Gauge-invariant TMD factorization for Drell-Yan hadronic tensor at small x

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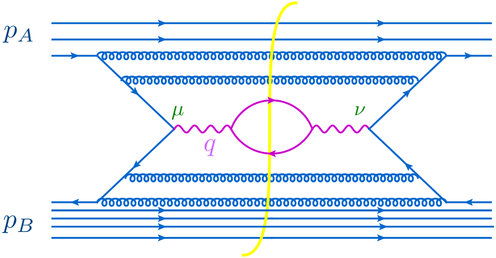
DY hadronic tensor for electromagnetic current

DY cross section is given by the product of leptonic tensor and hadronic tensor. The hadronic tensor $W_{\mu\nu}$ is defined as

$$W_{\mu\nu}(p_A, p_B, q) = \frac{1}{(2\pi)^4} \int d^4x \, e^{-iqx} \langle p_A, p_B | J_\mu(x) J_\nu(0) | p_A, p_B \rangle$$



p_A, p_B = hadron momenta, q = the momentum of DY pair, and J_μ is the electromagnetic or Z-boson current.

DY hadronic tensor for electromagnetic current

For unpolarized hadrons, the hadronic tensor $W_{\mu\nu}$ for EM current is parametrized by 4 functions, for example in Collins-Soper frame

$$W_{\mu\nu} = -\Big(g_{\mu\nu} - \frac{q_\mu q_\nu}{q^2}\Big)(W_T + W_{\Delta\Delta}) - 2X_\mu X_\nu W_{\Delta\Delta} + Z_\mu Z_\nu(W_L - W_T - W_{\Delta\Delta}) - (X_\mu Z_\nu + X_\nu Z_\mu)W_\Delta$$

where X, Z are unit vectors orthogonal to q and to each other

TMD representation for W_i

The hadronic tensor in the Sudakov region $q^2 \equiv Q^2 \gg q_\perp^2$ can be studied by TMD factorization. For example, functions W_T and $W_{\Delta\Delta}$ can be represented as

$$W_i = \sum_{\text{flavors}} e_f^2 \int d^2k_\perp \mathcal{D}^{(i)}_{f/A}(x_A, k_\perp) \mathcal{D}^{(i)}_{f/B}(x_B, q_\perp - k_\perp) C_i(q, k_\perp) + \text{power corrections} + \text{Y} - \text{terms} \quad (1)$$

- $\mathcal{D}_{f/A}(x_A, k_\perp)$ is the TMD density of a parton f in hadron A with fraction of momentum x_A and transverse momentum $k_\perp$,
- $\mathcal{D}_{f/B}(x, q_\perp - k_\perp)$ is a similar quantity for hadron B,
- $C_i(q, k)$ are determined by the cross section $\sigma(f\bar{f} \to \mu^+\mu^-)$ of production of DY pair of invariant mass q^2 in the scattering of two partons.

TMD representation for W_i

There is, however, a problem with Eq. (1) for the functions W_L and W_Δ.

W_T and $W_{\Delta\Delta}$ are determined by leading-twist quark TMDs, but W_Δ and W_L start from terms $\sim \frac{q_\perp}{Q}$ and $\sim \frac{q_\perp^2}{Q^2}$ determined by quark-quark-gluon TMDs.

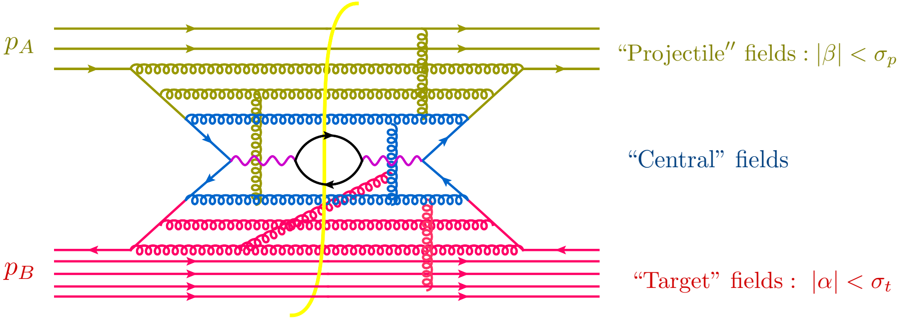
The power corrections $\sim \frac{q_\perp}{Q}$ were found more than two decades ago but there was no calculation of power corrections $\sim \frac{q_\perp^2}{Q^2}$ until recently.

