High energy $\pi\eta^{(\prime)}$ production and the double Regge exchange

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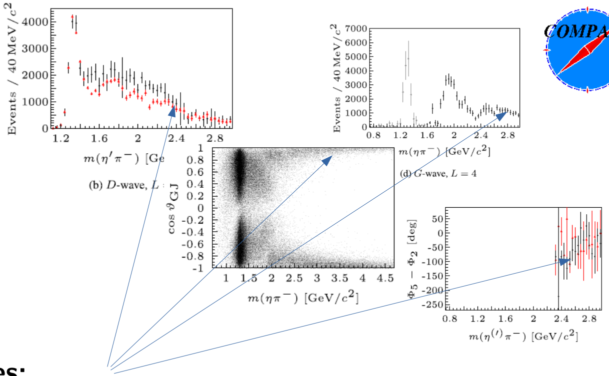
Motivation of the $\pi^-\eta^{(\prime)}$ channel study

- $\pi^-\eta^{(\prime)}$ pairs constitute a golden channel for the searches of hybrid exotics (odd partial waves are exotic)
- COMPASS experiment at CERN (Adolph et al. PLB 740, 2015) observed a strong forward-backward asymmetry in the $\pi^-\eta^{(\prime)}$ channels (stronger in the $\pi\eta'$ channel)
- the asymmetry is due to odd-even partial wave interference
- the strongest odd wave is the $P-$wave, which in the resonance region can be attributed to the π_1 hybrid
- relation between the high and low invariant mass region can be described in terms of Finite Energy Sum Rules (special kind of dispersion relations)

Experimental motivation



COMPASS

Objectives:

- Describe the $\pi^- \eta^{(\prime)}$ production above the resonance region.
- Identify dominant Regge exchanges and amplitude strengths.

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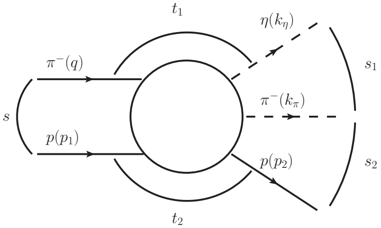
Kinematics

- The $2 \to 3$ reaction (upolarized) is determined by 5 kinematical variables
- In Regge analysis it is customary to choose 5 Lorentz invariants, eg: $s = (q + p_1)^2$, $t_1 = (q - k_\eta)^2$, $t_2 = (p_1 - p_2)^2$, $s_1 (k_\eta + k_\pi)^2$, $s_2 = (k_\pi + p_2)^2$.



- COMPASS analysis was performed in the Gottfried-Jackson frame, with θ and ϕ defining the direction of outgoing $\eta^{(\prime)}$
- In this frame $\cos\theta$ is related to t_1 and ϕ is related to s_2 (the other variables being s, s_1 and t_2).

Model vs. experimental data

- The COMPASS experiment operated with a fixed beam momentum of 191 GeV and t_2 was integrated in the region $t_2 \in [-1.0, -0.1]$ GeV^2.
- The invariant mass ($m = \sqrt{s_1}$) and angular dependent intensity was parametrized as:

$$I(m,\Omega)_{COMPASS} = \sum_{\epsilon=\pm 1} \left| \sum_{L,M} f^{\epsilon}_{LM}(m) \Psi^{\epsilon}_{LM}(\Omega) \right|^2 ,$$

where ϵ is the reflectivity ($\approx$ naturality in high energy limit), kept ϵ=+1 and

$$\Psi^{\epsilon=+1}_{LM}(\theta,\phi) = \sqrt{2} Y_{LM}(\theta,0) \sin(M\phi)$$
$$f^{\epsilon=+1}_{LM}(m) = \sqrt{I_{LM}}\, e^{i\phi_{LM}}$$

where the I_{LM} and ϕ_{LM} are the COMPASS experimental partial wave intensities and phaseshifts (determined relative to ϕ_{21}).

Fitting experimental data at large invariant masses

- The easiest approach: fit the model partial waves to experimental ones

But:

- COMPASS analysis was based on partial wave expansion truncated at $L = 6$ and $M = 1$, even though it converges slowly for the $\pi\eta^{(\prime)}$ invariant masses above the resonance region,
- The intensity was normalized to the number of events in the mass bin.

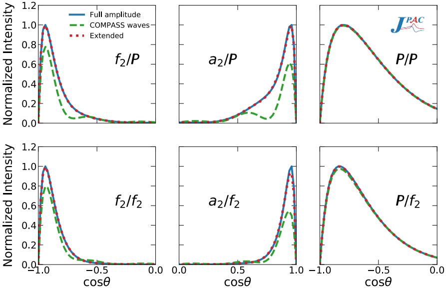
Therefore:

- The angular distribution constructed from the truncated partial wave expansion represents total yield.

