Covalent hadronic molecules from QCD sum rules

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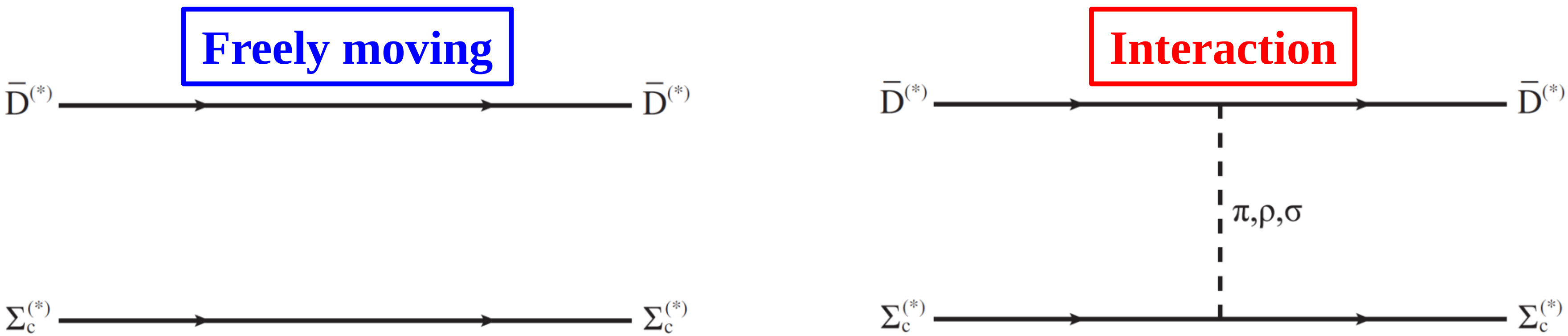
Contents

- Interactions at the quark level
- Our approach from QCD sum rules
- What do QCD sum rule results indicate?

Interactions at the hadron level (Example: $\bar{D}^{(*)}\Sigma_c^{(*)}$)



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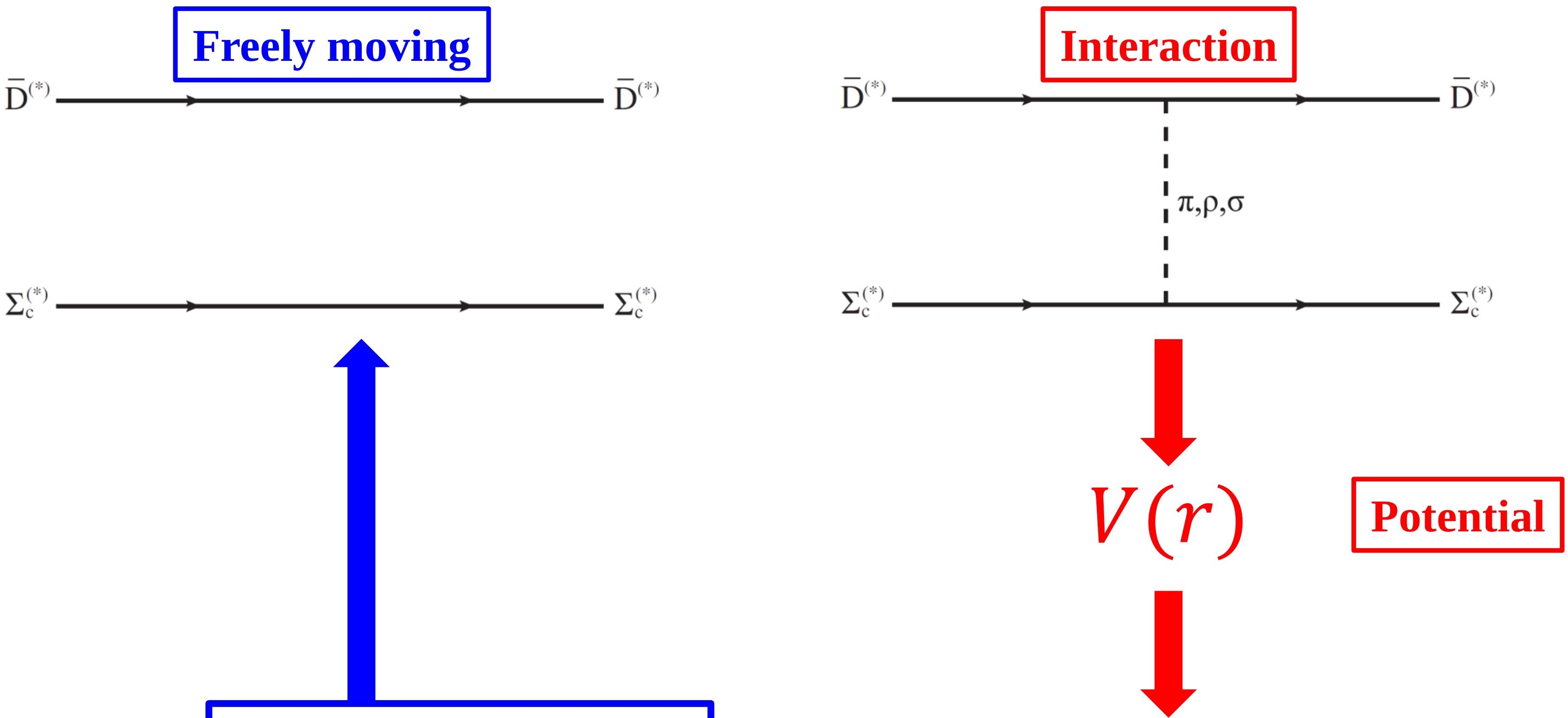
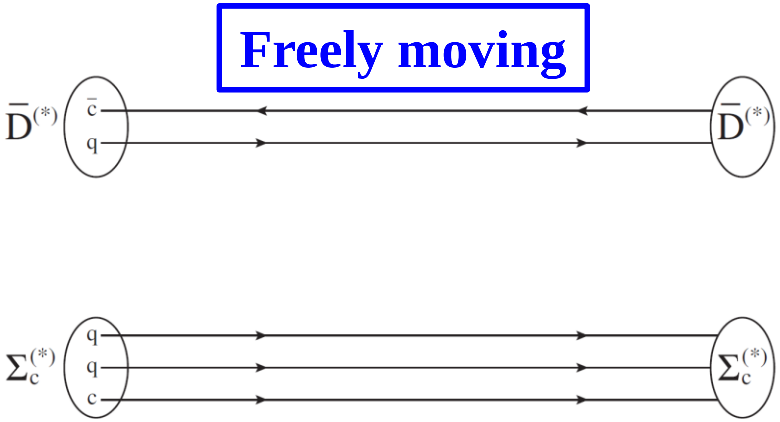
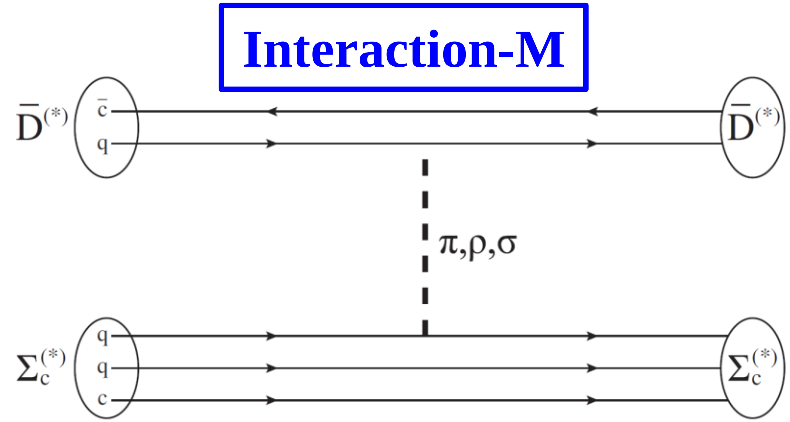


$$M_X = M_{\bar{D}^{(*)}} + M_{\Sigma_c^{(*)}} + \Delta M$$

Binding energy

Interactions at the quark level (Example: $\bar{D}^{(*)}\Sigma_c^{(*)}$)





