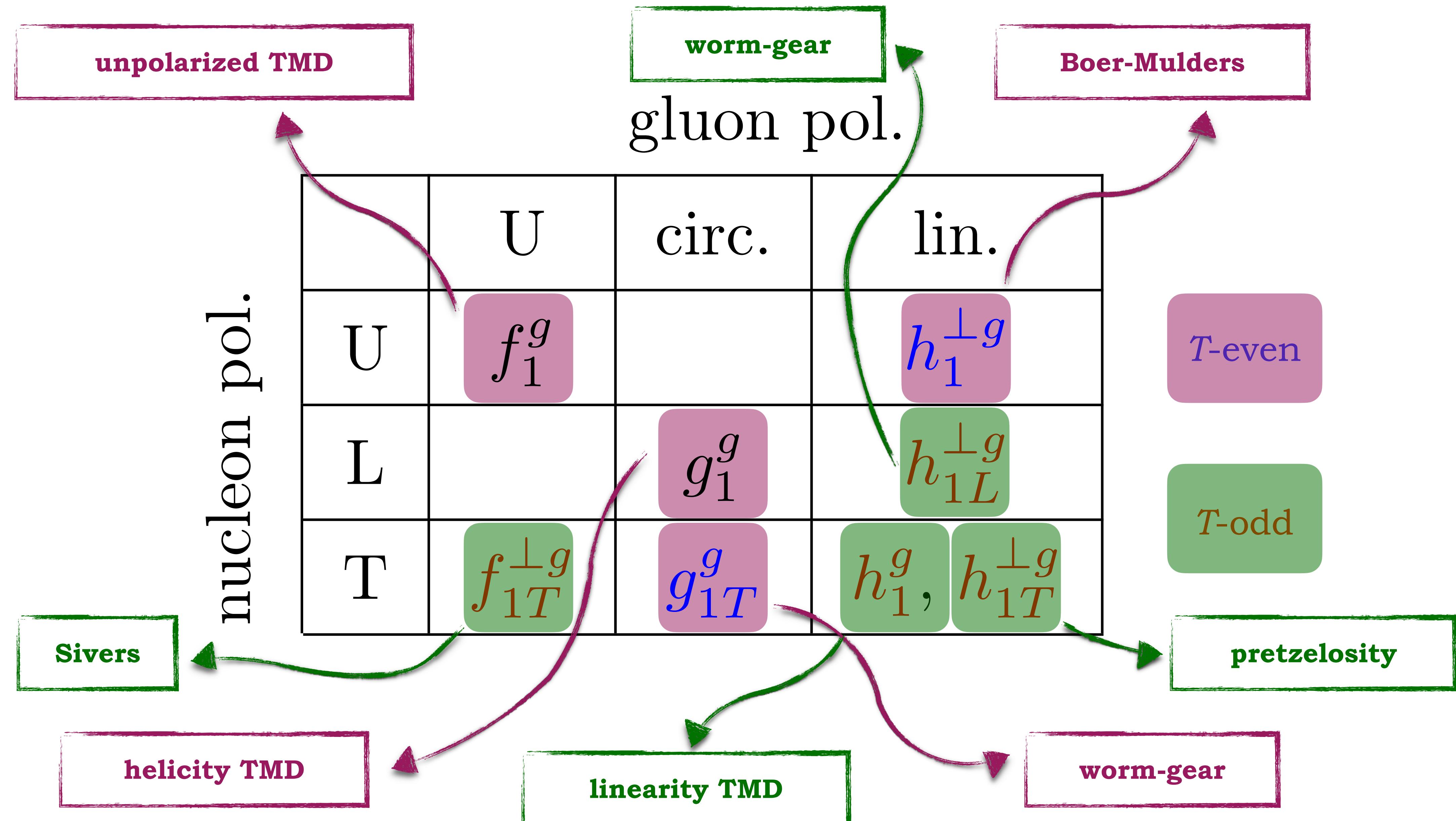
*T-odd*

nucleon pol.

# T-even and T-odd gluon TMD PDFs at twist-2

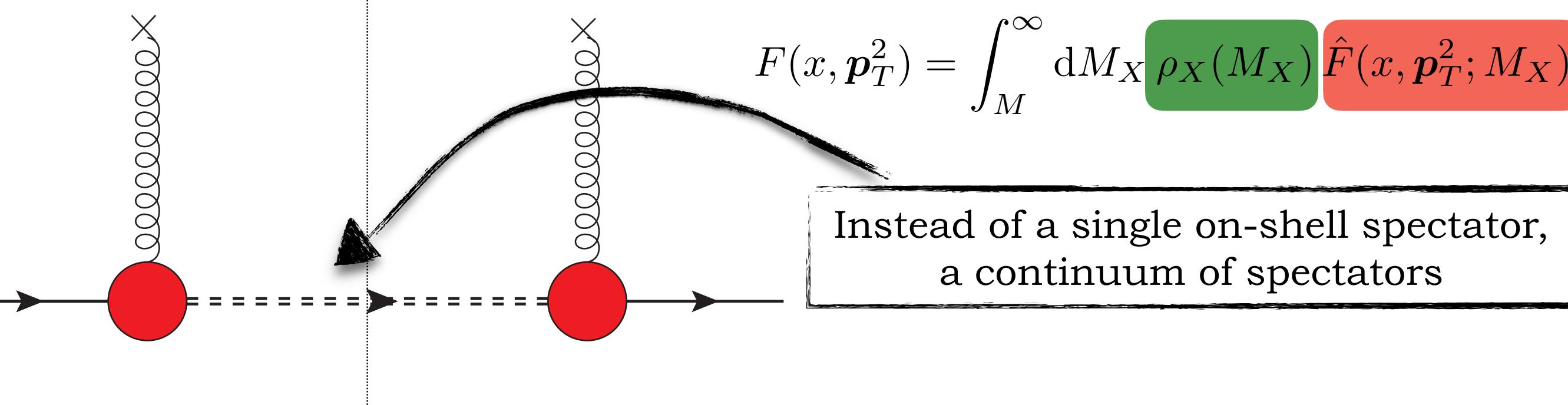


# T-even and T-odd gluon TMD PDFs at twist-2



# Our model at a glance

## Spectator-system spectral-mass function



Spectral function **learns** small- and moderate- $x$  info  
encoded in **NNPDF** collinear parametrizations

(NNPDF3.1sx + NNPDFpol1.1)

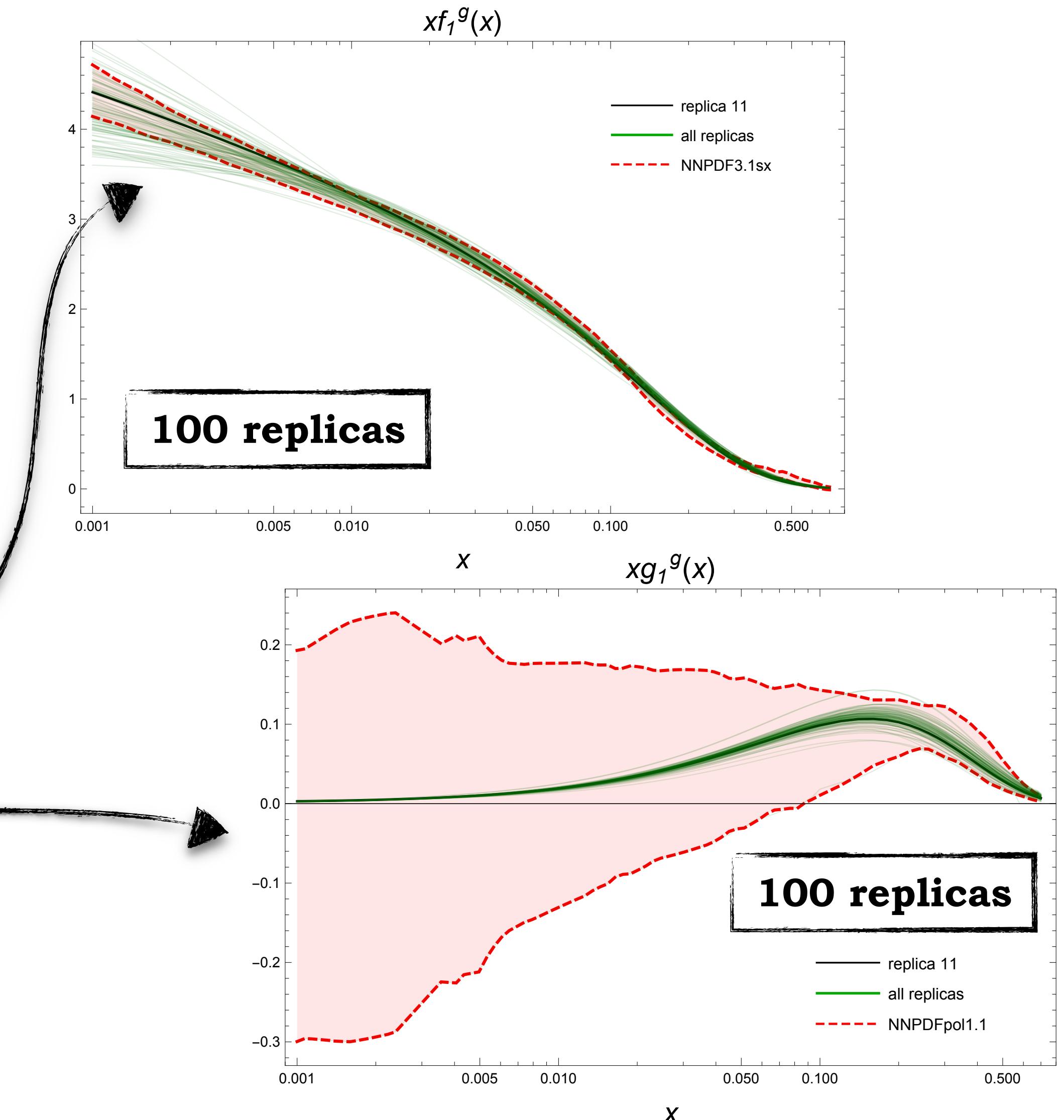
**Simultaneous fit** of  $f_1$  and  $g_1$  PDFs

Inclusion of small- $x$  resummation effects (**BFKL**)

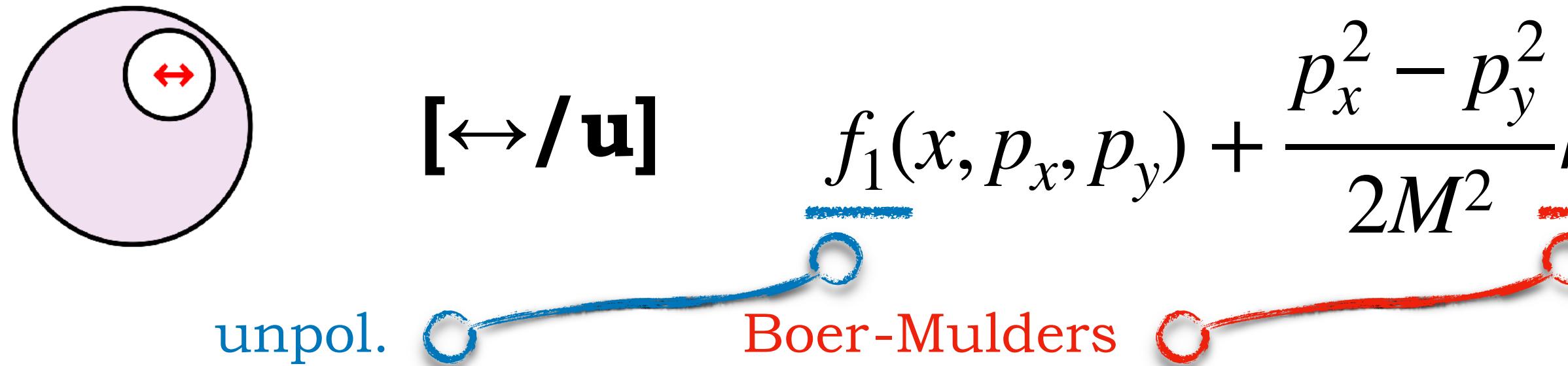
Calculation of all twist-2  $T$ -even gluon TMDs

## Link with collinear factorization

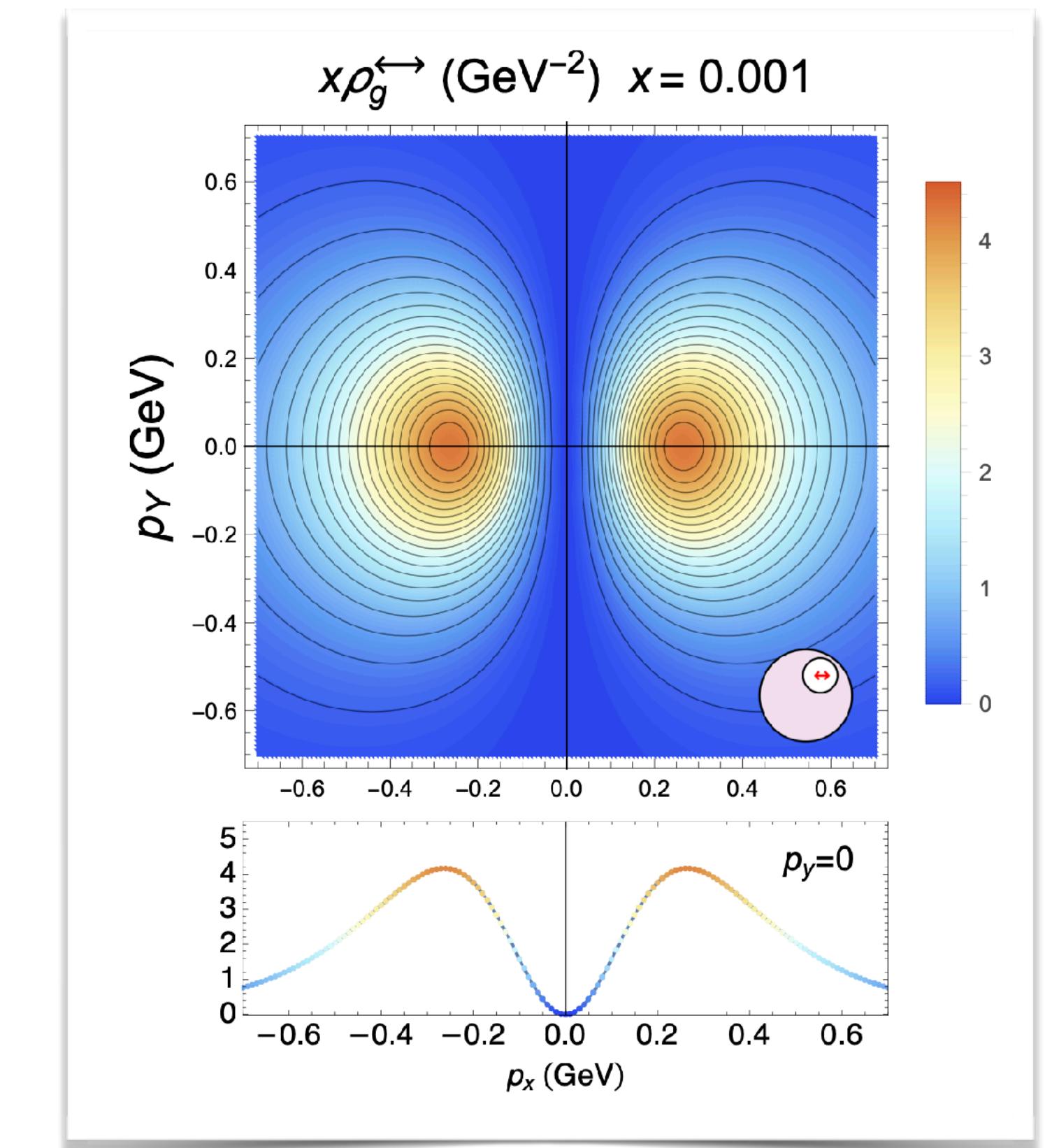
$p_T$ -integrated TMDs **have to** reproduce PDFs  
at the lowest scale ( $Q_0$ ) *before* evolution



# Boer-Mulders effect in unpolarized $pp$ collisions

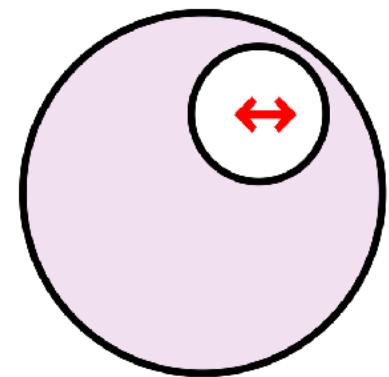


$$f_1(x, p_x, p_y) + \frac{p_x^2 - p_y^2}{2M^2} h_1^\perp(x, p_x, p_y)$$



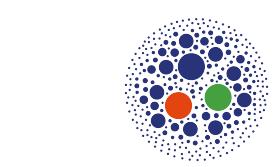
[A. Bacchetta, F.G.C., M. Radici, P. Taels (2020)]

# Boer-Mulders effect in unpolarized $pp$ collisions



$[\leftrightarrow / \mathbf{u}]$

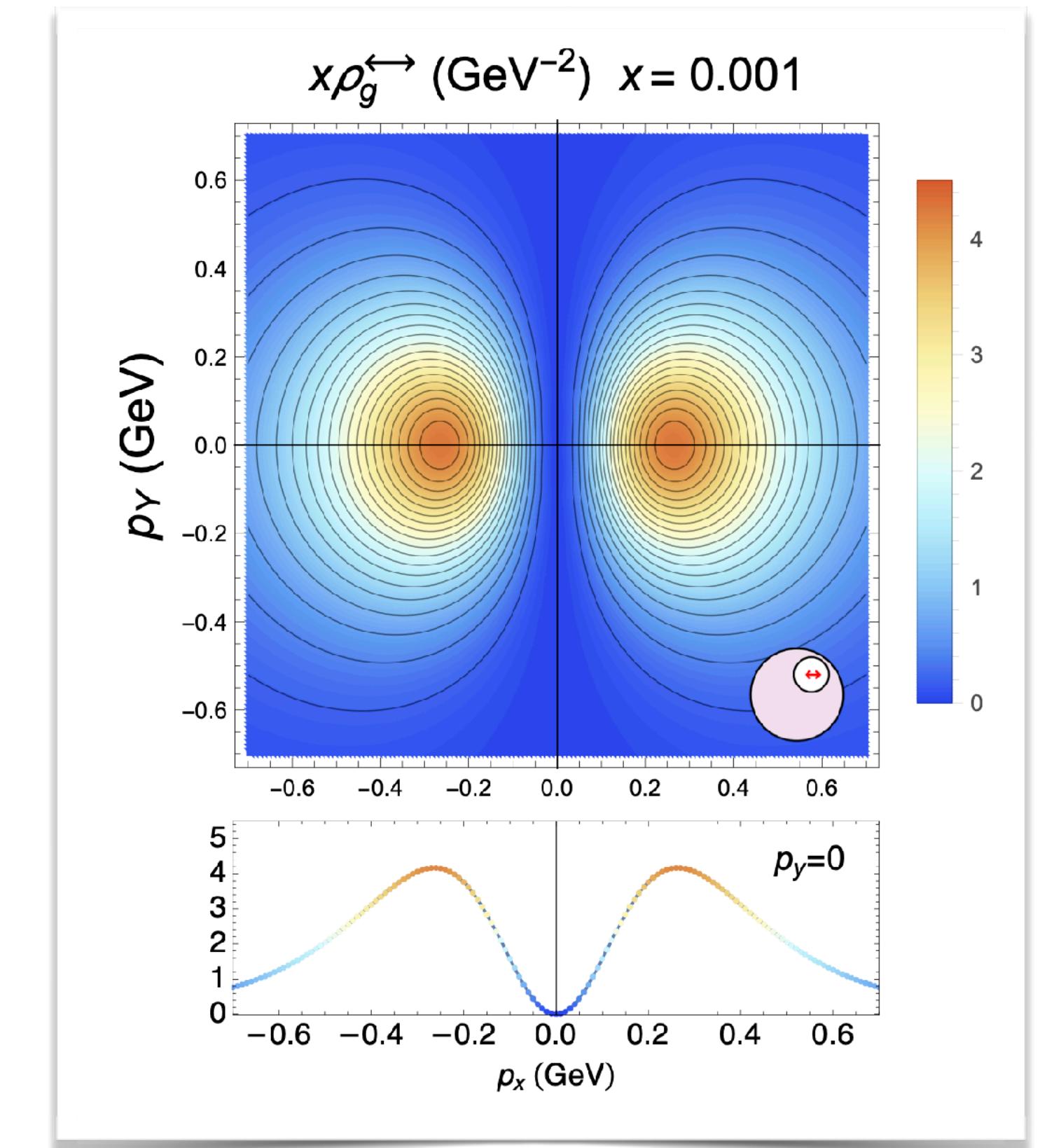
$$f_1(x, p_x, p_y) + \frac{p_x^2 - p_y^2}{2M^2} h_1^\perp(x, p_x, p_y)$$



(Pseudo)scalar Higgs  $p_T$ -distribution

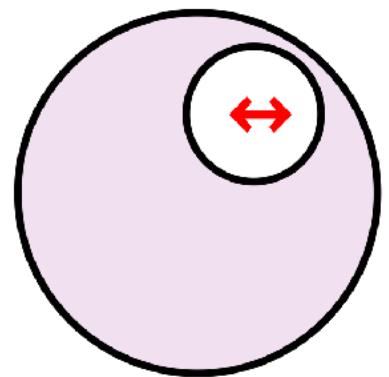
$$\begin{aligned} \frac{E d\sigma^{H(A)}}{d^3\vec{q}} \Big|_{q_T \ll m_H} &= \frac{\pi\sqrt{2}G_F}{128m_H^2 S} \left(\frac{\alpha_s}{4\pi}\right)^2 |\mathcal{A}_{H(A)}(\tau)|^2 \\ &\times \left( \mathcal{C}[f_1^g f_1^g] \pm \mathcal{C}[w_H h_1^{\perp g} h_1^{\perp g}] \right) + \mathcal{O}\left(\frac{q_T}{m_H}\right) \end{aligned}$$

🔗 [D. Boer, W.J. den Dunnen, C. Pisano, M. Schlegel, W. Vogelsang (2012)]  
 (Higgs+jet angular distributions) 🔗 [D. Boer, C. Pisano (2015)]



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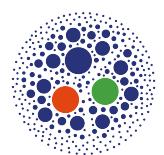
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$[ \leftrightarrow / \mathbf{u} ]$

$$\frac{f_1(x, p_x, p_y)}{\overline{u}} + \frac{p_x^2 - p_y^2}{2M^2} \frac{h_1^\perp(x, p_x, p_y)}{\overline{u}}$$

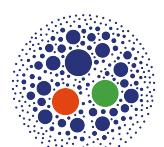
unpol.      Boer-Mulders



## (Pseudo)scalar Higgs $p_T$ -distribution

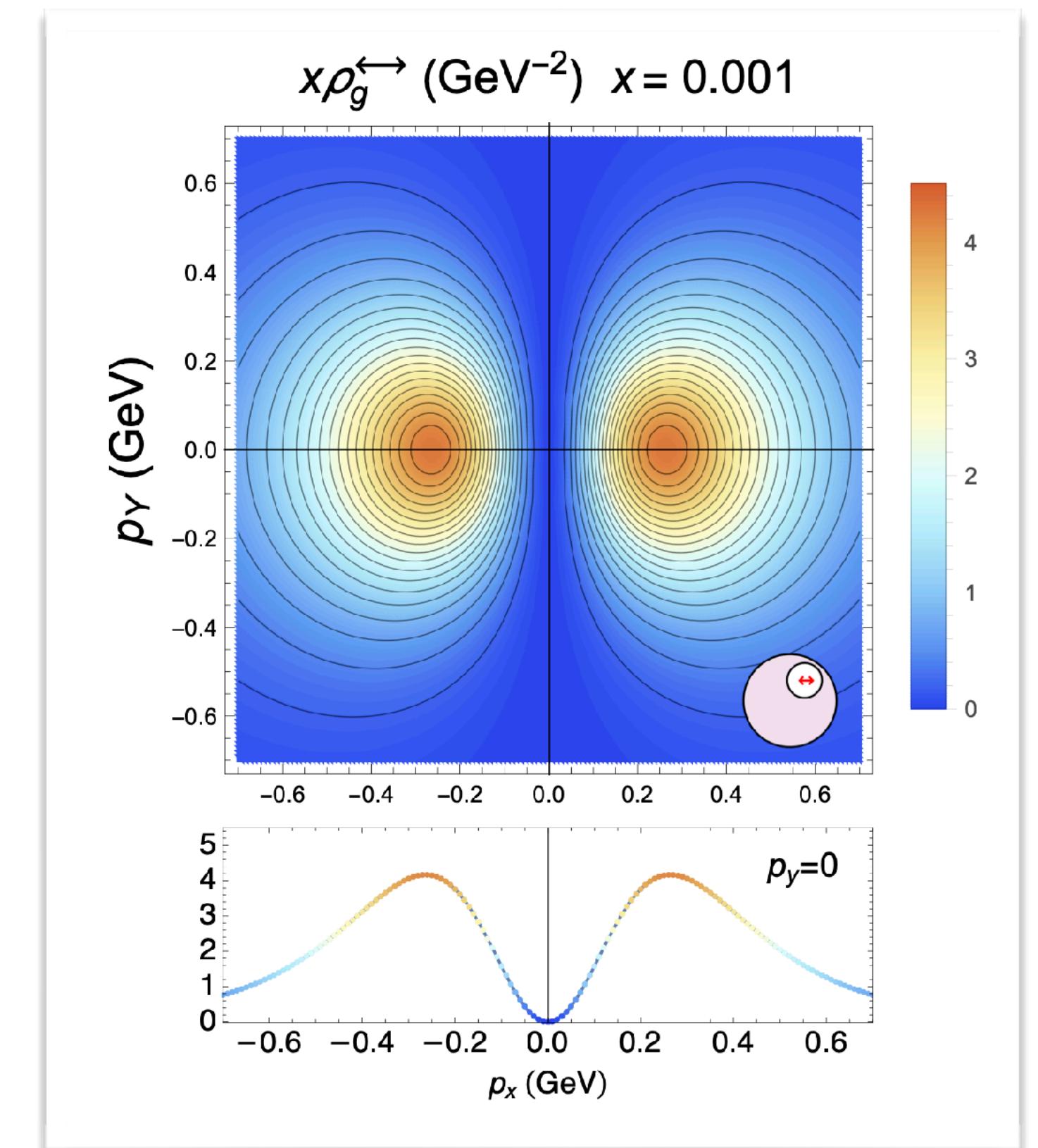
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## Model prediction at low- $x$

$$\frac{f_1^g(x, p_T^2)}{h_1^{\perp g}(x, p_T^2)} \underset{x \rightarrow 0^+}{\sim} \text{constant}$$