Fitting experimental data at large invariant masses

On the model side

- Model intensities for full amplitudes and partial wave amplitudes truncated at $L = 6$ differ considerably.



Recipe:

- Fit the mass and angular dependent intensity.

Fitting experimental data at large invariant masses

- Practically we fit the extended log likelihood (Barlow, Nucl.Instr.Meth. A 297, 1990)

$$\mathcal{L}_{\text{ext}} = \sum_{m_i} \int d\Omega \left[I_{JPAC}(m_i, \Omega) - I_{COMPASS}(m_i, \Omega) \log I_{JPAC}(m_i, \Omega) \right]$$

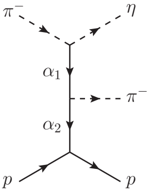
with model intensity defined as:

$$I_{JPAC}(m, z = \cos\theta, \phi) = q|T|^2 = q\,|\sum_{i=1}^{6} a_i T^i(m_{\pi\eta}, z, \phi)|^2$$

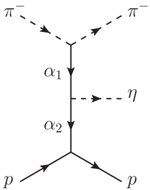
where $q = \lambda^{1/2}(m^2, m_\pi^2, m_\eta^2)/m$.

Double Regge exchange model

- Leading natural exchanges



Type I - fast η



Type II - fast π

Type of diagram	α_1	α_2
I (fast η)	α_{a_2}	$\alpha_{\boldsymbol{P}}$
	α_{a_2}	α_{f_2}
II (fast π)	α_{f_2}	$\alpha_{\boldsymbol{P}}$
	α_{f_2}	α_{f_2}
	$\alpha_{\boldsymbol{P}}$	$\alpha_{\boldsymbol{P}}$
	$\alpha_{\boldsymbol{P}}$	α_{f_2}

- Individual diagram strengths depend on the 6 coupling triples: $G_{a_2\pi\eta} G_{a_2\pi\boldsymbol{P}} G_{\boldsymbol{P}NN}$, $G_{a_2\pi\eta} G_{a_2\pi f_2} G_{f_2NN}$, $G_{f_2\pi\pi} G_{f_2\eta\boldsymbol{P}} G_{\boldsymbol{P}NN}$, $G_{f_2\pi\pi} G_{f_2\eta f_2} G_{f_2NN}$, $G_{\boldsymbol{P}\pi\pi} G_{\eta\boldsymbol{P}\boldsymbol{P}} G_{\boldsymbol{P}NN}$, $G_{\boldsymbol{P}\pi\pi} G_{f_2\eta\boldsymbol{P}} G_{f_2NN}$, wherein at least one coupling is unknown.
- Therefore we treat diagram strengths as parameters to be fitted.

General form of the amplitude

- Double reggeon exchange (we follow Shimada *et al.* NPB 142, 1978)

$$T = -K\Gamma(1-\alpha_1)\Gamma(1-\alpha_2)$$
$$\left[(\alpha' s)^{\alpha_1 - 1}(\alpha' s_2)^{\alpha_2-\alpha_1}\xi_1\xi_{21}\hat{V}_1 + (\alpha' s)^{\alpha_2-1}(\alpha' s_1)^{\alpha_1-\alpha_2}\xi_2\xi_{12}\hat{V}_2\right]$$

$\xi_i = \frac{1}{2}(\tau_i + e^{-i\pi\alpha_i})$ and $\xi_{ij} = \frac{1}{2}(\tau_i\tau_j + e^{-i\pi(\alpha_i-\alpha_j)})$ are signature factors.

$$\hat{V}_1(\alpha_1,\alpha_2,\eta) = \frac{\Gamma(\alpha_1-\alpha_2)}{\Gamma(1-\alpha_2)}\,{}_1F_1\left(1-\alpha_1, 1-\alpha_1+\alpha_2, -\kappa\right)$$
$$\hat{V}_2 = \hat{V}_1(\alpha_1 \leftrightarrow \alpha_2) \quad \text{with} \quad \kappa^{-1} \equiv s/(\alpha' s_1 s_2)$$

and $K = -4\sqrt{s_1}|q||k_\eta||p_2|\sin\theta_2\sin\theta\sin\phi$

- Both α_1 and α_2 are of 2^{++} type so only positive signature and naturality.
- Regge trajectories:

$$\alpha_{f_2}(t) = 0.47 + 0.89t \quad \alpha_{a_2}(t) = 0.53 + 0.90t \quad \alpha_{\mathbb{P}}(t) = 1.08 + 0.25t$$