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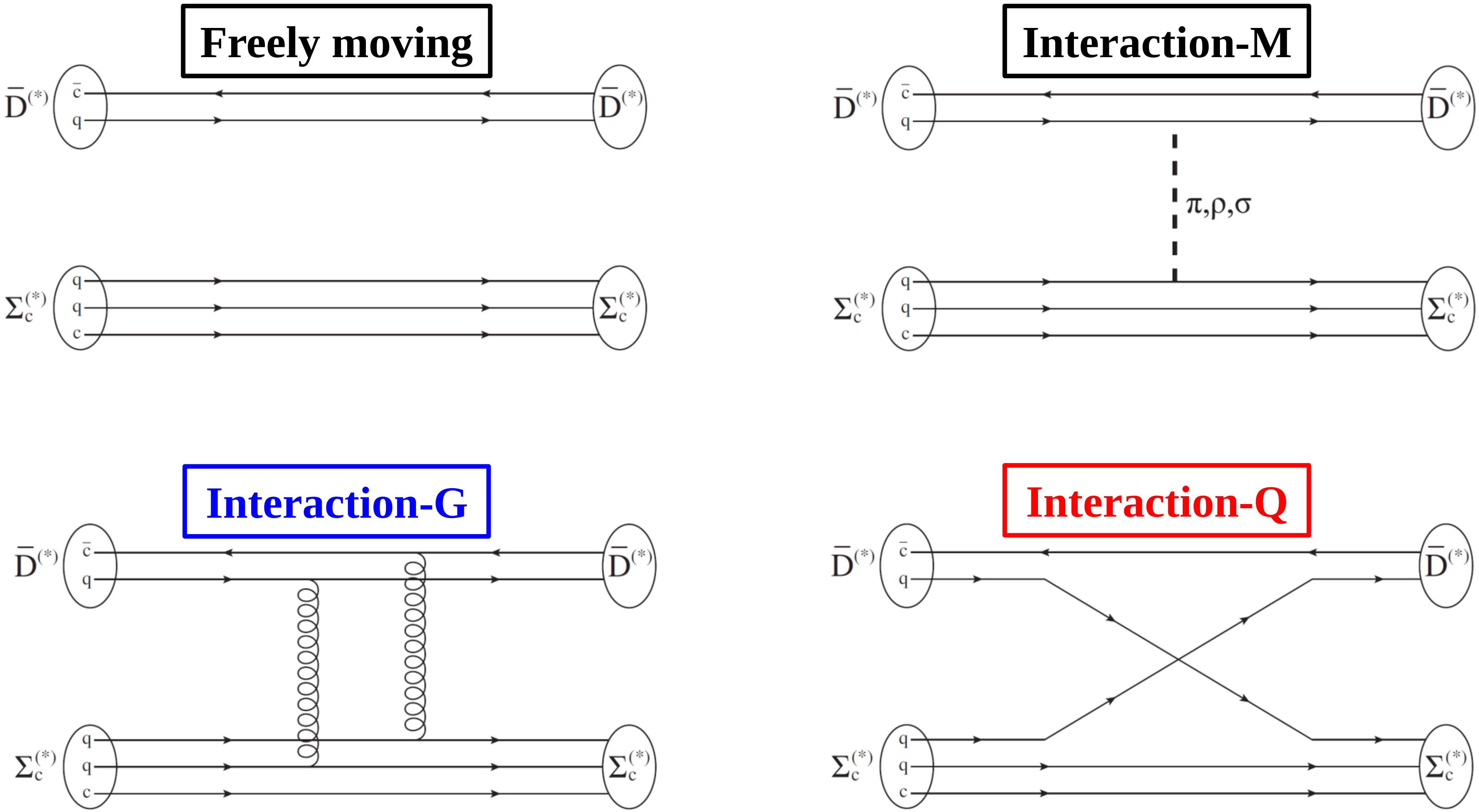

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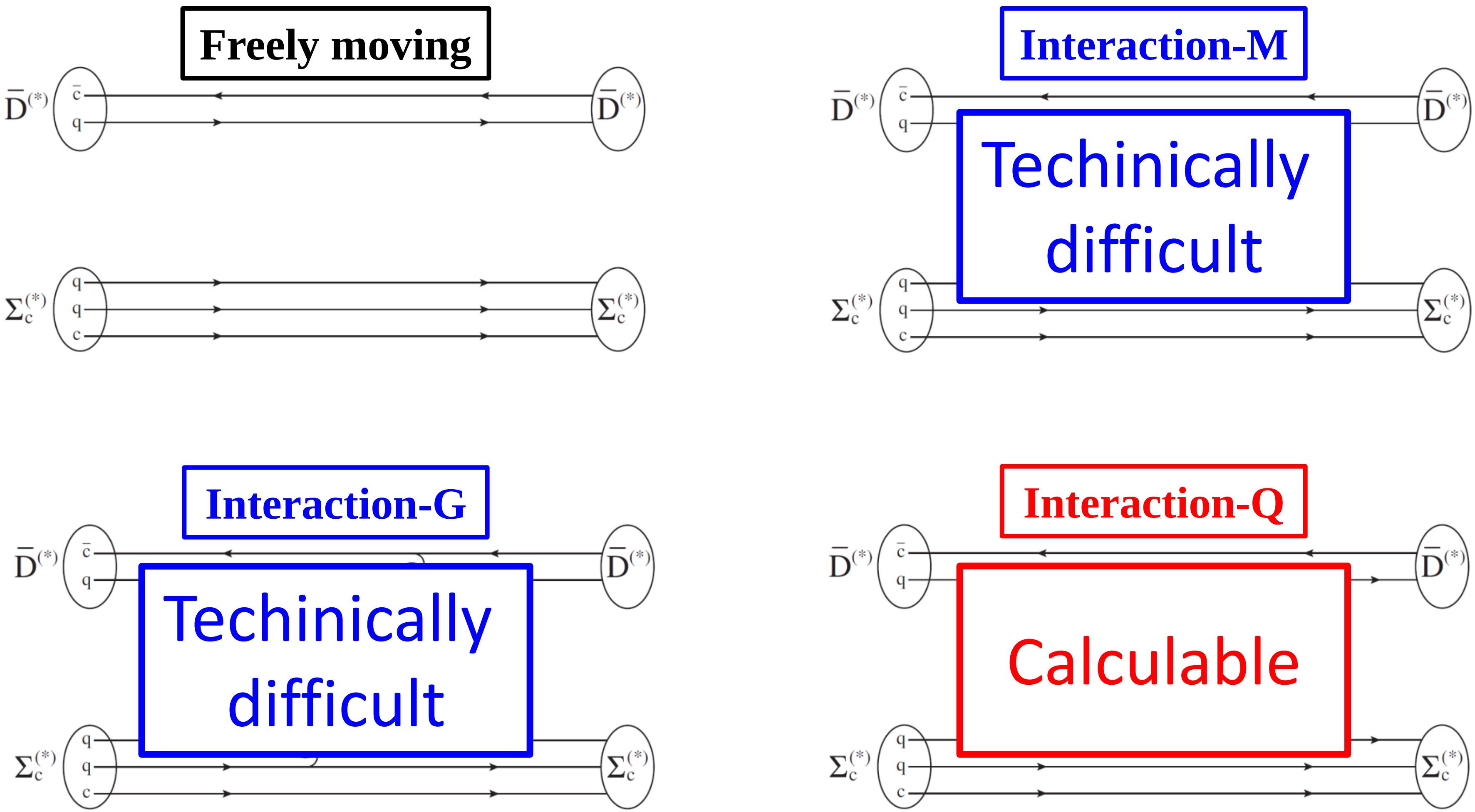

