Full NLO QCD corrections to Higgs-pair production in the Standard Model and beyond

PANIC Lisbon 2021 Conference [remote-only]

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Julien Baglio

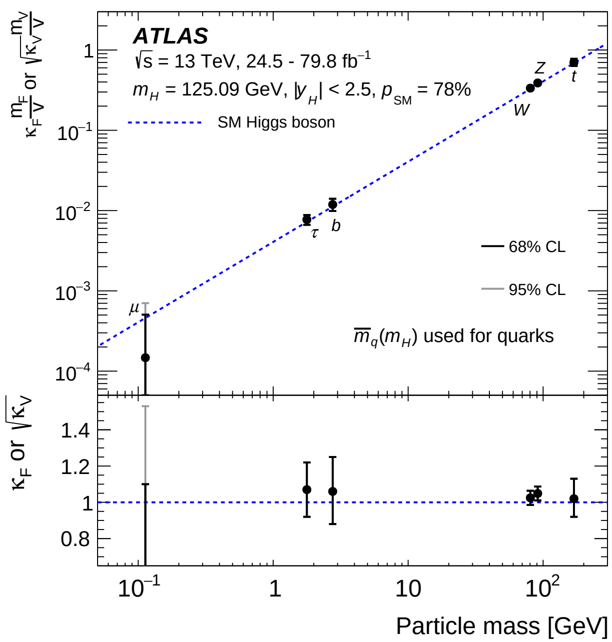
[with F. Campanario, S. Glaus, M. M. Mühlleitner, J. Ronca, and M. Spira,

EPJC 79 (2019) 459; JHEP 04 (2020) 181; Phys. Rev. D 103 (2021) 056002; arXiv:2021.XXXX (to appear)]

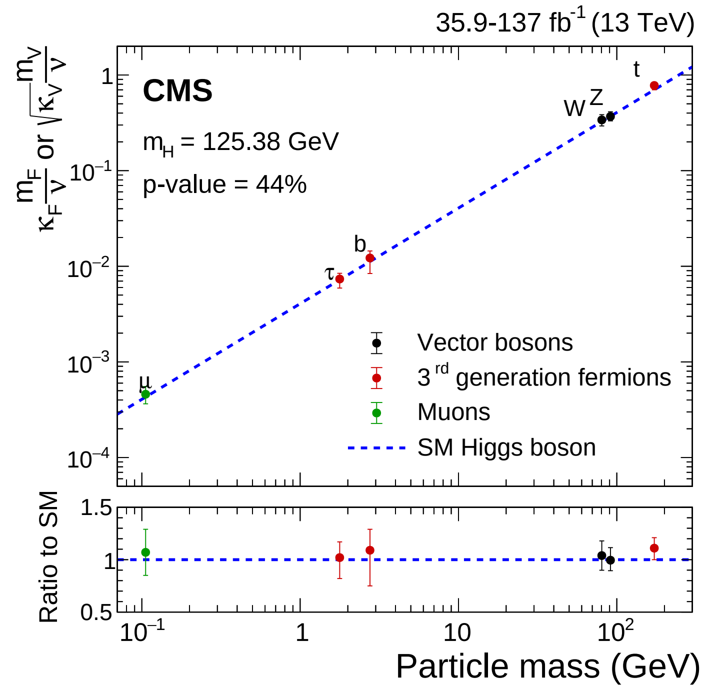
Higgs physics facts in 2021

4/7/2012: CERN presents the discovery of a bosonic particle with Higgs-boson-like properties [ATLAS, PLB 716 (2012) 1; CMS *ibid* 30]