Also, the leading-twist contribution is not EM gauge invariant.

TMD factorization from rapidity factorization (A. Tarasov and I.B.)

Sudakov variables:

$$p \;=\; \alpha p_1 + \beta p_2 + p_\perp, \qquad p_1 \simeq p_A, \;\; p_2 \simeq p_B, \;\; p_1^2 = p_2^2 = 0$$



The result of the integration over “central” fields in the background of projectile and target fields is a series of TMD operators made from projectile (or target) fields multiplied by powers of $\frac{1}{Q^2} \Rightarrow$ power corrections

Leading-N_c power corrections

Power corrections are $\sim$ leading twist $\times\Big(\frac{q_\perp}{Q}$ or $\frac{q_\perp^2}{Q^2}\Big)\times\big(1+\frac{1}{N_c}+\frac{1}{N_c^2}\big)$.

(Pleasant) surprise: most of the terms not suppressed by $\frac{1}{N_c}$ are determined by the leading-twist TMDs due to QCD equations of motion

Leading twist:

$$\frac{1}{8\pi^3 s}\int dx_- d^2x_\perp\ e^{-i\alpha sqrt\frac{s}{2}x_+ + i(k,x)_\perp}\ \langle p_A|\hat{\bar{\psi}}_f(x_-,x_\perp)\ \not{p}_2\hat{\psi}_f(0)|p_A\rangle\ =\ f_{1f}(\alpha,k_\perp^2)$$

Power correction:

$$\frac{1}{8\pi^3 s}\int dx_- dx_\perp\ e^{-i\alpha_q\sqrt{\frac{s}{2}}x_- + i(k,x)_\perp}$$

$$\times\ \langle p_A|\hat{\bar{\psi}}^f(x_-,x_\perp)\ \not{p}_2[\hat{A}_i(x_-,x_\perp) - i\gamma_5\hat{\tilde{A}}_i(x_-,x_\perp)]\hat{\psi}^f(0)|p_A A\rangle$$

$$=\ -k_i f_1(\alpha_q,k_\perp) + \alpha_q k_i\big[f_\perp(\alpha_q,k_\perp) + g^\perp(\alpha_q,k_\perp)\big],$$

(Mulders & Tangerman, 1996)

At small $\alpha_q\equiv x_A$ one can drop the second term

Result for $W_{\mu\nu}$ for unpolarized hadrons

Result:

$$W_{\mu\nu}(q) \;=\; W^1_{\mu\nu}(q) + W^2_{\mu\nu}(q)$$

The first, gauge-invariant, part is given by

$$W^1_{\mu\nu}(q) \;=\; W^{1F}_{\mu\nu}(q) + W^{1H}_{\mu\nu}(q),$$

$$W^{1F}_{\mu\nu}(q) \;=\; \sum_f e_f^2 W^{fF}_{\mu\nu}(q), \quad W^{fF}_{\mu\nu}(q) \;=\; \frac{1}{N_c}\int d^2k_\perp F^f(q,k_\perp)\mathcal{W}^F_{\mu\nu}(q,k_\perp),$$

$$W^{1H}_{\mu\nu}(q) \;=\; \sum_f e_f^2 W^{fH}_{\mu\nu}(q), \quad W^{fH}_{\mu\nu}(q) \;=\; \frac{1}{N_c}\int d^2k_\perp H^f(q,k_\perp)\mathcal{W}^H_{\mu\nu}(q,k_\perp)$$

where F^f and H^f are ($\alpha_q \equiv x_A$, $\beta_q \equiv x_B$)

$$F^f(q,k_\perp) \;=\; f_1^f(\alpha_q,k_\perp)\bar{f}_1^f(\beta_q,(q-k)_\perp) \;+\; f_1^f \leftrightarrow \bar{f}_1^f$$

$$H^f(q,k_\perp) \;=\; h_{1f}^\perp(\alpha_q,k_\perp)\bar{h}_{1f}^\perp(\beta_q,(q-k)_\perp) \;+\; h_{1f}^\perp \leftrightarrow \bar{h}_{1f}^\perp$$