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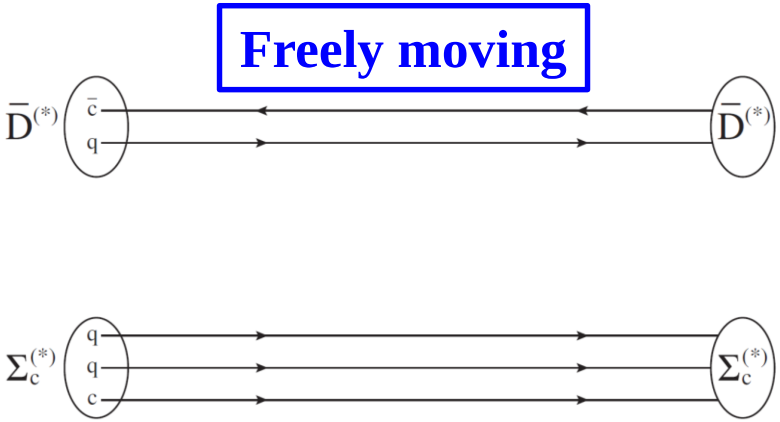
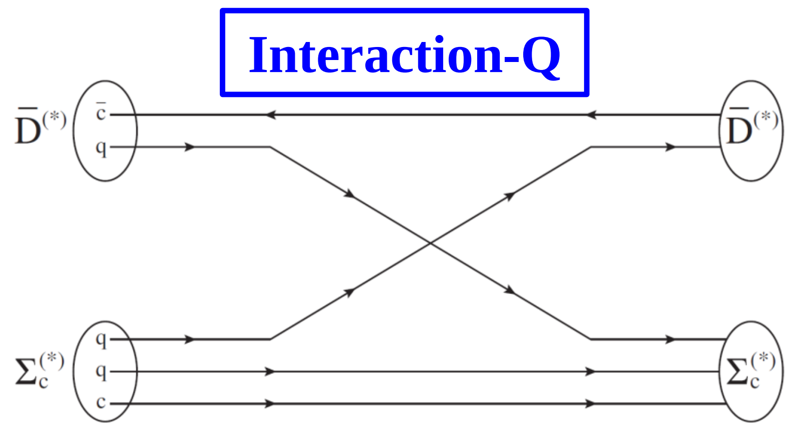


Calculable

Is it lucky or not that we are only capable of calculating Interaction-Q?

Calculable



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Four Examples:

$$J^{\bar{D}}(x) = \bar{c}_a(x)\gamma_5 q_a(x)$$
$$J^{\Sigma_c}(x) = \epsilon^{abc} q_a^T(x)\mathbb{C}\gamma^\mu q_b(x)\gamma_\mu\gamma_5 c_c(x)$$

- $D^-\Sigma_c^{++}$ $$J^{D^-\Sigma_c^{++}}(x) = J^{D^-}(x) \times J^{\Sigma_c^{++}}(x)$$

- $\bar{D}^0\Sigma_c^+$ $$J^{\bar{D}^0\Sigma_c^+}(x) = J^{\bar{D}^0}(x) \times J^{\Sigma_c^+}(x)$$

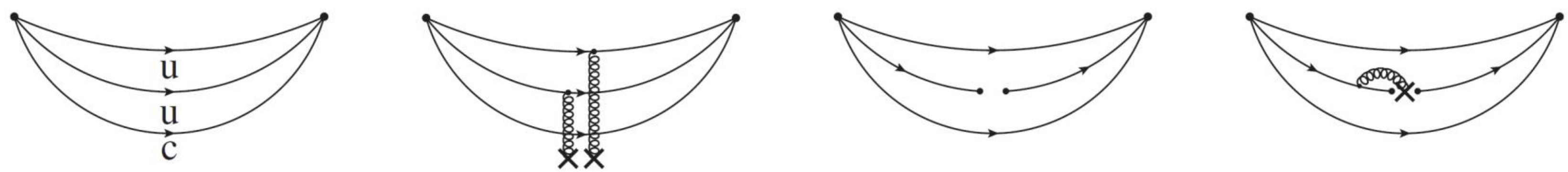
- $\bar{D}\Sigma_c$ of $I = 1/2$ $$J^{\bar{D}\Sigma_c}(x) = \sqrt{\frac{1}{3}} J^{\bar{D}^0\Sigma_c^+}(x) - \sqrt{\frac{2}{3}} J^{D^-\Sigma_c^{++}}(x)$$

- $\bar{D}\Sigma_c$ of $I = 3/2$ $$J^{\bar{D}\Sigma_c}_{I=3/2}(x) = \sqrt{\frac{2}{3}} J^{\bar{D}^0\Sigma_c^+}(x) + \sqrt{\frac{1}{3}} J^{D^-\Sigma_c^{++}}(x)$$