- Couplings compatible with a Higgs boson



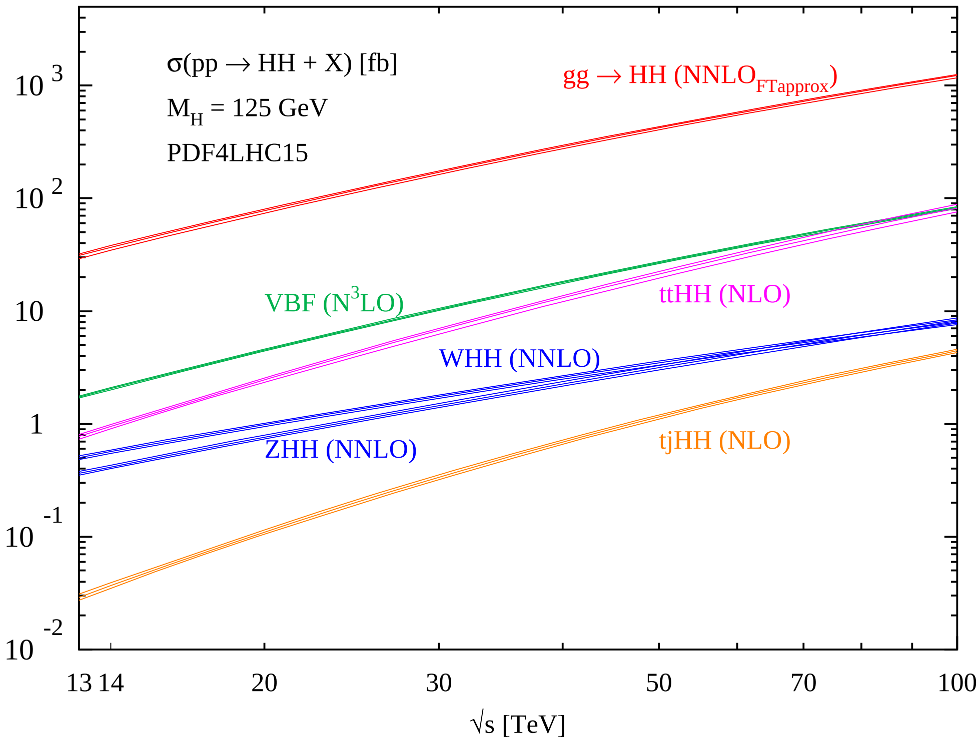
[ATLAS, PRD 101 (2020) 012002]



[CMS, JHEP 01 (2021) 148]

- **Still unknown: Higgs boson self-couplings**

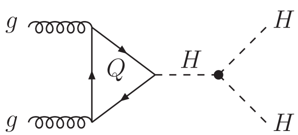
Overview of *HH* production channels

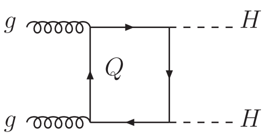


[from Di Micco *et al*, arXiv:1910.00012]

Gluon fusion the largest cross section

Leading order (LO) QCD already loop-induced



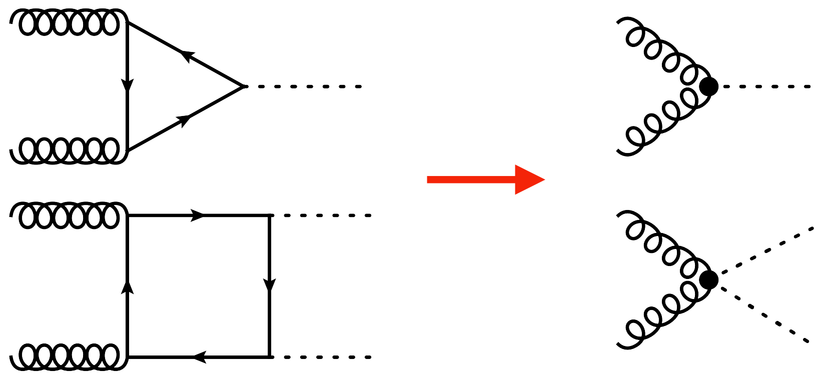


Heavy top-quark limit (HTL) calculation

Gluon fusion: main production channel, top-quark loops dominant [Eboli, Marques, Novaes, Natale, PLB 197 (1987) 269; Glover, van der Bij, NPB 309 (1988) 282; Dicus, Kao, Willenbrock, PLB 203 (1988) 457]

HTL $\equiv m_t \to +\infty$

$\to$ Effective tree-level *ggH* and *ggHH* couplings
$\to$ Reduce the number of loop by one at each perturbative order



- HTL valid for $\hat{s} \ll 4m_t^2$, but *HH* production threshold $4M_H^2 \leq \hat{s}$
  $\Rightarrow$ narrow energy range for which HTL is valid!
- **Born-improved NLO QCD HTL:** improve HTL result with

$$d\sigma_{\rm NLO} \simeq d\sigma_{\rm NLO}^{\rm HTL} \times \frac{d\sigma_{\rm LO}^{\rm full}}{d\sigma_{\rm LO}^{\rm HTL}}$$ [Dawson, Dittmaier, Spira, PRD 58 (1998) 115012]

