Four-quark operators and SU(3): from $K \to \pi\pi$ to $\emptyset \to \emptyset$ transitions

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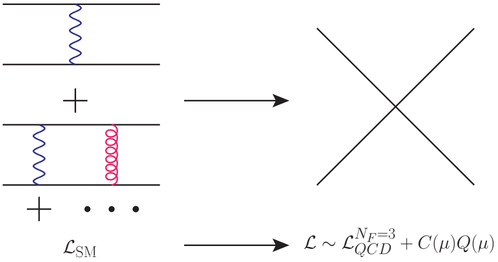
Four-quark operators in $K \to \pi\pi$

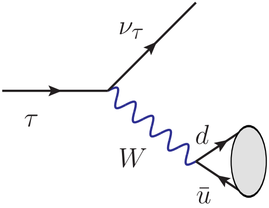
- K, π emerge from $\mathcal{L}_{\rm QCD}^{\rm N_F=3}$
- Rest of forces: small perturbations of $\mathcal{L} \sim \mathcal{L}_{\rm QCD}^{\rm N_F=3}$
- $\Delta S = 1$ process? $\mathcal{L}_{\rm EW}$: from $\mu \sim M_W$ to $\mu \lesssim m_c$

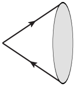


$$Q_1 = 4\,(\bar{s}_L^\alpha \gamma^\mu u_L^\beta)\,(\bar{u}_L^\beta \gamma_\mu d_L^\alpha), \ldots,\ Q_8 = 6\,(\bar{s}_L^\alpha \gamma^\mu d_L^\beta)\, e_q\,(\bar{q}_R^\beta \gamma_\mu q_R^\alpha) \ldots$$

Rev.Mod.Phys. 68 (1996) 1125-1144

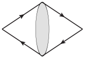
Four-quark operators in $\tau \to \nu_\tau + \mathrm{hadrons}$



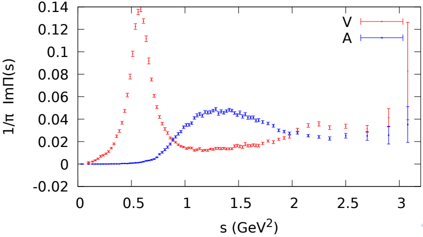


- Low energies: χpT
- But $m_\tau \sim 1.8\,\mathrm{GeV}$

$$\frac{d\Gamma}{ds} \sim \sum |M|^2 \sim L \cdot H$$

$\sum H \sim \sum$  $\sim \mathrm{Im}\Pi$

$$\Pi \sim \int d^4q\, e^{-iqx} \langle T(J(x)J(0)) \rangle$$



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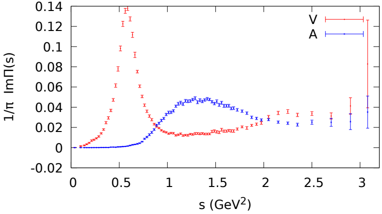
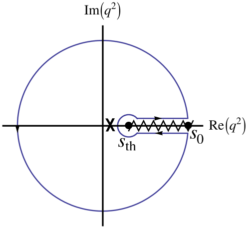
Four-quark operators in $\tau \to \nu_\tau + \text{hadrons}$



- $\Pi \sim \int d^4q\, e^{-iqx} \langle T(J(x)J(0)) \rangle$
- $Q^2 \gg \Lambda_{QCD} \to \Pi \sim \Pi^{OPE} = \frac{C_D \langle \mathcal{O}_D \rangle}{Q^d}$
- $\mathcal{O}_6 \to$ Four-quark operators



$$\underbrace{\int_{s_{\rm th}}^{s_0} \frac{ds}{s_0}\, \omega(s)\, \frac{1}{\pi}\, \mathrm{Im}\, \Pi(s)}_{\text{data}} + \underbrace{\frac{1}{2\pi i} \oint_{|s|=s_0} \frac{ds}{s_0}\, \omega(s)\, \Pi(s)}_{\sim \text{OPE}} = 2\frac{F_\pi^2}{s_0}\omega(M_\pi^2)$$