$\Pi^{\bar{D}}(x)$ *and* $\Pi^{\Sigma_c}(x)$

Π^{D^-}

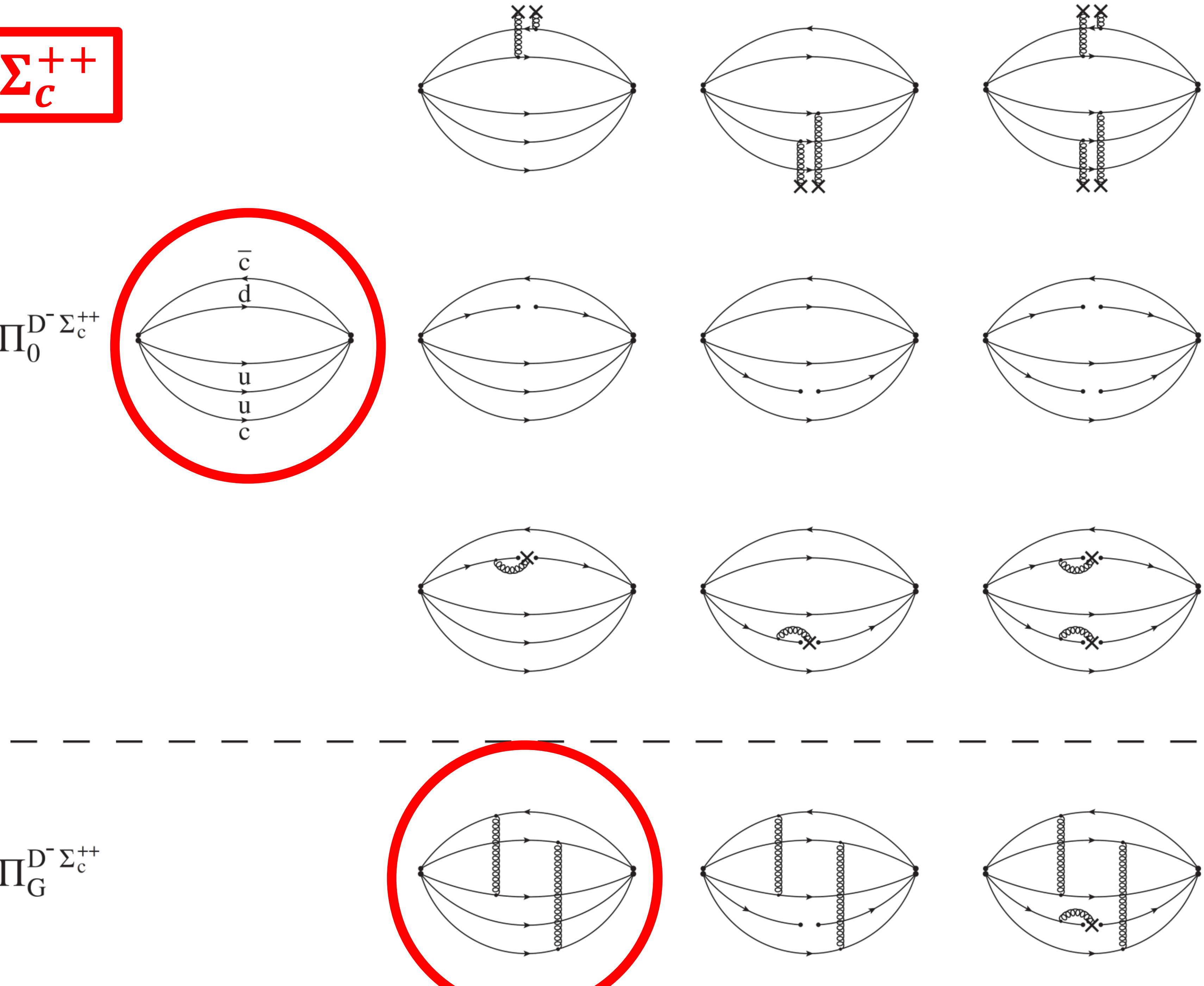
$\Pi^{\Sigma_c^{++}}$



➢ $D^{-}\Sigma_c^{++}$

$\Pi_0^{D^{-}\Sigma_c^{++}}$

$\Pi_G^{D^{-}\Sigma_c^{++}}$

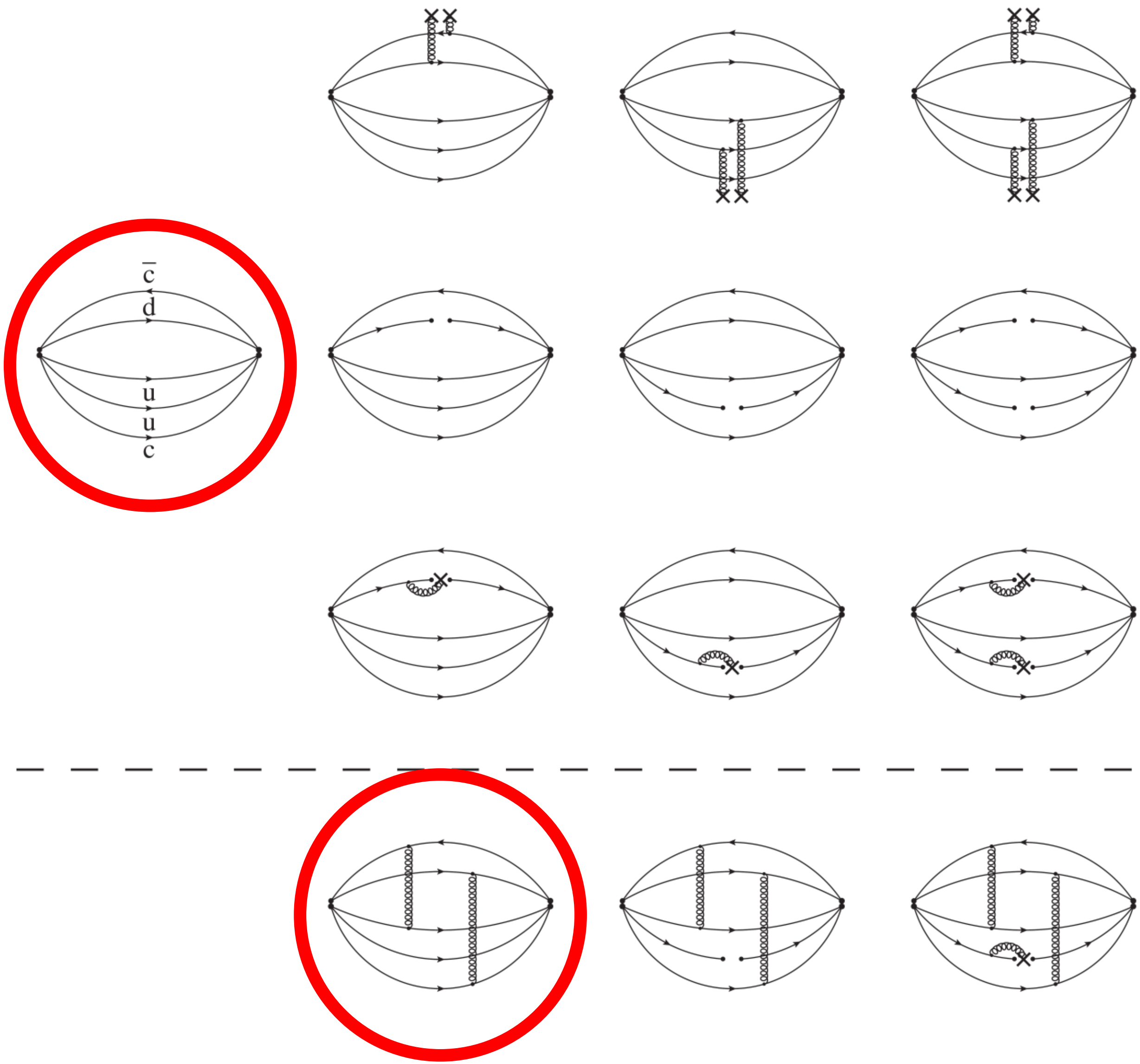


➢ $D^{-}\Sigma_c^{++}$

$$\Pi^{D^-\Sigma_c^{++}}(x) = \Pi_0^{D^-\Sigma_c^{++}}(x) + \Pi_G^{D^-\Sigma_c^{++}}(x)$$