Gluon fusion: Status in 2021

- **NLO QCD HTL (1-loop):** $+93\%$ correction [Dawson, Dittmaier, Spira, PRD 58 (1998) 115012]
- **NNLO QCD HTL (2-loop):** $+20\%$ for total xs [De Florian, Mazzitelli, PRL 111 (2013) 201801]
- **N^3LO QCD (3-loop):** $+3\%$ for total xs [Chen, Li, Shao, Wang, PLB 803 (2020) 135292]
- **Beyond HTL:**

  $\to$ NLOFT$_{\text{approx}}$, m_t-effects in real radiation: -10%
  [Frederix *et al*, PLB 732 (2014) 079, Maltoni, Vryonidou, Zaro, JHEP 11 (2014) 079]

  $\to$ $\mathcal{O}(1/m_t^{12})$ terms in virtuals: $\pm 10\%$ [see e.g. Grigo, Hoff, Steinhauser, NPB 900 (2015) 412]

  $\to$ **full NLO QCD**: -15% mass effects [Borowka *et al*, PRL 117 (2016) 012001 & JHEP 10 (2016) 107; **J.B.**, *et al*, EPJC 79 (2019) 459 & JHEP 04 (2020) 181]

  $\to$ **NNLO FT$_{\text{approx}}$:** full NLO QCD + NNLO HTL + NNLO exact reals
  $\Rightarrow$ $+10\%$ to $+20\%$ in distributions [Grazzini *et al*, JHEP 05 (2018) 059]

  $\to$ **N^3LO beyond HTL:** full NLO QCD convoluted with N^3LO HTL $\Rightarrow +3\%$ on top of the NNLO FT$_{\text{approx}}$ [Chen, Li, Shao, Wang, JHEP 03 (2020) 072]

  $\to$ **Various analytical approximations:** Padé approximants, p_T^2-expansion, high-energy expansion [Gröber, Maier, Rauh JHEP 03 (2018) 020; Bonciani, Degrassi, Giardino, Gröber, PRL 121 (2018) 162003; Davies, Mishima, Steinhauser, Wellmann, JHEP 01 (2019) 176; Wang *et al*, arXiv:2010.15649]

Full NLO QCD corrections to $gg \to HH$

Two independent calculations on the market:

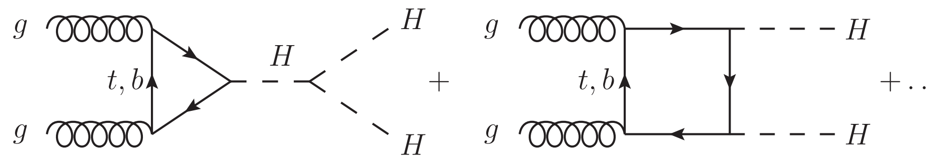
- Reduction to master integrals, sector decomposition, contour deformation [Borowka *et al*, PRL 117 (2016) 012001; JHEP 10 (2016) 107]
  - $\to$ Large mass effects in the tail up to $\sim -30\%$ w.r.t. HTL
  - $\to$ Born-improved HTL outside full NLO scale variation for $m_{HH} > 400$ GeV
  - $\to$ No quantitative statement on the top-quark scheme uncertainty

- Direct integration of the tensor integrals, integration-by-part, Richardson extrapolation [**J.B.**, Campanario, Glaus, Mühlleitner, Spira, Streicher, EPJC 79 (2019) 459; **J.B.**, Campanario, Glaus, Mühlleitner, Ronca, Spira, Streicher, JHEP 04 (2020) 181]
  - $\to$ **Same findings for the total cross section and invariant mass distributions**
  - $\to$ **Detailed study of the top-mass scale-and-scheme uncertainty**