$V - A, \omega = 1, s;\ s_0 \to \infty, m_q = 0$: Weinberg Sum Rules

Low-energy realization of QCD

$$\mathcal{L}^0_{\mathrm{QCD}} = -\frac{1}{4} G^a_{\mu\nu} G^{\mu\nu}_a + i\,\bar{q}_L \gamma^\mu D_\mu q_L + i\,\bar{q}_R \gamma^\mu D_\mu q_R, \quad q = (u\ d\ s)^T$$

- Invariant under $q_{L(R)} \to L(R) q_{L(R)}$; $L, R \in SU(3)$ matrices
- $SU(3)_L \times SU(3)_R \to SU(3)_V$
- $\mathcal{L}^0_{\mathrm{eff}}$? $U \to R U L^\dagger$. $U(\pi, K, \eta)$. Counting in derivatives

Add perturbations: external sources

- $\mathcal{L} = \mathcal{L}^0_{QCD} - \bar{q}_R \hat{M} q_L - \bar{q}_L \hat{M}^\dagger q_R + \ldots$ Eventually $\hat{M} = \mathrm{diag}(m_u, m_d, m_s)$
- Invariant if $\hat{M} \to R \hat{M} L^\dagger$.
- $\mathcal{L}_{\mathrm{eff}} = \frac{F^2}{4} \mathrm{Tr}(D_\mu U^\dagger D^\mu U + U^\dagger \chi + \chi^\dagger U) + \mathcal{O}(p^4)\,.\,\chi = 2 B_0 \hat{M}$
- $B_0(\mathcal{L}^0_{QCD})$ is not fixed by symmetries alone

Low-energy realization of four-quark operators

External sources for four-quark operators

$$\mathcal{L} \sim [t_L]^{jl}_{ik}\,(\bar{q}_L{}^i \gamma^\mu q_{Lj})\,(\bar{q}_L{}^k \gamma_\mu\, q_{Ll});\ [t_L]^{jl}_{ik} \to L^{\dagger}_{j'}{}^{j} L^{\dagger}_{l'}{}^{l}\, t^{j'l'}_{i'k'}\, L_i{}^{i'} L_k{}^{k'}$$

- Low-energy theory ($U \to RUL^\dagger$, $\chi \to R\chi L^\dagger$). Possible terms?

$$\text{e.g. } a\, L^{\mu ij} L_{\mu kl}\, t^{kl}_{ijL}, \text{ with } L_\mu \equiv i\, U^\dagger D_\mu U$$

- Full benefit of the symmetries: decompose into irreps

$$\mathcal{A} = \mathcal{A}^{aM}_{rN}\ e^{aM}_{rN}\ ;\ \mathcal{A}^{aM}_{rN} = A^{ij}_{kl}\ \left[e^{aM}_{rN}\right]^{kl}_{ij}$$

$$\text{e.g. } \left[e^{aS}_{8S}\right]^{jl}_{ik} = \frac{1}{\sqrt{40}}\left(\lambda^{a,j}_i \delta^l_k + \lambda^{a,l}_i \delta^j_k + \lambda^{a,j}_k \delta^l_i + \lambda^{a,l}_k \delta^j_i\right)$$

$$\mathcal{L}^S_8 = [t_L]^{jl}_{ik} \cdot [\bar{q}q\bar{q}q]^{ikS}_{8jl}$$

$$\mathcal{L}^S_8 = a^S_8\, \frac{F^4}{80} \left\{ [t_L]^{jl}_{ik}\, \mathrm{Tr}(\lambda^a L_\mu L^\mu) + [t_R]^{jl}_{ik}\, \mathrm{Tr}(\lambda^a R_\mu R^\mu) \right\} (\lambda^i_{a,j}\delta^k_l + \lambda^i_{a,l}\delta^k_j + \lambda^k_{a,j}\delta^i_l + \lambda^k_{a,l}\delta^i_j) + \mathcal{O}(p^4)$$

