Decay modes of the pseudoscalar glueball and its first excited state

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Introduction

- Quantum Chromodynamics (QCD)

- Symmetries of the QCD Lagrangian.

  if all quark massless then we have chiral symmetry

$$U(N_f)_r \times U(N_f)_l = SU(N_f)_r \times SU(N_f)_l \times U(1)_V \times U(1)_A$$

- Spontaneous breaking of chiral symmetry by quark condensates.
- Explicit breaking of global chiral symmetry by quark masses and chiral anomaly.

- Effective chiral models of (QCD).

Motivation

• Decay of the pseudoscalar glueball into scalar and pseudoscalar mesons.
• Decay of the pseudoscalar glueball into excited scalar and excited pseudoscalar mesons.
• Decay of the pseudoscalar glueball into the scalar glueball.

• Decay of the first excited pseudoscalar glueball into the pseudoscalar and the scalar glueball.
• Decay of the first excited pseudoscalar glueball into the pseudoscalar charmonium state.
• Decay of the first excited pseudoscalar glueball into scalar and pseudoscalar mesons and their first excited states.

Fields of the models

- **Mesons: quark-antiquark states ($q\bar{q}$)**

  (scalar, pseudoscalar, excited scalar and excited pseudoscalar quarkonia.)

Quantum number: J^{PC} → **Charge Conjugation**

TotalSpin ← J

P ↓ **Parity**

- **Glueballs: Scalar, pseudoscalar, first excited pseudoscalar glueballs**

Decays of the pseudoscalar glueball

A globally chirally invariant for three flavours

Interaction Lagrangian for the pseudoscalar glueball with scalar and pseudoscalar mesons

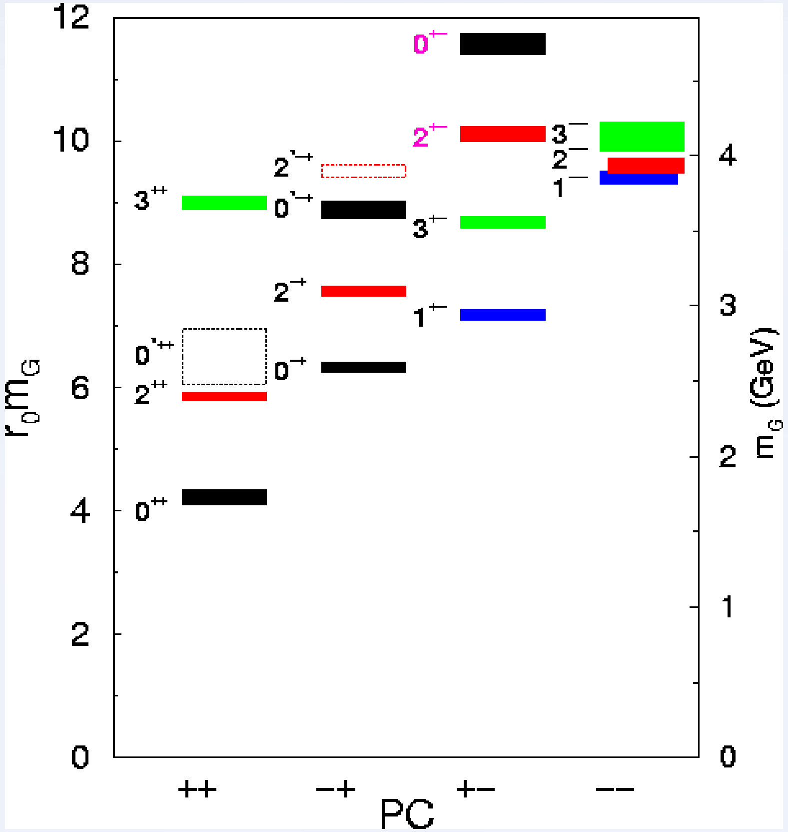
$$\mathcal{L}_{\tilde{G}}^{int} = i c_{\tilde{G}\Phi} \tilde{G} \left(\det\Phi - \det\Phi^{\dagger} \right)$$

where $c_{\tilde{G}\Phi}$ is a dimensionless coupling constant and Φ reads for three flavours, $N_f = 3$:

$$\Phi = (S^a + iP^a)\, t^a = \frac{1}{\sqrt{2}} \begin{pmatrix} \frac{(\sigma_N + a_0^0) + i(\eta_N + \pi^0)}{\sqrt{2}} & a_0^+ + i\pi^+ & K_S^+ + iK^+ \\ a_0^- + i\pi^- & \frac{(\sigma_N - a_0^0) + i(\eta_N - \pi^0)}{\sqrt{2}} & K_S^0 + iK^0 \\ K_S^- + iK^- & \bar{K}_S^0 + i\bar{K}^0 & \sigma_S + i\eta_S \end{pmatrix}$$