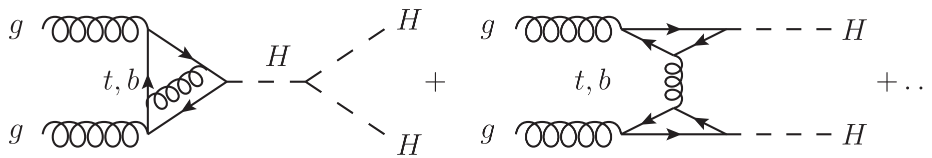
Focus on the second method

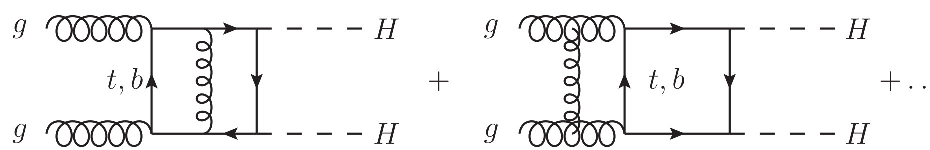
Overview of the calculation

$$\sigma_{\rm NLO}(pp\to HH+X) = \sigma_{\rm LO} + \Delta\sigma^{(1)}_{\rm virt} + \Delta\sigma^{(2)}_{\rm virt} + \Delta\sigma_{gg} + \Delta\sigma_{gq} + \Delta\sigma_{q\bar{q}}$$

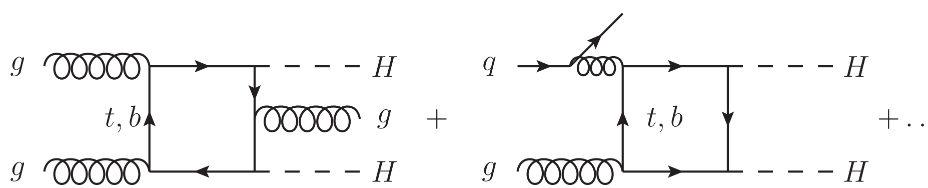


- **1-loop LO $\sigma_{\rm LO}$:**
- **2-loop triangle + 1-particle reducible $\Delta\sigma^{(1)}_{\rm virt}$:**





- **2-loop box $\Delta\sigma^{(2)}_{\rm virt}$:**



- **1-loop reals $\Delta\sigma_{ij}$:**

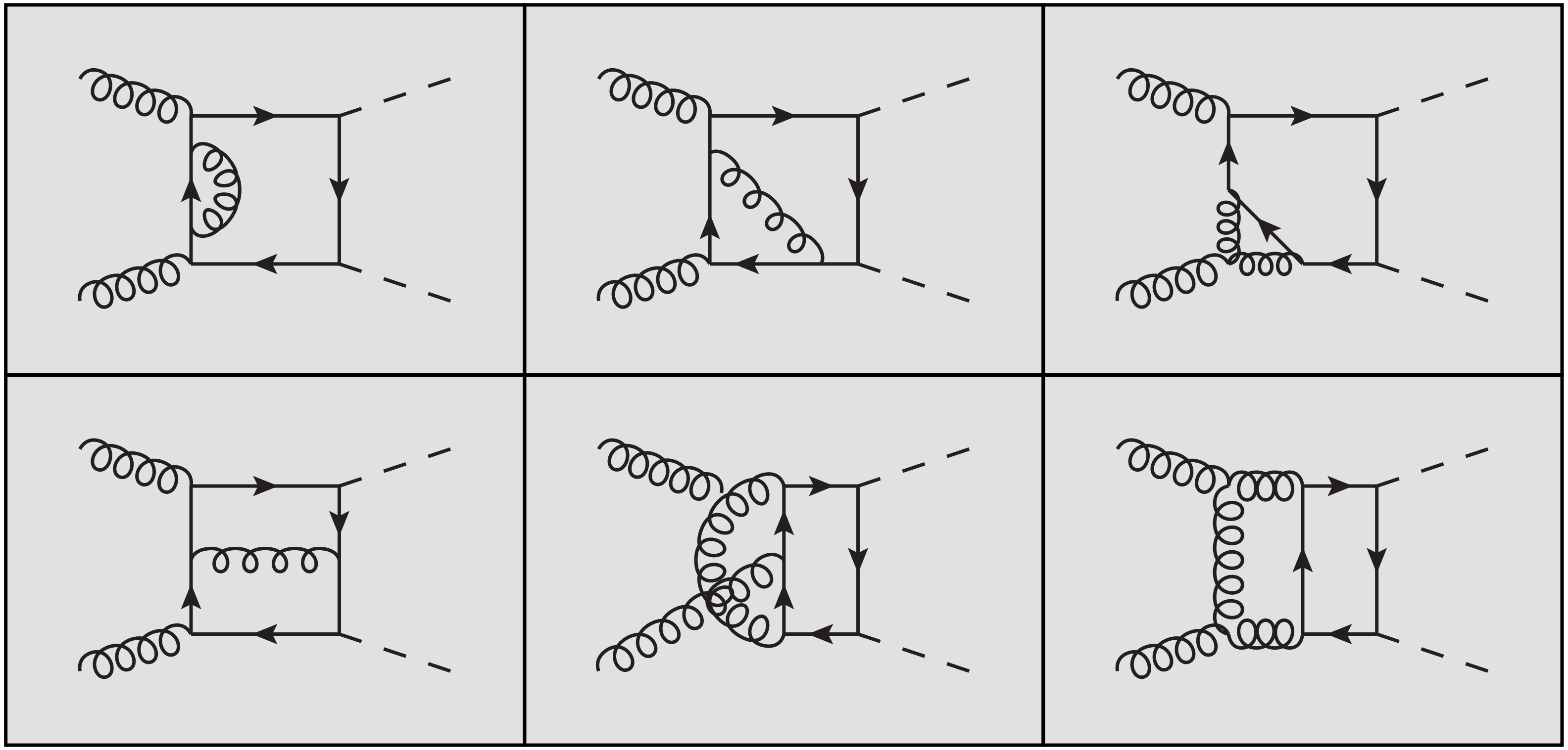
Technical setup for the virtual corrections