Fitting procedure

1. Full amplitude (a_i to be fitted):

$$T(s, s_1, t_2, \cos\theta, \phi; a_i) = K \sum_{i \in I, II} a_i \tilde{T}_i(\alpha_1(t_1), \alpha_2(t_2); s, s_1, t_2, \cos\theta, \phi)$$

2. Model intensity:

$$I_{JPAC}(m, z = \cos\theta, \phi) = q|T|^2 = q\, |\sum_{i=1}^{6} a_i T^i(m_{\pi\eta}, z, \phi)|^2$$

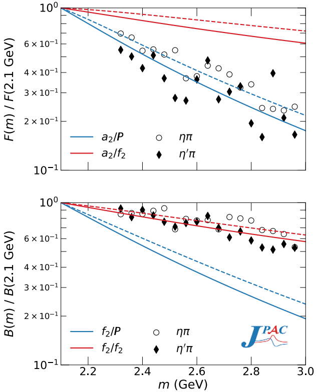
3. Extended log likelihood fit:

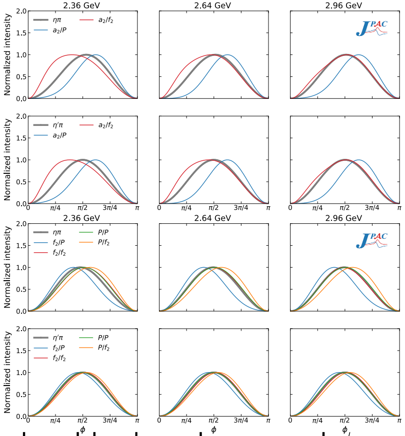
$$\mathcal{L}_{\text{ext}} = \sum_{m_i} \int d\Omega \left[I_{JPAC}(m_i, \Omega) - I_{COMPASS}(m_i, \Omega) \log I_{JPAC}(m_i, \Omega) \right]$$

Fit results

Minimal set of amplitudes

Fitting the full set of 6 amplitudes is statistically inconclusive.





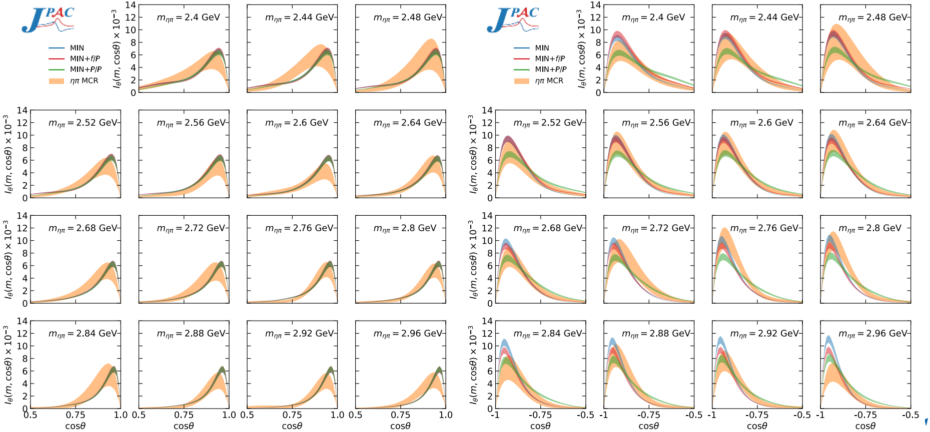
From the analysis of the forward and backward mass and ϕ distributions we infer that the minimal set of amplitudes should include $a_2/\boldsymbol{P}$, a_2/f_2 and f_2/f_2.

Parameter values from the fits

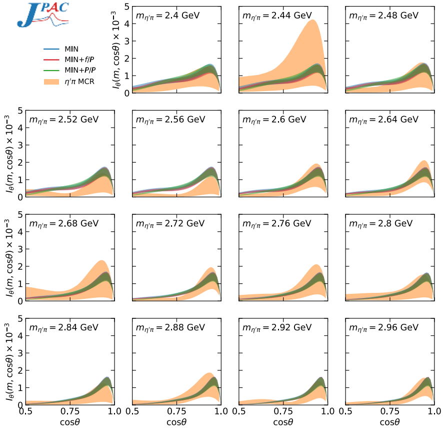
- To make the parameter evaluation stable we can fit at most 4 amplitudes.
- We exclude $\boldsymbol{P}/f_2$ from all fits as it may disrupt the ϕ distribution.