🔗 [A. Bacchetta, F.G.C., M. Radici, P. Taels (2020)]

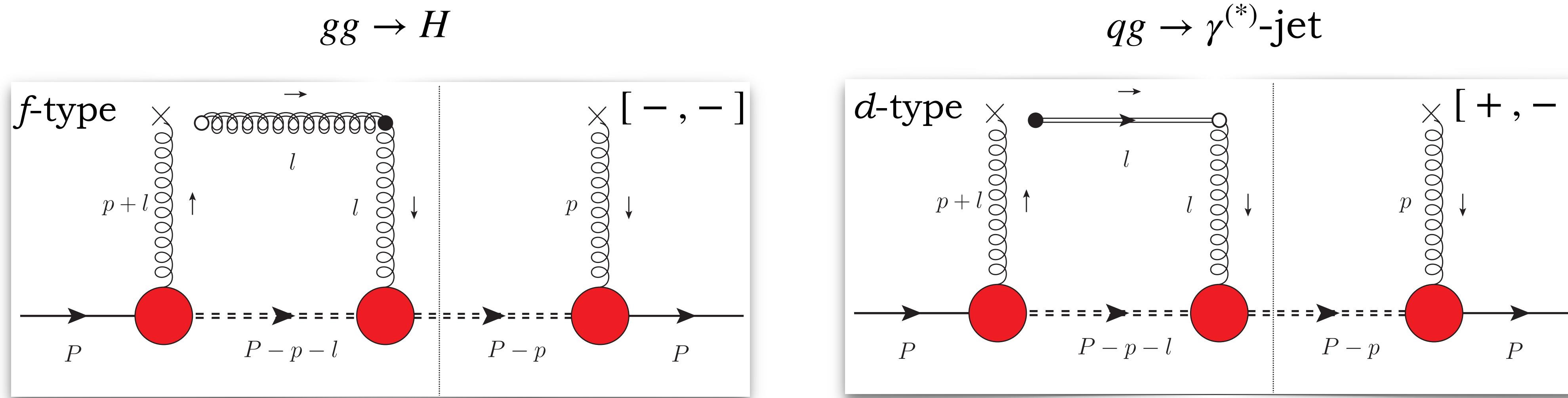
**...towards twist-2  
T-odd gluon TMDs**

# **T-odd gluon TMDs in a spectator model**

- No residual gluon-spectator interaction at tree level
- *Interference with one-gluon exchange (eikonal)*

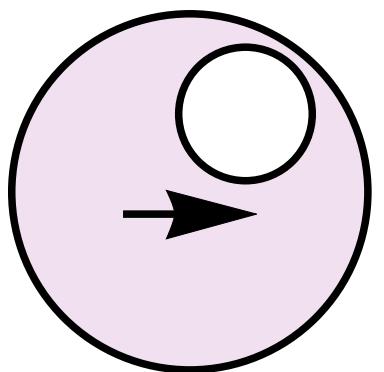
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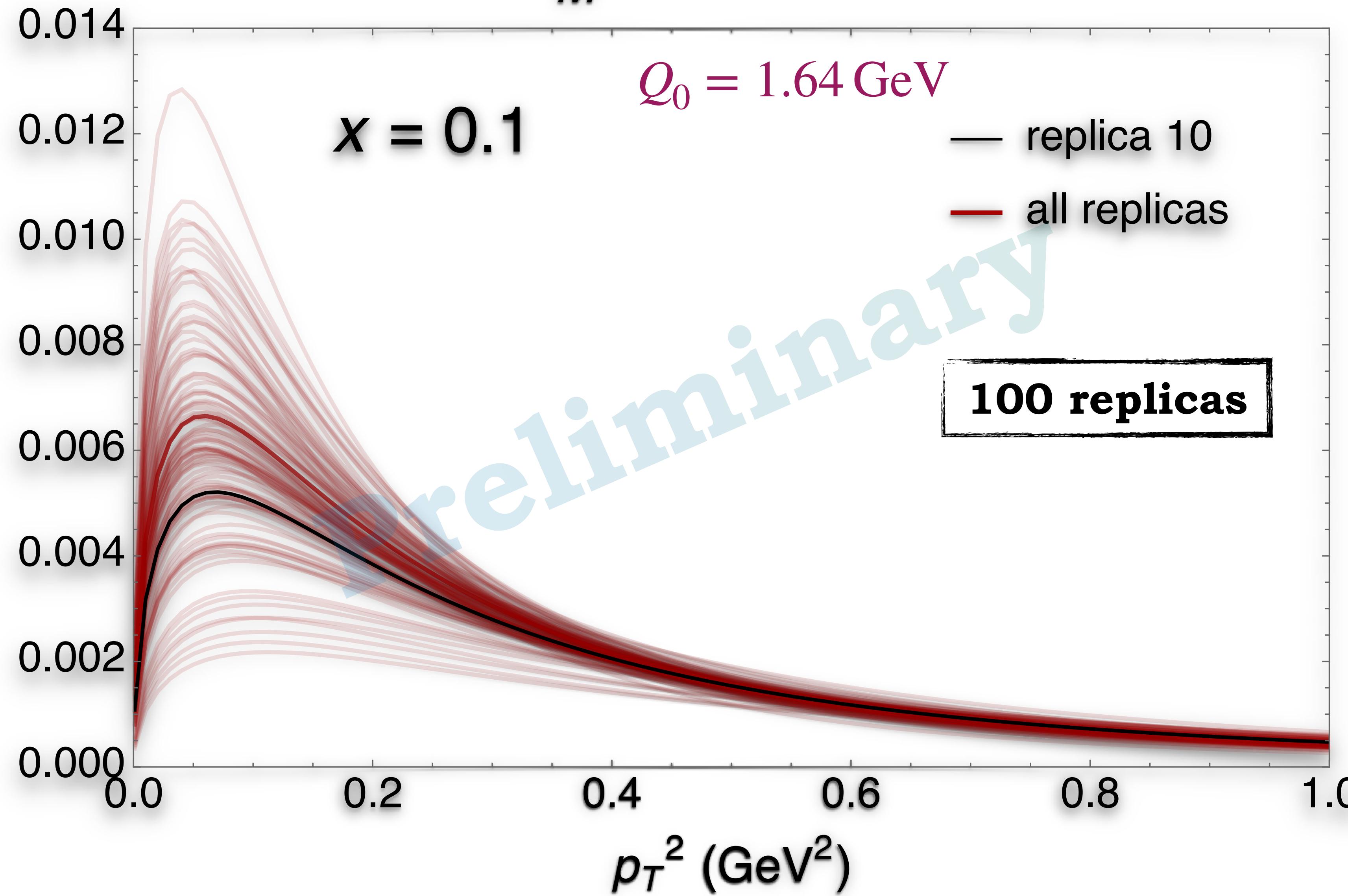


- Leading-twist one-gluon-exchange of the gauge-link operator
- Sensitivity to WW/DP structures
- Calculation of **Sivers** function *underway!*

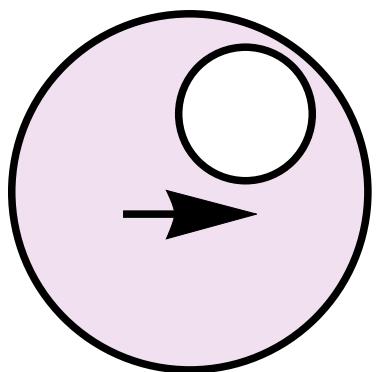
# *f*-type Sivers gluon TMD



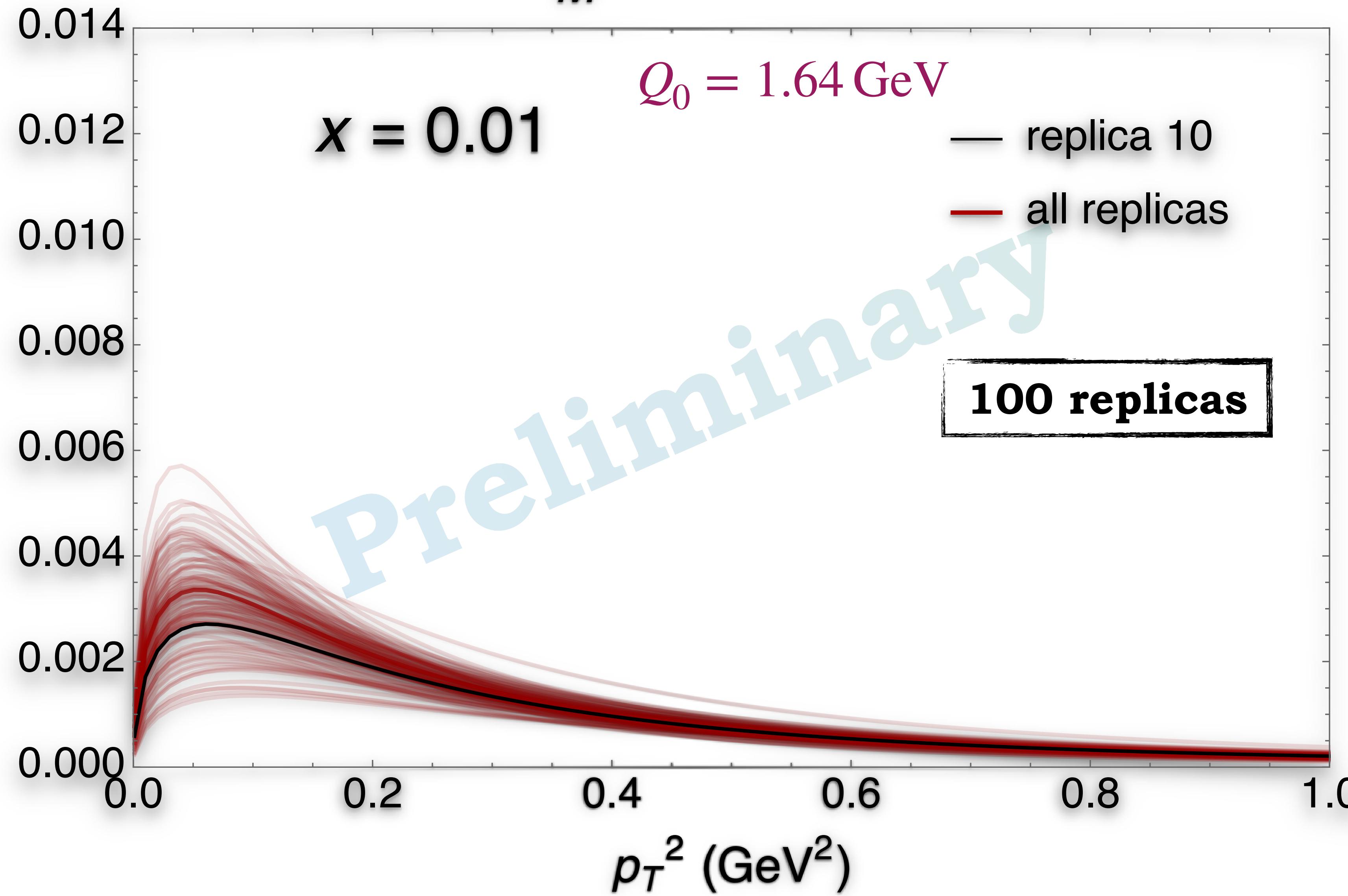
$$x \frac{p_T}{M} f_{1\tau}^{\perp[+,+]}(x, p_T^2)$$



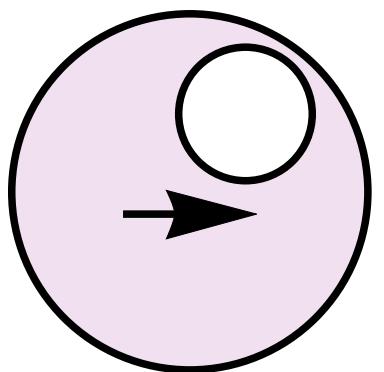
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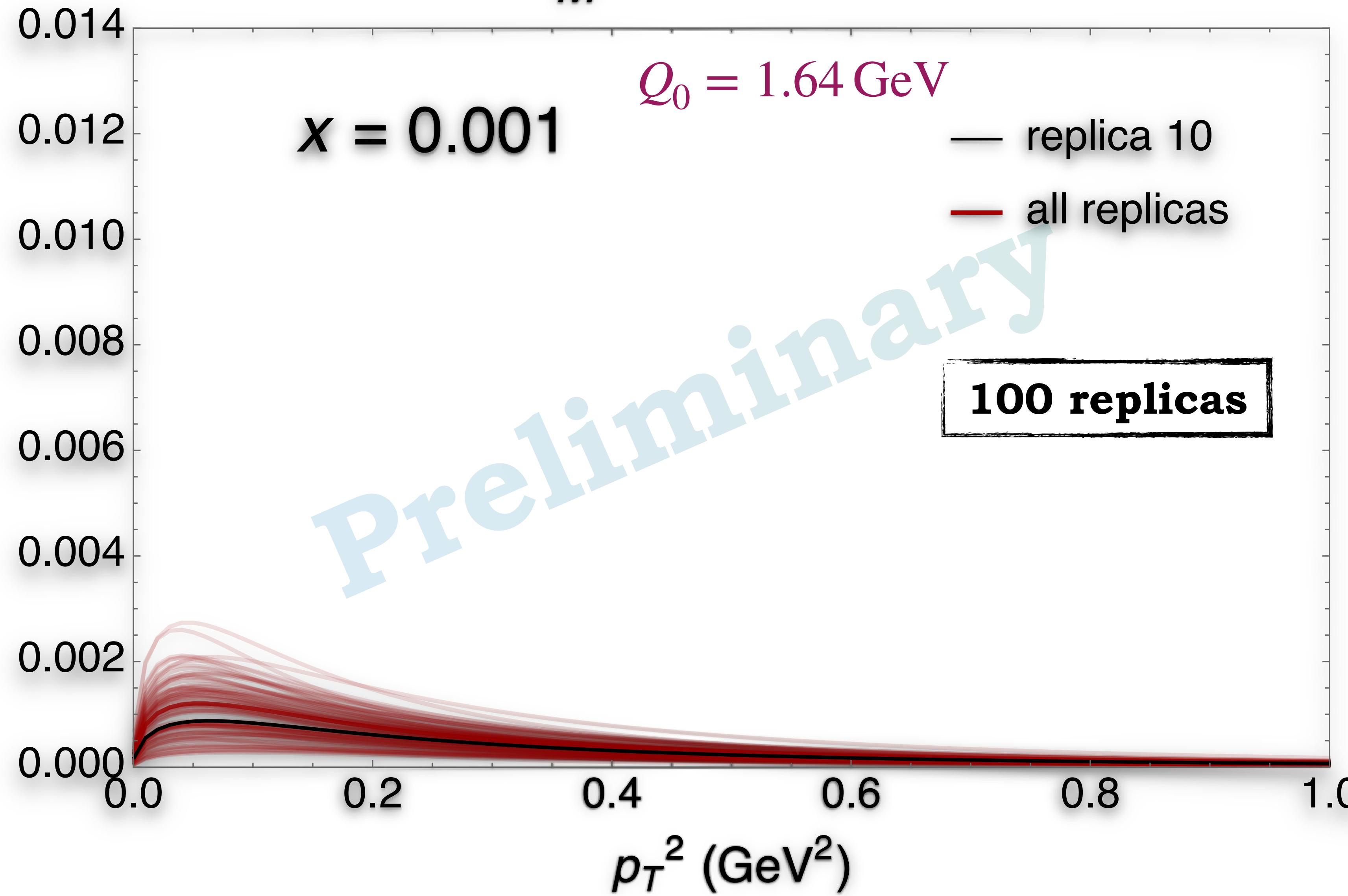
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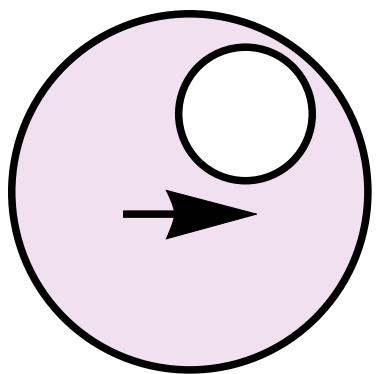
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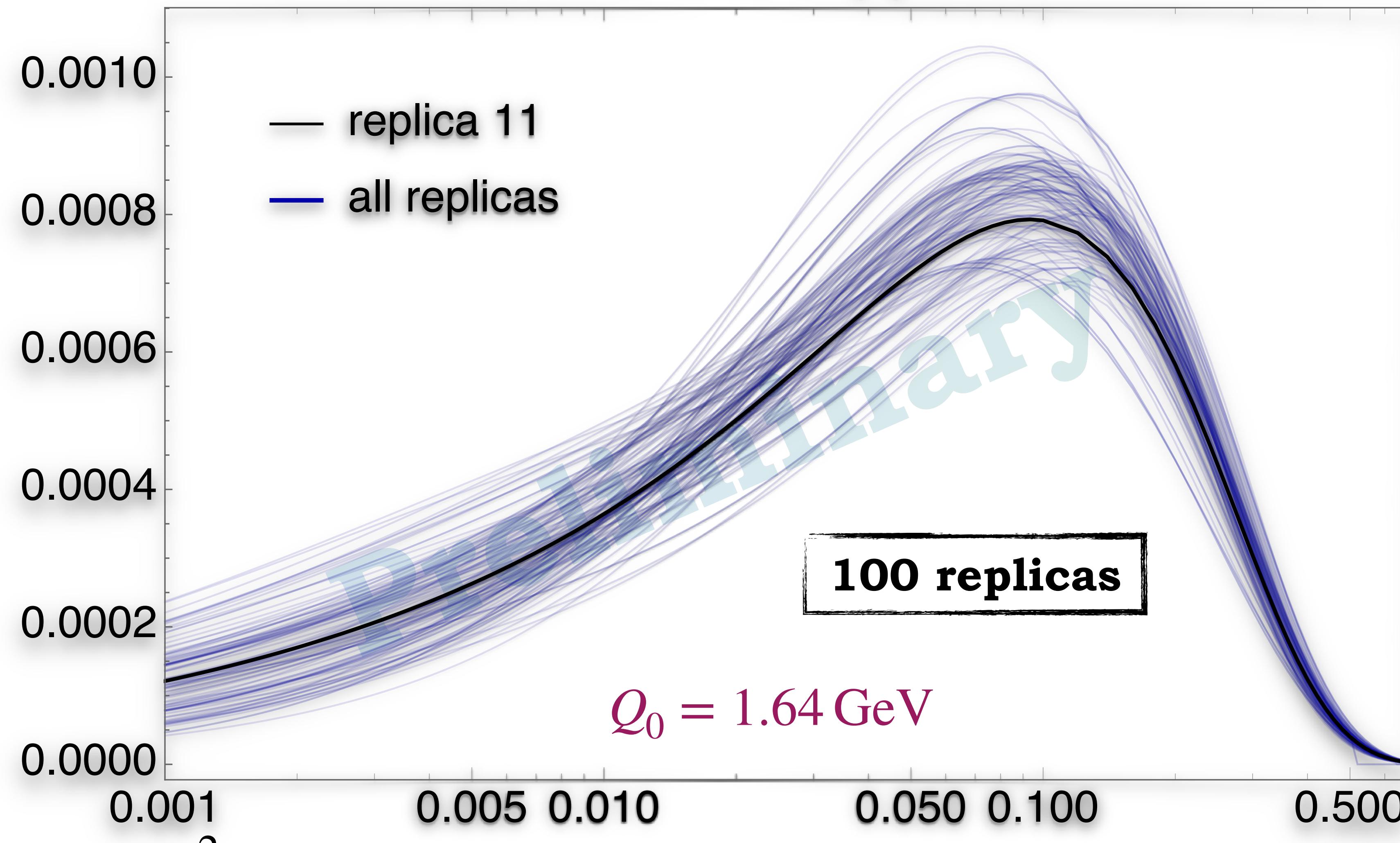
$$x \frac{p_T}{M} f_{1\perp}^{\left[+,+\right]}(x, p_T^2)$$



# *f*-type Qiu-Sterman twist-3 gluon PDF



$xf_{1T}^{\perp(f)}(x)$



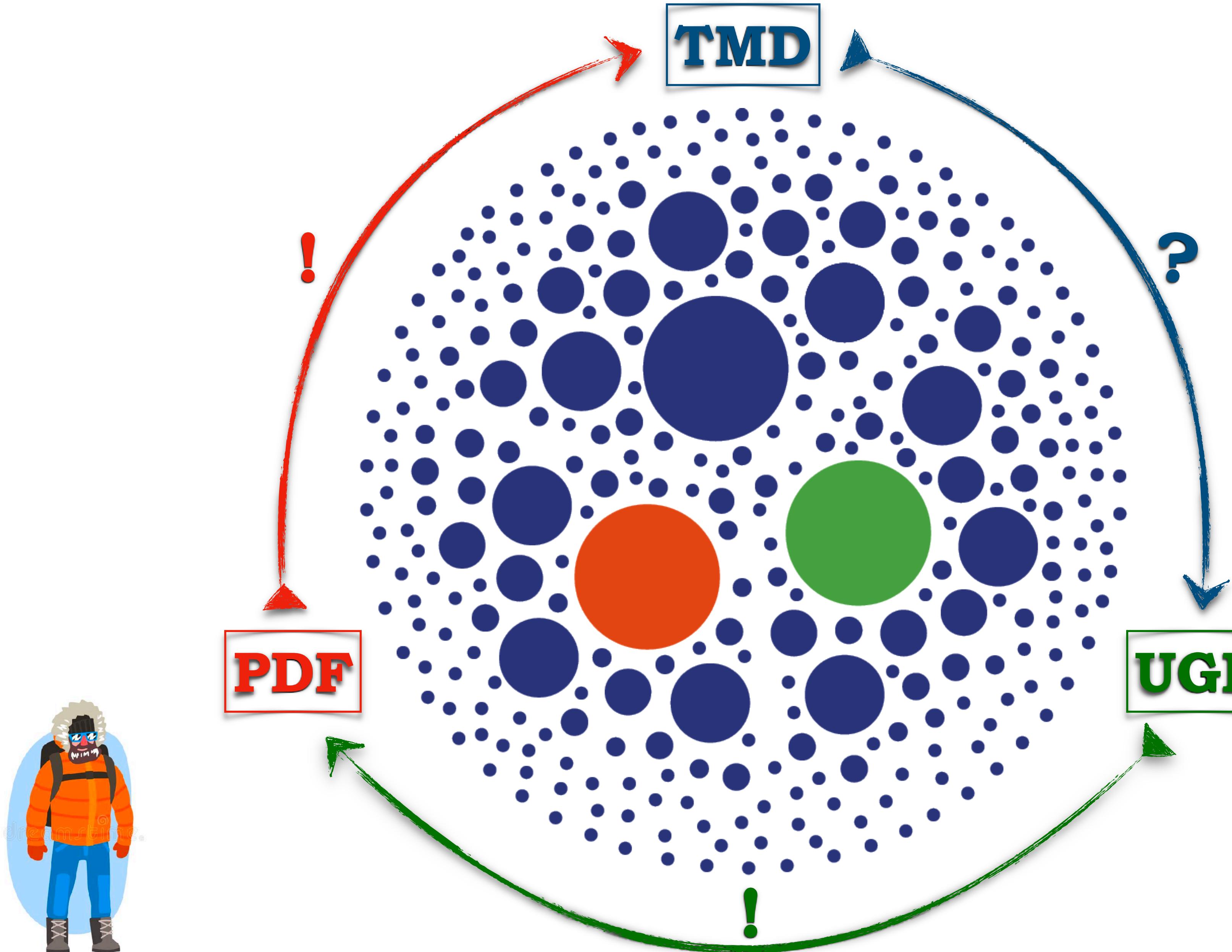
# Checkpoints and further steps

- Systematic calculation of all twist-2  $T$ -even gluon TMDs
- Spectral mass to catch small- and large- $x$  effects
- Simultaneous fit** of  $f_1$  and  $g_1$  PDFs via **replica method**

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- Twist-2  $T$ -odd gluon TMDs (**Sivers**, etc.) in progress!
- Pheno: **spin asymmetries**, **pseudodata** and **impact studies**
- Evolution: extension to quark TMDs in the same framework
- Explorative studies on gauge-link sensitivity and factorization
- Studies on GPD and small- $x$  UGD sectors