Low-energy realization of $K \to \pi\pi$

$$\mathcal{L}^{\Delta S=1} = G \sum_{i=1}^{10} C_i(\mu)\, Q_i(\mu) \text{ Rev.Mod.Phys. 68 (1996) 1125-1144}$$
$$Q_1 = 4\,(\bar{s}_L^{\alpha}\gamma^{\mu} u_L^{\beta})\,(\bar{u}_L^{\beta}\gamma_{\mu} d_L^{\alpha}),\ldots,\; Q_8 = 6\,(\bar{s}_L^{\alpha}\gamma^{\mu} d_L^{\beta})\, e_q\,(\bar{q}_R^{\beta}\gamma_{\mu} q_R^{\alpha})\ldots\,.$$

Particularize t to match the short-distance lagrangian and

$$\mathcal{L}_{\rm eff}^{\Delta S=1} = G F^4 \left\{ g_{27}\left(L_2^{\mu 3} L_{\mu 1}^{1} + \tfrac{2}{3}\, L_2^{\mu 1} L_{\mu 1}^{3} \right) + g_8\, \mathrm{Tr}(\lambda L^{\mu} L_{\mu}) + e^2 g_8\, g_{\rm ewk}\, F^2\, \mathrm{Tr}(\lambda U^{\dagger} Q U) \right\}$$

Recover well-known chiral lagrangians with

$$\begin{aligned}
g_{27} &= \frac{3}{5}\, a_{27}(\mu) \left(C_1 + C_2 + \frac{3}{2}\, C_9 + \frac{3}{2}\, C_{10} \right)(\mu)\,, \\
g_8 &= \frac{1}{10}\, a_8^S(\mu)\, (C_1 + C_2 + 5\, C_3 + 5\, C_4 - C_9 - C_{10})(\mu) \\
&\quad -\frac{1}{2}\, a_8^A(\mu)\, (C_1 - C_2 + C_3 - C_4 + C_9 - C_{10})(\mu) \\
&\quad +4\, a_{LR}^{\delta\delta}(\mu) \left(C_5 + \frac{C_6}{N_c} \right)(\mu) + 8\, a_{LR}^{\lambda\lambda}(\mu)\, C_6(\mu)\,, \\
e^2 g_8\, g_{\rm ewk} &= 6 \left\{ a_{88}^{\delta\delta}(\mu) \left(C_7 + \frac{C_8}{N_c} \right)(\mu) + 2\, a_{88}^{\lambda\lambda}(\mu)\, C_8(\mu) \right\}\,.
\end{aligned}$$

NLO? Loops contributions fully fixed by EFT. Counterterms estimated using large-N_c

Low-energy realization of vacuum condensates

$$\Pi^{\rm OPE}(q^2) = \sum_{i,D} \frac{C_{i,D}(q^2,\mu)\,\langle \mathcal{O}_{i,D}(\mu)\rangle}{(-q^2)^{D/2}}$$

$$\mathcal{O}_{6V-A} = [t^{\lambda\lambda}_{LR}]^{jl}_{ik}\,(\bar{q}^i_L \gamma^\mu T^a q_{Lj})\,(\bar{q}^k_R \gamma_\mu T^a q_{Rl}) + \cdots\,;\,[t^{\lambda\lambda}_{LR}]^{jl}_{ik} \approx 8\pi\alpha_s\,(\lambda^{1,j}_{L,i}\lambda^{1,l}_{R,k} + \lambda^{2,j}_{L,i}\lambda^{2,l}_{R,k})$$

$$\langle \mathcal{O}^d_{6,V-A}(\mu)\rangle = 32\pi\alpha_s(\mu)\,F^6 a^{\lambda\lambda}_{88}(\mu) + \mathcal{O}(p^2,\alpha_s^2)$$