$\Pi_0^{D^-\Sigma_c^{++}}$

$\Pi_G^{D^-\Sigma_c^{++}}$



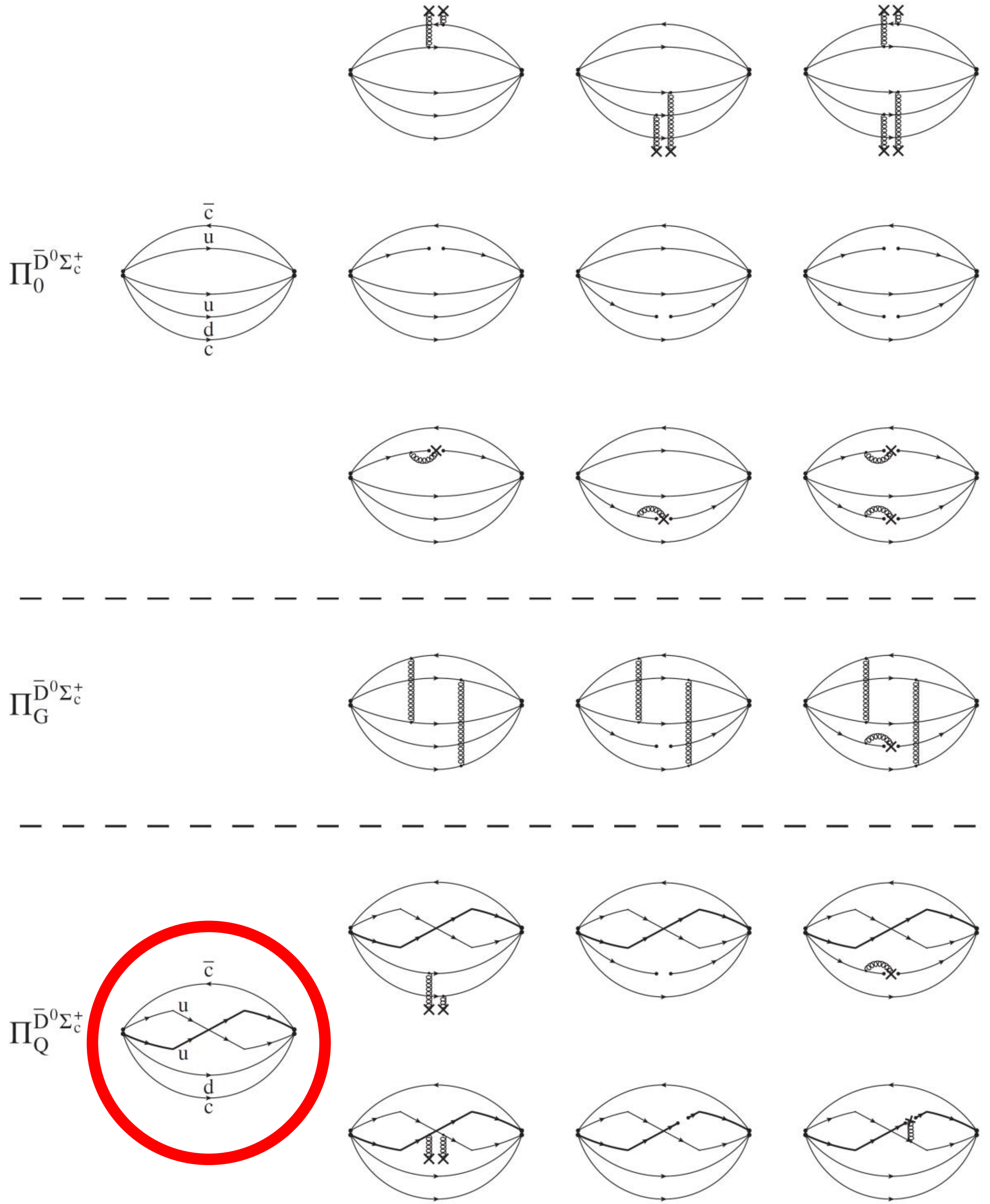
Freely moving

$$\Pi_0^{D^-\Sigma_c^{++}}(x) = \Pi^{D^-}(x) \times \Pi^{\Sigma_c^{++}}(x)$$

Interaction-G

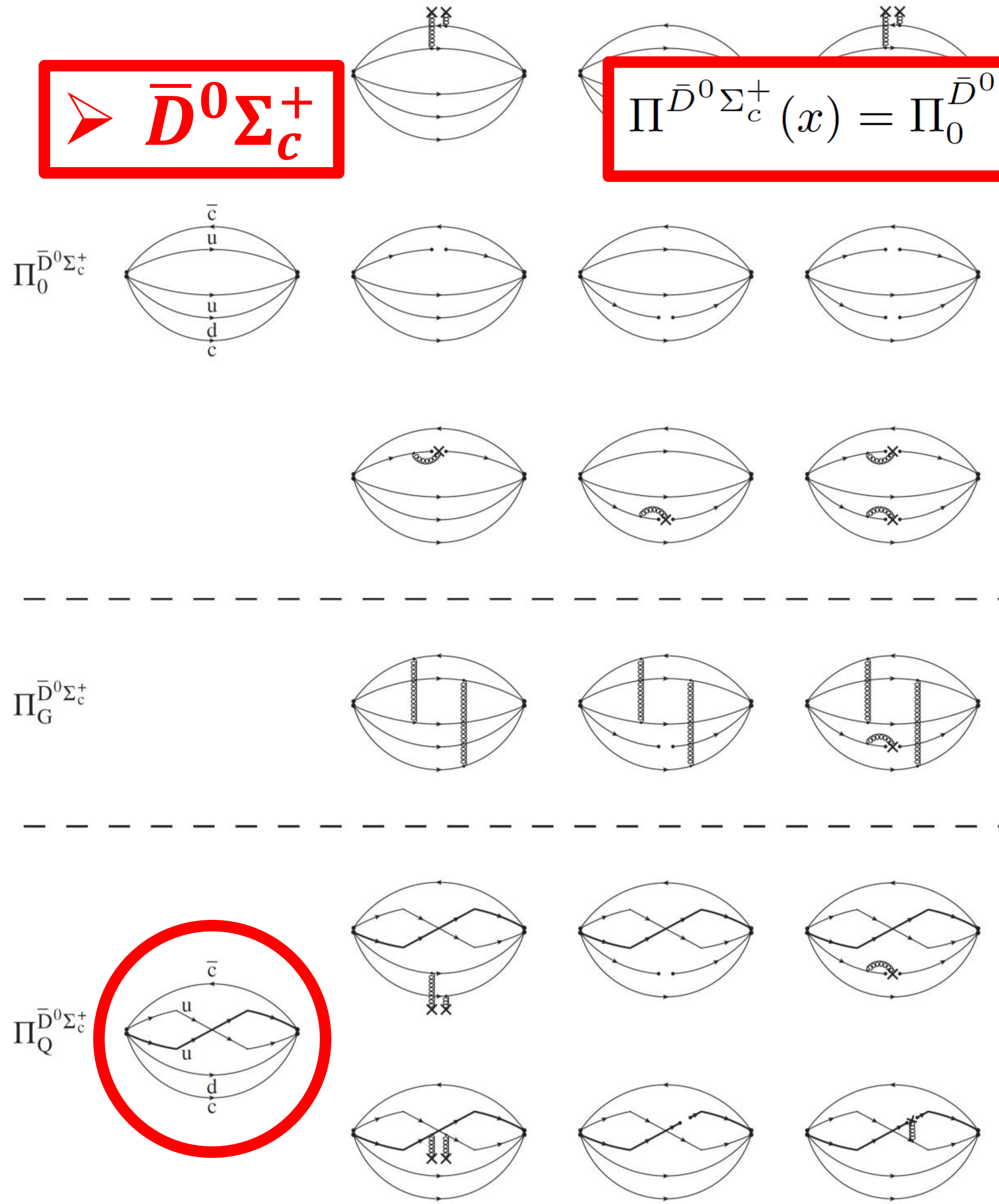
Techinically difficult

➢ $\bar{D}^0\Sigma_c^+$



➤ $\bar{D}^0\Sigma_c^+$

$$\Pi^{\bar{D}^0\Sigma_c^+}(x) = \Pi_0^{\bar{D}^0\Sigma_c^+}(x) + \Pi_G^{\bar{D}^0\Sigma_c^+}(x) + \Pi_Q^{\bar{D}^0\Sigma_c^+}(x)$$



Freely moving

$$\Pi_0^{\bar{D}^0\Sigma_c^+}(x) = \Pi^{\bar{D}^0}(x) \times \Pi^{\Sigma_c^+}(x)$$

Interaction-G

Techinically difficult

Interaction-Q

Calculable

Four Examples:

$$\Pi_0^{\bar{D}\Sigma_c}(x) = \Pi^{\bar{D}}(x) \times \Pi^{\Sigma_c}(x)$$

- $D^-\Sigma_c^{++}$

$$\Pi^{D^-\Sigma_c^{++}}(x) = \Pi_0^{\bar{D}\Sigma_c}(x) + \Pi_G^{\bar{D}\Sigma_c}(x)$$

- $\bar{D}^0\Sigma_c^+$

$$\Pi^{\bar{D}^0\Sigma_c^+}(x) = \Pi_0^{\bar{D}\Sigma_c}(x) + \Pi_G^{\bar{D}\Sigma_c}(x) - \Pi_Q^{\bar{D}\Sigma_c}(x)$$