Glueballs

Lattice QCD calculation



[C. Morningstar and M. J. Peardon, AIP Conf. Proc. 688, 220 (2004) [arXiv:nucl-th/0309068]];

The pseudoscalar glueball $\breve{G} \equiv |gg\rangle$ at the border within light and heavy

$$M_{\tilde{G}} = 2.6\ \text{GeV},\ J^{PC} = 0^{-+},\ I = 0.$$

The first excited pseudoscalar Glueball

$$M_{\tilde{G}'} = 3.7\,\text{GeV}\ ,\ J^{PC} = 0^{*-+}.$$

Two experiments related to our work:

1. PANDA experiment at FAIR facility.
   It will be capable to scan the mass region above 2.5 GeV.

2. BESIII experiment.
   The resonance X(2370) could be a pseudoscalar glueball with a mass 2.37 GeV.

Predictions for a pseudoscalar glueball

- Predict branching ratios for decays into three pseudoscalar mesons $\tilde{G} \to PPP$.

Quantity	Case (i): $M_{\tilde{G}} = 2.6$ GeV	Case (ii): $M_{\tilde{G}} = 2.37$ GeV
$\Gamma_{\tilde{G}\to KK\eta}/\Gamma^{tot}_{\tilde{G}}$	0.049	0.043
$\Gamma_{\tilde{G}\to KK\eta'}/\Gamma^{tot}_{\tilde{G}}$	0.019	0.011
$\Gamma_{\tilde{G}\to \eta\eta\eta}/\Gamma^{tot}_{\tilde{G}}$	0.016	0.013
$\Gamma_{\tilde{G}\to \eta\eta\eta'}/\Gamma^{tot}_{\tilde{G}}$	0.0017	0.00082
$\Gamma_{\tilde{G}\to \eta\eta'\eta'}/\Gamma^{tot}_{\tilde{G}}$	0.00013	0
$\Gamma_{\tilde{G}\to KK\pi}/\Gamma^{tot}_{\tilde{G}}$	0.47	0.47
$\Gamma_{\tilde{G}\to \eta\pi\pi}/\Gamma^{tot}_{\tilde{G}}$	0.16	0.17
$\Gamma_{\tilde{G}\to \eta'\pi\pi}/\Gamma^{tot}_{\tilde{G}}$	0.095	0.090

The decay of the pseudoscalar glueball into three pions vanishes:

$$\Gamma_{\tilde{G}\to\pi\pi\pi} = 0$$

W. I. Eshraim; S. Janowski; F. Giacosa; D. H. Rischke. Phys.Rev. D87 (2013) 054036 [arXiv: 1208.6474 [hep-ph].

Predict branching ratios for decays into a scalar and a pseudoscalar meson

$$\tilde{G} \rightarrow PS$$

Quantity	Case (i): $M_{\tilde{G}} = 2.6$ GeV	Case (ii): $M_{\tilde{G}} = 2.37$ GeV
$\Gamma_{\tilde{G}\rightarrow KK_S}/\Gamma^{tot}_{\tilde{G}}$	0.060	0.070
$\Gamma_{\tilde{G}\rightarrow a_0\pi}/\Gamma^{tot}_{\tilde{G}}$	0.083	0.10
$\Gamma_{\tilde{G}\rightarrow \eta\sigma_N}/\Gamma^{tot}_{\tilde{G}}$	0.0000026	0.0000030
$\Gamma_{\tilde{G}\rightarrow \eta'\sigma_N}/\Gamma^{tot}_{\tilde{G}}$	0.039	0.026
$\Gamma_{\tilde{G}\rightarrow \eta\sigma_S}/\Gamma^{tot}_{\tilde{G}}$	0.012 (0.015)	0.0094 (0.017)
$\Gamma_{\tilde{G}\rightarrow \eta'\sigma_S}/\Gamma^{tot}_{\tilde{G}}$	0 (0.0082)	0 (0)

Could be measured by 

whereas

$$K_s = K_0^*(1430), a_0 = a_0(1450), \sigma_N \approx f_0(1370), \sigma_S \approx f_0(1710)$$

The full width of the pseudoscalar glueball is expected to be small

W. I. Eshraim; S. Janowski; F. Giacosa; D. H. Rischke. Phys.Rev. D87 (2013) 054036 [arXiv: 1208.6474 [hep-ph]; W.I. Eshraim and S. Janowski, PoS ConfinementX 118, (2012) [arXiv:1301.3345 [hep-ph]]; W.I. Eshraim and S. Janowski, J. Phys.Conf. Ser. 426, 012018 (2013) [arXiv:1211.7323 [hep-ph]].

