A Common Origin of Muon g-2, B-Meson Anomalies, and Fermion Mass Hierarchies

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PANIC, September 5th 2021

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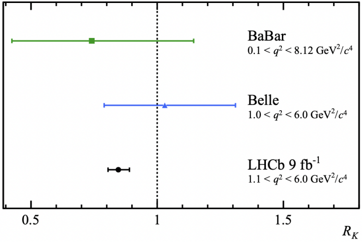
FOR FUNDAMENTAL PHYSICS

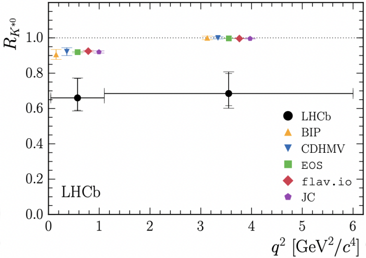
$b \to s\ell^+\ell^-$ anomalies

$$R_{K^{(*)}} = \frac{\mathrm{BR}(B \to K^{(*)}\mu^+\mu^-)}{\mathrm{BR}(B \to K^{(*)}e^+e^-)}$$

- LHCb measurements of $R_K^{[1,6]}$, $R_{K^*}^{[1.1,6]}$, and $R_{K^*}^{[0.045,1.1]}$ deviate from SM by 3.1σ, 2.5σ, and 2.3σ, respectively
- Average ATLAS, CMS, and LHCb $B_s \to \mu^+\mu^-$ branching ratio deviate from SM by 2σ
  Altmannshofer, Stangl [2103.13370]
- Angular observables in $B \to K^*\mu^+\mu^-$ and branching ratios in $B \to K^{(*)}\mu^+\mu^-$ and $B_s \to \phi\mu^+\mu^-$
- Consistent picture emerges in the EFT (primarily a left-handed current): global 3.9σ significance for NP hypothesis
  Lancierini *et al.* [2104.05631]

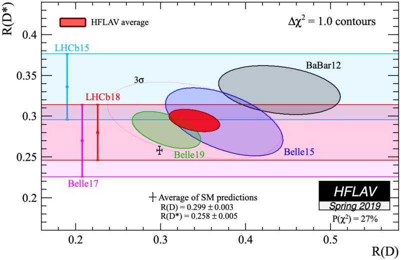


Aaij *et al.* [2103.11769]



Aaij *et al.* [1705.05802]

$b \to c\tau\nu$ anomalies



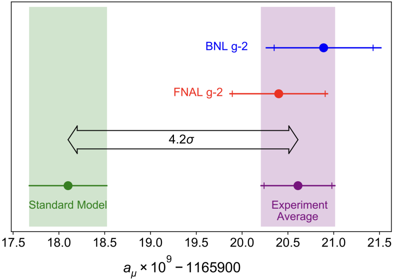
HFLAV [1909.12524]

- $b \to c\ell(\tau)\nu$ occurs a tree-level in the SM. Phase space difference between heavy and light leptons.
- Construct a clean observable: QCD uncertainties largely cancel in the ratio

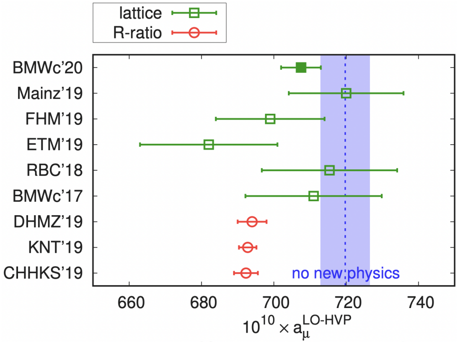
$$R_{D^{(*)}} = \frac{\mathrm{BR}(B \to D^{(*)}\tau\nu)}{\mathrm{BR}(B \to D^{(*)}\ell\nu)}, \quad \ell = e, \mu$$

- The combined BaBar, Belle, and LHCb measurements deviate from SM prediction by 3.1σ.

$(g-2)_\mu$ anomaly



Abi *et al.* [2104.03281]



Borsany *et al.* [2002.12347]

- First measurement of the Fermilab Muon g-2 Experiment is compatible with the Brookhaven experiment. Combined 4.2σ discrepancy with the Muon g-2 Theory Initiative. Aoyama *et al.* [2006.04822]
- HVP is the dominant error of the SM prediction. Lattice results (BMWc) in potential disagreement with the data-driven calculations (R-ratio) used in SM prediction.