Benchmark

- $\bar{D}\Sigma_c$ of $I = 1/2$

$$\Pi_{I=1/2}^{\bar{D}\Sigma_c}(x) = \Pi_0^{\bar{D}\Sigma_c}(x) + \Pi_G^{\bar{D}\Sigma_c}(x) + \Pi_Q^{\bar{D}\Sigma_c}(x)$$

- $\bar{D}\Sigma_c$ of $I = 3/2$

$$\Pi_{I=3/2}^{\bar{D}\Sigma_c}(x) = \Pi_0^{\bar{D}\Sigma_c}(x) + \Pi_G^{\bar{D}\Sigma_c}(x) - 2\Pi_Q^{\bar{D}\Sigma_c}(x)$$

Four Examples:

$$\Pi_0^{\bar{D}\Sigma_c}(x) = \Pi^{\bar{D}}(x) \times \Pi^{\Sigma_c}(x)$$

Neglecting Π_G

- $D^-\Sigma_c^{++}$ $\quad \Pi^{D^-\Sigma_c^{++}}(x) = \Pi_0^{\bar{D}\Sigma_c}(x) + \Pi_G^{\bar{D}\Sigma_c}(x)$
- $\bar{D}^0\Sigma_c^+$ $\quad \Pi^{\bar{D}^0\Sigma_c^+}(x) = \Pi_0^{\bar{D}\Sigma_c}(x) + \Pi_G^{\bar{D}\Sigma_c}(x) - \Pi_Q^{\bar{D}\Sigma_c}(x)$
- $\bar{D}\Sigma_c$ of $I = 1/2$ $\quad \Pi_{I=1/2}^{\bar{D}\Sigma_c}(x) = \Pi_0^{\bar{D}\Sigma_c}(x) + \Pi_G^{\bar{D}\Sigma_c}(x) + \Pi_Q^{\bar{D}\Sigma_c}(x)$
- $\bar{D}\Sigma_c$ of $I = 3/2$ $\quad \Pi_{I=3/2}^{\bar{D}\Sigma_c}(x) = \Pi_0^{\bar{D}\Sigma_c}(x) + \Pi_G^{\bar{D}\Sigma_c}(x) - 2\Pi_Q^{\bar{D}\Sigma_c}(x)$

same $\quad$ **different**

Our QCD sum rule approach ($\bar{D}\Sigma_c$ of $I = 1/2$)

Quark-Level:

$$\Pi^{\bar{D}\Sigma_c}_{I=1/2}(x) = \Pi^{\bar{D}\Sigma_c}_0(x) + \Pi^{\bar{D}\Sigma_c}_Q(x)$$

Our QCD sum rule approach ($\bar{D}\Sigma_c$ of $I = 1/2$)

Quark-Level:

$$\Pi^{\bar{D}\Sigma_c}_{I=1/2}(x) = \Pi^{\bar{D}\Sigma_c}_0(x) + \Pi^{\bar{D}\Sigma_c}_Q(x)$$

Hadron-Level:

$$M_X = M_{\bar{D}} + M_{\Sigma_c} + \Delta M = M_0 + \Delta M$$

$$\Pi(q^2) = \frac{f_X^2}{M_X^2 - q^2} + \cdots$$

$$\approx \frac{f_X^2}{M_0^2 - q^2} - \frac{2M_0 f_X^2}{(M_0^2 - q^2)^2}\Delta M + \cdots$$

Our QCD sum rule approach ($\bar{D}\Sigma_c$ of $I = 1/2$)

Quark-Level:

$$\Pi^{\bar{D}\Sigma_c}_{I=1/2}(x) = \Pi^{\bar{D}\Sigma_c}_{0}(x) + \Pi^{\bar{D}\Sigma_c}_{Q}(x)$$

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$$\approx \frac{f_X^2}{M_0^2 - q^2} - \frac{2M_0 f_X^2}{(M_0^2 - q^2)^2}\Delta M + \cdots$$

Quark-Hadron Duality:

$$-\frac{2M_0}{M_B^2}\Delta M = \frac{\Pi_Q}{\Pi_0}$$

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Four Examples:

- $\boldsymbol{D^-\Sigma_c^{++}}$ $\Delta M^{D^-\Sigma_c^{++}} = 0$ **no interaction**
- $\boldsymbol{\bar{D}^0\Sigma_c^{+}}$ $\Delta M^{\bar{D}^0\Sigma_c^{+}} = +95$ MeV **repulsive**
- $\boldsymbol{\bar{D}\Sigma_c}$ of $\boldsymbol{I = 1/2}$ $\boldsymbol{\Delta M^{\bar{D}\Sigma_c}_{I=1/2} = -95}$ **MeV** **attractive**
- $\boldsymbol{\bar{D}\Sigma_c}$ of $\boldsymbol{I = 3/2}$ $\Delta M^{\bar{D}\Sigma_c}_{I=3/2} = +190$ MeV **repulsive**

More Examples:

- $\bar{D}^{(*)}\Lambda_c$:

$$\Delta M^{\bar{D}^{(*)}\Lambda_c} > 0$$

- $D^{(*)}\bar{D}^{(*)}$:

$$\Delta M^{D^{(*)}\bar{D}^{(*)}} = 0$$

- $\bar{D}^{(*)}\Sigma_c^{(*)}$:

$$\Delta M^{\bar{D}\Sigma_c}_{I=1/2,J=1/2} = -95 \text{ MeV},$$

$$\Delta M^{\bar{D}^*\Sigma_c}_{I=1/2,J=3/2} = -89 \text{ MeV},$$

$$\Delta M^{\bar{D}\Sigma_c^*}_{I=1/2,J=3/2} = -86 \text{ MeV},$$

$$\Delta M^{\bar{D}^*\Sigma_c^*}_{I=1/2,J=5/2} = -107 \text{ MeV},$$

- $D^{(*)}\bar{K}^*$:

$$\Delta M^{D\bar{K}^*}_{I=0,J=1} = -180 \text{ MeV}$$

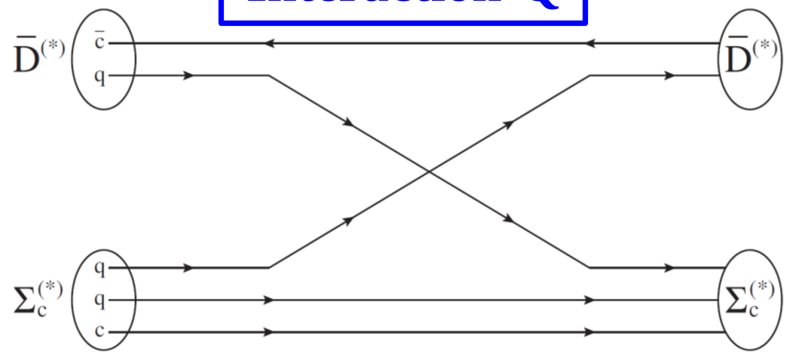
$$\Delta M^{D^*\bar{K}^*}_{I=0,J=2} = -119 \text{ MeV}$$