Gauge-invariant structures $q^\mu \mathcal{W}^F_{\mu\nu} = q^\mu \mathcal{W}^H_{\mu\nu} = 0$

$$\begin{aligned}
&\mathcal{W}^F_{\mu\nu}(q,k_\perp)\\
&= -g^\perp_{\mu\nu} + \frac{1}{Q^2_\parallel}(q^\parallel_\mu q^\perp_\nu + q^\parallel_\nu q^\perp_\mu) + \frac{q^2_\perp}{Q^4_\parallel} q^\parallel_\mu q^\parallel_\nu + \frac{\tilde{q}_\mu \tilde{q}_\nu}{Q^2_\parallel}[q^2_\perp - 4(k,q-k)_\perp]\\
&- \Big[\frac{\tilde{q}_\mu}{Q^2_\parallel}\Big(g^\perp_{\nu i} - \frac{q^\parallel_\nu q_i}{Q^2_\parallel}\Big)(q-2k)^i_\perp + \mu \leftrightarrow \nu\Big]
\end{aligned}$$

$$\tilde{q} \equiv x_A p_1 - x_B p_2$$

$$\begin{aligned}
&m^2 \mathcal{W}^H_{\mu\nu}(q,k_\perp)\\
&= -k^\perp_\mu (q-k)^\perp_\nu - k^\perp_\nu (q-k)^\perp_\mu - g^\perp_{\mu\nu}(k,q-k)_\perp + 2\frac{\tilde{q}_\mu \tilde{q}_\nu - q^\parallel_\mu q^\parallel_\nu}{Q^4_\parallel} k^2_\perp (q-k)^2_\perp\\
&- \Big(\frac{q^\parallel_\mu}{Q^2_\parallel}\big[k^2_\perp (q-k)^\perp_\nu + k^\perp_\nu (q-k)^2_\perp\big] + \frac{\tilde{q}_\mu}{Q^2_\parallel}\big[k^2_\perp (q-k)^\perp_\nu - k^\perp_\nu (q-k)^2_\perp\big] + \mu \leftrightarrow \nu\Big)\\
&- \frac{\tilde{q}_\mu \tilde{q}_\nu + q^\parallel_\mu q^\parallel_\nu}{Q^4_\parallel}\big[q^2_\perp - 2(k,q-k)_\perp\big](k,q-k)_\perp - \frac{q^\parallel_\mu \tilde{q}_\nu + \tilde{q}_\mu q^\parallel_\nu}{Q^4_\parallel}(2k-q,q)_\perp (k,q-k)_\perp
\end{aligned}$$

Logarithmic estimates of angular coefficients

Take $s = 8$ TeV, $Q = 90$ GeV and $q_\perp = 20$ GeV where $x_A, x_B \sim 0.1$ and power corrections are small but sizable.

The differential cross section of DY process is parametrized as

$$\left(\frac{d\sigma}{d^4q}\right)^{-1}\frac{d\sigma}{d\Omega d^4q} \;=\; \frac{3}{4\pi(\lambda+3)}\left(1+\lambda\cos^2\theta+\mu\sin 2\theta\cos\phi+\frac{\nu}{2}\sin^2\theta\cos 2\phi\right)$$

Logarithmic estimates of angular coefficients

$$1-\lambda \;=\; 2\frac{W_L}{W_T+W_L} \;\simeq\; 2\frac{1+2\frac{\ln Q^2/q_\perp^2}{\ln q_\perp^2/m^2}}{\frac{Q^2}{q_\perp^2}-\frac{1}{2}+2\frac{\ln Q^2/q_\perp^2}{\ln q_\perp^2/m^2}} \;\simeq\; 0.19$$

$$\nu \;=\; \frac{2W_{\Delta\Delta}}{W_T+W_L} \;\simeq\; \frac{1}{\frac{Q^2}{q_\perp^2}-\frac{1}{2}+2\frac{\ln Q^2/q_\perp^2}{\ln q_\perp^2/m^2}} \;\simeq\; 0.05$$

$\mu \;=\; \frac{W_\Delta}{W_T+W_L}, = 0$ if we use factorization models for TMDs.