Strong constraints from LFV

The lepton mass hierarchy gives generic expectations to the lepton Yukawa:

$$\mathcal{L}_{\mathrm{EFT}} \supset -y_e^{ij}\, \bar{\ell}_{\mathrm{L}}^i H e_{\mathrm{R}}^j, \qquad y_e^{ij} \sim \begin{pmatrix} y_e & \sqrt{y_e y_\mu} & \sqrt{y_e y_\tau} \\ & y_\mu & \sqrt{y_\mu y_\tau} \\ & & y_\tau \end{pmatrix}$$

Flavor basis

Assume a NP explanation of $(g-2)_\mu$ with best fit $C_{\mu\mu}$: e.g. Calibbi et al. [2104.03296]

$$\mathcal{L}_{\mathrm{EFT}} \supset -e\, v\, C_{e\gamma}^{ij}\, \bar{e}_{\mathrm{L}}^i \sigma^{\mu\nu} e_{\mathrm{R}}^j F_{\mu\nu}, \qquad C_{e\gamma}^{ij} \sim \begin{pmatrix} \lesssim 10^{-1} & \lesssim 2\cdot 10^{-5} & \lesssim 1/4 \\ & 1 & \lesssim 1/4 \\ & & \lesssim 2\cdot 10^{5} \end{pmatrix} C_{\mu\mu}$$

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Mass basis

Radiative models of the muon mass: e.g. Baker, Cox, Volkas [2103.13401]

$$\begin{aligned} y_e^{\text{eff}} &= y_e^{(0)} + y_e^{(1)} + \ldots \\ C_{e\gamma}^{\text{eff}} &= 0 + C_{e\gamma}^{(1)} + \ldots \end{aligned}$$

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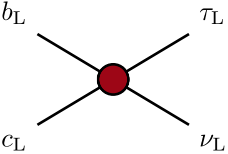
Share spurion structure

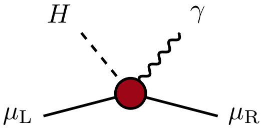
$$C_{e\gamma}^{(1)} \sim \frac{y_e^{(1)}}{\Lambda_{\mathrm{NP}}^2} \implies \Lambda_{\mathrm{NP}} = \text{few TeV}$$

Hints to a combined explanation

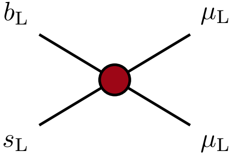
Low-energy effective theory for the anomalies:



$$\frac{1}{(3\,\mathrm{TeV})^2}$$



$$\frac{e}{16\pi^2}\,\frac{1}{(10\,\mathrm{TeV})^2}$$



$$\frac{1}{(40\,\mathrm{TeV})^2}$$

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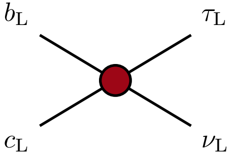
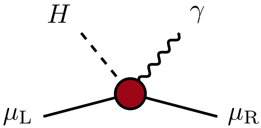


$$\frac{1}{(3\,\mathrm{TeV})^2}$$

$\mathbf{3}_q \to \mathbf{2}_q\,\mathbf{3}_\ell\,\mathbf{3}_\ell$



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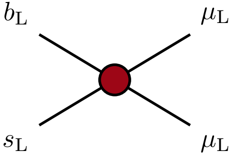

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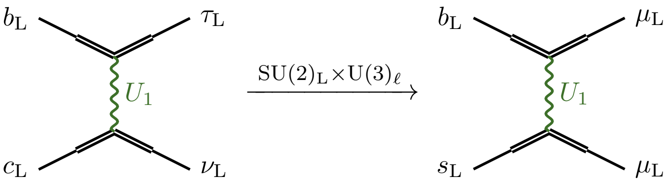
$\mathbf{3}_q \to \mathbf{2}_q\,\mathbf{2}_\ell\,\mathbf{2}_\ell$

- 2^{nd} generation coupling suppressed by ~ 0.1. Related to CKM flavor structure of the SM?
- All three anomalies point to a *common scale* $\Lambda_{\mathrm{NP}} \sim \mathrm{few\ TeV}$

Pati-Salam-like vector LQ for combined b anomaly explanation:

e.g. Di Luzio, Greljo, Nardecchia [1708.08450]; Greljo, Stefanek [1802.04274]; Crivellin, Greub, Saturino [1807.02068]; Di Luzio et al. [1808.00942];...