Relation with kaons equivalent to Phys.Lett.B 478 (2000) 172-184

- NLO in α_s also known Phys.Lett.B 522 (2001) 245-256
- NLO in the chiral counting:
  1. $\sim m_q \ln m_q$. Only relevant piece at $m_q \ll$. Fully fixed by EFT
  2. $\sim m_q$. Counterterms: estimated using large-N_c

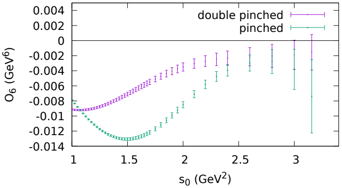
$$\langle \mathcal{O}^d_{6,V-A}(\mu)\rangle = 32\pi\alpha_s(\mu)\,F^4_\pi \left\{ F^2 a^{\lambda\lambda}_{88}(\mu)\left[1 + \frac{\alpha_s(\mu)}{2\pi}A_8 + \frac{\alpha_s(\mu)}{2\pi}B_8 \log\left(\frac{-q^2}{\mu^2}\right)\right] + F^2 a^{\delta\delta}_{88}(\mu)\,\frac{\alpha_s(\mu)}{8\pi}\left[A_1 + B_1 \log\left(\frac{-q^2}{\mu^2}\right)\right]\right\}\left\{1 - \frac{16 M^2_\pi}{F^2_\pi}\left(L_5 - \frac{8}{3}L_8\right)\right\}$$

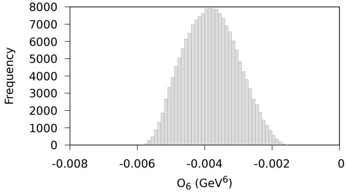
Determination of $a_8^{\lambda\lambda}$ from τ data

$$\langle \mathcal{O}^d_{6,V-A}(\mu)\rangle \approx 32\pi\alpha_s(\mu)\, F_\pi^4 F^2 a_{88}^{\lambda\lambda}(\mu) \left[1 + \frac{\alpha_s(\mu)}{2\pi} A_8 + \frac{\alpha_s(\mu)}{2\pi} B_8 \log\left(\frac{-q^2}{\mu^2}\right)\right] \cdot \left\{1 - \frac{16 M_\pi^2}{F_\pi^2}\left(L_5 - \frac{8}{3} L_8\right)\right\}.$$

$$\underbrace{\int_{s_{\rm th}}^{s_0} \frac{ds}{s_0}\, \omega(s)\, \frac{1}{\pi}\, \mathrm{Im}\, \Pi(s)}_{\rm data} + \underbrace{\frac{1}{2\pi i} \oint_{|s|=s_0} \frac{ds}{s_0}\, \omega(s)\, \Pi(s)}_{\sim{\rm OPE}} = 2\frac{F_\pi^2}{s_0}\omega(M_\pi^2)$$

$\Pi^{OPE} \to \langle \mathcal{O}^d_{6,V-A}(\mu)\rangle$





$$F^2\, a_{88}^{\lambda\lambda}(m_\tau) = (-1.15 \pm 0.30_{\,\langle\mathcal{O}_6\rangle} \pm 0.11_{\,\rm pert})\, \mathrm{GeV}^2$$

Consequence for $(\varepsilon'/\varepsilon)_{\mathrm{EWP}}$

$$(\varepsilon'/\varepsilon)^{(I=2)}_{\mathrm{EWP}} = \frac{1}{\sqrt{2}|\varepsilon|}\,\frac{\mathrm{Im}\,(A_2)^{\mathrm{EWP}}}{\mathrm{Re}\,(A_0)}$$
$$\mathrm{Im}\,(A_2)^{\mathrm{EWP}}_{Q_i} = G\;\mathrm{Im}\,[C_i(\mu)]\,\langle Q_i(\mu)\rangle_2$$

At $\sim 1\,\mathrm{GeV}$ dominated by Q_8:

$$\langle Q_8(\mu)\rangle_2 \approx -4F_\pi\,\Theta_{\delta_2}\left[2\,F^2 a^{\lambda\lambda}_{88}(\mu)\right]\left(1+\Delta^L_R A^{(g)}_{3/2}+\Delta^C_{Q_8,2}\right)$$