# Mapping the proton content

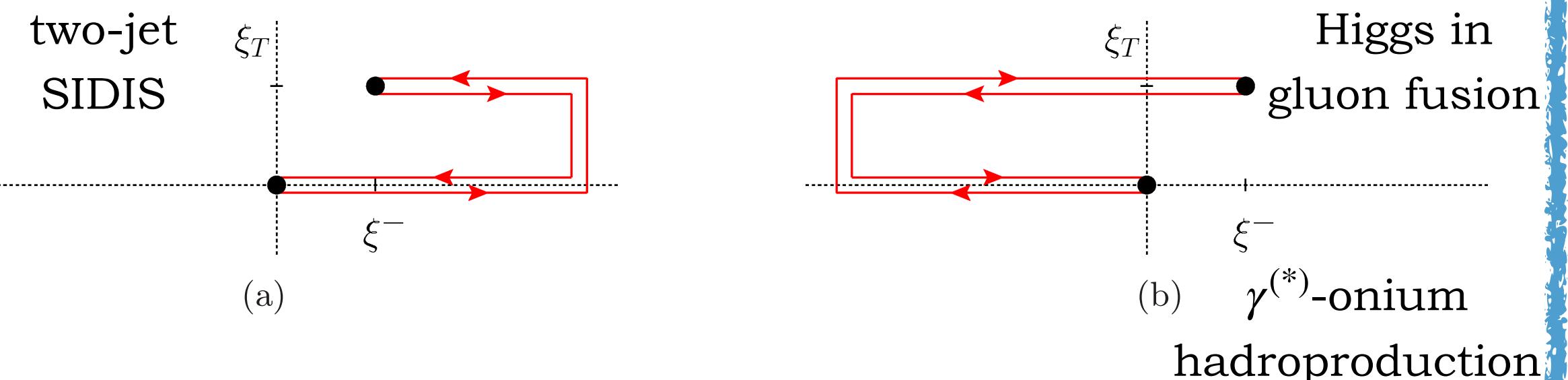


**Backup  
slides**

# Accessing WW and DP gluon TMDs

## Weiszäcker-Williams (WW)

(a) [ + , + ] or (b) [ - , - ]



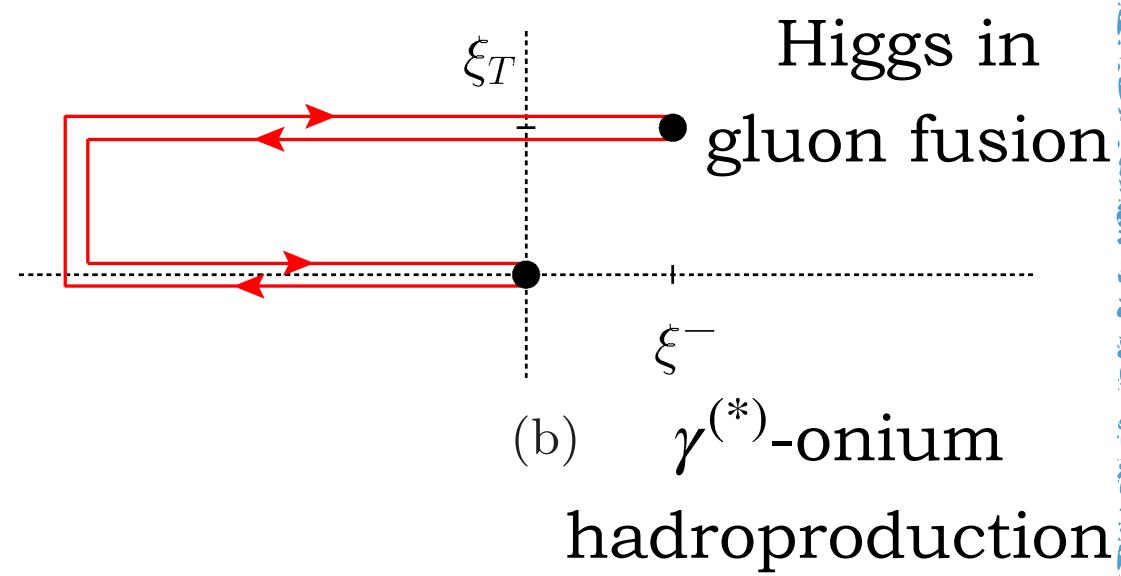
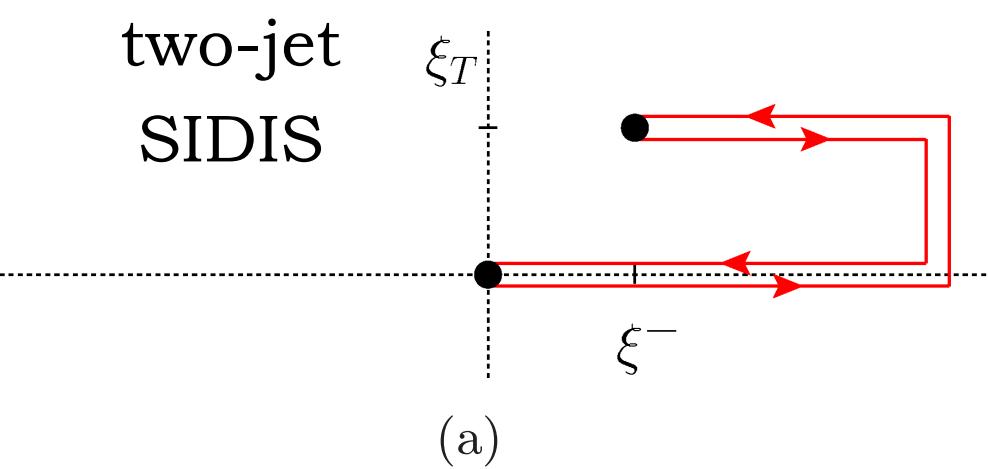
- \* Color flow annihilated within final/initial state
- \*  $f$ -type gluon TMDs  $\rightarrow f^{abc}$  color structure
- \* Modified universality:

$$f_1^{[+,+]} = f_1^{[-,-]},$$
$$f_{1T}^{\perp[+,+]} = -f_{1T}^{\perp[-,-]}$$
- \* Phenomenology: Higgs, quarkonia or  $\gamma\gamma$  in  $pp$ , two-jet SIDIS, heavy-quark pair SIDIS

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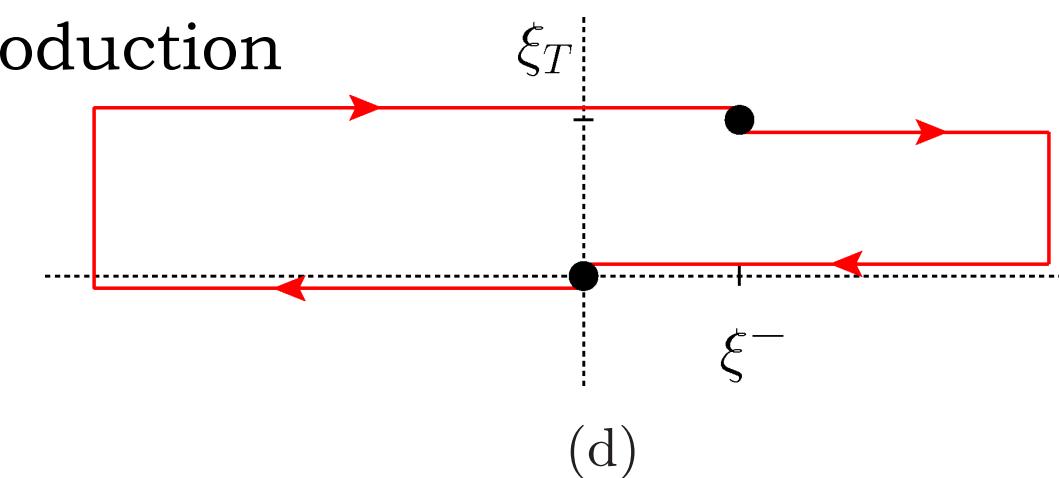
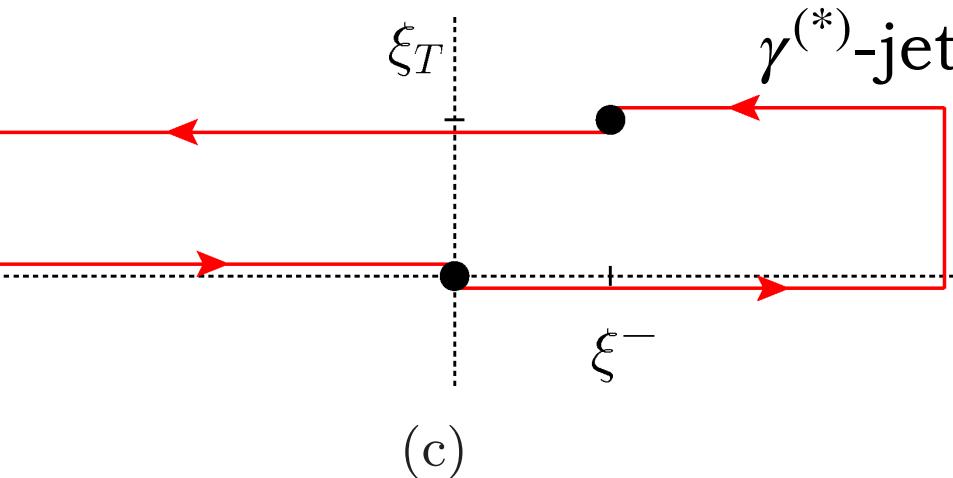
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(c) [ + , - ] or (d) [ - , + ]



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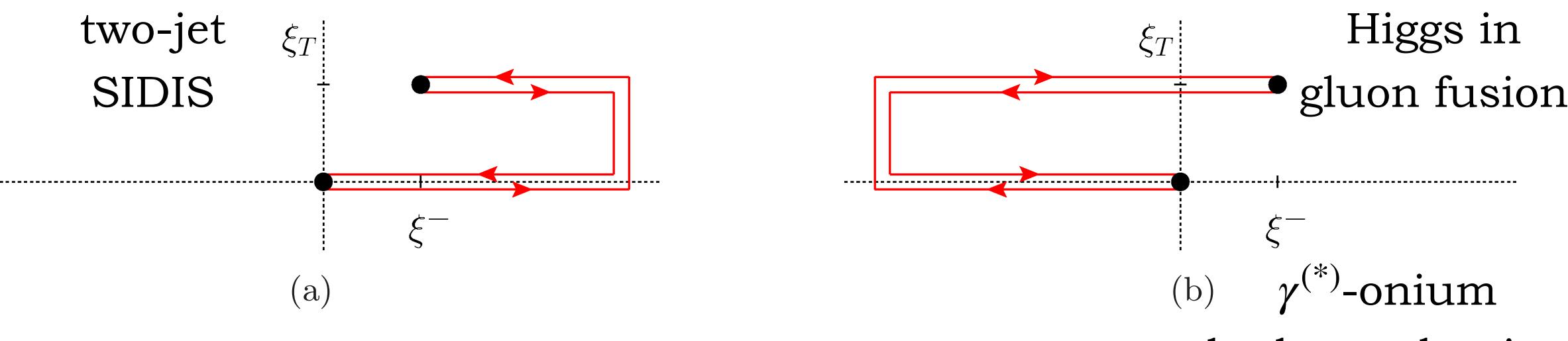
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- \* Phenomenology: single hadron or  $\gamma^{(*)}$ -jet hadroproduction, SIDIS or Drell-Yan (subleading)

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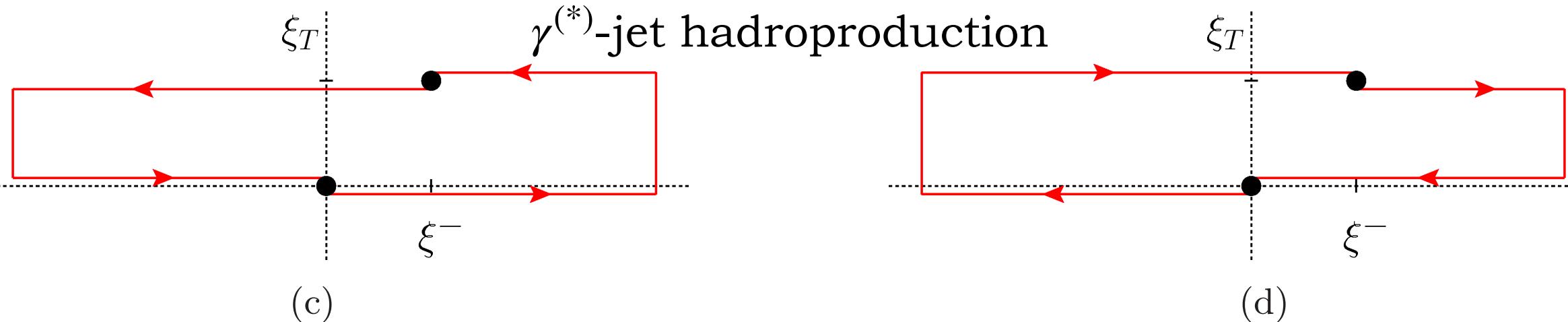
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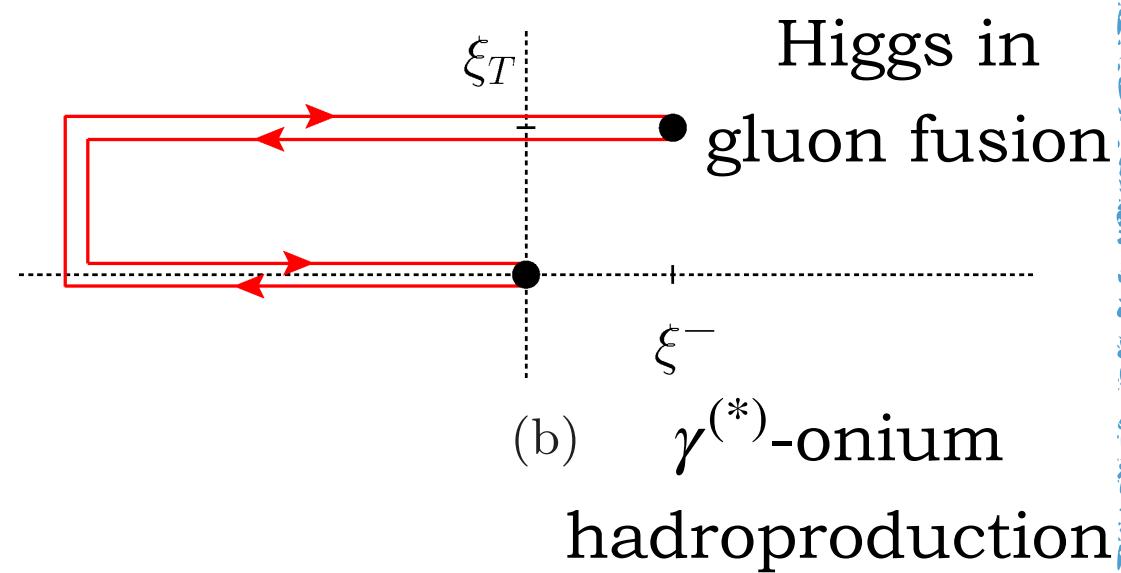
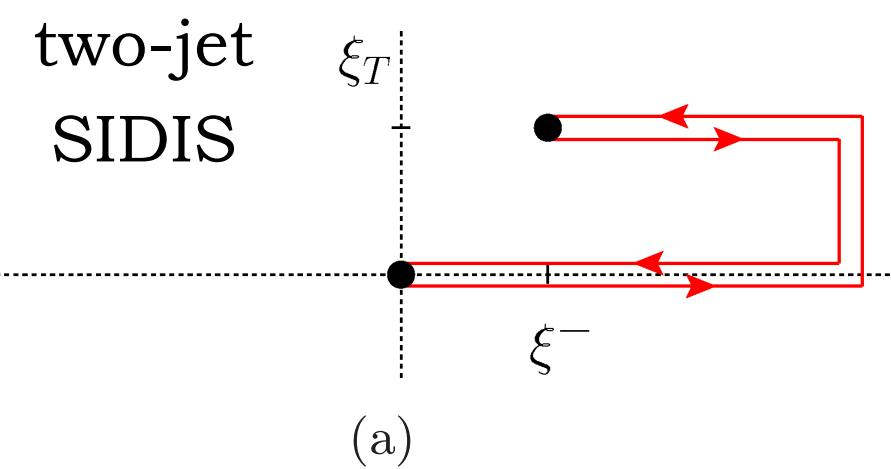
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Gauge link  $\rightarrow$  two main independent sets of TMDs, **not related** to each other

# Accessing WW and DP gluon TMDs

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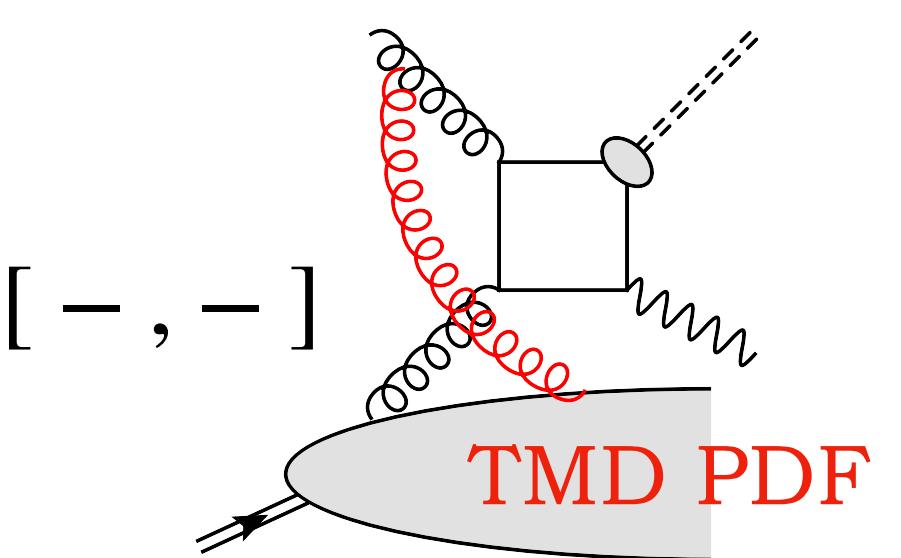
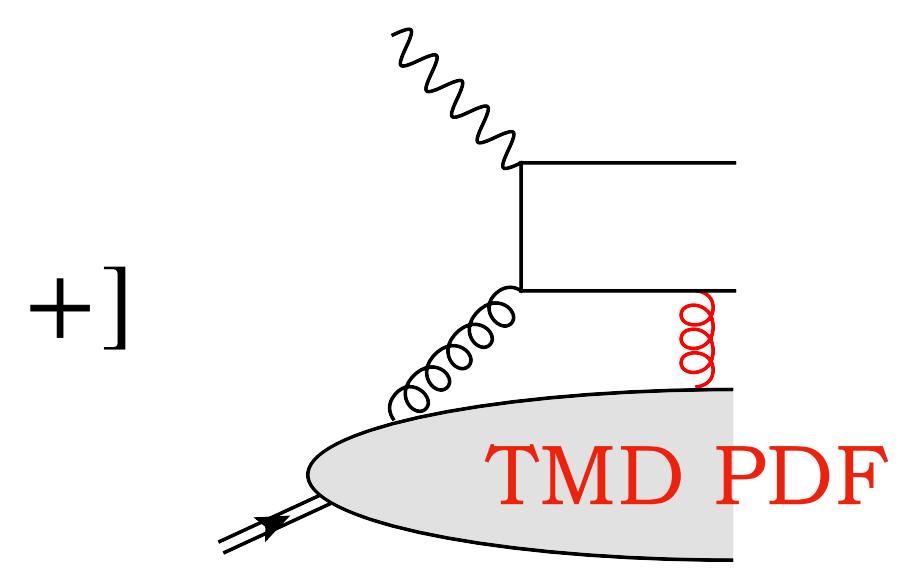
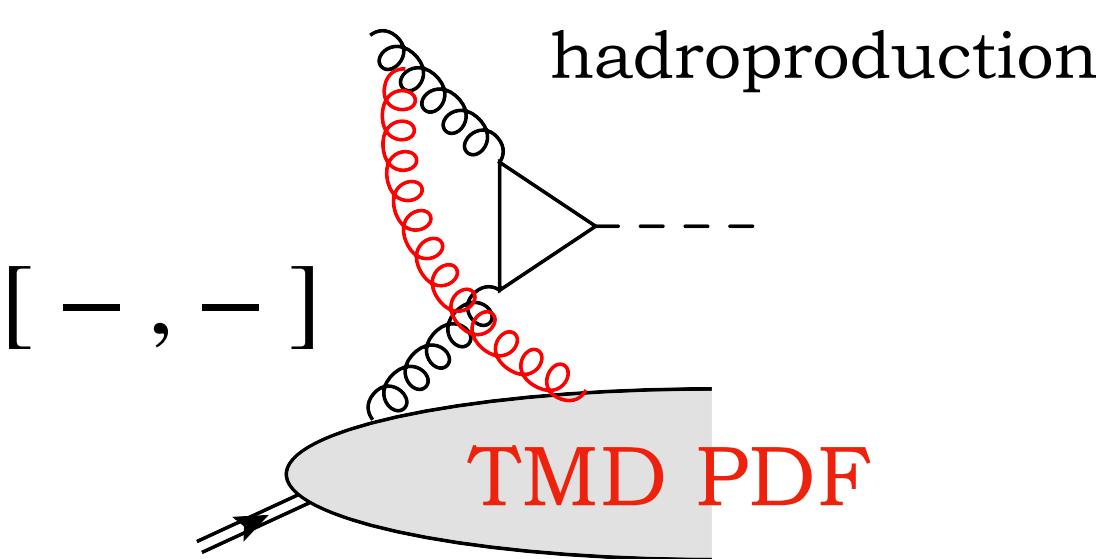
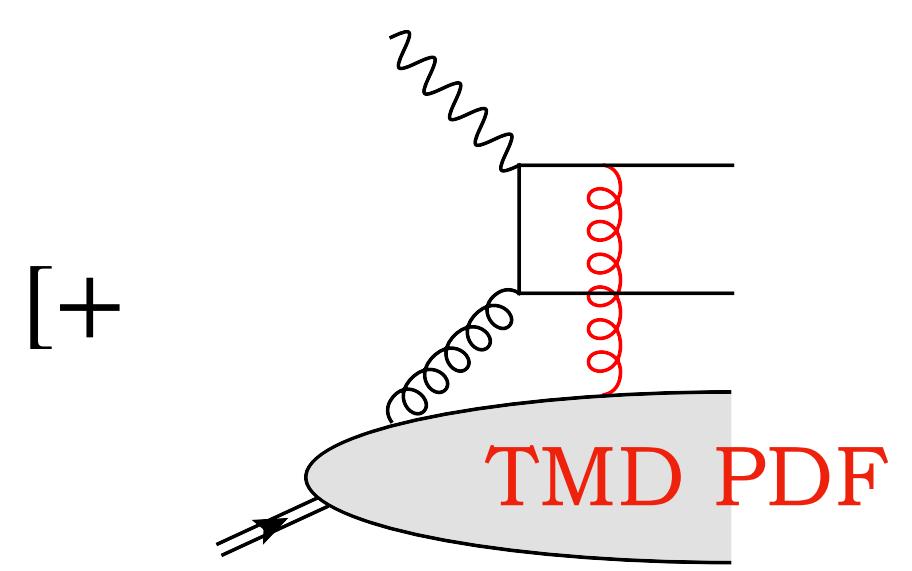
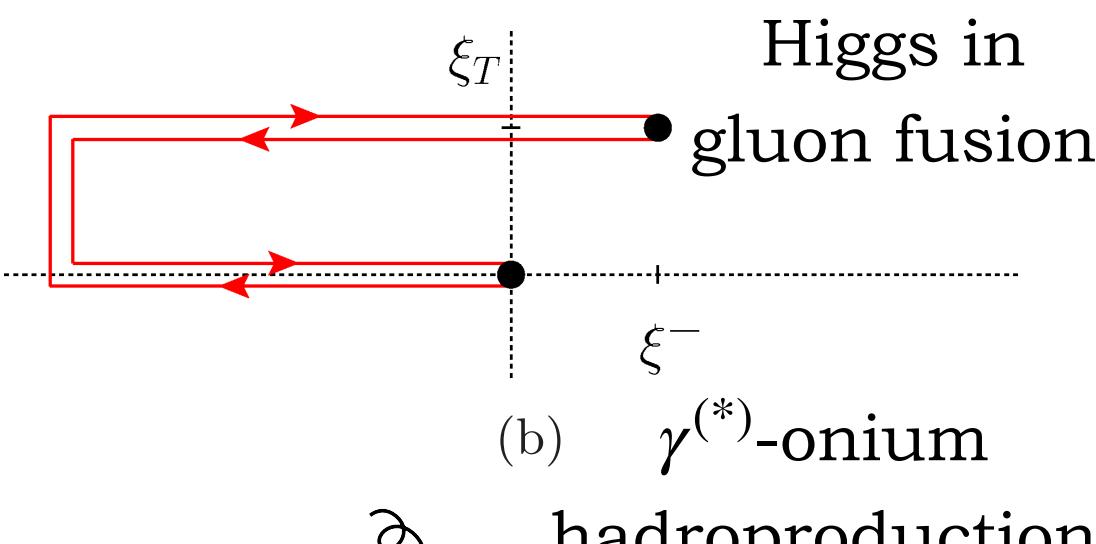
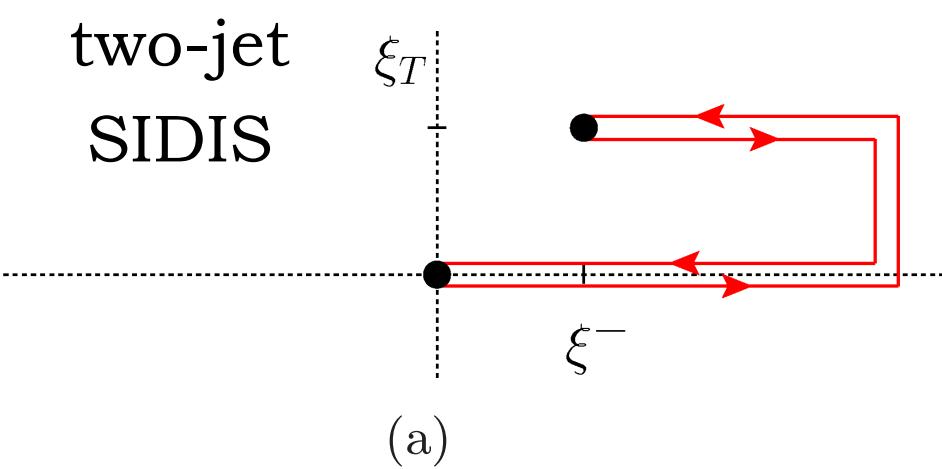
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# Accessing WW and DP gluon TMDs