- Triangle from single Higgs, 1-particle reducible analytically calculated [see also Degrassi, Giardino, Gröber, EPJC 76 (2016) 411] and can be obtained from $H \to Z\gamma$ calculation
- Classification of the **47 2-loop tensor box diagrams** into 6 topologies (+ corresponding fermion-flow reversed diagrams)
- With dimensional regularization $D = 4 - 2\epsilon$:
  $\to$ UV divergences extracted with end-point subtractions
  $\to$ infrared divergences extracted with suitable analytical subtraction term in the integrand
  $\to$ Threshold at $\hat{s} = m_{HH}^2 = 4m_t^2$: analytical continuation with

$$m_t^2 \to m_t^2\,(1 - i\tilde{\epsilon})\,, \quad \tilde{\epsilon} \ll 1$$

  Richardson extrapolation to obtain the narrow-width limit $\tilde{\epsilon} \to 0$ for m_t
  $\to$ Numerical instabilities: use integration-by-parts to reduce the power of denominators
- **Perform Feynman parametrization**
  $\to$ 6-dimensional integrals to be (numerically) evaluated

2-loop virtual box corrections



Putting everything together

- Numerical integration performed with `VEGAS` on a cluster, $\hat{t}$-integration
- Real corrections calculated with `FeynArts` and `FormCalc`
- **Final hadronic result:**

$$\Delta\hat{\sigma}_{\text{virt}} = \int d\Phi_{2\to 2}\left[(\delta_{\alpha_s} + \delta_g + \delta_{m_t} + \delta_{\mathrm{IR}} + \mathcal{M}^{\Box}_{\text{virt}})(\mathcal{M}_{\mathrm{LO}})^*\right] + \Delta\hat{\sigma}^{\triangle}_{\text{virt}} + \Delta\hat{\sigma}^{\mathrm{1PR}}_{\text{virt}}$$

With $Q^2 = m^2_{HH}$:

$$Q^2\frac{d\Delta\sigma_{\text{virt}}}{dQ^2} = \tau\frac{d\mathcal{L}^{gg}}{d\tau}\Delta\hat{\sigma}_{\text{virt}}(Q^2)\Big|_{\tau=\frac{Q^2}{s}}$$