Approximately the same λ and ν values as in analysis of LHC data by Lambertsen and Vogelsang

Z-boson production at LHC

The relevant terms of the Lagrangian for quark fields ψ^f are

$$\mathcal{L}_Z = e\int d^4x\, \mathcal{J}_\mu Z^\mu(x), \qquad \mathcal{J}_\mu = c_e\bar{e}(a_e - \gamma_5)e - \sum_{\text{flavors}} c_f\bar{\psi}^f\gamma_\mu(a_f - \gamma_5)\psi^f$$

where

$$c_{u,c} = \frac{1}{4c_W s_W}, \quad a_{u,c} = 1 - \frac{8}{3}s_W^2, \quad c_{d,s} = -\frac{1}{4c_W s_W}, \quad a_{d,s} = 1 - \frac{4}{3}s_W^2,$$
$$c_e = \frac{1}{4c_W s_W}, \quad a_e = 1 - 4s_W^2, \qquad c_W \equiv \cos\theta_W, \quad s_W \equiv \sin\theta_W.$$

$$d\sigma = \frac{e^4}{16\pi^2 s N_c}\frac{dQ^2}{Q^2}dY d^2q_\perp d\Omega_l\, \mathbb{W}(q,l,l')$$

l, l' - lepton momenta

Angular coefficients of Z-boson production

In CMS and ATLAS experiments $s = 8$ TeV, $Q = 80 - 100$ GeV and $Q_\perp$ varies from 0 to 120 GeV.

Our analysis is valid at $Q_\perp = 10 - 30$ GeV and $Y \simeq 0$ ($x_A \sim x_B \sim 0.1$) so that power corrections are small but sizable.

Angular distribution of DY leptons in the Collins-Soper frame ($c_\phi \equiv \cos\phi$, $s_\phi \equiv \sin\phi$ etc.)

$$\frac{d\sigma}{dQ^2 dy d\Omega_l} = \frac{3}{16\pi}\frac{d\sigma}{dQ^2 dy}\Big[(1+c_\theta^2) + \frac{A_0}{2}(1-3c_\theta^2) + A_1 s_{2\theta}c_\phi + \frac{A_2}{2}s_\theta^2 c_{2\phi} + A_3 s_\theta c_\phi + A_4 c_\theta + A_5 s_\theta^2 s_{2\phi} + A_6 s_{2\theta}s_\phi + A_7 s_\theta s_\phi\Big]$$

Result with $\frac{1}{Q^2}$, large-N_c and "f_1" accuracy

$$\mathbb{W}(q,l,l') = c_e^2 c_f^2 \frac{Q^4}{|m_Z^2 - Q^2|^2 + \Gamma_Z^2 m_Z^2}$$
$$\times \sum_f \Big\{(a_e^2+1)(a_f^2+1)\Big(\Big[\mathcal{W}^{\rm Ff} - \frac{Q_\perp^2}{2Q^2}(\mathcal{W}^{\rm Ff} - \mathcal{W}_L^{\rm Ff})\Big](1+\cos^2\theta)$$
$$+ \frac{Q_\perp^2}{2Q^2}\mathcal{W}_L^{\rm Ff}(1-3\cos^2\theta) + \frac{Q_\perp}{Q}\mathcal{W}_1^{\rm Ff}\sin 2\theta\cos\phi + \frac{Q_\perp^2}{2Q^2}\mathcal{W}^{\rm Ff}\sin^2\theta\cos 2\phi\Big]\Big)$$
$$+ 8a_e a_f\Big[\frac{Q_\perp}{Q}\mathcal{W}_3^{\rm F\mathit{f}}\sin\theta\cos\phi + \mathcal{W}_4^{\rm F\mathit{f}}\cos\theta\Big]\Big\}$$

$$\mathcal{W}^{\rm F\mathit{f}}(q) = \int d^2k_\perp F^f(q,k_\perp), \qquad \mathcal{W}_L^{\rm Ff}(q) = \int dk_\perp \frac{(q-2k)_\perp^2}{q_\perp^2} F^f(q,k_\perp)$$
$$\mathcal{W}_1^{\rm Ff}(q) = \int d^2k_\perp \frac{(q,q-2k)_\perp}{q_\perp^2} F^f(q,k_\perp)$$
$$\mathcal{W}_3^{\rm Ff}(q) = \int d^2k_\perp \frac{(q,q-2k)_\perp}{q_\perp^2} \mathcal{F}^f(q,k_\perp), \qquad \mathcal{W}_4^{\rm Ff}(q) = \int d^2k_\perp \mathcal{F}^f(q,k_\perp),$$
$$\mathcal{F}^f(q,k_\perp) = f_1^f(\alpha_q,k_\perp)\bar{f}_1^f(\beta_q,(q-k)_\perp) - f_1^f \leftrightarrow \bar{f}_1^f$$