ΔM is actually not the binding energy

- **Local operators:**

$$V(r=0)=\Delta M$$

Interaction-Q



$V(r)$ **Potential**

ΔM **Binding energy**

ΔM is actually not the binding energy

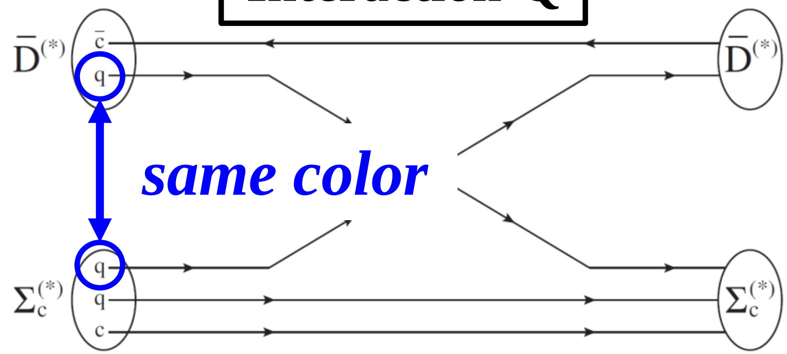
- **Local operators:**

$$V(r=0)=\Delta M$$

- **Color-unconfined Interaction-Q:**

$$V(r\to\infty)\to 0$$

Interaction-Q



$\downarrow$

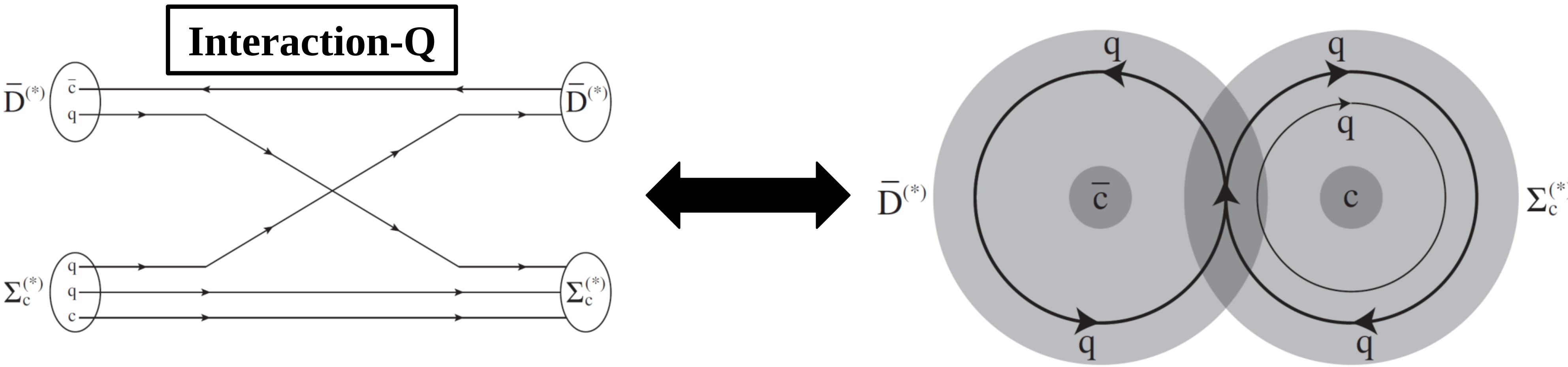
$V(r)$ **Potential**

$\downarrow$

ΔM **Binding energy**

We do not solve this potential, but seek a model-independent picture.

Covalent hadronic molecules



Our results indicate:

the light-quark-exchange Interaction-Q is attractive

when the shared light quarks are totally antisymmetric

so that obey the Pauli principle.

Four Examples:

- $D^{-}[\bar{c}_1 d_2]\Sigma_c^{++}[u_3 u_4 c_5]$
- $\bar{D}^0\Sigma_c^{+}$
- $\bar{D}\Sigma_c$ of $I = 1/2$
- $\bar{D}\Sigma_c$ of $I = 3/2$

no quarks exchanged

↓

no interaction-Q

Four Examples:

- $D^{-}\Sigma_c^{++}$
- $\bar{D}^0[\bar{c}_1 u_2]\Sigma_c^{+}[u_3 d_4 c_5]$
- $\bar{D}\Sigma_c$ of $I = 1/2$
- $\bar{D}\Sigma_c$ of $I = 3/2$

	color	flavor	spin	orbital	total
$u_2 \leftrightarrow u_3$	**S**	**S**	**S**	**S**	**S**

repulsive

Four Examples:

- $\boldsymbol{D^-\Sigma_c^{++}}$
- $\boldsymbol{\bar{D}^0\Sigma_c^{+}}$
- $\boldsymbol{\bar{D}\Sigma_c}$ of $\boldsymbol{I = 1/2}$
- $\boldsymbol{\bar{D}[\bar{c}_1 q_2]\Sigma_c[q_3 q_4 c_5]}$ of $\boldsymbol{I = 3/2}$

	color	flavor	spin	orbital	total
$q_2 \leftrightarrow q_3$	**S**	**S**	**S**	**S**	**S**

repulsive

Four Examples:

- $D^{-}\Sigma_c^{++}$
- $\bar{D}^{0}\Sigma_c^{+}$
- $\bar{D}[\bar{c}_1 q_2]\Sigma_c[q_3 q_4 c_5]$ of $I = 1/2$
- $\bar{D}\Sigma_c$ of $I = 3/2$

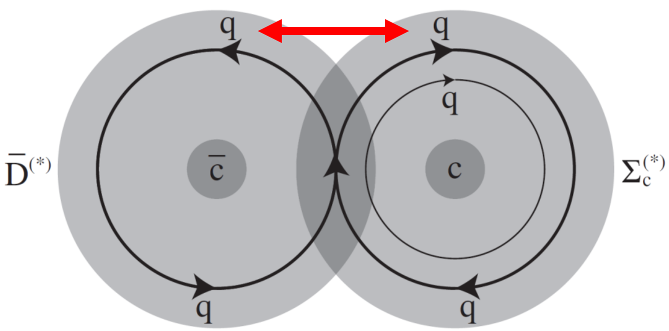
	color	flavor	spin	orbital	total
$q_2 \leftrightarrow q_3$	**S**	**A**	**S**	**S**	**A**
$q_2 \leftrightarrow q_4$	**A**	**S**	**S**	**S**	**A**
$q_3 \leftrightarrow q_4$	**A**	**S**	**S**	**S**	**A**