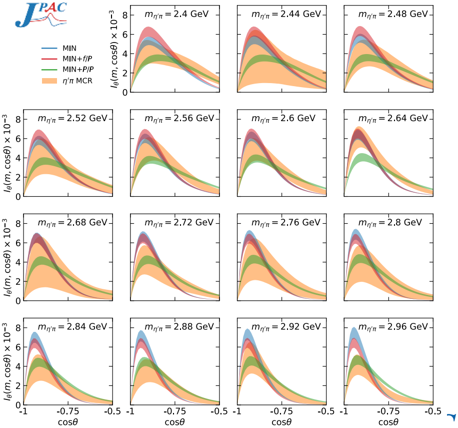
Channel		MIN		MIN+$f/\boldsymbol{P}$		MIN+$\boldsymbol{P}/\boldsymbol{P}$	
		MVR	MCR	MVR	MCR	MVR	MCR
$\eta\pi$	$\mathcal{L}\times 10^{-4}$	-22.8	-21.9 ± 0.9	-22.7	-22.0 ± 0.9	-22.8	-22.1 ± 0.8
	$c_{a_2 \boldsymbol{P}}$	0.29	0.42 ± 0.03	0.28	0.40 ± 0.04	0.29	0.36 ± 0.04
	$c_{a_2 f_2}$	3.67	3.3 ± 0.4	3.70	3.4 ± 0.4	3.59	3.8 ± 0.4
	$c_{f_2 \boldsymbol{P}}$	—	—	-0.20	-0.30 ± 0.05	—	—
	$c_{f_2 f_2}$	-11.82	-11.0 ± 0.3	-8.99	-6.6 ± 0.7	-10.86	-8.9 ± 0.4
	$c_{\boldsymbol{PP}}$	—	—	—	—	0.0073	0.0135 ± 0.002
$\eta'\pi$	$\mathcal{L}\times 10^{-4}$	-11.7	-10.9 ± 1.0	-11.7	-11.0 ± 1.0	-11.8	-11.4 ± 1.0
	$c_{a_2 \boldsymbol{P}}$	0.16	0.37 ± 0.07	0.16	0.34 ± 0.05	0.19	0.35 ± 0.05
	$c_{a_2 f_2}$	1.50	0.4 ± 0.6	1.51	0.7 ± 0.5	1.22	0.6 ± 0.5
	$c_{f_2 \boldsymbol{P}}$	—	—	-0.21	-0.29 ± 0.03	—	—
	$c_{f_2 f_2}$	-11.42	-11.0 ± 0.5	-7.73	-5.5 ± 0.7	-9.01	-7.1 ± 0.6
	$c_{\boldsymbol{PP}}$	—	—	—	—	0.012	0.018 ± 0.002

Forward and backward peaks for $\pi\eta$



Forward and backward peaks for $\pi\eta'$



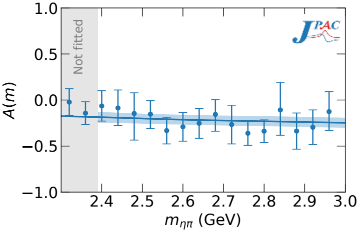


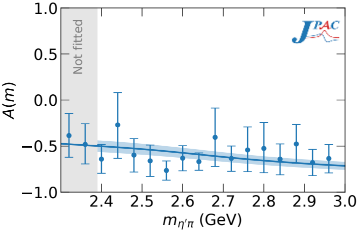
Forward-backward asymmetry

Define the forward-backward asymmetry as:

$$A(m) \equiv \frac{F(m) - B(m)}{F(m) + B(m)},$$

with $$F(m) \equiv \int_0^1 \mathrm{d}\cos\theta \, I_\theta(m, \cos\theta), \quad B(m) \equiv \int_{-1}^0 \mathrm{d}\cos\theta \, I_\theta(m, \cos\theta)$$





Summary

- We found that $a_2/\boldsymbol{P}$, a_2/f_2 , f_2/f_2 and either $f_2\boldsymbol{P}$ or $\boldsymbol{P}/\boldsymbol{P}$ (the data do not show clear preference for either exchange) are required to describe $\pi\eta$ intensity.
- To describe $\pi\eta'$ intensity the $a_2/\boldsymbol{P}$, a_2/f_2 , f_2/f_2 and $\boldsymbol{P}/\boldsymbol{P}$ exchanges are necessary. Glue rich exchange points toward π_1 hybrid production.
- Quite surprisingly, lower f_2 exchange is necessary to describe COMPASS data,

Thank you for your attention