A globally chirally invariant for three flavours

Interaction Lagrangian for the pseudoscalar glueball with (pseudo)scalar and excited (pseudo)scalar mesons

$$\mathcal{L}^{int}_{\tilde{G}\Phi\Phi_E} = c_{\tilde{G}\Phi\Phi_E}\,\tilde{G}\left[\left(det\Phi - det\Phi_E^\dagger\right)^2 + \left(det\Phi^\dagger - det\Phi_E\right)^2\right] ,$$

W. I. Eshraim, [arXiv: 1902.11148 [hep-ph]]

Where $c_{\breve{G}\Phi\Phi E}$ is a dimensionless coupling constant and Φ_E reads for three flavours, $N_f = 3$

$$\Phi_E = (S_E^a + iP_E^a)t^a = \frac{1}{\sqrt{2}}\begin{pmatrix} \frac{(\sigma_{NE}+a_{0E}^0)+i(\eta_{NE}+\pi_E^0)}{\sqrt{2}} & a_{0E}^+ + i\pi_E^+ & K_{SE}^+ + iK_E^+ \\ a_{0E}^- + i\pi_E^- & \frac{(\sigma_{NE}-a_{0E}^0)+i(\eta_{NE}-\pi_E^0)}{\sqrt{2}} & K_{SE}^0 + iK_E^0 \\ K_{SE}^- + iK_E^- & \bar{K}_{SE}^0 + i\bar{K}_E^0 & \sigma_{SE} + i\eta_{SE} \end{pmatrix} .$$

D. Parganlija and F. Giacosa, Eur.Phys.J. C77 (2017) no.7, 450 [arXiv:1612.09218 [hep-ph]].

The assignment of the excited (pseudo)scalar mesons

Model state	IJ^P	Assignment
σ_N^E	00^+	$f_0(1790)$
η_N^E	00^-	$\eta(1295)$
η_S^E	00^-	$\eta(1440)$
σ_S^E	00^+	Possible overlap with $f_0(2020)/f_0(2100)$ to be discussed as a model consequence
a_0^E	10^+	Possible overlap with $a_0(1950)$ to be discussed as a model consequence
π^E	10^-	Possible overlap with $\pi(1300)$ to be discussed as a model consequence
$K_0^{\star E}$	$\frac{1}{2}0^+$	Possible overlap with $K_0^\star(1950)$ to be discussed as a model consequence
K^E	$\frac{1}{2}0^-$	Possible overlap with $K(1460)$ to be discussed as a model consequence

D. Parganlija and F. Giacosa, Eur.Phys.J. C77 (2017) no.7, 450 [arXiv:1612.09218 [hep-ph]].

Branching ratios for the two- and three-body decays of the pseudoscalar glueball

Quantity	The theoretical result [MeV]
$\Gamma_{\tilde{G}\to\eta\eta}/\Gamma^{tot}_{\tilde{G}\Phi\Phi_E}$	0.002
$\Gamma_{\tilde{G}\to\eta\eta'}/\Gamma^{tot}_{\tilde{G}\Phi\Phi_E}$	0.440
$\Gamma_{\tilde{G}\to\eta'\eta'}/\Gamma^{tot}_{\tilde{G}\Phi\Phi_E}$	0.249
$\Gamma_{\tilde{G}\to\eta_{SE}\eta}/\Gamma^{tot}_{\tilde{G}\Phi\Phi_E}$	0.0085
$\Gamma_{\tilde{G}\to\eta_{NE}\eta}/\Gamma^{tot}_{\tilde{G}\Phi\Phi_E}$	0.0289
$\Gamma_{\tilde{G}\to\eta_{NE}\eta'}/\Gamma^{tot}_{\tilde{G}\Phi\Phi_E}$	0.2082
$\Gamma_{\tilde{G}\to\pi\pi\sigma_{SE}}/\Gamma^{tot}_{\tilde{G}\Phi\Phi_E}$	0.00016
$\Gamma_{\tilde{G}\to a_0\pi\eta}/\Gamma^{tot}_{\tilde{G}\Phi\Phi_E}$	0.0011
$\Gamma_{\tilde{G}\to\pi\pi f_0(1370)}/\Gamma^{tot}_{\tilde{G}\Phi\Phi_E}$	0.0405
$\Gamma_{\tilde{G}\to\pi\pi f_0(1500)}/\Gamma^{tot}_{\tilde{G}\Phi\Phi_E}$	0.0209
$\Gamma_{\tilde{G}\to\pi\pi f_0(1710)}/\Gamma^{tot}_{\tilde{G}\Phi\Phi_E}$	0.0003
$\Gamma_{\tilde{G}\to KK f_0(1370)}/\Gamma^{tot}_{\tilde{G}\Phi\Phi_E}$	0.000045