## Weiszäcker-Williams (WW)

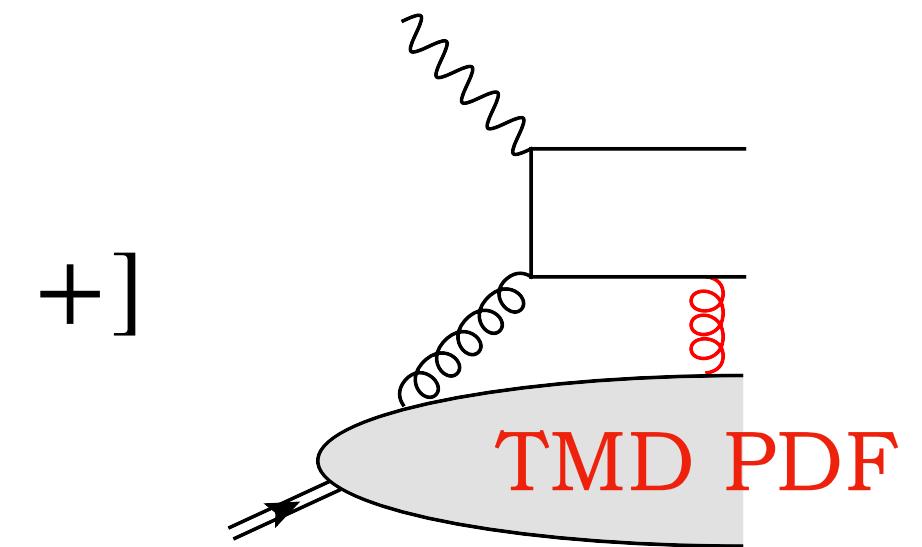
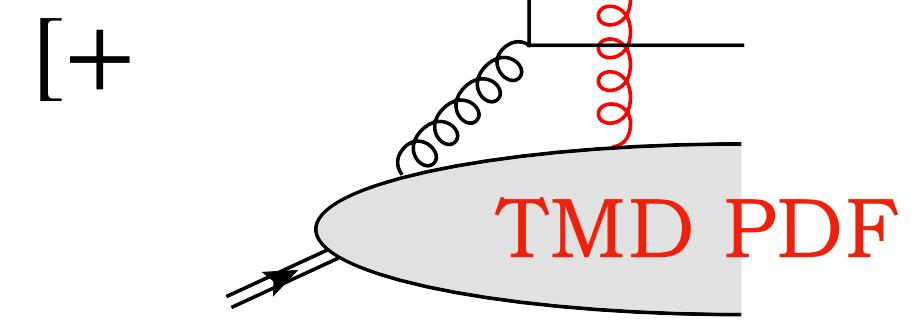
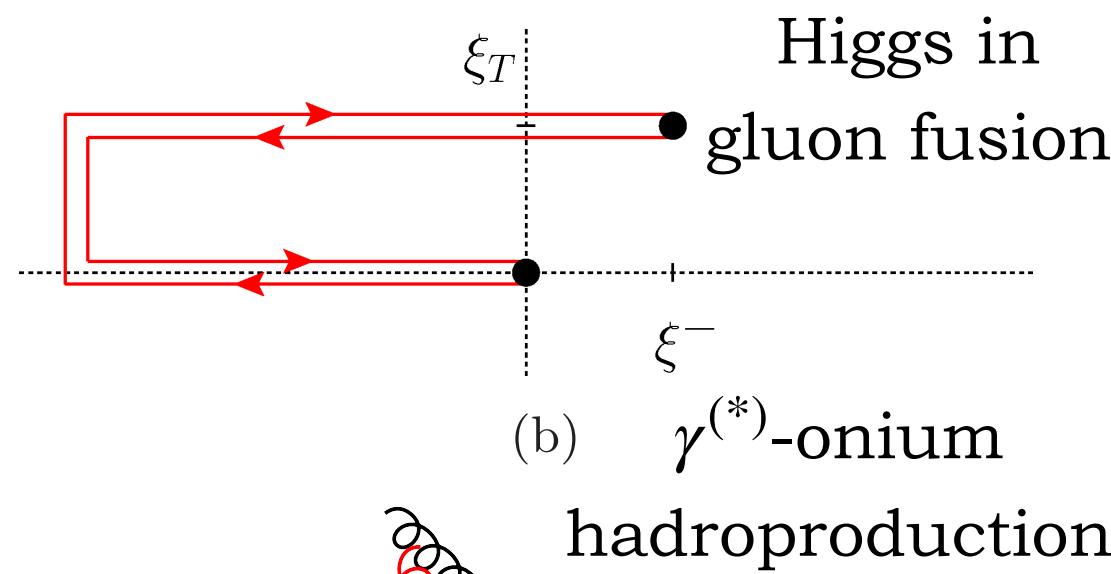
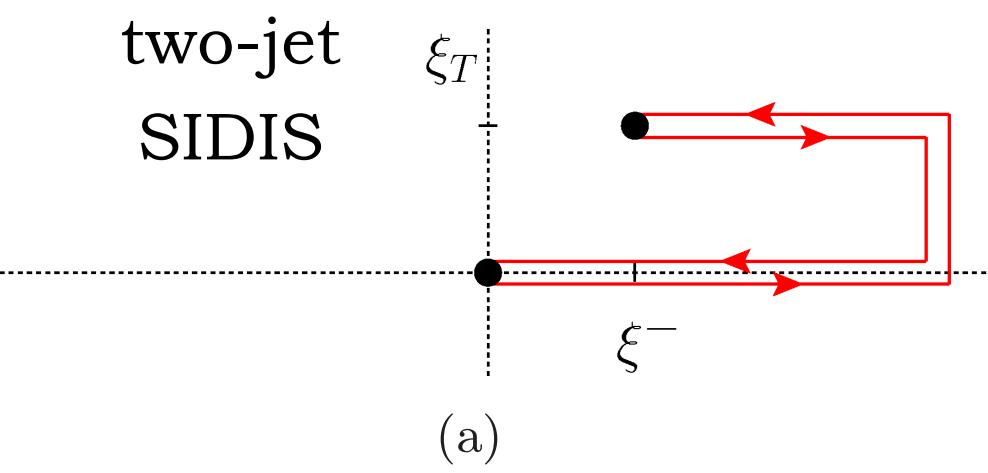
(a) [ + , + ] or (b) [ - , - ]



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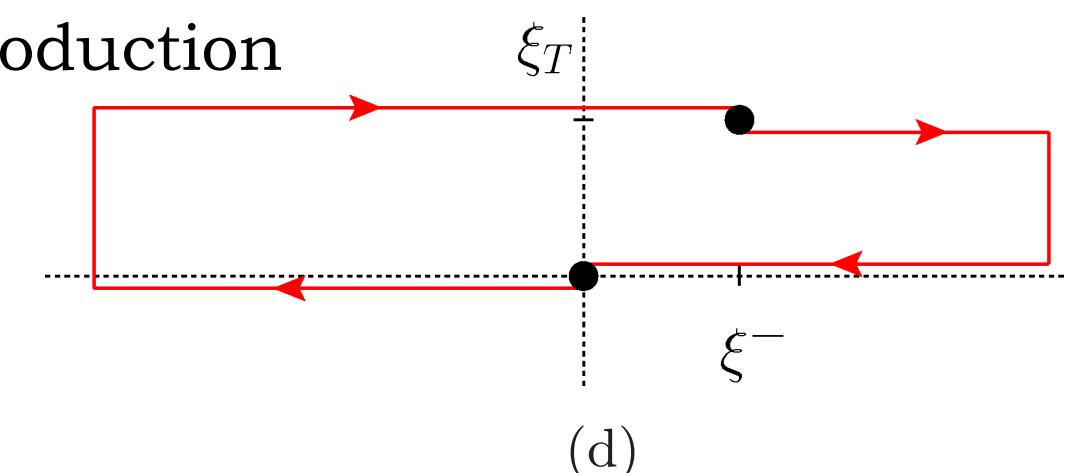
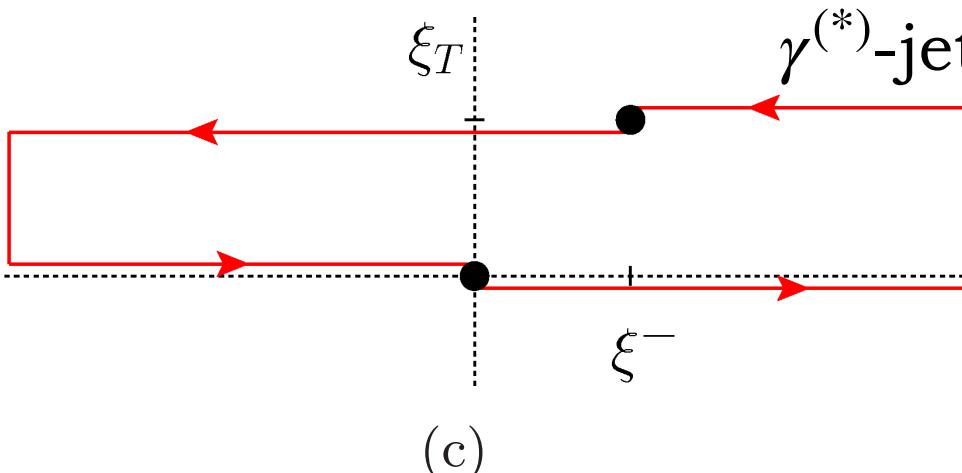
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(a) [ + , + ] or (b) [ - , - ]



## Dipole (DP)

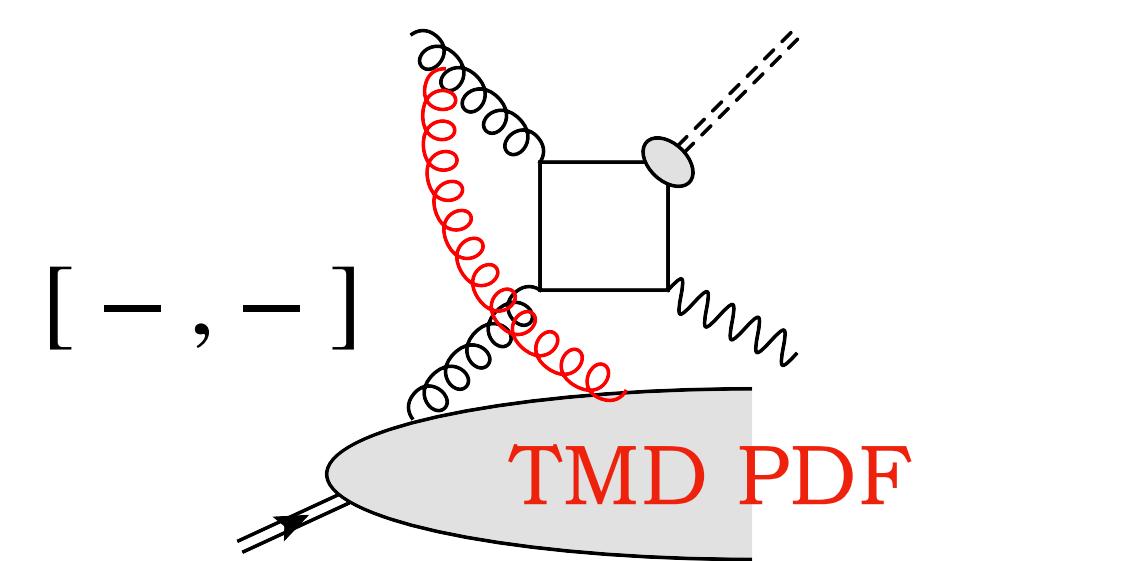
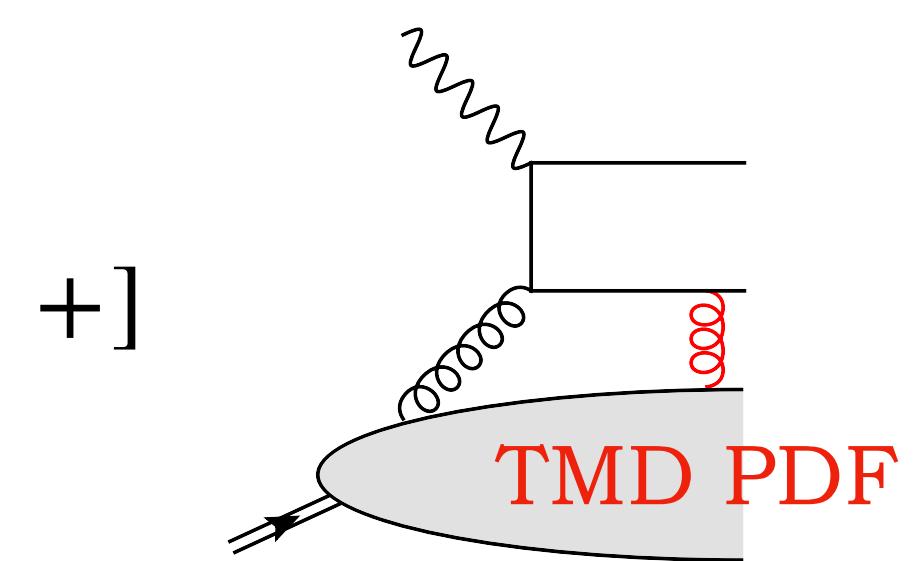
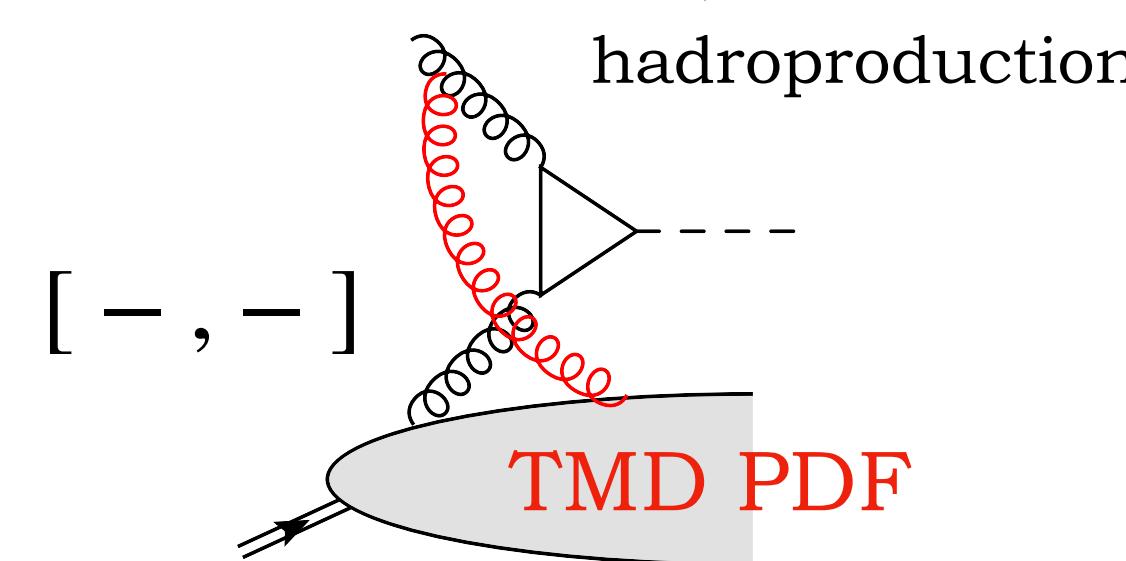
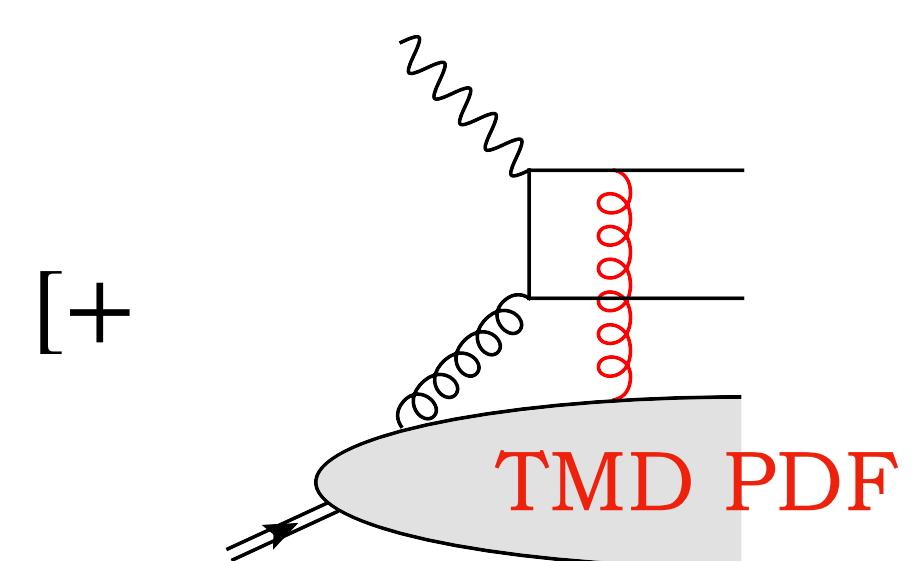
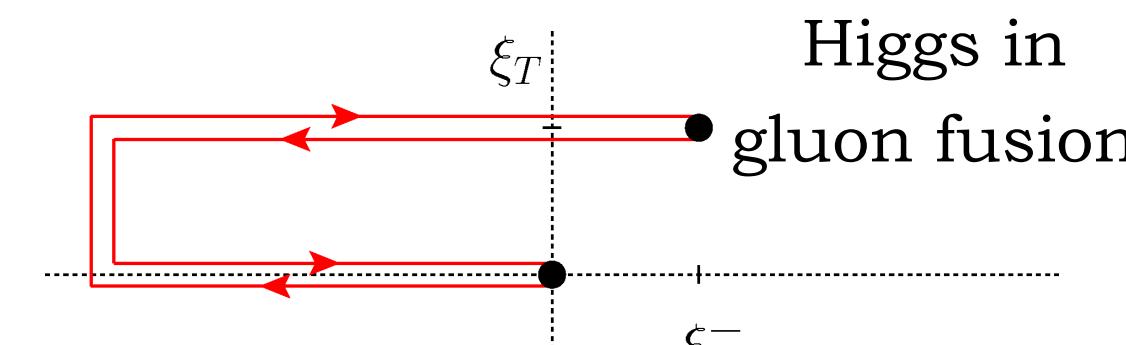
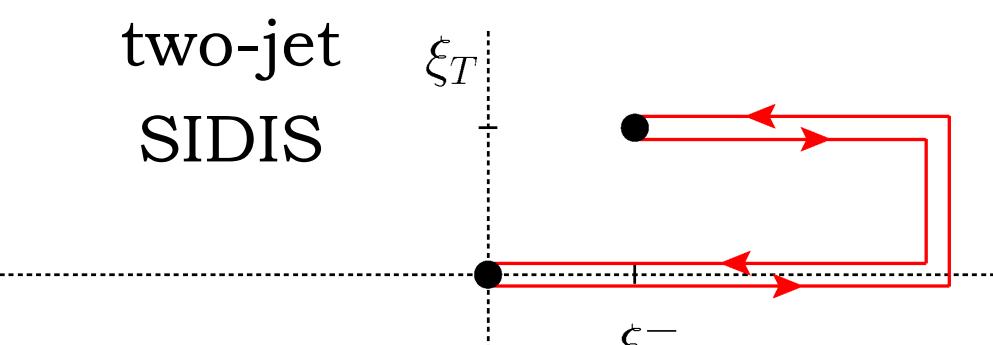
(c) [ + , - ] or (d) [ - , + ]



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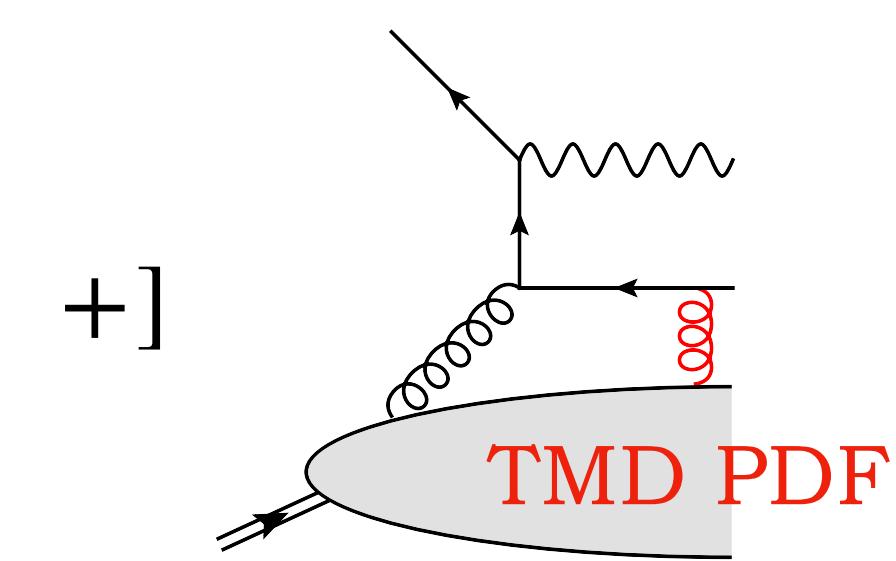
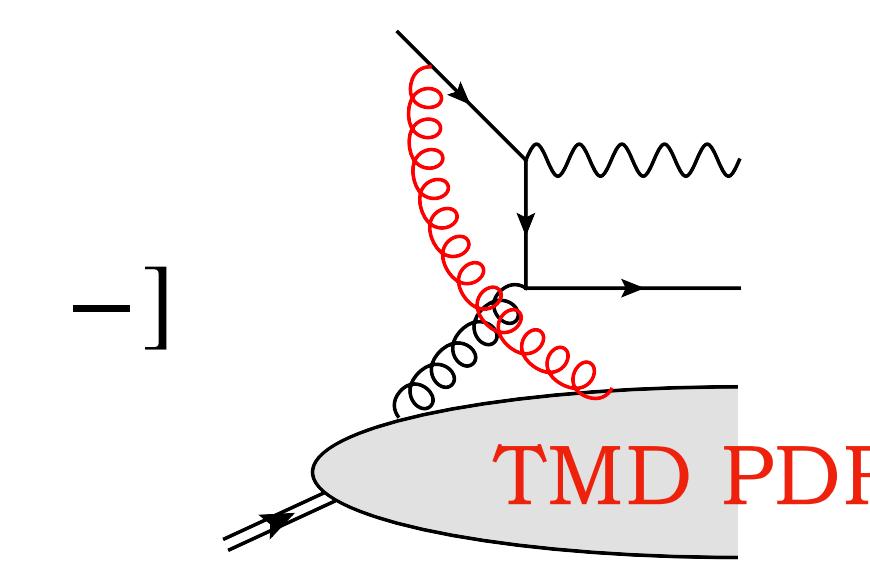
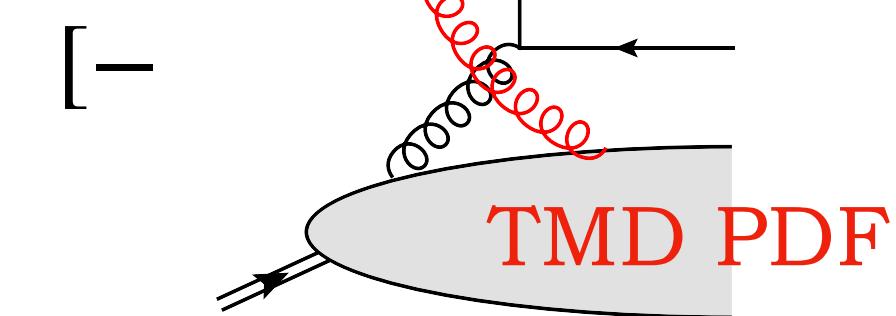
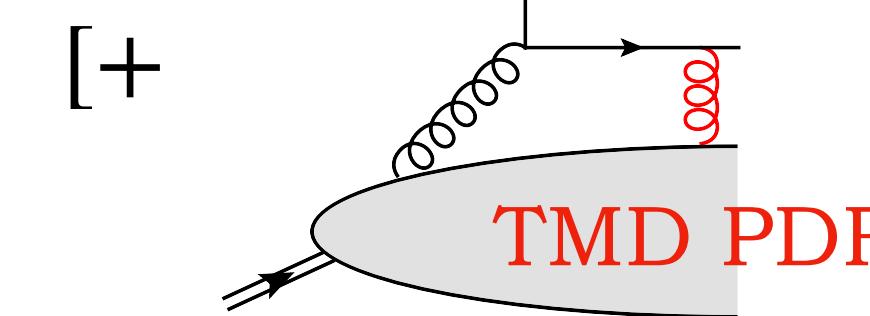
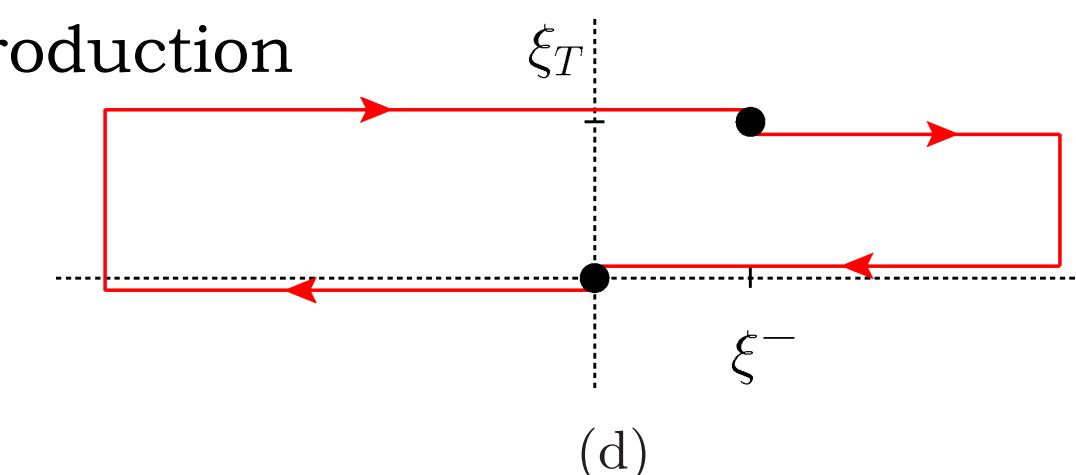
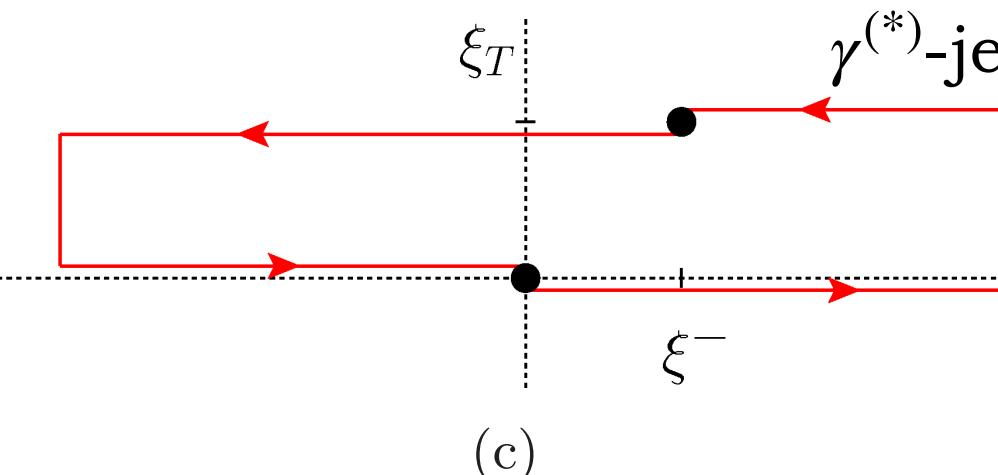
## Weiszäcker-Williams (WW)