Comparison with LHC results

$$\mathbb{W} \sim \sum_f \mathcal{W}^{\rm Ff}\Big\{(a_e^2+1)(a_f^2+1)\Big(\Big[1-\frac{Q_\perp^2}{2m_Z^2}+\frac{Q_\perp^2}{2m_Z^2}\frac{\mathcal{W}_L^{\rm Ff}}{\mathcal{W}^{\rm Ff}}\Big](1+\cos^2\theta)$$
$$+\frac{Q_\perp^2}{2m_Z^2}\frac{\mathcal{W}_L^{\rm Ff}}{\mathcal{W}^{\rm Ff}}(1-3\cos^2\theta)+\frac{Q_\perp}{m_Z}\frac{\mathcal{W}_1^{\rm Ff}}{\mathcal{W}^{\rm Ff}}\sin 2\theta\cos\phi+\frac{Q_\perp^2}{2m_Z^2}\sin^2\theta\cos 2\phi\Big]\Big)$$
$$+8a_ea_f\Big[\frac{Q_\perp}{m_Z}\frac{\mathcal{W}_3^{\rm Ff}}{\mathcal{W}^{\rm Ff}}\sin\theta\cos\phi+\frac{\mathcal{W}_4^{\rm Ff}}{\mathcal{W}^{\rm Ff}}\cos\theta\Big]\Big\}$$

We can easily estimate A_0 and A_2 which depend on $\frac{\mathcal{W}_L^{\rm Ff}}{\mathcal{W}^{\rm Ff}}$.

Logarithmic estimate of $\frac{\mathcal{W}_L^{\rm Ff}}{\mathcal{W}^{\rm Ff}}$: if $k_\perp^2 \gg m_N^2$ we can approximate

$$f_1(x,k_\perp^2) \simeq \frac{f(x)}{k_\perp^2} \quad \Rightarrow \quad F(q,k_\perp) \simeq \frac{f(\alpha_q)\bar{f}(\beta_q)+f\leftrightarrow\bar{f}}{k_\perp^2(q-k)_\perp^2}$$

Performing integration over $k_\perp$ in logarithmical approximation, one obtains

$$\frac{\mathcal{W}_L^{\rm Ff}}{\mathcal{W}^{\rm Ff}} \simeq 1+2\frac{\ln m_z^2/Q_\perp^2}{\ln Q_\perp^2/m^2}$$

Comparison of A_0 with LHC results

Logarithmic estimate of A_0

$$\frac{\mathcal{W}_L^{\rm Ff}}{\mathcal{W}^{\rm Ff}} \simeq 1+2\frac{\ln m_z^2/Q_\perp^2}{\ln Q_\perp^2/m^2} \quad \Rightarrow \quad A_0 = \frac{Q_\perp^2}{m_z^2}\frac{1+2\frac{\ln m_z^2/Q_\perp^2}{\ln Q_\perp^2/m^2}}{1+\frac{Q_\perp^2}{m_z^2}\frac{\ln m_z^2/Q_\perp^2}{\ln Q_\perp^2/m^2}} \qquad (*)$$

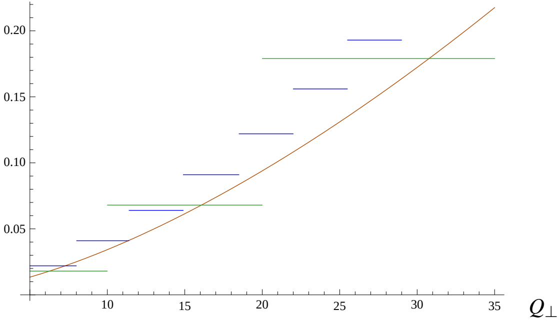


Figure : Comparison of prediction $(*)$ with lines depicting angular coefficient A_0 in bins of $Q_\perp$ and $Y < 1$ from CMS (arXiv:1504.03512) and ATLAS (arXiv1606.00689)