W. I. Eshraim [arXiv: 1902.11148 [hep-ph]]

Decays of the first excited pseudoscalar glueball

Interaction Lagrangain for the excited pseudoscalar glueball

with a pseudoscalar glueball and the ordinary scalar and pseudoscalar mesons

$$\mathcal{L}^{int}_{\tilde{G}\tilde{G}'} = c_{\tilde{G}\tilde{G}'} \tilde{G}\tilde{G}' \, Tr\left(\Phi^{\dagger}\Phi\right)$$

with a scalar glueball and the pseudo(scalar) mesons

$$\mathcal{L}^{int}_{\tilde{G}G} = i c_{\tilde{G}G\Phi} \tilde{G} G \left(\det\Phi - \det\Phi^{\dagger}\right)$$

with scalar and pseudoscalar mesons

$$\mathcal{L}^{int}_{\tilde{G}\Phi} = i c_{\tilde{G}\Phi} \tilde{G} \left(\det\Phi - \det\Phi^{\dagger}\right)$$

Walaa I. Eshraim, Stefan Schramm, Phys.Rev. D95 (2017) 014028 [arXiv:1606.02207 [hep-ph]].

Results

Branching ratios for the decay of the excited pseudoscalar glueball into the pseudoscalar glueball

Quantity	The theoretical result
$\Gamma_{\tilde{G}\to\tilde{G}'KK}/\Gamma^{tot}_{\tilde{G}}$	0.0277
$\Gamma_{\tilde{G}\to\tilde{G}'\pi\pi}/\Gamma^{tot}_{\tilde{G}}$	0.9697
$\Gamma_{\tilde{G}\to\tilde{G}'\eta\eta'}/\Gamma^{tot}_{\tilde{G}}$	0.0026
$\Gamma_{\tilde{G}\to\tilde{G}'\eta\eta}/\Gamma^{tot}_{\tilde{G}}$	0.000012

The branching ratio for the decay of the excited pseudoscalar glueball into charmonium state

$$\Gamma_{\tilde{G}\to\eta_C\pi\pi}/\Gamma^{tot}_{\tilde{G}_3} = 0.001$$

Could be measured by BESIII and PANDA!

Walaa I. Eshraim, Stefan Schramm, Phys.Rev. D95 (2017) 014028 [arXiv:1606.02207 [hep-ph]].

Branching ratios for the decays of the excited pseudoscalar glueball into *PS* and scalar-isoscalar states as well as η and η'