(a) [ + , + ] or (b) [ - , - ]



## Dipole (DP)

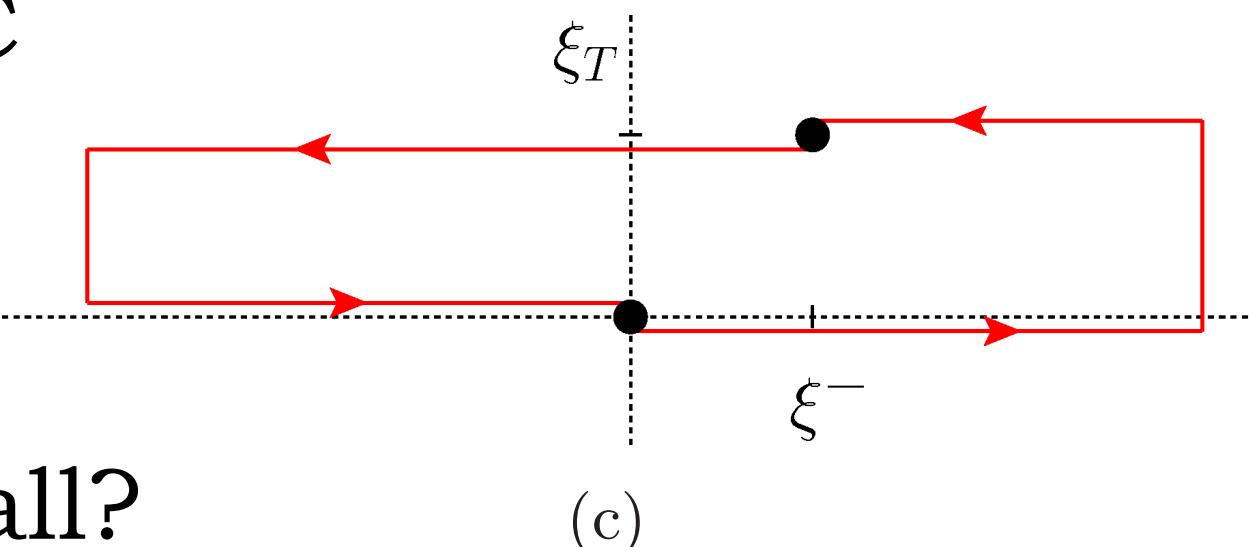
(c) [ + , - ] or (d) [ - , + ]



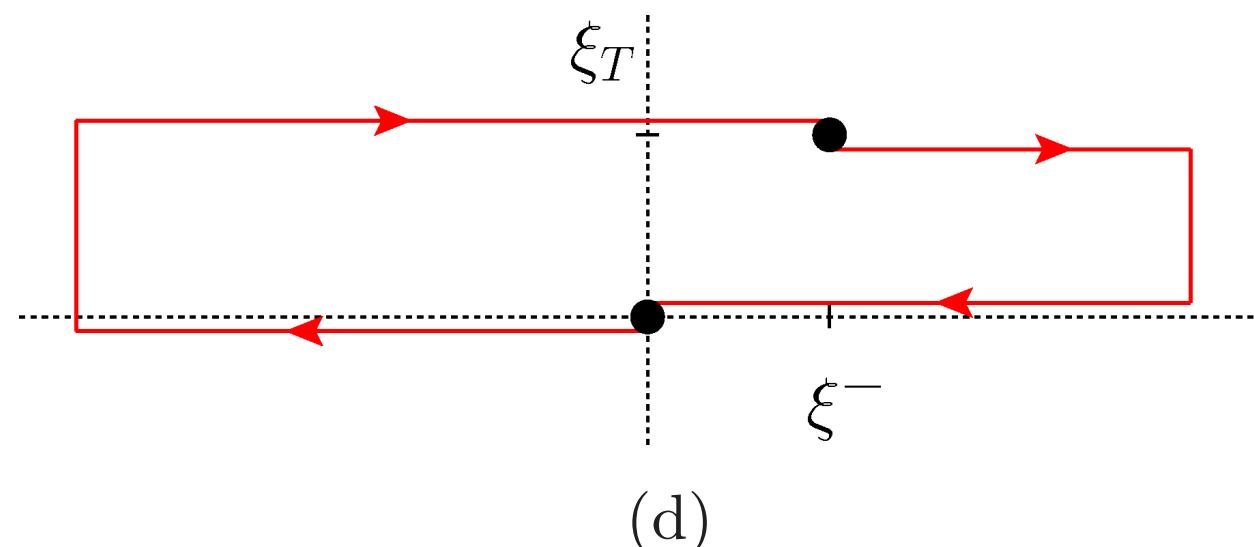
# Dihadron hadroproduction and factorization breaking

- \* Proof of factorization violation  [T. J. Rogers, P. J. Mulders (2010)]

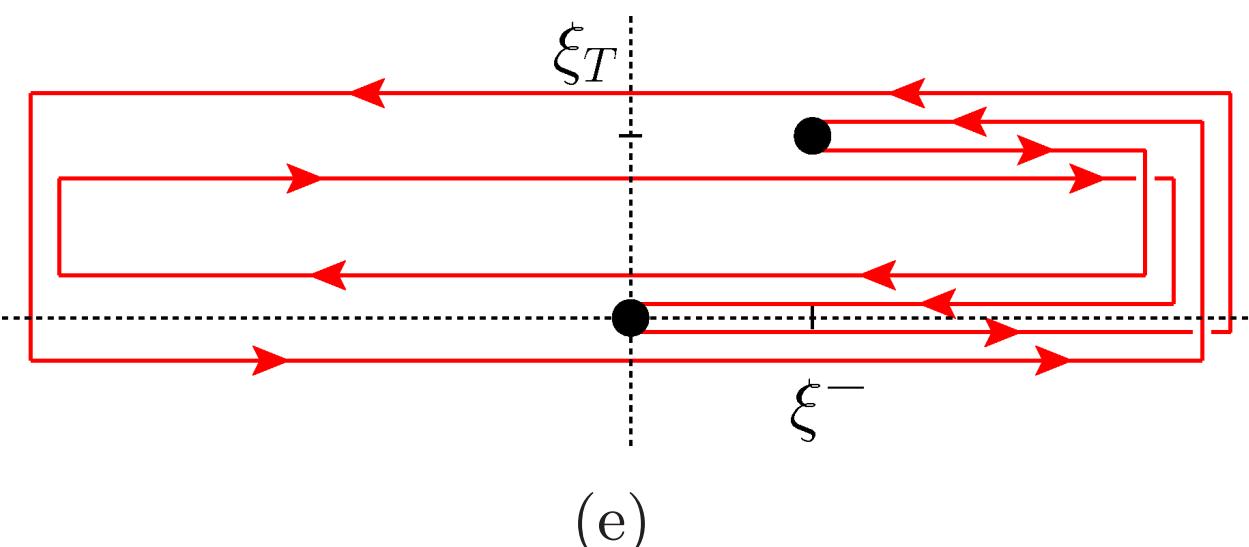
- \* Assumed factorization in SCET and CGC



- \* Significance of low- $x$  studies

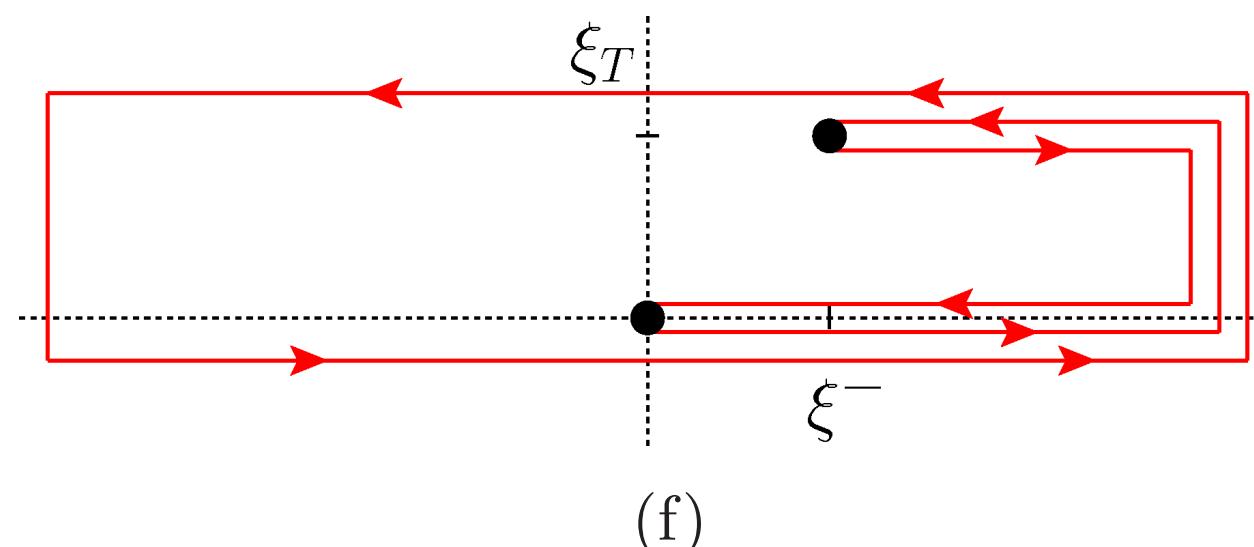


- \* Size of factorization-breaking effects small?



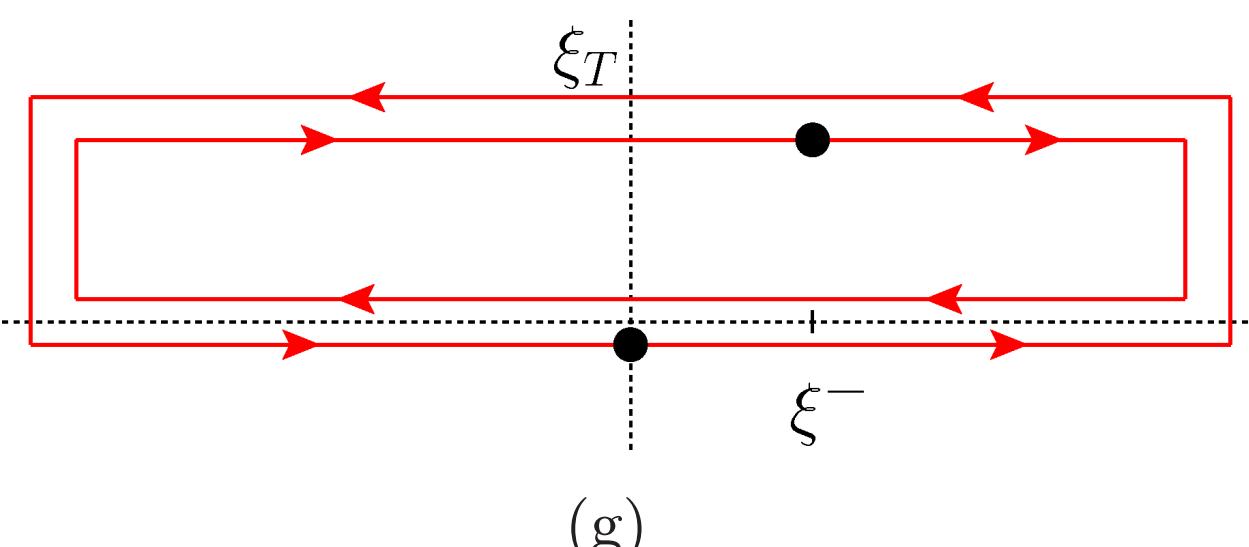
- \* DP TMDs:

(c)  $[+, -]$  and (d)  $[-, +]$

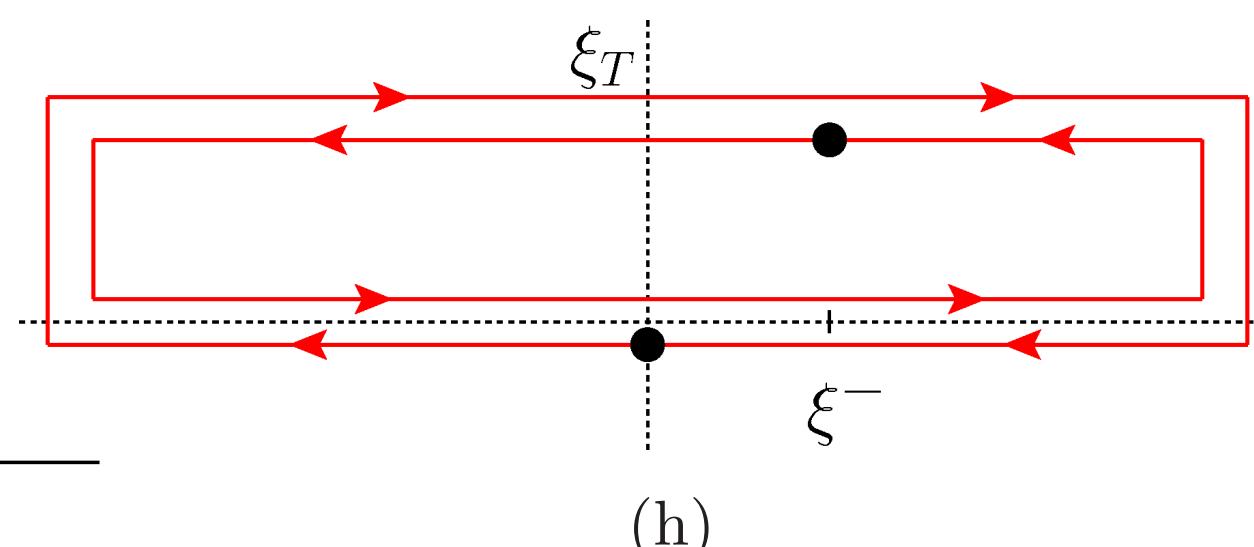
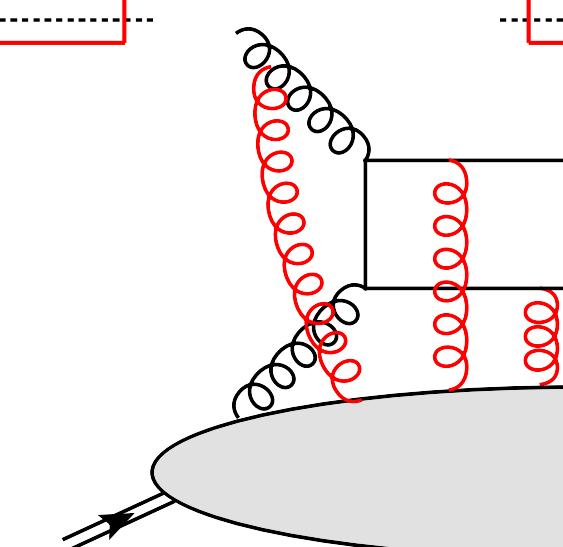


- \* Appearance of new gauge **loop links**:

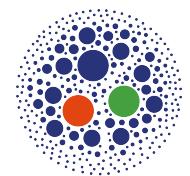
(e)  $[+ \square, + \square]$ , (f)  $[+, + \square]$ ,



(g)  $[\square, \square]$ , and (h)  $[\square, \square]$

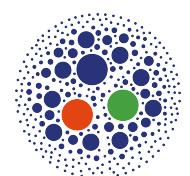
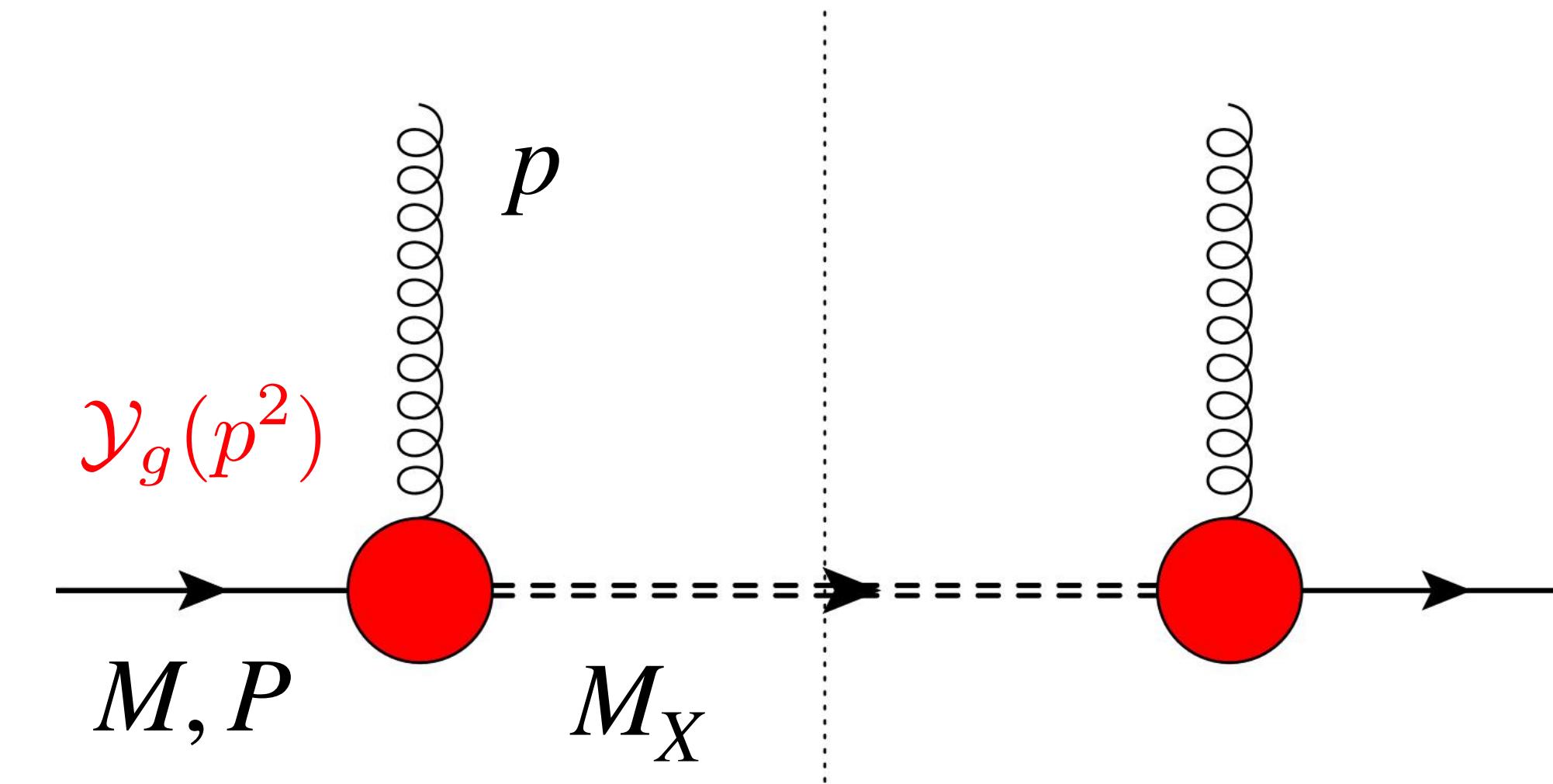


# Assumptions of the model



## Effective vertex

Lowest Fock state:  
**tri-quark spectator**  
on-shell and  
with mass  $M_X$



## Spin-1/2 spectator (gluon)

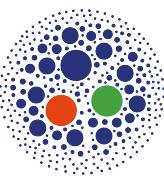
$$\Phi_g = \frac{1}{2(2\pi)^3(1-x)P^+} Tr \left[ (\not{P} + M) \frac{1 + \gamma^5 \$}{2} G_{\mu\rho}^*(p) G^{\nu\sigma}(p) \gamma_g^{\rho*} \gamma_{g\sigma}(\not{P} - \not{p} + M) \right]$$

$$\gamma_g^\mu = g_1(p^2) \gamma^\mu + i \frac{g_2(p^2)}{2M} \sigma^{\mu\nu} p_\nu$$



mimics proton form factors  
(conserved EM current  
of a free nucleon)

# Assumptions of the model



## Link with collinear factorization

$p_T$ -integrated TMDs **have to** reproduce PDFs at the lowest scale ( $Q_0$ ) *before* evolution

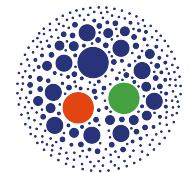


## Dipolar form factor(s)

$$g_{1,2}(p^2) = \kappa_{1,2} \frac{p^2}{|p^2 - \Lambda_X^2|^2}$$

1. Cancels singularity of gluon propagator
2. Suppresses effects of high  $p_T$
3. Compensates log divergences arising from  $p_T$ -integration
4. Adds three more parameters:  $\kappa_{1,2}$  and  $\Lambda_X$

# Assumptions of the model



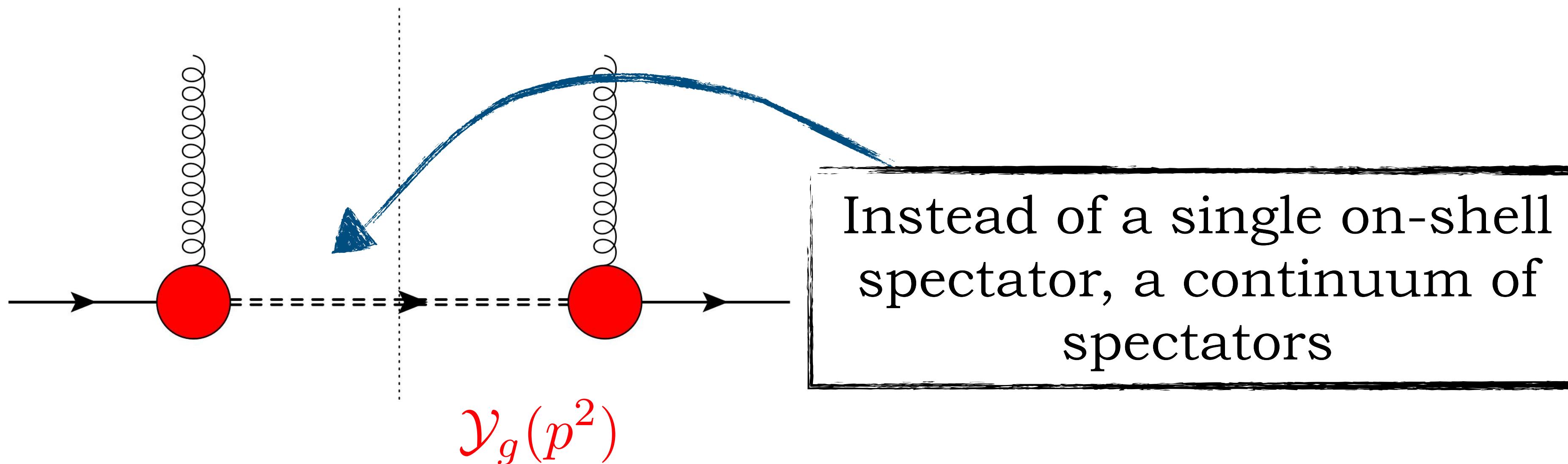
## Spectator-system spectral-mass function

$$F(x, \mathbf{p}_T^2) = \int_M^\infty dM_X \rho_X(M_X) \hat{F}(x, \mathbf{p}_T^2; M_X)$$

spectral-mass function

spectator-model TMD

[Inspired by G.R. Goldstein, J.O.G. Hernandez, S. Liuti (2011)]