attractive

Possibly-existing covalent hadronic molecules

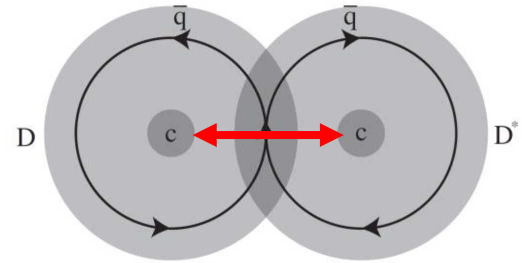
	$\vert\bar{Q}q,\frac{1}{2}0^-\rangle$	$\vert\bar{Q}q,\frac{1}{2}1^-\rangle$	$\vert Q[qq],0\frac{1}{2}^+\rangle$	$\vert Q\{qq\},1\frac{1}{2}^+\rangle$	$\vert Q\{qq\},1\frac{3}{2}^+\rangle$	$\vert Q[sq],\frac{1}{2}\frac{1}{2}^+\rangle$	$\vert Q\{sq\},\frac{1}{2}\frac{1}{2}^+\rangle$	$\vert Q\{sq\},\frac{1}{2}\frac{3}{2}^+\rangle$
$\vert\bar{Q}'q,\frac{1}{2}0^-\rangle$	$\vert(0)0^+\rangle$	$\vert(0)1^+\rangle$ ($\checkmark$)	–	$\vert(\frac{1}{2})\frac{1}{2}^-\rangle$ ($\checkmark$)	$\vert(\frac{1}{2})\frac{3}{2}^-\rangle$ ($\checkmark$)	$\vert(0)\frac{1}{2}^-\rangle$	$\vert(0)\frac{1}{2}^-\rangle$	$\vert(0)\frac{3}{2}^-\rangle$
$\vert\bar{Q}'q,\frac{1}{2}1^-\rangle$		$\vert(0)0^+\rangle$ (?) $\vert(1)0^+\rangle$ (??) $\vert(0)1^+\rangle$ (?) $\vert(1)1^+\rangle$ (??) $\vert(0)2^+\rangle$ ($\checkmark$)	–	$\vert(\frac{1}{2})\frac{1}{2}^-\rangle$ (?) $\vert(\frac{3}{2})\frac{1}{2}^-\rangle$ (??) $\vert(\frac{1}{2})\frac{3}{2}^-\rangle$ ($\checkmark$) $\vert(\frac{3}{2})\frac{3}{2}^-\rangle$ (??)	$\vert(\frac{1}{2})\frac{1}{2}^-\rangle$ (?) $\vert(\frac{3}{2})\frac{1}{2}^-\rangle$ (??) $\vert(\frac{1}{2})\frac{3}{2}^-\rangle$ (?) $\vert(\frac{3}{2})\frac{3}{2}^-\rangle$ (??) $\vert(\frac{1}{2})\frac{5}{2}^-\rangle$ ($\checkmark$)	$\vert(0)\frac{1}{2}^-\rangle$ $\vert(0)\frac{3}{2}^-\rangle$	$\vert(0)\frac{1}{2}^-\rangle$ $\vert(1)\frac{1}{2}^-\rangle$ $\vert(0)\frac{3}{2}^-\rangle$ $\vert(1)\frac{3}{2}^-\rangle$	$\vert(0)\frac{1}{2}^-\rangle$ $\vert(1)\frac{1}{2}^-\rangle$ $\vert(0)\frac{3}{2}^-\rangle$ $\vert(1)\frac{3}{2}^-\rangle$ $\vert(0)\frac{5}{2}^-\rangle$
$\vert Q[qq],0\frac{1}{2}^+\rangle$			–	–	–	–	–	–
$\vert Q\{qq\},1\frac{1}{2}^+\rangle$				$\vert(0)1^+\rangle$ $\vert(1)0/1^+\rangle$ $\vert(2)0/1^+\rangle$	$\vert(0)1/2^+\rangle$ $\vert(1)1/2^+\rangle$ $\vert(2)1^+\rangle$	$\vert(\frac{1}{2})0/1^+\rangle$	$\vert(\frac{1}{2})0/1^+\rangle$ $\vert(\frac{3}{2})0/1^+\rangle$	$\vert(\frac{1}{2})1/2^+\rangle$ $\vert(\frac{3}{2})1/2^+\rangle$
$\vert Q\{qq\},1\frac{3}{2}^+\rangle$					$\vert(0)1/2/3^+\rangle$ $\vert(1)0/1/2^+\rangle$ $\vert(2)0/1^+\rangle$	$\vert(\frac{1}{2})1/2^+\rangle$	$\vert(\frac{1}{2})1/2^+\rangle$ $\vert(\frac{3}{2})1/2^+\rangle$	$\vert(\frac{1}{2})0/1/2/3^+\rangle$ $\vert(\frac{3}{2})0/1/2^+\rangle$

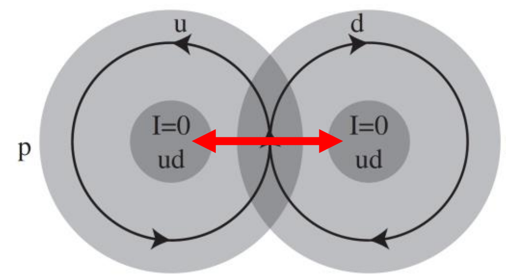
A toy model



	color	flavor	spin	orbital	total
$q \leftrightarrow q'$	**S**	**A**	**S**	**S**	**A**

attractive bond *A*~30 MeV



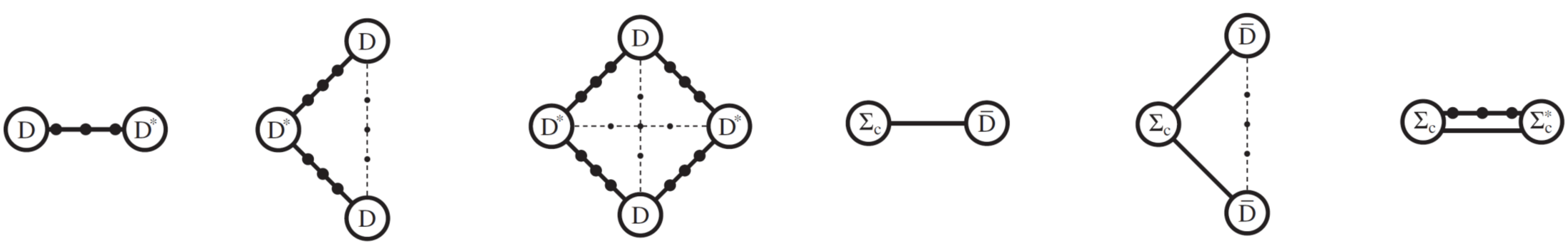


repulsive bond *R*~17 MeV

residual energy *ε*~6 MeV

Binding energies of some possibly-existing covalent hadronic molecules

Molecules	Binding energies	Molecules	Binding energies
^{2}H, $D^*D^{(*)}/\bar{B}^*\bar{B}^{(*)}$	1 MeV	$D^{(*)}\bar{B}^{(*)}$	18 MeV
^{3}H/^{3}He, $D^*D^{(*)}D^{(*)}/\bar{B}^*\bar{B}^{(*)}\bar{B}^{(*)}$	8 MeV	$D^{(*)}D^{(*)}\bar{B}^{(*)}/D^{(*)}\bar{B}^{(*)}\bar{B}^{(*)}$	42 MeV
^{4}He, $D^*D^*D^{(*)}D^{(*)}/\bar{B}^*\bar{B}^*\bar{B}^{(*)}\bar{B}^{(*)}$	28 MeV	$D^*D^{(*)}D^{(*)}\bar{B}^{(*)}/D^{(*)}\bar{B}^{(*)}\bar{B}^{(*)}\bar{B}^*$	62 MeV
		$D^{(*)}D^{(*)}\bar{B}^{(*)}\bar{B}^{(*)}$	96 MeV
$\Sigma_c^{(*)}\Sigma_c^{(*)}/\Sigma_b^{(*)}\Sigma_b^{(*)}$	31 MeV	$\Sigma_c^{(*)}\Sigma_b^{(*)}$	48 MeV
		$\bar{D}^{(*)}\Sigma_c^{(*)}/\bar{D}^{(*)}\Sigma_b^{(*)}/B^{(*)}\Sigma_c^{(*)}/B^{(*)}\Sigma_b^{(*)}$	18 MeV
		$\bar{D}^{(*)}\bar{D}^{(*)}\Sigma_c^{(*)}$	42 MeV



DD^* DDD^* DDD^*D^* $\bar{D}\Sigma_c$ $\bar{D}\bar{D}\Sigma_c$ $\Sigma_c\Sigma_c^*$

Summary

- We systematically examine Feynman diagrams corresponding to some hadronic molecules, and propose **a possible binding mechanism induced by shared light quarks**, and study it via **QCD sum rules**.

- Our results indicate the **covalent hadronic molecule picture**:

  the light-quark-exchange Interaction-Q is attractive when the shared light quarks are totally antisymmetric so that obey the Pauli principle.

- We build **a toy model** to formulize this picture with **the unique feature**:

  binding energies of $D\bar{B}^*/D^*\bar{B}$ hadronic molecules are much larger than those of $DD^*/\bar{B}\bar{B}^*$ ones, while $\bar{D}\Sigma_c/\bar{D}\Sigma_b/B\Sigma_c/B\Sigma_b$ hadronic molecules have similar binding energies.

Comments are appreciate!

A long logic chain:

Interaction-Q?

modified quark-hadron duality?

$V(r=0)=\Delta M$?

the covalent picture?

the toy model?

。。。 。。。

Thank you very much!