Case (i):$\mathcal{L}^{int}_{\tilde{G}G}$	The theoretical result	Case (ii):$\mathcal{L}^{int}_{\tilde{G}\Phi}$	The theoretical result
$\Gamma_{\tilde{G}\to a_0\pi}/\Gamma^{tot}_{\tilde{G}_2}$	0.0325	$\Gamma_{\tilde{G}\to a_0\pi}/\Gamma^{tot}_{\tilde{G}_3}$	0.0313
$\Gamma_{\tilde{G}\to KK_S}/\Gamma^{tot}_{\tilde{G}_2}$	0.032	$\Gamma_{\tilde{G}\to KK_S}/\Gamma^{tot}_{\tilde{G}_3}$	0.001
$\Gamma_{\tilde{G}\to\eta f_0(1370)}/\Gamma^{tot}_{\tilde{G}_2}$	0.00004	$\Gamma_{\tilde{G}\to\eta f_0(1370)}/\Gamma^{tot}_{\tilde{G}_3}$	0.0014
$\Gamma_{\tilde{G}\to\eta' f_0(1370)}/\Gamma^{tot}_{\tilde{G}_2}$	0.048	$\Gamma_{\tilde{G}\to\eta' f_0(1370)}/\Gamma^{tot}_{\tilde{G}_3}$	0.031
$\Gamma_{\tilde{G}\to\eta f_0(1500)}/\Gamma^{tot}_{\tilde{G}_2}$	0.0068	$\Gamma_{\tilde{G}\to\eta f_0(1500)}/\Gamma^{tot}_{\tilde{G}_3}$	0.0067
$\Gamma_{\tilde{G}\to\eta' f_0(1500)}/\Gamma^{tot}_{\tilde{G}_2}$	0.0219	$\Gamma_{\tilde{G}\to\eta' f_0(1500)}/\Gamma^{tot}_{\tilde{G}_3}$	0.0214
$\Gamma_{\tilde{G}\to\eta f_0(1710)}/\Gamma^{tot}_{\tilde{G}_2}$	0.0008	$\Gamma_{\tilde{G}\to\eta f_0(1710)}/\Gamma^{tot}_{\tilde{G}_3}$	0.0007
$\Gamma_{\tilde{G}\to\eta' f_0(1710)}/\Gamma^{tot}_{\tilde{G}_2}$	0.001	$\Gamma_{\tilde{G}\to\eta' f_0(1710)}/\Gamma^{tot}_{\tilde{G}_3}$	0.001

Could be measured by BESIII and PANDA!

Walaa I. Eshraim, Stefan Schramm, Phys.Rev. D95 (2017) 014028 [arXiv:1606.02207 [hep-ph]].

Branching ratios for the decays of the excited pseudoscalar glueball into scalar-isoscalar states and (pseudo)scalar mesons