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$$\rho_X(M_X; \{X^{(\text{pars})}\} \equiv \{A, B, a, b, C, D, \sigma\}) = \mu^{2a} \left[ \frac{A}{B + \mu^{2b}} + \frac{C}{\pi\sigma} e^{-\frac{(M_X - D)^2}{\sigma^2}} \right]$$

low- $x$  (high- $\mu^2$ ) tail  $\propto (a - b)$

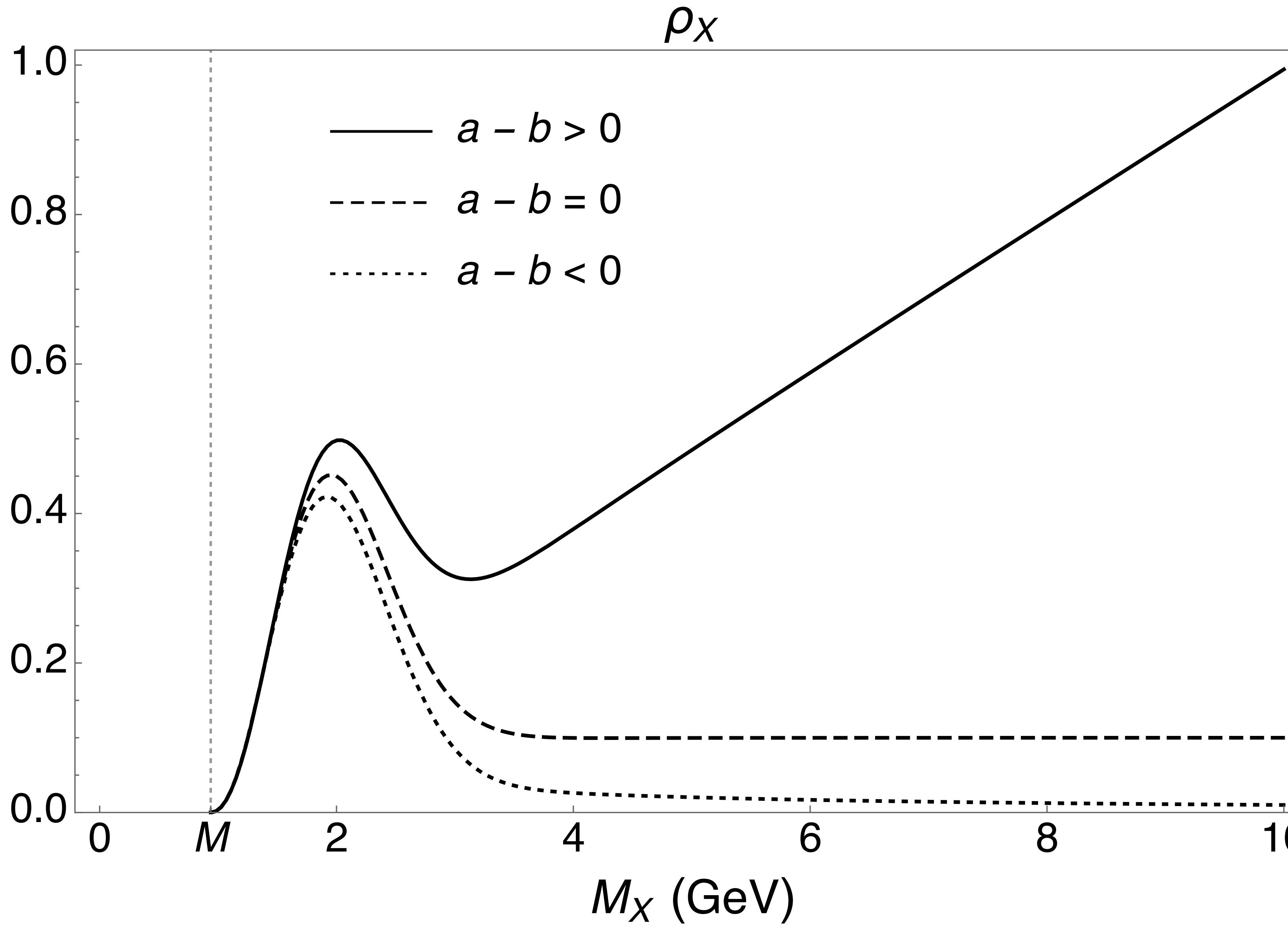
$q\bar{q}$  contributions energetically available at large  $M_X$

$$\mu^2 = M_X^2 - M^2$$

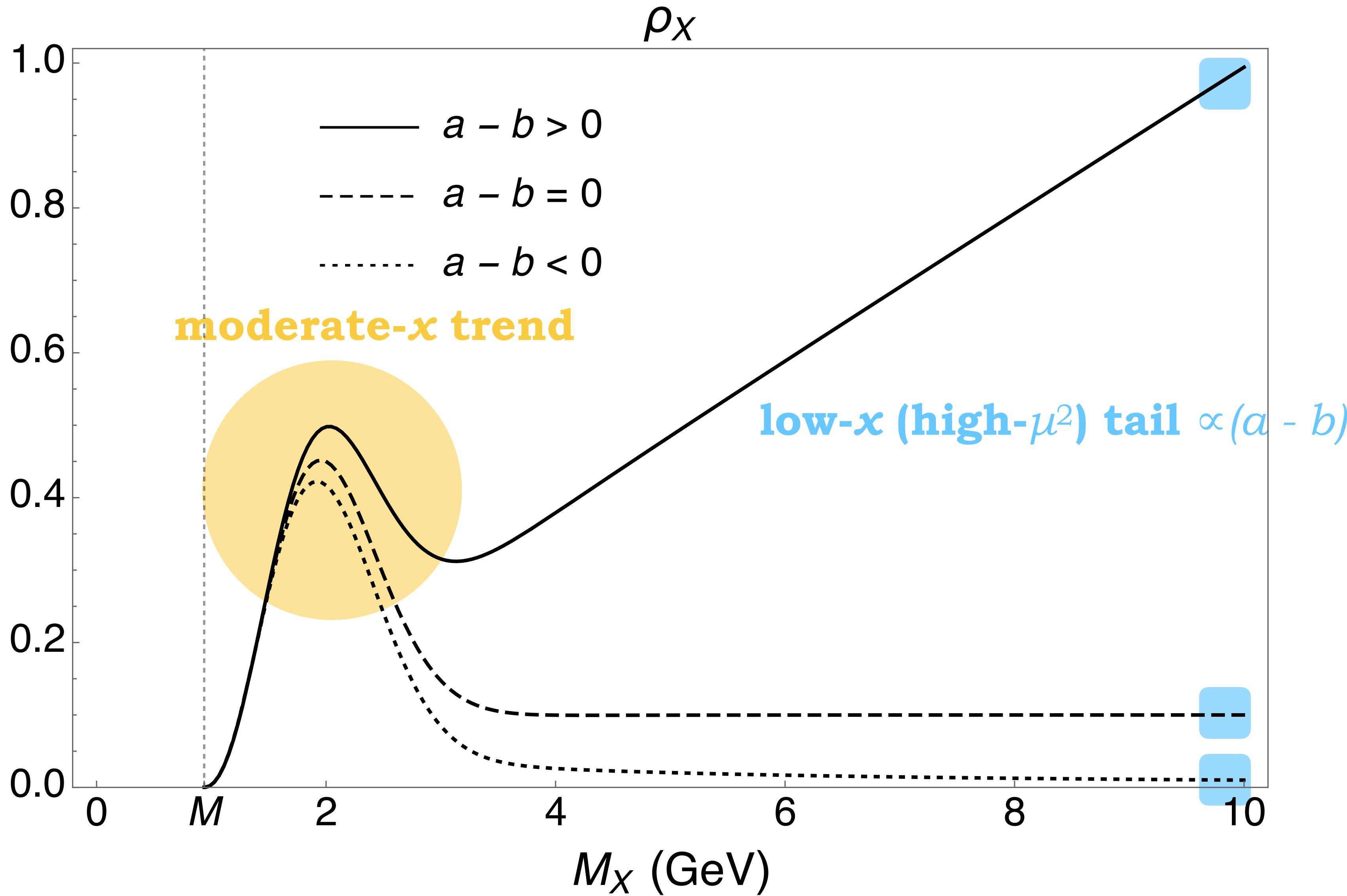
moderate- $x$  trend

pure tri-quark contribution at low  $M_X$

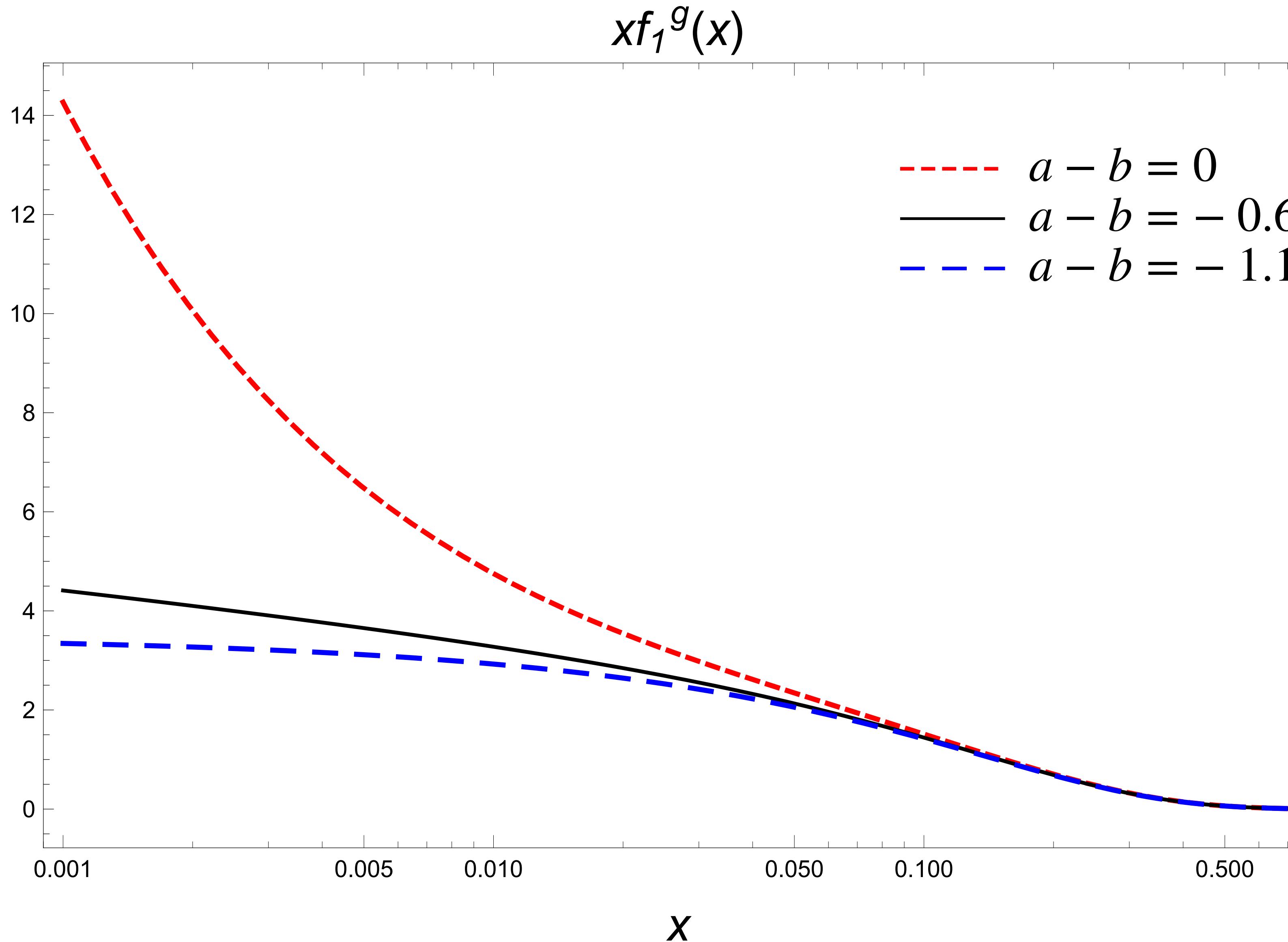
# Spectral function vs $(a - b)$



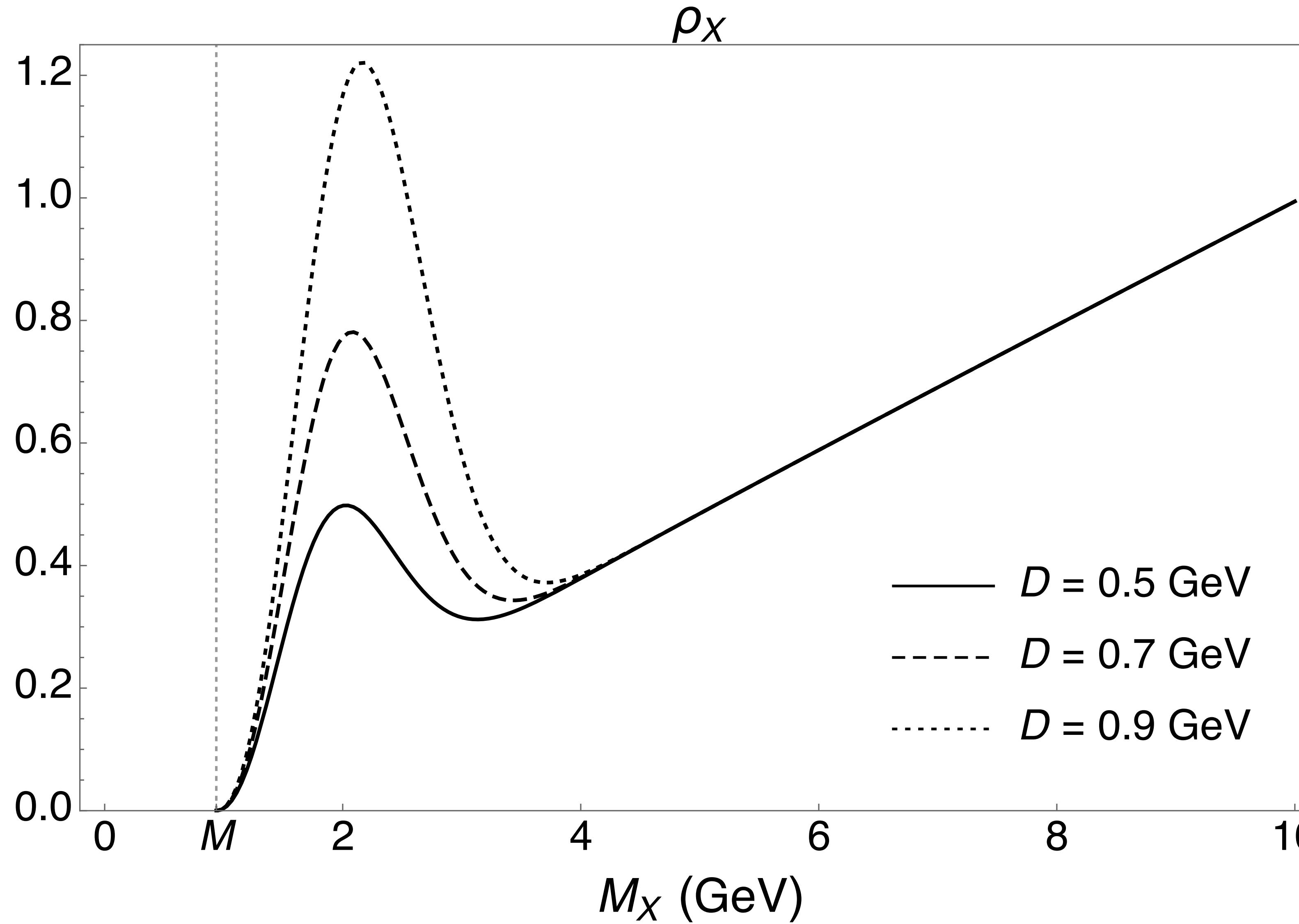
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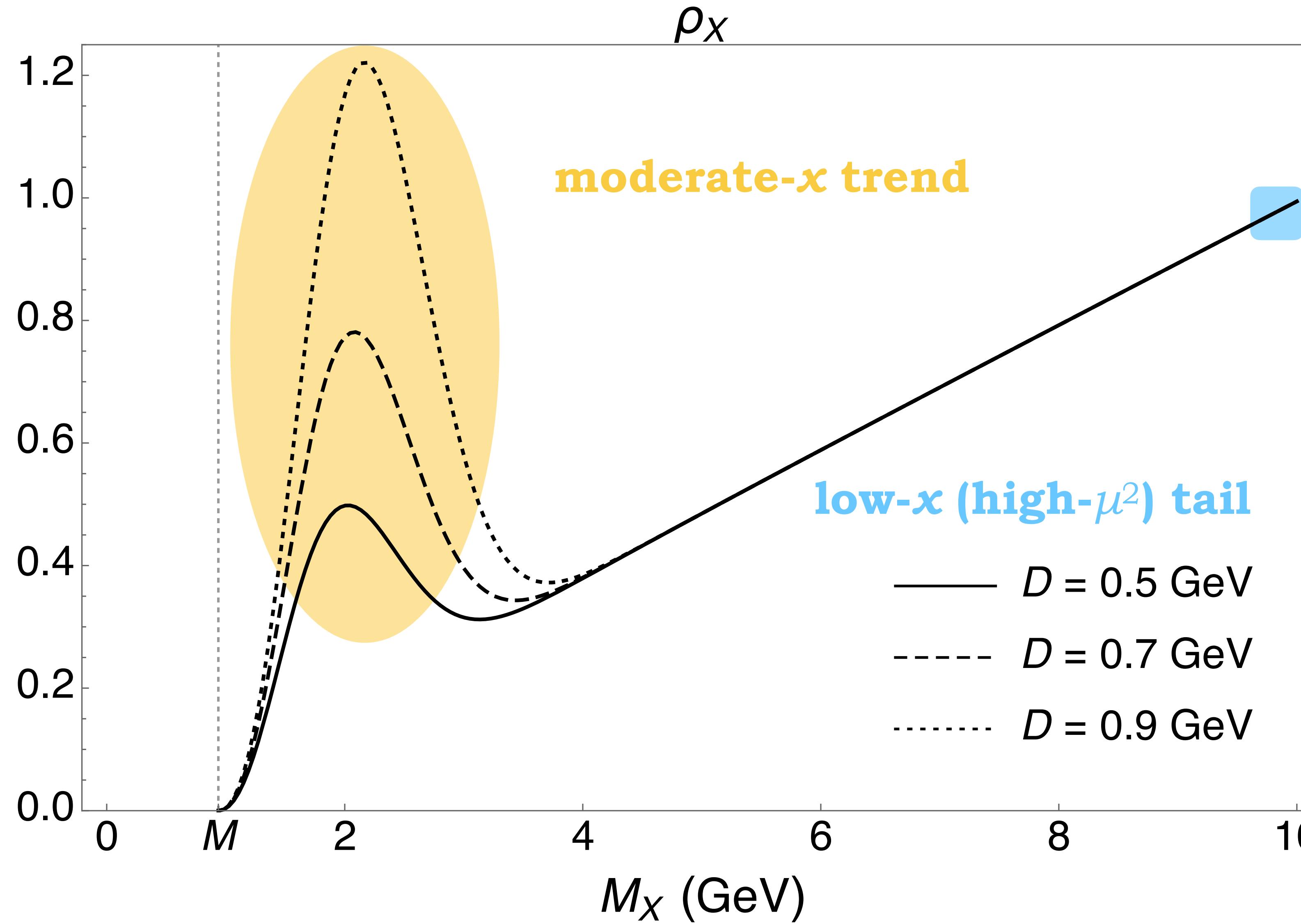
# $xf_1$ collinear PDF vs $(a - b)$