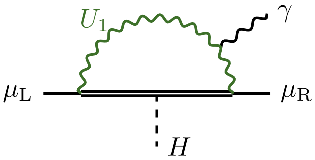


Realized in flavor non-universal 4321 models with vector-like fermions:

$$\underbrace{\overbrace{\mathrm{SU}(4)\times\mathrm{SU}(3)'}^{\mathrm{SU}(3)_c}\times\mathrm{SU}(2)_\mathrm{L}\times\mathrm{U}(1)_X}_{\mathrm{U}(1)_Y} \xrightarrow{\text{SSB}} \mathrm{SU}(3)_c\times\mathrm{SU}(2)_\mathrm{L}\times\mathrm{U}(1)_Y + \{Z', G', U_1\}$$

U_1 problems in $(g-2)_\mu$

With a right-handed lepton current, we could have



Mixing of SM and vector-like fermions

$$\sim y_H (a_\mu^{\rm exp} - a_\mu^{\rm th}) \left(\frac{g_4 \cdot 2\,\mathrm{TeV}}{M_{U_1}}\right)^2 \left(\frac{s_{\ell_2}}{0.2}\right) \left(\frac{s_{e_2}}{0.05}\right)$$

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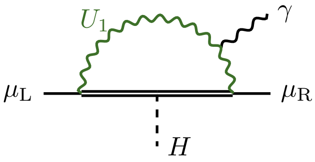

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Tuning is required, and LFV might be an unavoidable problem:

$$\delta y_\mu = y_H s_\ell s_\mu \sim 10 y_\mu^{\mathrm{SM}}, \qquad C_{\mu\tau} \sim C_{\mu\mu} s_{\ell_2}^{-1}$$

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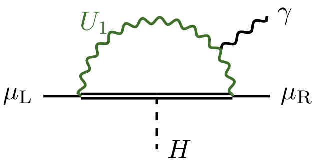

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Incorporate radiative lepton masses in a 4321 model:

A model of all three anomalies with partial explanation of the mass hierarchy

A new universal 4321 model

Fields	SU(4)	SU(3)′	SU(2)$_L$	U(1)$_X$	Z_2	Flavor
q_L^i	$\mathbf{1}$	$\mathbf{3}$	$\mathbf{2}$	$1/6$	$+$	$\mathbf{3}_q$
u_R^i	$\mathbf{1}$	$\mathbf{3}$	$\mathbf{1}$	$2/3$	$+$	$\mathbf{3}_u$
d_R^i	$\mathbf{1}$	$\mathbf{3}$	$\mathbf{1}$	$-1/3$	$-$	$\mathbf{3}_d$
ℓ_L^i	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{2}$	$-1/2$	$+$	$\mathbf{3}_\ell$
e_R^i	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	-1	$-$	$\mathbf{3}_\ell$
$\chi_{L,R}^i$	$\mathbf{4}$	$\mathbf{1}$	$\mathbf{2}$	0	$+$	$\mathbf{3}_\chi$
H	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{2}$	$1/2$	$+$	$\mathbf{1}$
Ω_1	$\mathbf{4}$	$\mathbf{1}$	$\mathbf{1}$	$1/2$	$+$	$\mathbf{1}$
Ω_3	$\mathbf{4}$	$\bar{\mathbf{3}}$	$\mathbf{1}$	$-1/6$	$+$	$\mathbf{1}$
Ω_{15}	$\mathbf{15}$	$\mathbf{1}$	$\mathbf{1}$	0	$+$	$\mathbf{1}$
Π_e	$\mathbf{4}$	$\mathbf{1}$	$\mathbf{2}$	1	$-$	$\mathbf{1}$
Π_d	$\mathbf{4}$	$\bar{\mathbf{3}}$	$\mathbf{2}$	$1/3$	$-$	$\mathbf{1}$

Universal 4321 for b-anomalies

Doublets for down-type masses

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Ω_3	$\mathbf{4}$	$\bar{\mathbf{3}}$	$\mathbf{1}$	$-1/6$	$+$	$\mathbf{1}$
Ω_{15}	$\mathbf{15}$	$\mathbf{1}$	$\mathbf{1}$	0	$+$	$\mathbf{1}$
Π_e	$\mathbf{4}$	$\mathbf{1}$	$\mathbf{2}$	1	$-$	$\mathbf{1}$
Π_d	$\mathbf{4}$	$\bar{\mathbf{3}}$	$\mathbf{2}$	$1/3$	$-$	$\mathbf{1}$

Universal 4321 for b-anomalies

Doublets for down-type masses

Predictive flavor structure

Forbids tree-level down-type masses (softly broken)

Radiative muon mass

Lepton-flavored spurions $\eta_\ell = (\mathbf{3}_\ell,\, \bar{\mathbf{3}}_\chi)$ and $\eta_e = (\mathbf{3}_\chi,\, \bar{\mathbf{3}}_\ell)$ are identified:

$$\mathcal{L}_{\text{yuk}} \supset -\eta_\ell^{ij} \overline{\ell}_{\mathrm{L}}^{i} \Omega_1 \chi_{\mathrm{R}}^{j} - \eta_e^{ij} \overline{\chi}_{\mathrm{L}}^{i} \Pi_e e_{\mathrm{R}}^{j} \,+\, \text{h.c.}\,, \qquad \eta_e \sim \eta_\ell^\dagger$$