Case (i): $\mathcal{L}^{int}_{\tilde{G}G}$	The theoretical result	Case (ii): $\mathcal{L}^{int}_{\tilde{G}\Phi}$	The theoretical result
$\Gamma_{\tilde{G}\to\eta\pi\pi}/\Gamma^{tot}_{\tilde{G}_2}$	0.095	$\Gamma_{\tilde{G}\to\eta\pi\pi}/\Gamma^{tot}_{\tilde{G}_3}$	0.1376
$\Gamma_{\tilde{G}\to\eta'\pi\pi}/\Gamma^{tot}_{\tilde{G}_2}$	0.111	$\Gamma_{\tilde{G}\to\eta'\pi\pi}/\Gamma^{tot}_{\tilde{G}_3}$	0.1069
$\Gamma_{\tilde{G}\to a_0 K K_S}/\Gamma^{tot}_{\tilde{G}_2}$	0.0026	$\Gamma_{\tilde{G}\to a_0 K K_S}/\Gamma^{tot}_{\tilde{G}_3}$	0.0025
$\Gamma_{\tilde{G}\to\eta a_0 a_0}/\Gamma^{tot}_{\tilde{G}_2}$	0.0001	$\Gamma_{\tilde{G}\to\eta a_0 a_0}/\Gamma^{tot}_{\tilde{G}_3}$	0.0001
$\Gamma_{\tilde{G}\to a_0\pi f_0(1370)}/\Gamma^{tot}_{\tilde{G}_2}$	0.0003	$\Gamma_{\tilde{G}\to a_0\pi f_0(1370)}/\Gamma^{tot}_{\tilde{G}_3}$	0.0003
$\Gamma_{\tilde{G}\to a_0\pi f_0(1500)}/\Gamma^{tot}_{\tilde{G}_2}$	0.0034	$\Gamma_{\tilde{G}\to a_0\pi f_0(1500)}/\Gamma^{tot}_{\tilde{G}_3}$	0.0032
$\Gamma_{\tilde{G}\to a_0\pi f_0(1710)}/\Gamma^{tot}_{\tilde{G}_2}$	0.0001	$\Gamma_{\tilde{G}\to a_0\pi f_0(1710)}/\Gamma^{tot}_{\tilde{G}_3}$	0.0001
$\Gamma_{\tilde{G}\to\eta f_0^2(1370)}/\Gamma^{tot}_{\tilde{G}_2}$	0.0003	$\Gamma_{\tilde{G}\to\eta f_0^2(1370)}/\Gamma^{tot}_{\tilde{G}_3}$	0.001
$\Gamma_{\tilde{G}\to\eta' f_0^2(1370)}/\Gamma^{tot}_{\tilde{G}_2}$	0.03×10^{-6}	$\Gamma_{\tilde{G}\to\eta' f_0^2(1370)}/\Gamma^{tot}_{\tilde{G}_3}$	0.006×10^{-6}
$\Gamma_{\tilde{G}\to\eta f_0^2(1500)}/\Gamma^{tot}_{\tilde{G}_2}$	0.00004	$\Gamma_{\tilde{G}\to\eta f_0^2(1500)}/\Gamma^{tot}_{\tilde{G}_3}$	0.00001
$\Gamma_{\tilde{G}\to\eta f_0(1370) f_0(1500)}/\Gamma^{tot}_{\tilde{G}_2}$	0.00003	$\Gamma_{\tilde{G}\to\eta f_0(1370) f_0(1500)}/\Gamma^{tot}_{\tilde{G}_3}$	0.0001
$\Gamma_{\tilde{G}\to\eta f_0(1370) f_0(1710)}/\Gamma^{tot}_{\tilde{G}_2}$	3.798×10^{-6}	$\Gamma_{\tilde{G}\to\eta f_0(1370) f_0(1710)}/\Gamma^{tot}_{\tilde{G}_3}$	7.25×10^{-6}
$\Gamma_{\tilde{G}\to K K_S f_0(1370)}/\Gamma^{tot}_{\tilde{G}_2}$	0.0025	$\Gamma_{\tilde{G}\to K K_S f_0(1370)}/\Gamma^{tot}_{\tilde{G}_3}$	0.0025
$\Gamma_{\tilde{G}\to K K_S f_0(1500)}/\Gamma^{tot}_{\tilde{G}_2}$	0.00013	$\Gamma_{\tilde{G}\to K K_S f_0(1500)}/\Gamma^{tot}_{\tilde{G}_3}$	0.00013
$\Gamma_{\tilde{G}\to K K_S f_0(1710)}/\Gamma^{tot}_{\tilde{G}_2}$	6.2×10^{-6}	$\Gamma_{\tilde{G}\to K K_S f_0(1710)}/\Gamma^{tot}_{\tilde{G}_3}$	4.75×10^{-6}
$\Gamma_{\tilde{G}\to K K\eta}/\Gamma^{tot}_{\tilde{G}_2}$	0.0668	$\Gamma_{\tilde{G}\to K K\eta}/\Gamma^{tot}_{\tilde{G}_3}$	0.0643
$\Gamma_{\tilde{G}\to K K\eta'}/\Gamma^{tot}_{\tilde{G}_2}$	0.045	$\Gamma_{\tilde{G}\to K K\eta'}/\Gamma^{tot}_{\tilde{G}_3}$	0.044
$\Gamma_{\tilde{G}\to K_S K_S\eta}/\Gamma^{tot}_{\tilde{G}_2}$	0.0002	$\Gamma_{\tilde{G}\to K_S K_S\eta}/\Gamma^{tot}_{\tilde{G}_3}$	0.0002
$\Gamma_{\tilde{G}\to\eta^3}/\Gamma^{tot}_{\tilde{G}_2}$	0.024	$\Gamma_{\tilde{G}\to\eta^3}/\Gamma^{tot}_{\tilde{G}_3}$	0.0233
$\Gamma_{\tilde{G}\to\eta'^3}/\Gamma^{tot}_{\tilde{G}_2}$	0.0048	$\Gamma_{\tilde{G}\to\eta'^3}/\Gamma^{tot}_{\tilde{G}_3}$	0.0046
$\Gamma_{\tilde{G}\to\eta'\eta^2}/\Gamma^{tot}_{\tilde{G}_2}$	0.005	$\Gamma_{\tilde{G}\to\eta'\eta^2}/\Gamma^{tot}_{\tilde{G}_3}$	0.0048
$\Gamma_{\tilde{G}\to\eta'^2\eta}/\Gamma^{tot}_{\tilde{G}_2}$	0.0035	$\Gamma_{\tilde{G}\to\eta'^2\eta}/\Gamma^{tot}_{\tilde{G}_3}$	0.0034
$\Gamma_{\tilde{G}\to K K\pi}/\Gamma^{tot}_{\tilde{G}_2}$	0.489	$\Gamma_{\tilde{G}\to K K\pi}/\Gamma^{tot}_{\tilde{G}_3}$	0.471
$\Gamma_{\tilde{G}\to K_S K_S\pi}/\Gamma^{tot}_{\tilde{G}_2}$	0.002	$\Gamma_{\tilde{G}\to K_S K_S\pi}/\Gamma^{tot}_{\tilde{G}_3}$	0.0057

Walaa I. Eshraim, Stefan Schramm, Phys.Rev. D95 (2017) 014028 [arXiv:1606.02207 [hep-ph]].