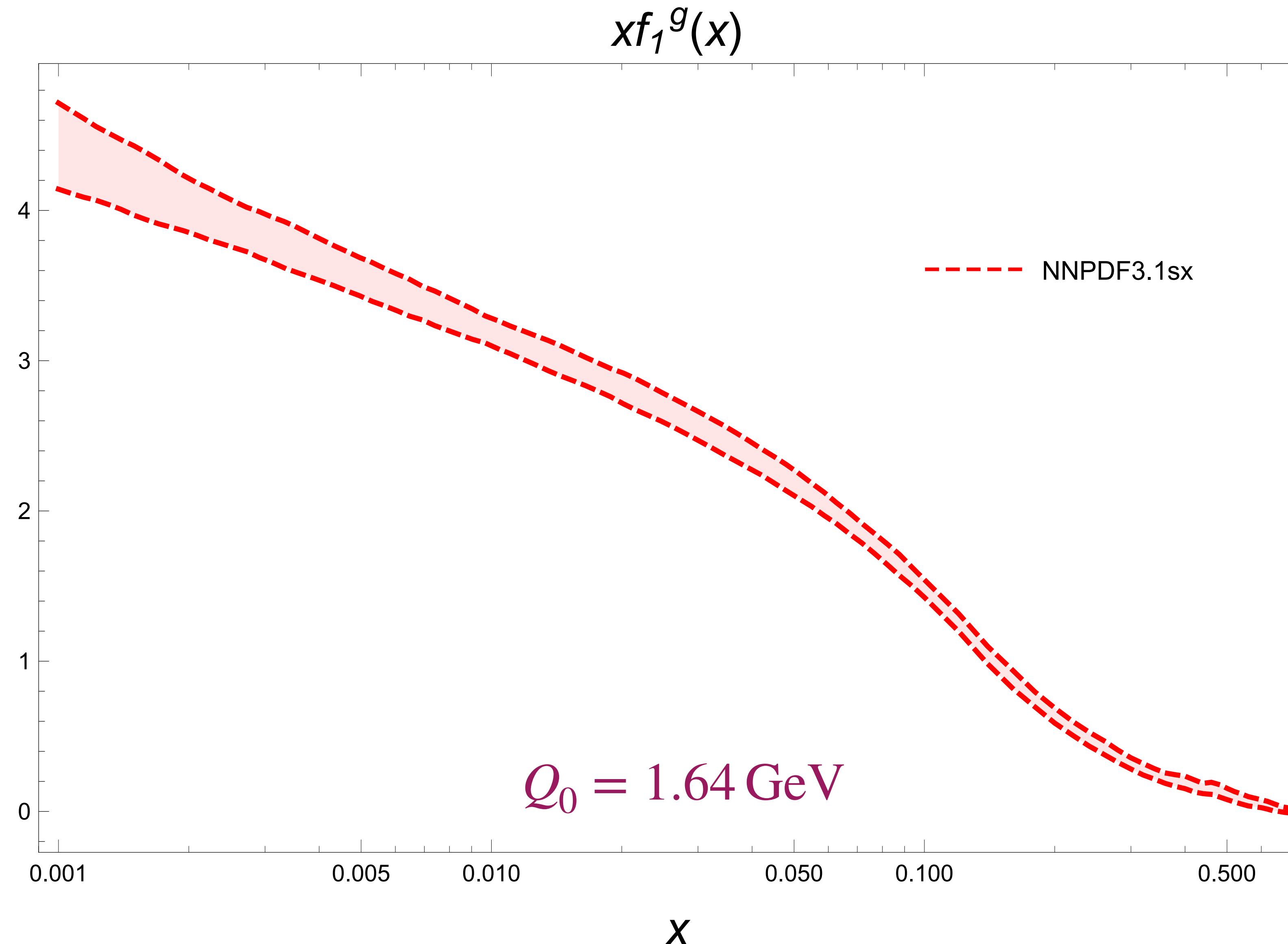
# Spectral function vs $D$



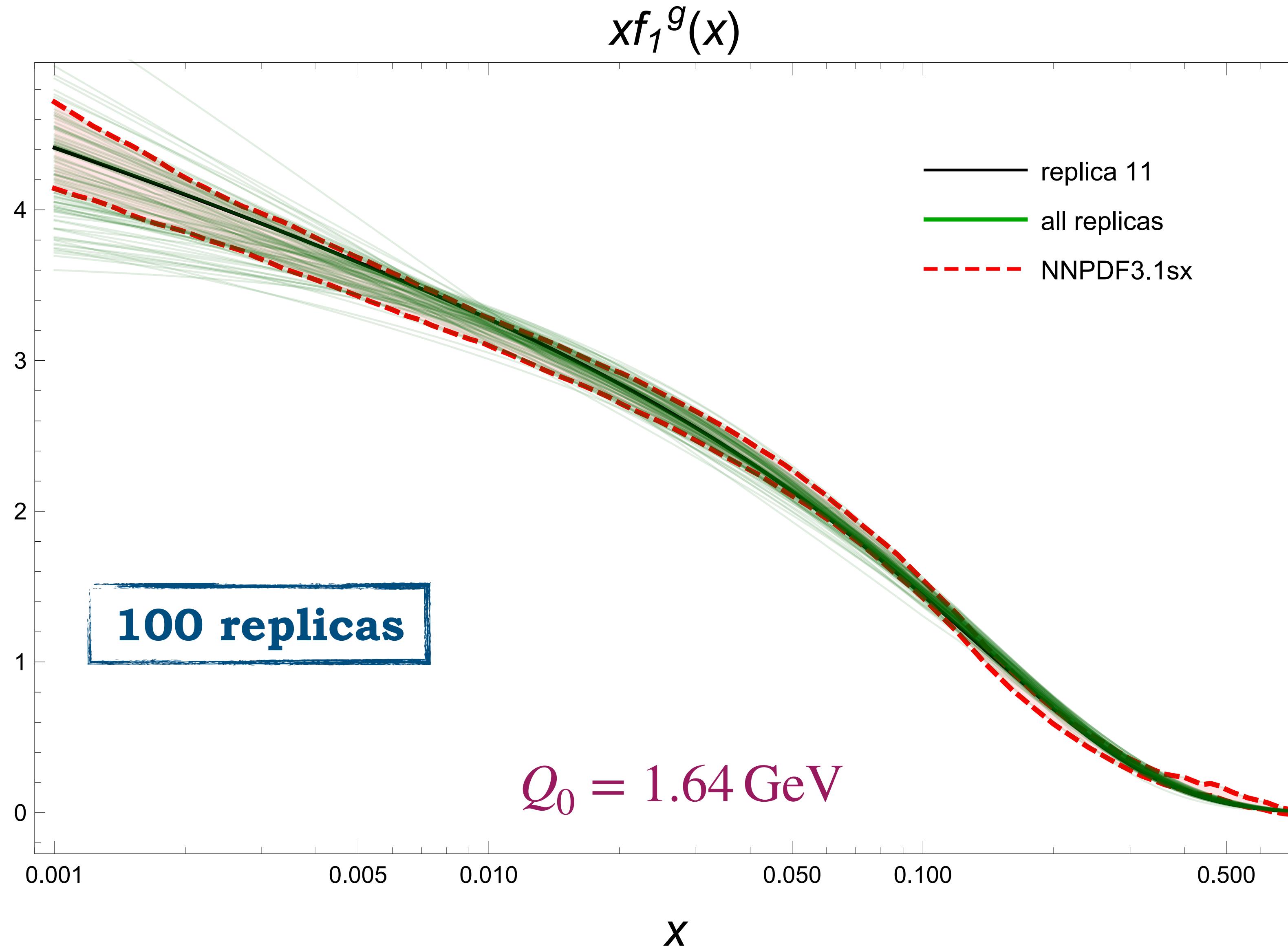
# Spectral function vs $D$



# Unpolarized gluon PDF

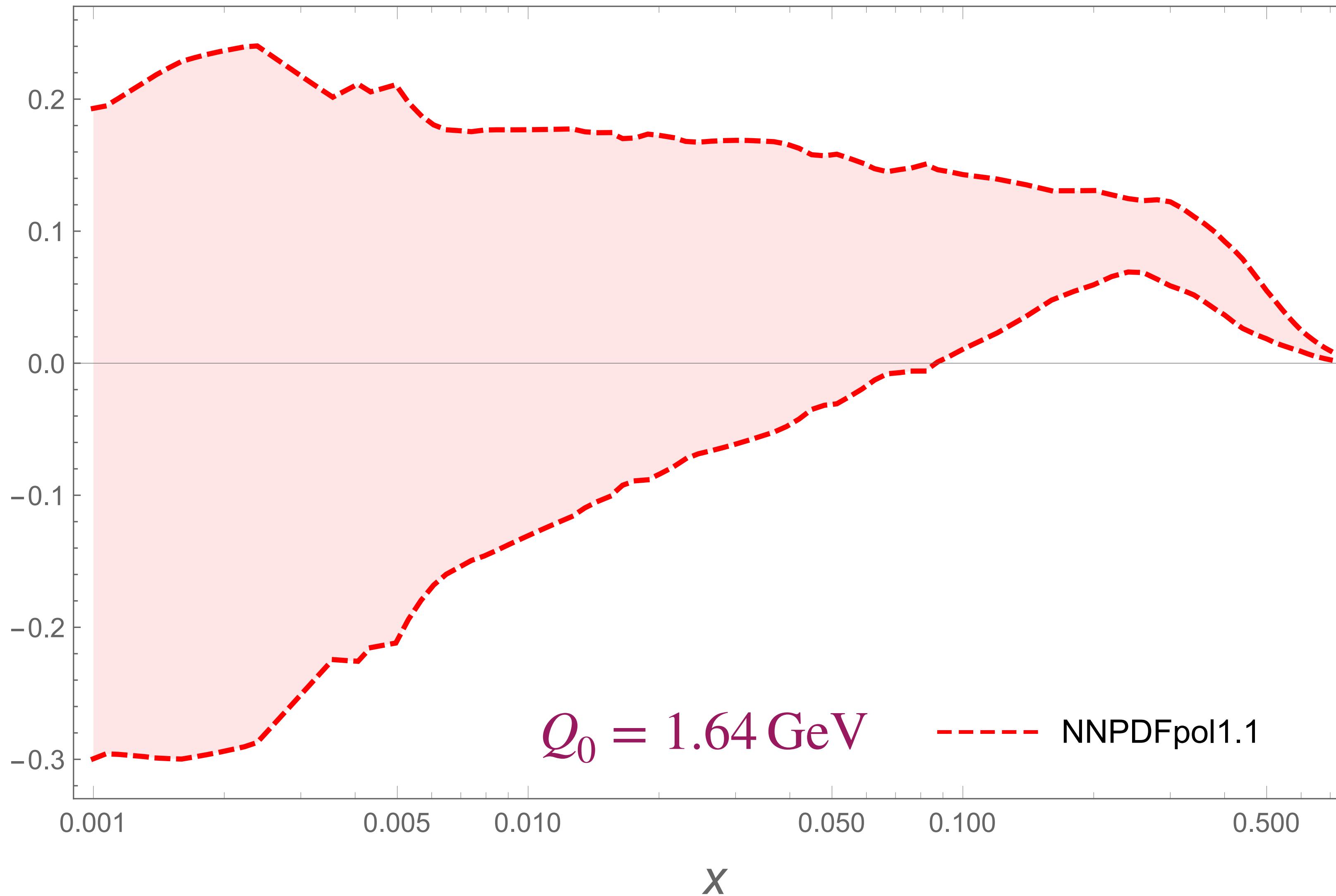


# Unpolarized gluon PDF



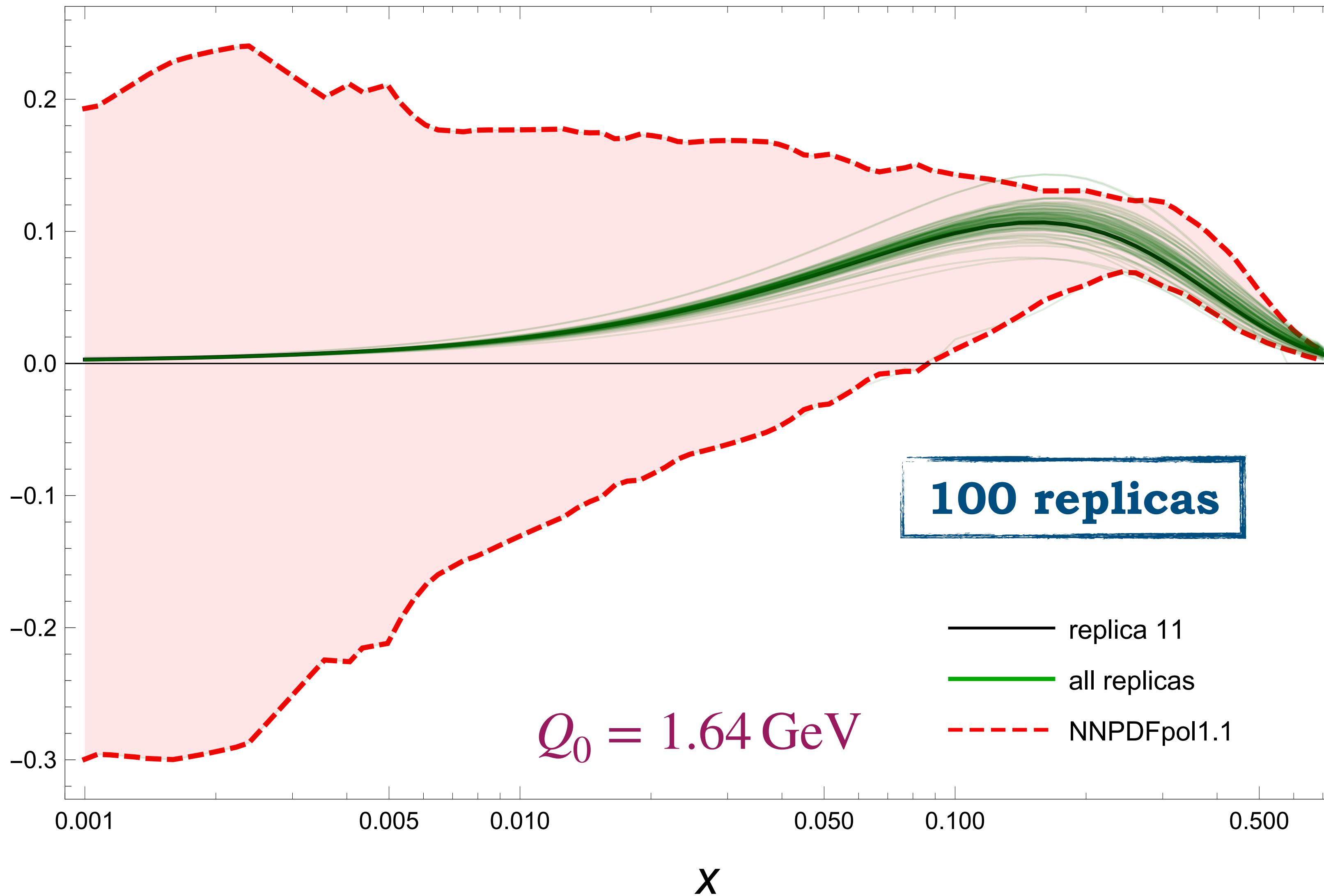
# Helicity gluon PDF

$xg_1^g(x)$



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# Fit specifics

$$\chi^2/\text{d.o.f.} = 0.54 \pm 0.38$$

no **overlearning**, just large errors for  $g_1$

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$$\langle x \rangle_g = \int_0^1 dx x f_1^g(x, Q_0)$$

$$S_g = \frac{1}{2} \langle 1 \rangle_{\Delta g} = \int_0^1 dx g_1^g(x, Q_0)$$

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Our model @  $Q_0 = 1.64$  GeV

$$\langle x \rangle_g = 0.424(9)$$

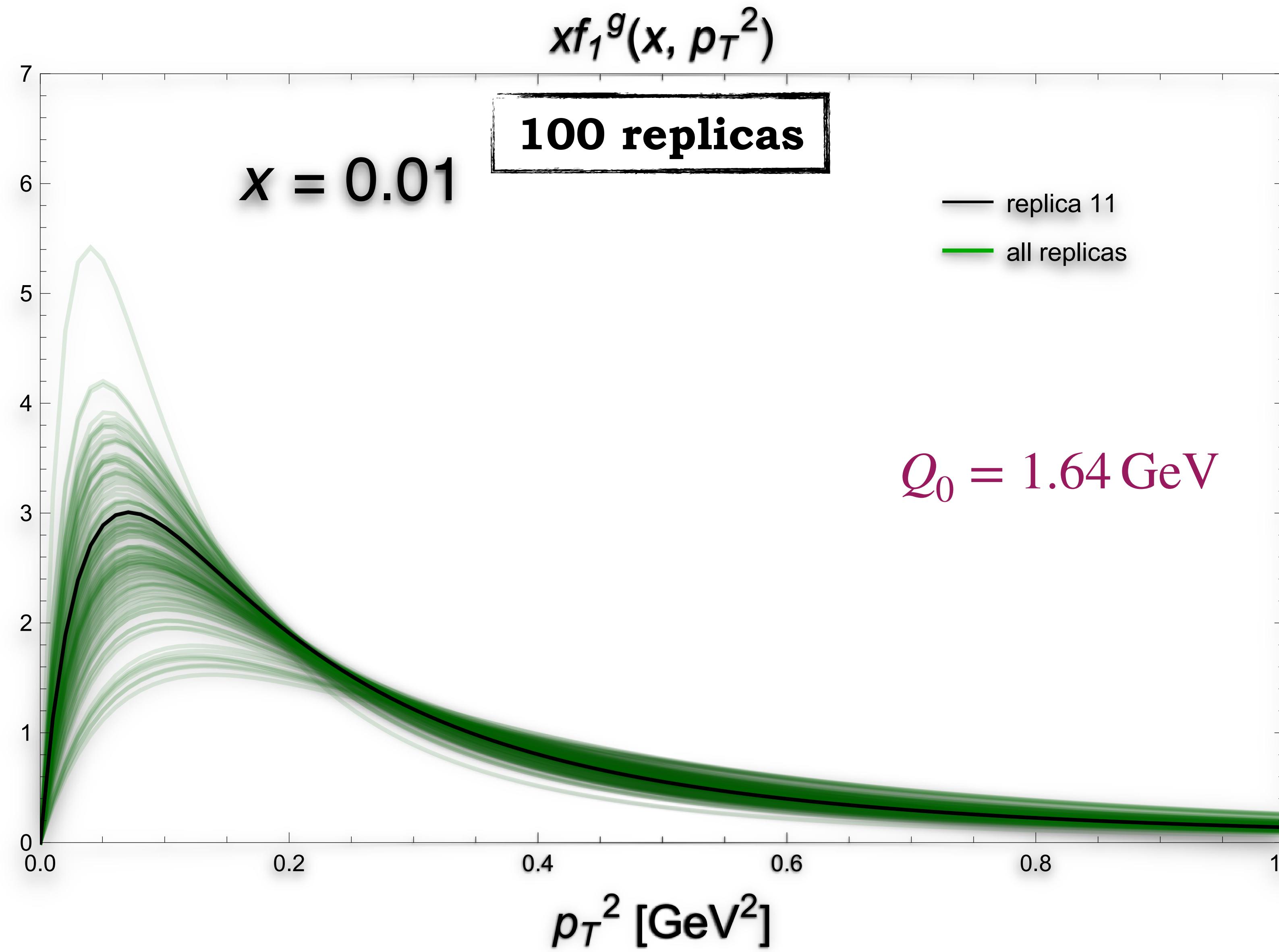
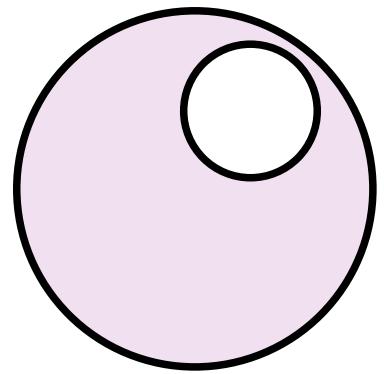
$$\langle S \rangle_g = 0.159(11)$$

Lattice @  $Q_0 = 2$  GeV

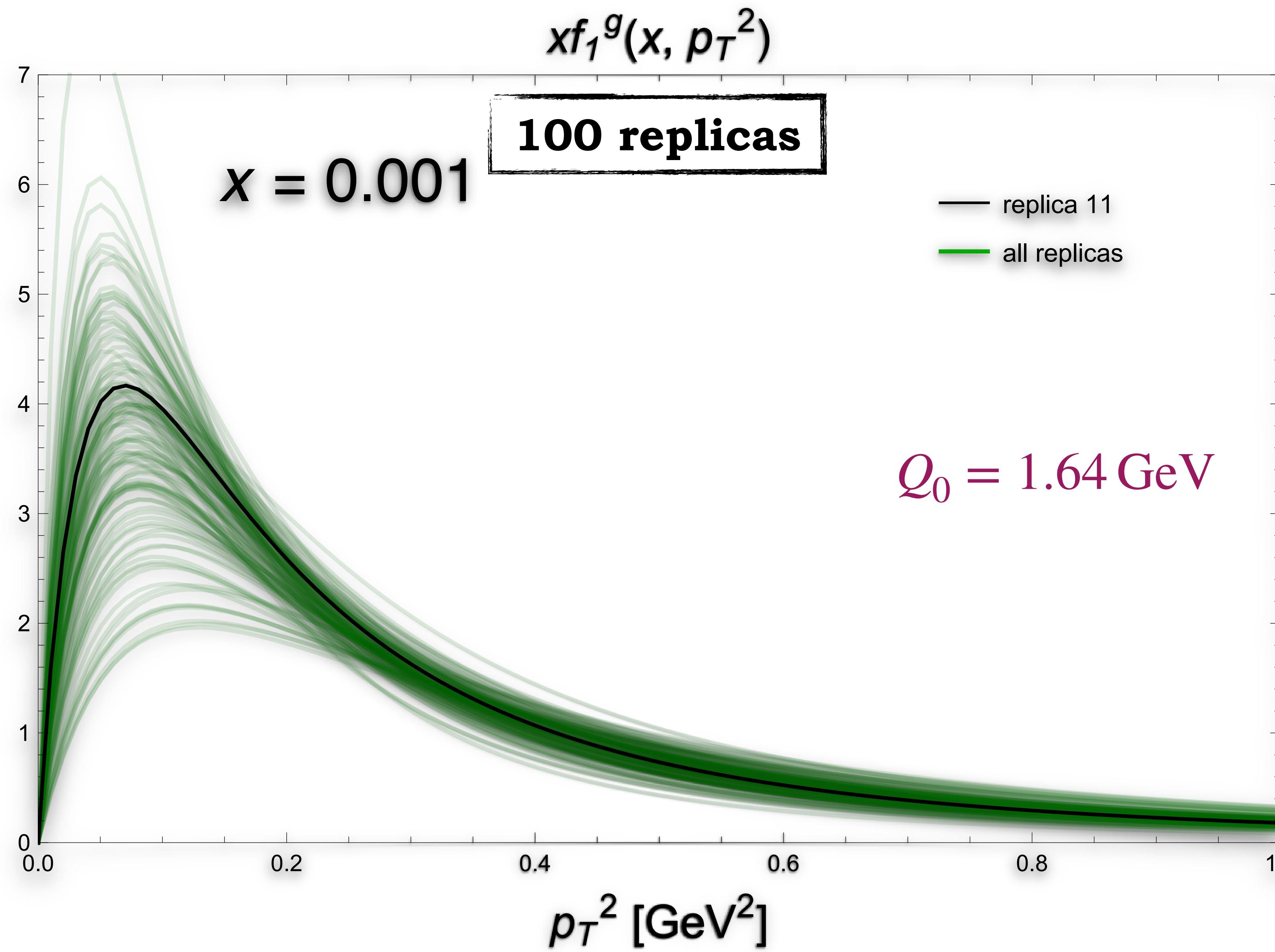
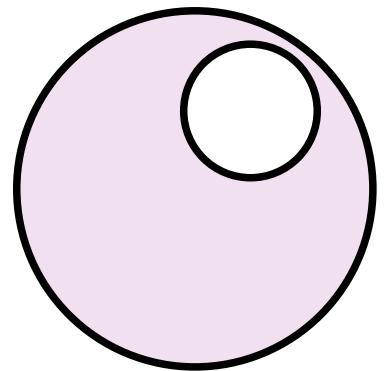
$$\langle x \rangle_g = 0.427(92)$$

$$\langle J \rangle_g = 0.187(46)$$

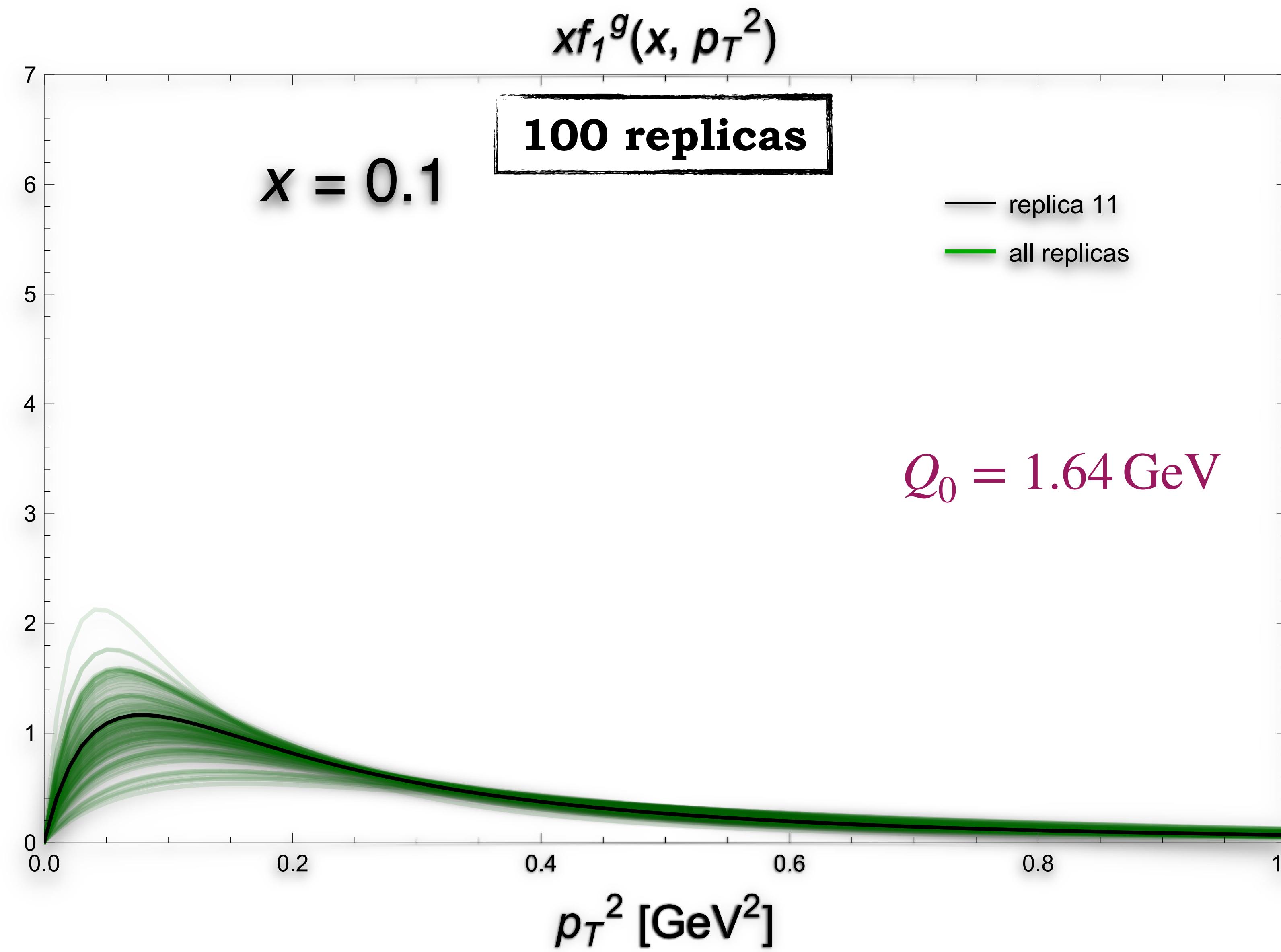
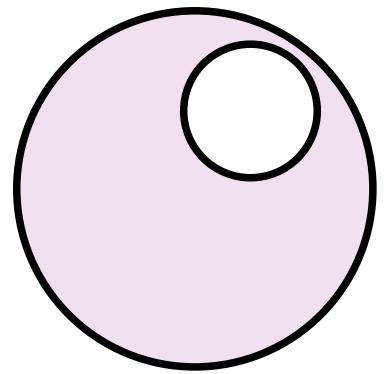
# Unpolarized gluon TMD



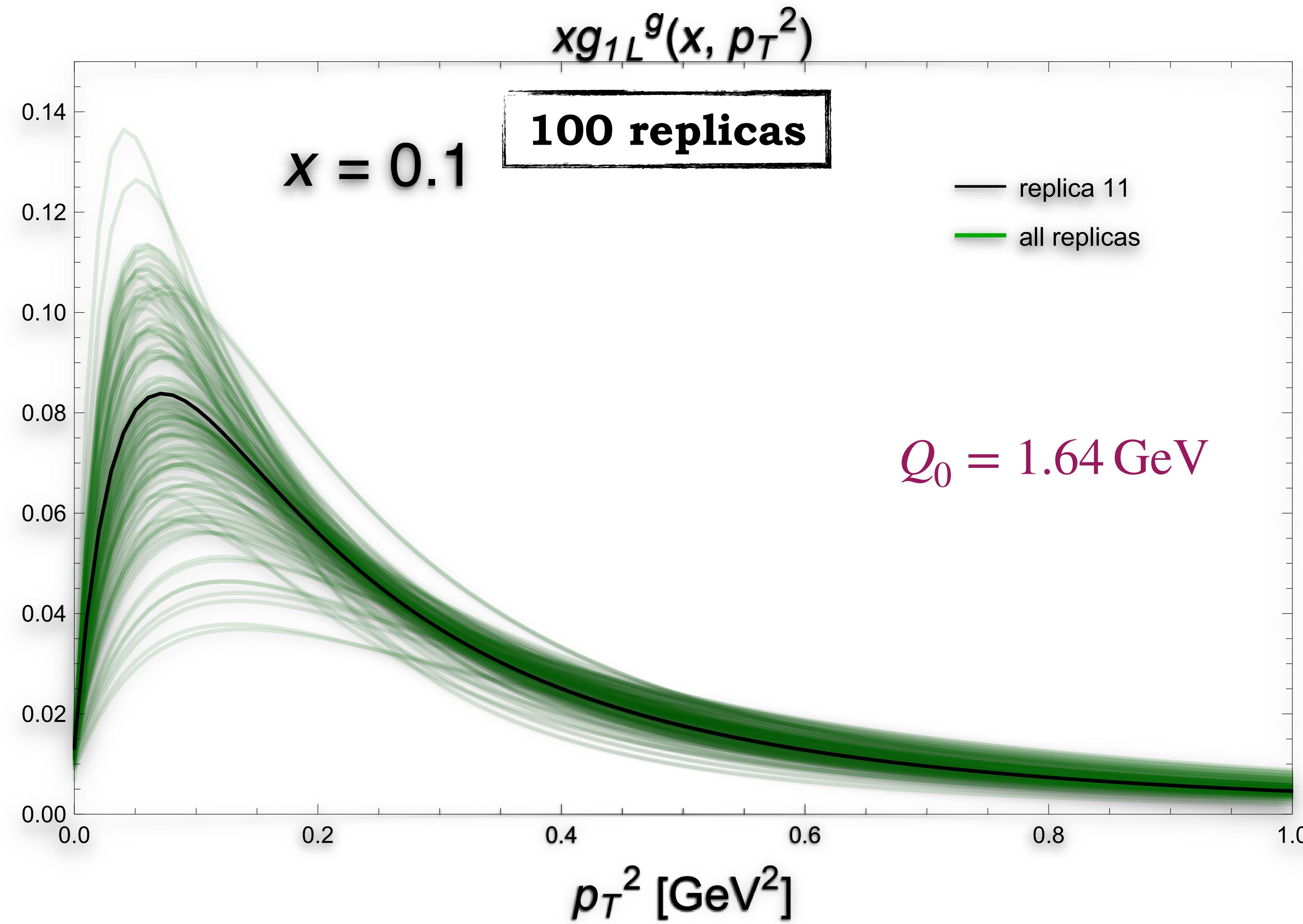
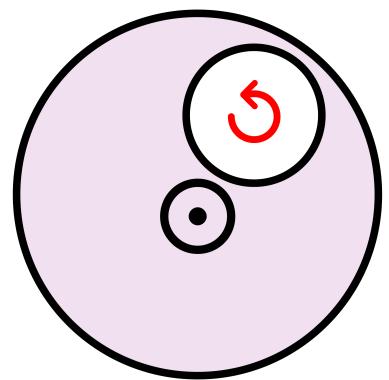
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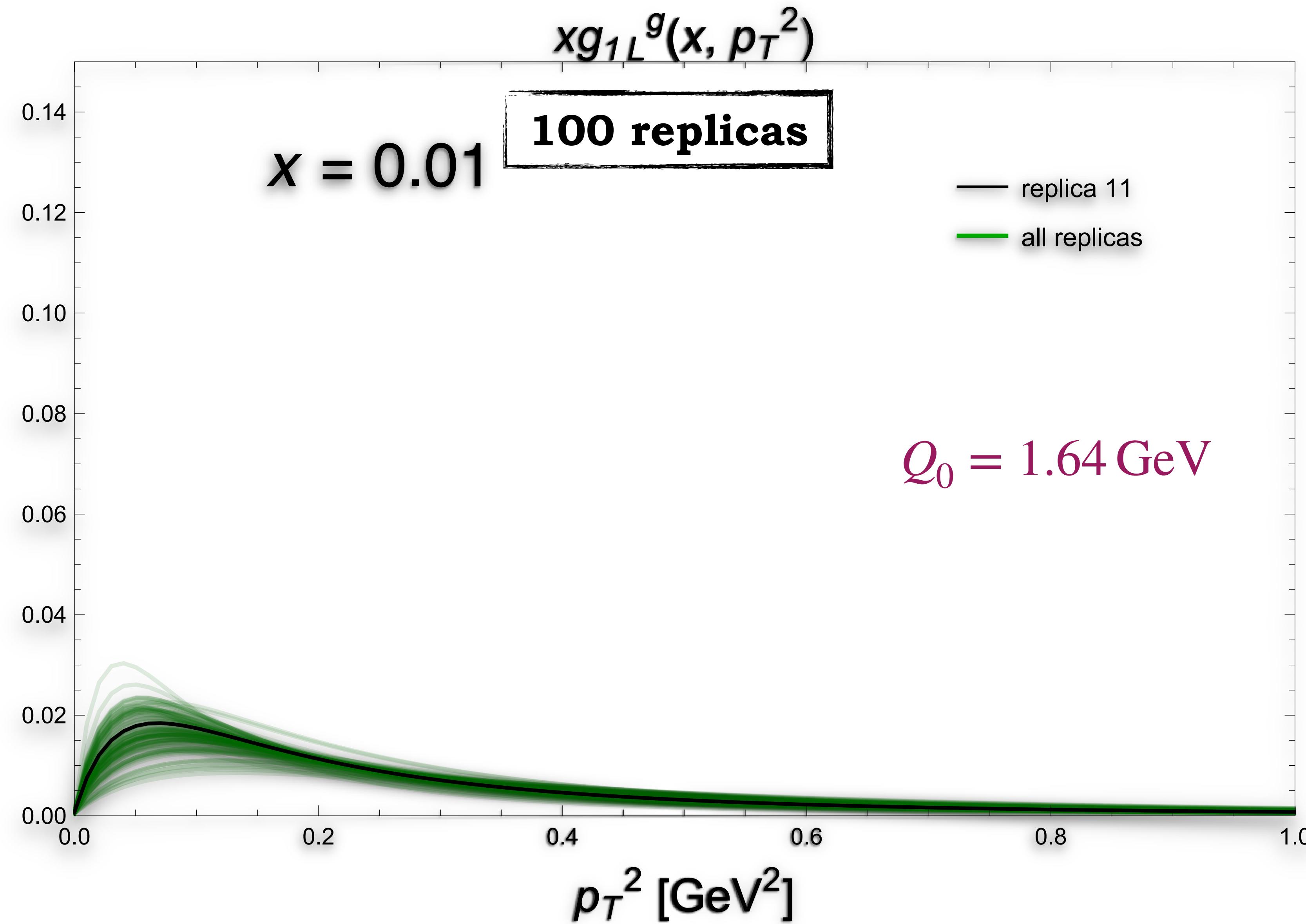
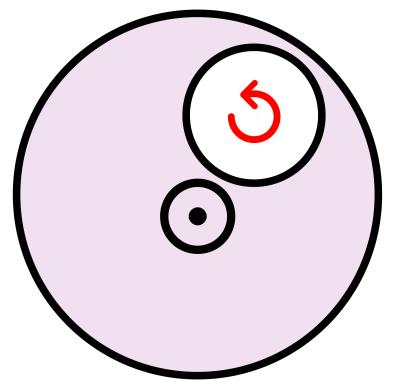
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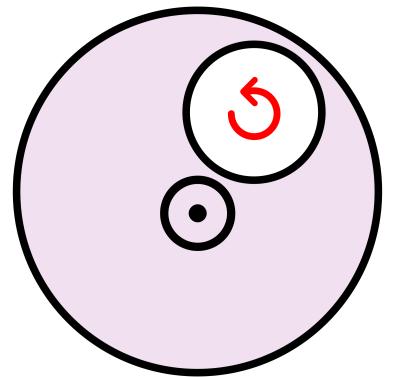
# Helicity gluon TMD