$\frac{d\mathcal{L}^{gg}}{d\tau} \equiv$ gluon parton density

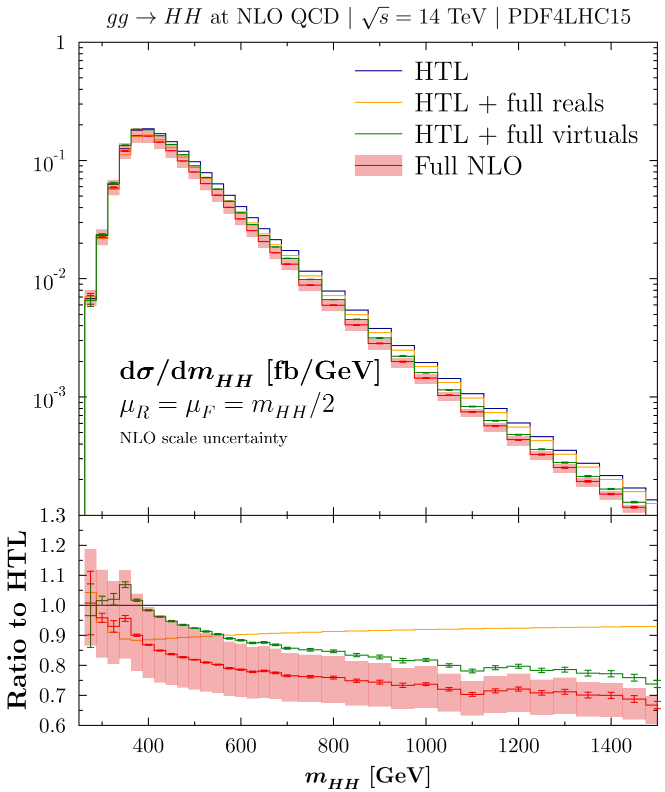
$$Q^2\frac{d\sigma_{\mathrm{NLO}}}{dQ^2} = Q^2\frac{d\sigma_{\mathrm{HPAIR}}}{dQ^2} + Q^2\frac{d\Delta\sigma_{\text{virt}}}{dQ^2} + Q^2\frac{d\Delta\sigma_{\text{reals}}}{dQ^2}$$

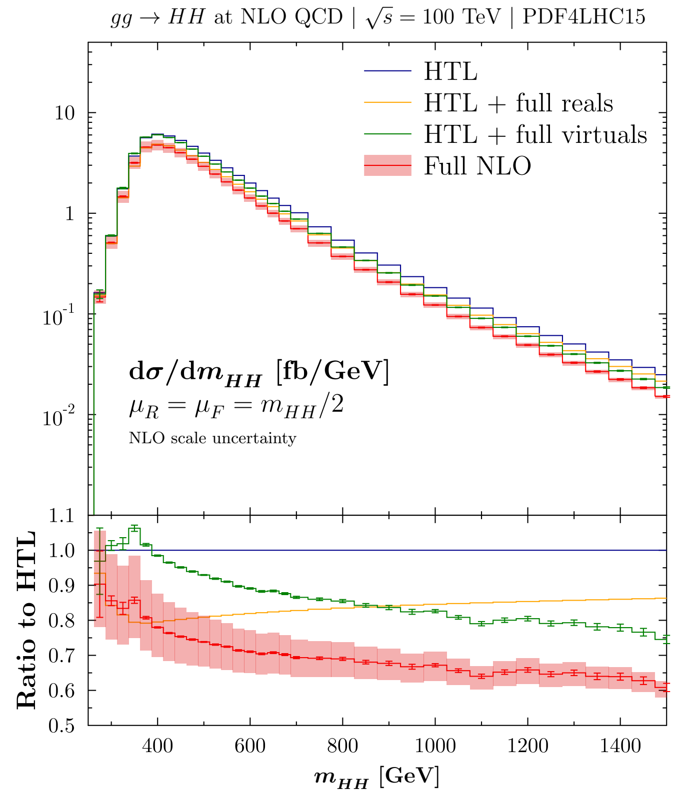
HTL hadronic result calculated with `HPAIR` [Spira, 1996]