Could be measured by BESIII and PANDA!

Interaction Lagrangain for the excited pseudoscalar glueball

with excited scalar and excited pseudoscalar mesons

$$\mathcal{L}^{int}_{\tilde{G}^*\phi_E} = i c_{\tilde{G}^*\Phi_E} \tilde{G}^* \left(\det\Phi_E - \det\Phi_E^\dagger \right) ,$$

with the ordinary (pseudo)scalar mesons and the excited (pseudo)scalar mesons

$$\mathcal{L}^{int}_{\tilde{G}^*\Phi\Phi_E} = c_{\tilde{G}^*\Phi\Phi_E} \tilde{G}^* \left[\left(det\Phi - det\Phi_E^\dagger \right)^2 + \left(det\Phi^\dagger - det\Phi_E \right)^2 \right]$$

Branching ratios for the two-body decays of the excited pseudoscalar glueball

Case (i): $\mathcal{L}^{int}_{\tilde{G}^*\Phi_E}$	The theoretical result [MeV]
$\Gamma_{\tilde{G}^*\to a_{0E}\pi_E}/\Gamma^{tot}_{\tilde{G}^*\Phi_E}$	0.0367
$\Gamma_{\tilde{G}^*\to K_E K_{SE}}/\Gamma^{tot}_{\tilde{G}^*\Phi_E}$	0.223
$\Gamma_{\tilde{G}^*\to \eta_{NE}\sigma_{NE}}/\Gamma^{tot}_{\tilde{G}^*\Phi_E}$	0.105
$\Gamma_{\tilde{G}^*\to \eta_{NE}\sigma_{SE}}/\Gamma^{tot}_{\tilde{G}^*\Phi_E}$	0.147
$\Gamma_{\tilde{G}^*\to \eta_{SE}\sigma_{NE}}/\Gamma^{tot}_{\tilde{G}^*\Phi_E}$	0.159

W. I. Eshraim, Phys.Rev.D 100 (2019) 9, 096007 [arXiv: 1902.11148 [hep-ph]].

Branching ratios for the two- and three-body decays of the excited pseudoscalar glueball

Quantity	The theoretical result [MeV]
$\Gamma_{\tilde{G}\to\eta\eta}/\Gamma^{tot}_{\tilde{G}\Phi\Phi_E}$	0.002
$\Gamma_{\tilde{G}\to\eta\eta'}/\Gamma^{tot}_{\tilde{G}\Phi\Phi_E}$	0.440
$\Gamma_{\tilde{G}\to\eta'\eta'}/\Gamma^{tot}_{\tilde{G}\Phi\Phi_E}$	0.249
$\Gamma_{\tilde{G}\to\eta_{S\,E}\eta}/\Gamma^{tot}_{\tilde{G}\Phi\Phi_E}$	0.0085
$\Gamma_{\tilde{G}\to\eta_{N\,E}\eta}/\Gamma^{tot}_{\tilde{G}\Phi\Phi_E}$	0.0289
$\Gamma_{\tilde{G}\to\eta_{N\,E}\eta'}/\Gamma^{tot}_{\tilde{G}\Phi\Phi_E}$	0.2082
$\Gamma_{\tilde{G}\to\pi\pi\sigma_{S\,E}}/\Gamma^{tot}_{\tilde{G}\Phi\Phi_E}$	0.00016
$\Gamma_{\tilde{G}\to a_0\pi\eta}/\Gamma^{tot}_{\tilde{G}\Phi\Phi_E}$	0.0011
$\Gamma_{\tilde{G}\to\pi\pi f_0(1370)}/\Gamma^{tot}_{\tilde{G}\Phi\Phi_E}$	0.0405
$\Gamma_{\tilde{G}\to\pi\pi f_0(1500)}/\Gamma^{tot}_{\tilde{G}\Phi\Phi_E}$	0.0209
$\Gamma_{\tilde{G}\to\pi\pi f_0(1710)}/\Gamma^{tot}_{\tilde{G}\Phi\Phi_E}$	0.0003
$\Gamma_{\tilde{G}\to KKf_0(1370)}/\Gamma^{tot}_{\tilde{G}\Phi\Phi_E}$	0.000045