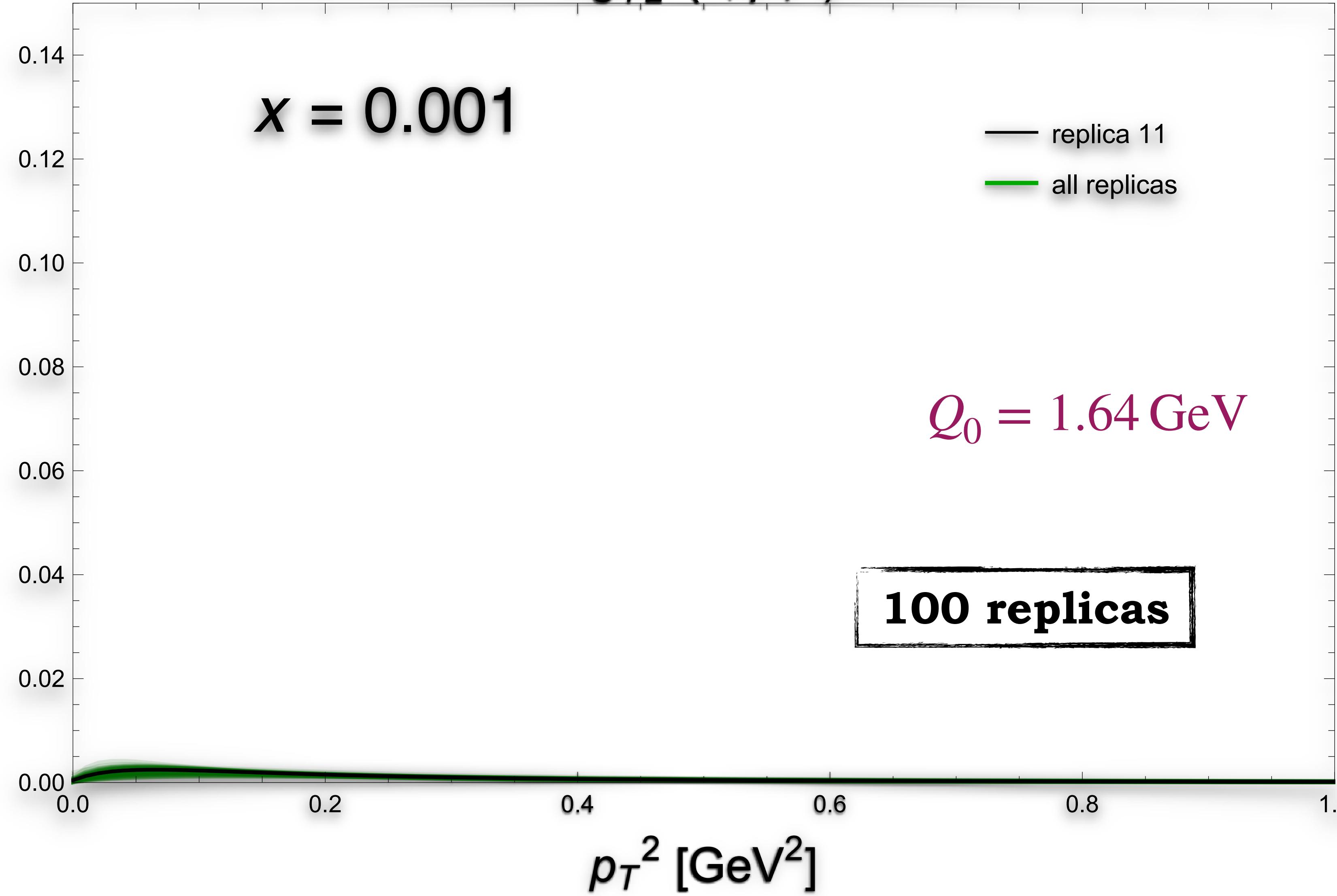
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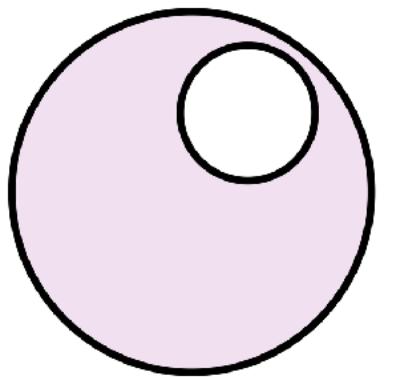
# Helicity gluon TMD



$x g_{1L}^g(x, p_T^2)$



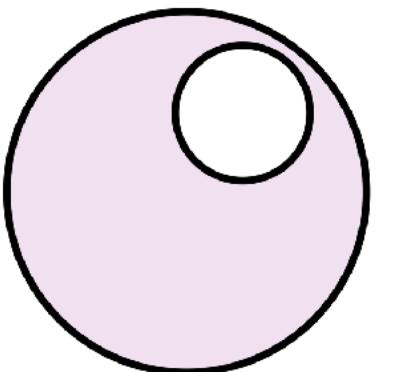
# $\rho$ -densities



**Unpolarized [u/u]**

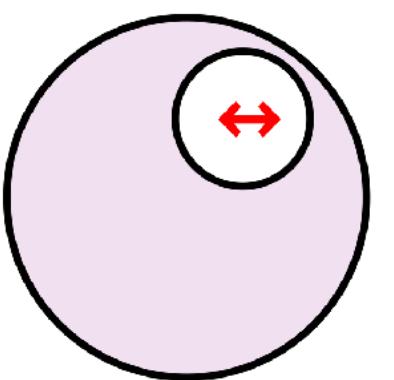
$$f_1(x, p_x, p_y)$$

# $\rho$ -densities



**Unpolarized [u/u]**

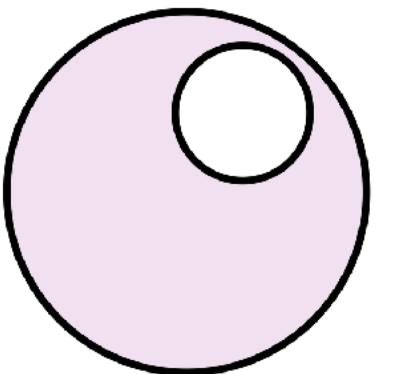
$$f_1(x, p_x, p_y)$$



**Boer-Mulders [↔/u]**

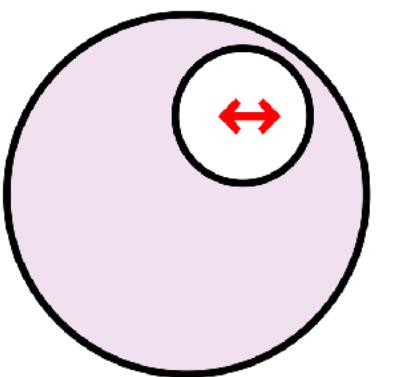
$$f_1(x, p_x, p_y) + \frac{p_x^2 - p_y^2}{2M^2} h_1^\perp(x, p_x, p_y)$$

# $\rho$ -densities



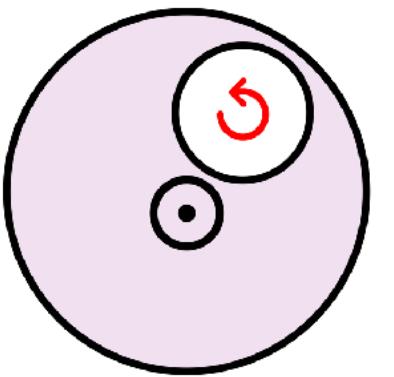
**Unpolarized** [u/u]

$$f_1(x, p_x, p_y)$$



**Boer-Mulders** [ $\leftrightarrow/u$ ]

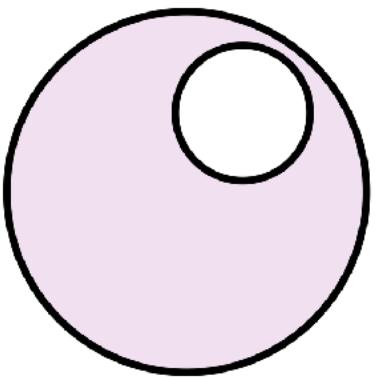
$$f_1(x, p_x, p_y) + \frac{p_x^2 - p_y^2}{2M^2} h_1^\perp(x, p_x, p_y)$$



**Helicity** [ $\cup/+$ ]

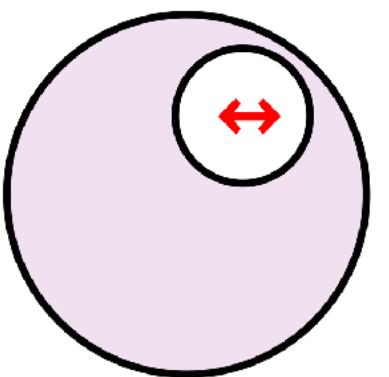
$$\frac{1}{2} \left[ f_1(x, p_x, p_y) + g_{1L}(x, p_x, p_y) \right]$$

# $\rho$ -densities



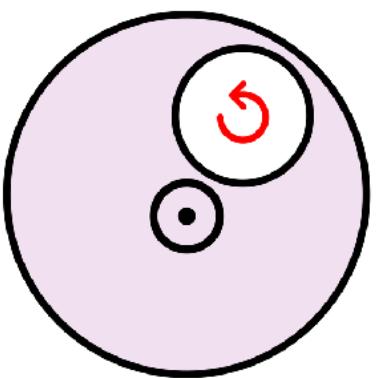
**Unpolarized** [u/u]

$$f_1(x, p_x, p_y)$$



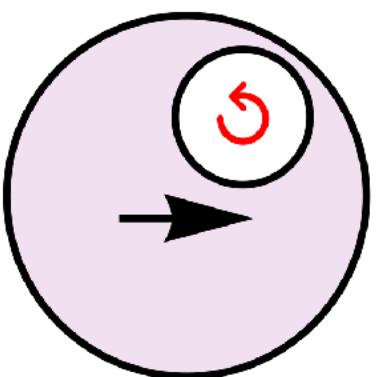
**Boer-Mulders** [ $\leftrightarrow/u$ ]

$$f_1(x, p_x, p_y) + \frac{p_x^2 - p_y^2}{2M^2} h_1^\perp(x, p_x, p_y)$$



**Helicity** [ $\cup/+$ ]

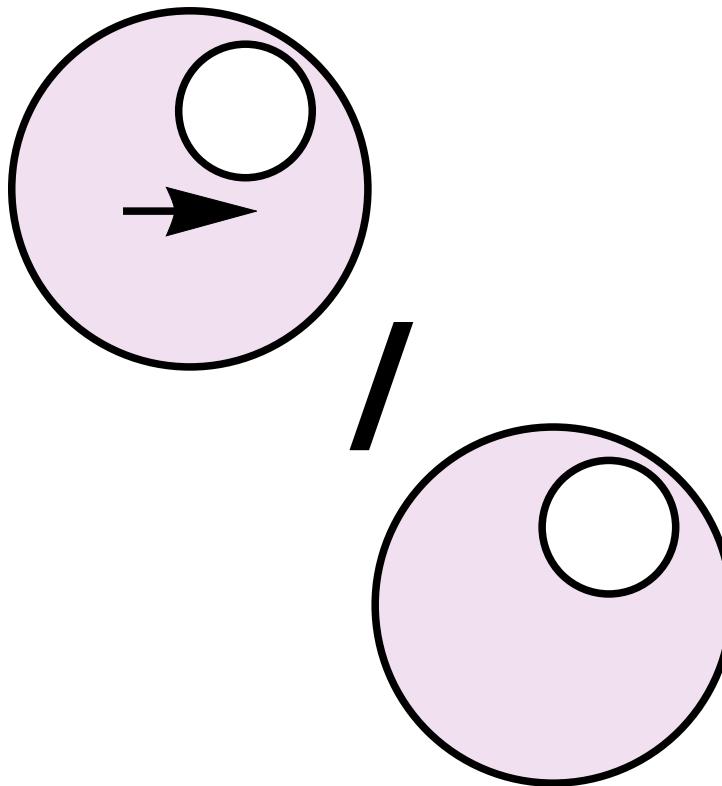
$$\frac{1}{2} \left[ f_1(x, p_x, p_y) + g_{1L}(x, p_x, p_y) \right]$$



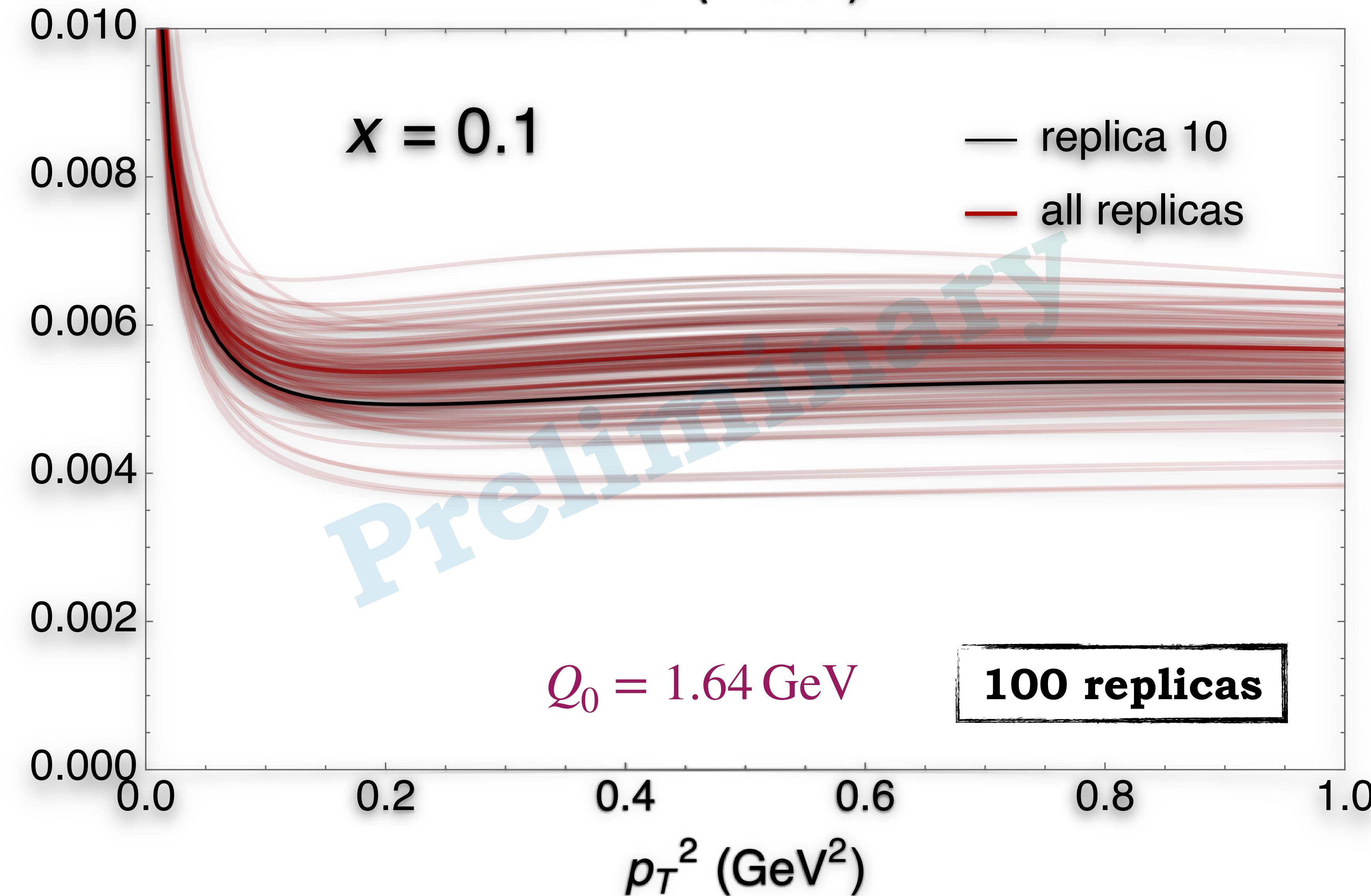
**Worm-gear** [ $\cup/\rightarrow$ ]

$$f_1(x, p_x, p_y) - \frac{p_x}{M} g_{1T}(x, p_x, p_y)$$

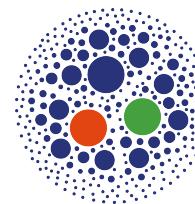
# *f*-type Sivers / unpol.



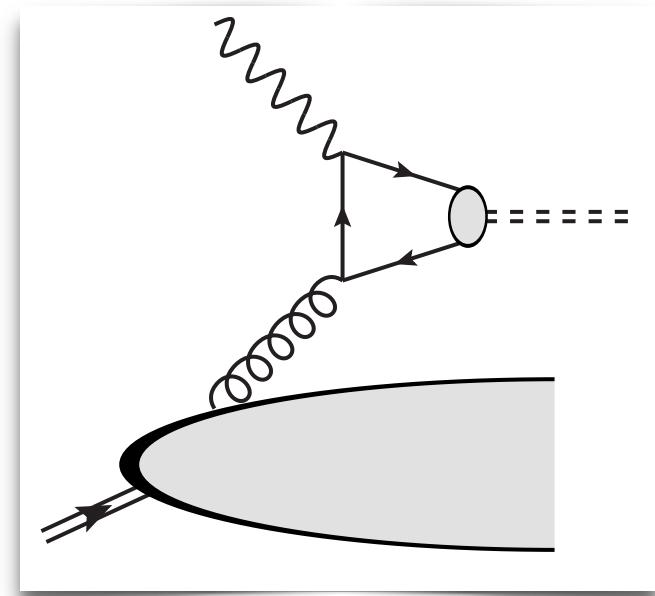
$$\frac{\frac{p_T}{M} f_1 \tau^{\perp[+,+]}(x, p_T^2)}{f_1^g(x, p_T^2)}$$



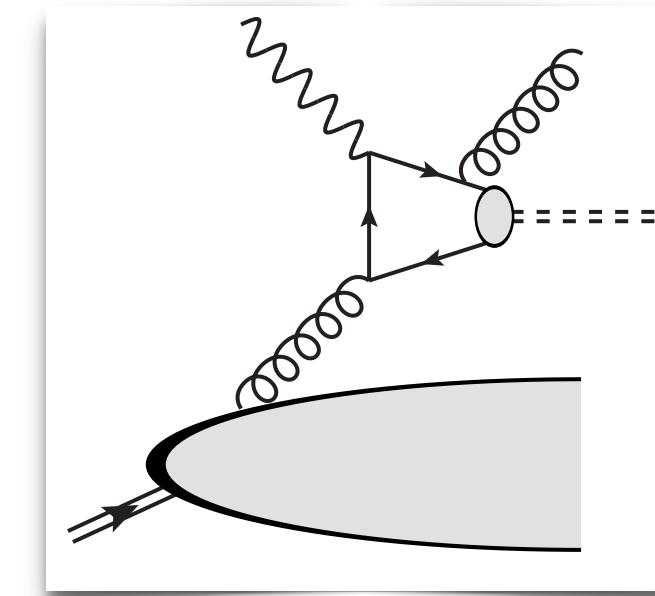
# Hadronic structure and quarkonia



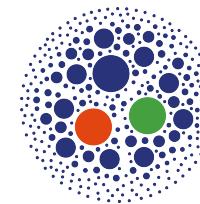
**SIDIS**



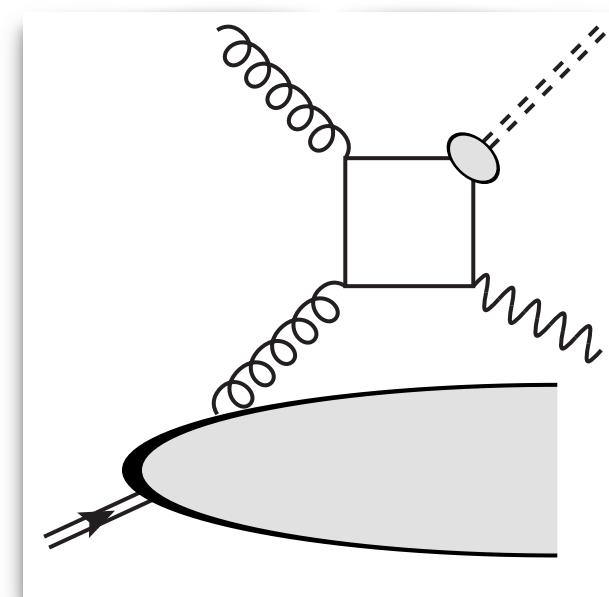
CO



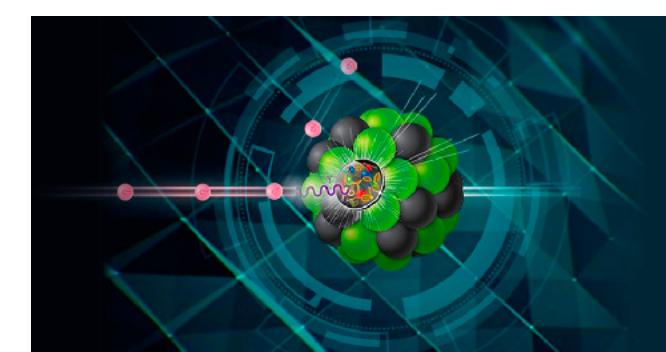
CO + CS



**Hadroproduction**



CO + CS



EIC Yellow Report Document

🔗 (EICUG website)

🔗 [EICUG [arXiv:2103.05419]]

On the physics potential to study the gluon content of proton and deuteron at NICA SPD



🔗 [NICA Collaboration [arXiv:2011.15005]]

Perspectives for quarkonium studies at the high-luminosity LHC

🔗 (QAT 2021 Workshop)

🔗 [Quarkonia As Tools Collaboration [arXiv:2012.14161]]

**Backup**