Using $F^2\,a^{\lambda\lambda}_{88}$ from τ data:

$$(\varepsilon'/\varepsilon)^{(2)}_{\mathrm{EWP}} = \left(-4.5\pm 1.5_{\,a^{\lambda\lambda}_{88}}\pm 0.9_{\Delta_L}\pm 0.5_{\Delta_C}\right)\cdot 10^{-4} = (-4.5\pm 1.8)\cdot 10^{-4}$$

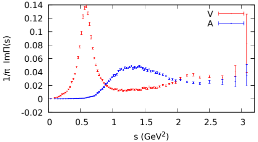
Fit to lattice data

$\mu_0 = 1$ GeV Fit to Phys.Rev.D 102 (2020) 5, 054509

	$a_{27}(\mu_0)$	$a_8^S(\mu_0)$	$a_8^A(\mu_0)$	$a_{LR}^{\delta\delta}(\mu_0)$	$a_{LR}^{\lambda\lambda}(\mu_0)$	$F^2 a_{88}^{\delta\delta}(\mu_0)$	$F^2 a_{88}^{\lambda\lambda}(\mu_0)$
Lattice	0.64 (10)	-0.2 (2.4)	2.7 (5)	-0.48 (41)	-1.17 (26)	-0.22 (12)	-0.68 (11) GeV^2
K, τ data	0.622 (43)						-0.78 (22) GeV^2
Large N_C	1	1	1	0	-1.06	0	-0.70 GeV^2

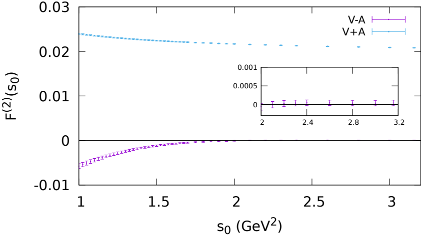
- Large-N_c (at $\mu \sim \mu_0$) provides the correct hierarchy
- Failure of large-N_c in explaining $\Delta I = 1/2$ rule: dominated by underestimation of a_8^A
- Complemented by overestimation of a_{27}. Expected from fixing the coupling from determinations of $\lim_{m_q \to 0} B_K$ (e.g. JHEP 03 (2006) 048)
- Large-N_c works better for couplings with nonzero anomalous dimension at $N_c \to \infty$, confirming reliability of $(\varepsilon'/\varepsilon)_{\rm chpt}$

Lattice fit to improve predictive power in τ



$$F^{(2)}_{V\pm A}(s_0) \equiv \int_{s_{\rm th}}^{s_0} \frac{ds}{s_0}\left(1-\frac{s}{s_0}\right)^2 \frac{1}{\pi}\,{\rm Im}\,\Pi_{V\pm A}(s) \pm 2\,\frac{F_\pi^2}{s_0}\left(1-\frac{M_\pi^2}{s_0}\right)^2 - \frac{\langle \mathcal{O}^d_{6,V\pm A}(s_0)\rangle'}{s_0^3}$$

- $\langle \mathcal{O}^d_{6,V-A}(s_0)\rangle'$ determined from lattice fit
- Other non-perturbative effects completely negligible at $s_0 \sim m_\tau^2$
- Strong prediction: at the current experimental precision ($\lesssim 1\%$): $F^{(2)}_{V-A}(s_0 \sim m_\tau^2) = 0$



$\sqrt{2}F_\pi = (130.9 \pm 0.8)\,\mathrm{MeV}$

Conclusions

- EFT knowledge of low-energy QCD $\rightarrow$ rigorous relations between very different observables involving four-quark operators

- Comprehensive analysis on how to find and use them to
  1. Extract $(\varepsilon'/\varepsilon)^{(2)}_{\mathrm{EWP}} = (-4.5 \pm 1.8) \cdot 10^{-4}$
  2. Assess, combining with lattice data, the accuracy of large-N_c estimates of different matrix elements
  3. Improve the predictive power in the τ sector