Comparison of A_2 with LHC results

Logarithmic estimate of A_2

$$\frac{\mathcal{W}_L^{\rm Ff}}{\mathcal{W}^{\rm Ff}} \simeq 1+2\frac{\ln m_z^2/Q_\perp^2}{\ln Q_\perp^2/m^2} \quad \Rightarrow \quad A_2 = \frac{Q_\perp^2}{m_z^2}\frac{1}{1+\frac{Q_\perp^2}{m_z^2}\frac{\ln m_z^2/Q_\perp^2}{\ln Q_\perp^2/m^2}} \qquad (**)$$

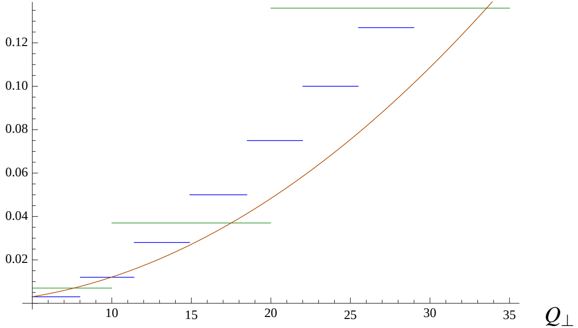


Figure : Comparison of prediction $(**)$ with lines depicting angular coefficient A_2 in bins of $Q_\perp$ and $Y < 1$ from CMS (arXiv:1504.03512) and ATLAS (arXiv1606.00689)

Qualitative checks

$$\mathbb{W} \sim \sum_f r^f \mathcal{W}^{\rm Ff}\Big\{1 + \cos^2\theta + \frac{Q_\perp^2}{2m_Z^2}\frac{\mathcal{W}_L^{\rm Ff}}{\mathcal{W}^{\rm Ff} r^f}(1 - 3\cos^2\theta)$$

$$+ \frac{Q_\perp}{m_Z}\frac{\mathcal{W}_1^{\rm Ff}}{\mathcal{W}^{\rm Ff} r^f}\sin 2\theta\cos\phi + \frac{Q_\perp^2}{2m_Z^2 r^f}\sin^2\theta\cos 2\phi]$$

$$+ \frac{8a_e a_f}{(a_e^2+1)(a_f^2+1)}\Big[\frac{Q_\perp}{m_Z}\frac{\mathcal{W}_3^{\rm Ff}}{\mathcal{W}^{\rm Ff} r^f}\sin\theta\cos\phi + \frac{\mathcal{W}_4^{\rm Ff}}{\mathcal{W}^{\rm Ff} r^f}\cos\theta\Big]\Big\}$$

$$r^f \equiv 1 - \frac{Q_\perp^2}{2m_Z^2} + \frac{Q_\perp^2}{2m_Z^2}\frac{\mathcal{W}_L^{\rm Ff}}{\mathcal{W}^{\rm Ff}}$$

Qualitative checks:

- Factorization of TMD $f_1(x, k_\perp^2) \simeq f(x) f(k_\perp^2) \Rightarrow \mathcal{W}_1^{Ff}(q) = 0$ $\Rightarrow A_1$ is smaller than A_2
- A_4 does not depend on $Q_\perp$ and increases with rapidity
- A_3 is smaller than A_4
- A_5, A_6, A_7 are order of magnitude smaller than A_0, A_2, A_4

1. Conclusions
   - The Drell-Yan hadronic tensor is calculated in the Sudakov region $s \gg Q^2 \gg q_\perp^2$ in the tree approximation with $\frac{1}{Q^2}$ accuracy.
   - In the leading order in N_c the higher-twist quark-quark-gluon TMDs reduce to leading-twist TMDs due to QCD equation of motion.
   - The resulting hadronic tensor for unpolarized hadrons is (EM) gauge-invariant and depends on two leading-twist TMDs: f_1 responsible for total DY cross section, and Boer-Mulders function $h_1^\perp$.
   - Results for angular coefficients of Z-boson production seem to agree with LHC measurements at corresponding kinematics.
2. Outlook
   - Rapidity factorization at the one-loop level.

Thank you for attention!