W. I. Eshraim, Phys.Rev.D 100 (2019) 9, 096007 [arXiv: 1902.11148 [hep-ph]].

Branching ratios for the two-body decays of the excited pseudoscalar glueball

Case (i): $\mathcal{L}^{int}_{\tilde{G}^*\Phi\Phi_E}$	The theoretical result [MeV]
$\Gamma_{\tilde{G}^*\to\eta\eta}/\Gamma^{tot}_{\tilde{G}e^*\Phi\Phi_E}$	7.399×10^{-7}
$\Gamma_{\tilde{G}^*\to\eta\eta'}/\Gamma^{tot}_{\tilde{G}^*\Phi\Phi_E}$	0.00019
$\Gamma_{\tilde{G}^*\to\eta'\eta'}/\Gamma^{tot}_{\tilde{G}\Phi\Phi_E}$	0.00013
$\Gamma_{\tilde{G}^*\to\eta_{NE}\eta_{NE}}/\Gamma^{tot}_{\tilde{G}^*\Phi\Phi_E}$	0.000068
$\Gamma_{\tilde{G}^*\to\eta_{SE}\eta_{SE}}/\Gamma^{tot}_{\tilde{G}^*\Phi\Phi_E}$	7.16×10^{-6}
$\Gamma_{\tilde{G}^*\to\eta_{SE}\eta}/\Gamma^{tot}_{\tilde{G}^*\Phi\Phi_E}$	5.13×10^{-6}
$\Gamma_{\tilde{G}^*\to\eta_{SE}\eta_{NE}}/\Gamma^{tot}_{\tilde{G}^*\Phi\Phi_E}$	0.000044
$\Gamma_{\tilde{G}^*\to\eta_{SE}\eta'}/\Gamma^{tot}_{\tilde{G}^*\Phi\Phi_E}$	0.000062
$\Gamma_{\tilde{G}^*\to\eta_{NE}\eta}/\Gamma^{tot}_{\tilde{G}^*\Phi\Phi_E}$	0.000015
$\Gamma_{\tilde{G}^*\to\eta_{NE}\eta'}/\Gamma^{tot}_{\tilde{G}^*\Phi\Phi_E}$	0.00019
$\Gamma_{\tilde{G}^*\to\sigma_{NE}\sigma_{NE}}/\Gamma^{tot}_{\tilde{G}^*\Phi\Phi_E}$	0.000011
$\Gamma_{\tilde{G}^*\to f_0(1370)\sigma_{SE}}/\Gamma^{tot}_{\tilde{G}^*\Phi\Phi_E}$	8.2×10^{-6}
$\Gamma_{\tilde{G}^*\to f_0(1500)\sigma_{SE}}/\Gamma^{tot}_{\tilde{G}^*\Phi\Phi_E}$	9.18×10^{-8}
$\Gamma_{\tilde{G}^*\to f_0(1370)\sigma_{NE}}/\Gamma^{tot}_{\tilde{G}^*\Phi\Phi_E}$	0.00003
$\Gamma_{\tilde{G}^*\to f_0(1500)\sigma_{NE}}/\Gamma^{tot}_{\tilde{G}^*\Phi\Phi_E}$	0.000022
$\Gamma_{\tilde{G}^*\to f_0(1710)\sigma_{NE}}/\Gamma^{tot}_{\tilde{G}^*\Phi\Phi_E}$	6.54×10^{-7}
$\Gamma_{\tilde{G}^*\to f_0(1370)f_0(1370)}/\Gamma^{tot}_{\tilde{G}^*\Phi\Phi_E}$	0.000012
$\Gamma_{\tilde{G}^*\to f_(1500)f_0(1500)}/\Gamma^{tot}_{\tilde{G}^*\Phi\Phi_E}$	7.99×10^{-7}
$\Gamma_{\tilde{G}^*\to f_(1710)f_0(1710)}/\Gamma^{tot}_{\tilde{G}^*\Phi\Phi_E}$	2.81×10^{-10}
$\Gamma_{\tilde{G}^*\to f_0(1370)f_0(1500)}/\Gamma^{tot}_{\tilde{G}^*\Phi\Phi_E}$	0.000027
$\Gamma_{\tilde{G}^*\to f_0(1370)f_0(1710)}/\Gamma^{tot}_{\tilde{G}^*\Phi\Phi_E}$	4.71×10^{-7}
$\Gamma_{\tilde{G}^*\to f_0(1500)f_0(1710)}/\Gamma^{tot}_{\tilde{G}^*\Phi\Phi_E}$	1.99×10^{-6}

W. I. Eshraim, Phys.Rev.D 100 (2019) 9, 096007 [arXiv: 1902.11148 [hep-ph]].