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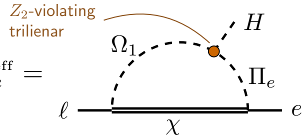
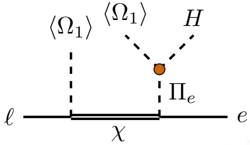
Approximate Z_2 symmetry forbids lepton-Higgs coupling and ensures alignment of the Yukawa contributions

$y_e^{\text{eff}} =$



$+$



$+ \ldots \;\sim\; \eta_\ell\, \eta_\ell^\dagger$

Lepton–Higgs Yukawas are suppressed: $y_{\tau(b)} \sim y_t/100$

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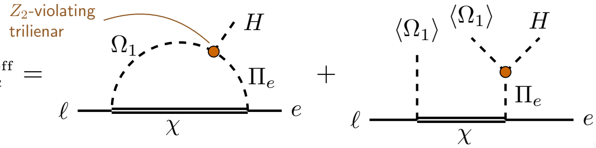
$y_e^{\text{eff}} =$  $+ \dots \;\sim\; \eta_\ell\, \eta_\ell^\dagger$

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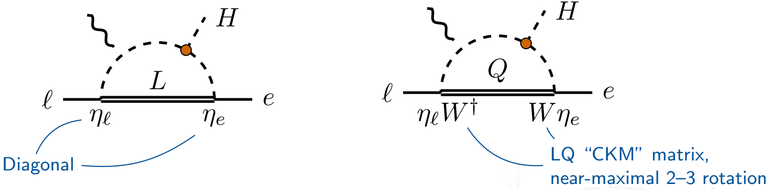
Approximate prediction of compositeness of the leptons from lepton masses:

$$t_{\ell_i} = \frac{\eta_\ell^i \langle\Omega_1\rangle}{M_L} \implies \frac{t_{\ell_2}^2}{t_{\ell_3}^2} \sim \frac{m_\mu}{m_\tau},$$

in excellent agreement with $(s_{\ell_3},\, s_{\ell_2}) \simeq (0.8,\, 0.3)$ required for b-anomalies.

Lepton dipole

- Size of the dipole fits well with Δa_μ given the radiative lepton Yukawa.
- LFV stems from the colored components of $\chi = (W\, Q,\, L)$:



- A GIM mechanism in the flavor-violating loops further suppresses LFV at the order of magnitude level.
- Likely in Belle-II discovery range for $\tau \to \mu\gamma$ (preliminary).

A unified model

This extension lends itself well to embedding in a larger symmetry,

e.g. Fuentes-Marín, Stangl [2004.11376]

$$\mathrm{SU}(4) \times \mathrm{SU}(3)' \times \mathrm{SU}(2)_\mathrm{L} \times \mathrm{U}(1)_X \subseteq \mathrm{SU}(4) \times \mathrm{SU}(4)' \times \mathrm{SU}(2)_\mathrm{L} \times \mathrm{SU}(2)_\mathrm{R},$$

with field content

Fields	$\mathrm{SU}(4)$	$\mathrm{SU}(4)'$	$\mathrm{SU}(2)_\mathrm{L}$	$\mathrm{SU}(2)_\mathrm{R}$
ψ_L^i	$\mathbf{1}$	$\mathbf{4}$	$\mathbf{2}$	$\mathbf{1}$
ψ_R^i	$\mathbf{1}$	$\mathbf{4}$	$\mathbf{1}$	$\mathbf{2}$
$\chi_{\mathrm{L,R}}^i$	$\mathbf{4}$	$\mathbf{1}$	$\mathbf{2}$	$\mathbf{1}$
$\mathcal{H}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{2}$	$\bar{\mathbf{2}}$
Ω	$\mathbf{4}$	$\bar{\mathbf{4}}$	$\mathbf{1}$	$\mathbf{1}$
Ω_{15}	$\mathbf{15}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$
Π	$\mathbf{4}$	$\bar{\mathbf{4}}$	$\mathbf{2}$	$\bar{\mathbf{2}}$

Conclusions and outlook

- $b \to s\ell\ell$, $b \to c\tau\nu$, and $(g-2)_\mu$ anomalies all point to the scale of a few TeV, once accounting for a CKM-like flavor structure.
- The b-anomalies are explained nicely with the vector LQ of 4321 models.
- The $(g-2)_\mu$ can be incorporated and LFV suppressed when structure is added to realize radiative lepton masses $\to$ new class of 4321 models.
- This structure can also accounts for the SM mass hierarchies, $y_{\tau(b)} \sim y_t/100$.

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Thank you for listening