- **Input parameters: can be freely chosen!** PDG values for M_W and M_Z,

$M_H = 125$ GeV, $m_t = 172.5$ GeV, $G_F = 1.1663787 \times 10^{-5}$ GeV^{-2}

SM differential cross section

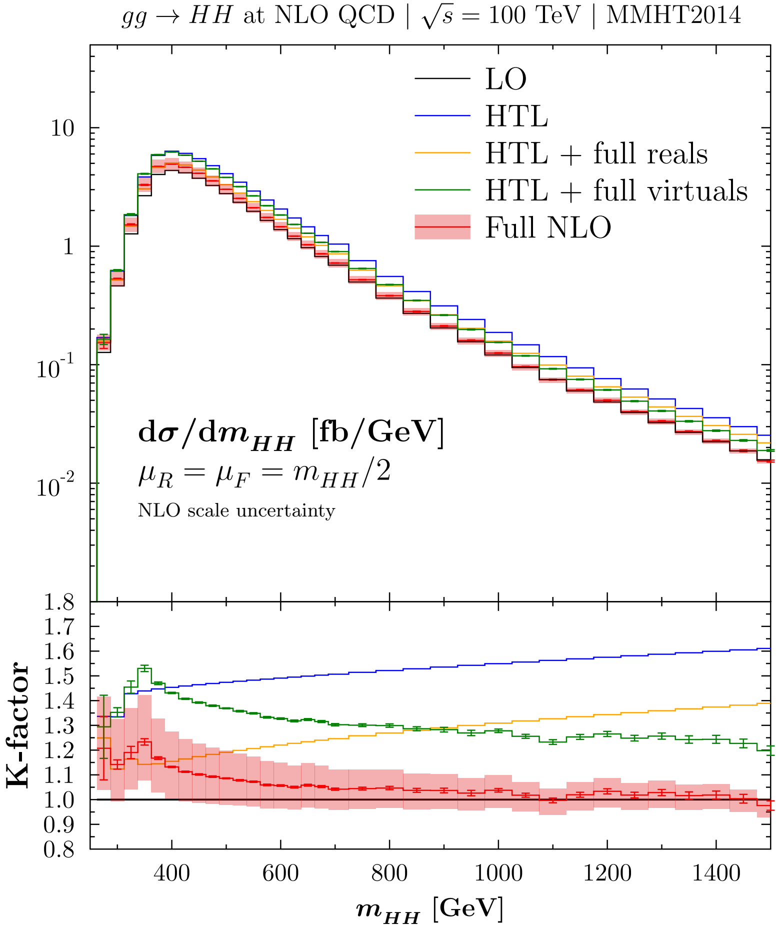




- Mass effects in the real corrections $\sim -10\%$ as in [Maltoni, Vryonidou, Zaro, JHEP 11 (2014) 079]
- Mass effects in the virtual corrections $\sim -25\%$ at $m_{HH} = 1$ TeV
- HTL results outside the scale variation band (in red) of the full results

Total cross section with scale uncertainty

Differential xs with K-factors



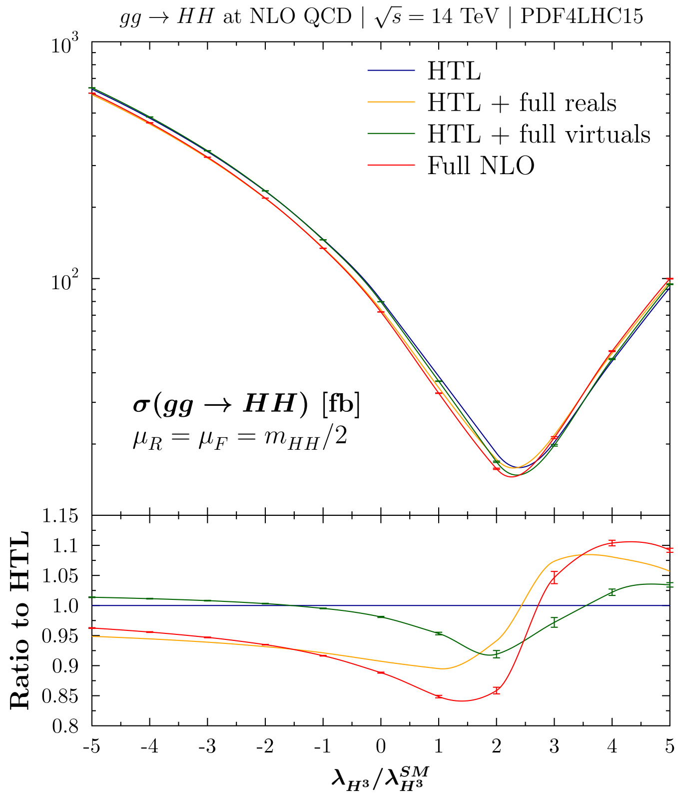
$$K = \frac{\sigma^{\mathrm{NLO}}}{\sigma^{\mathrm{LO}}}$$

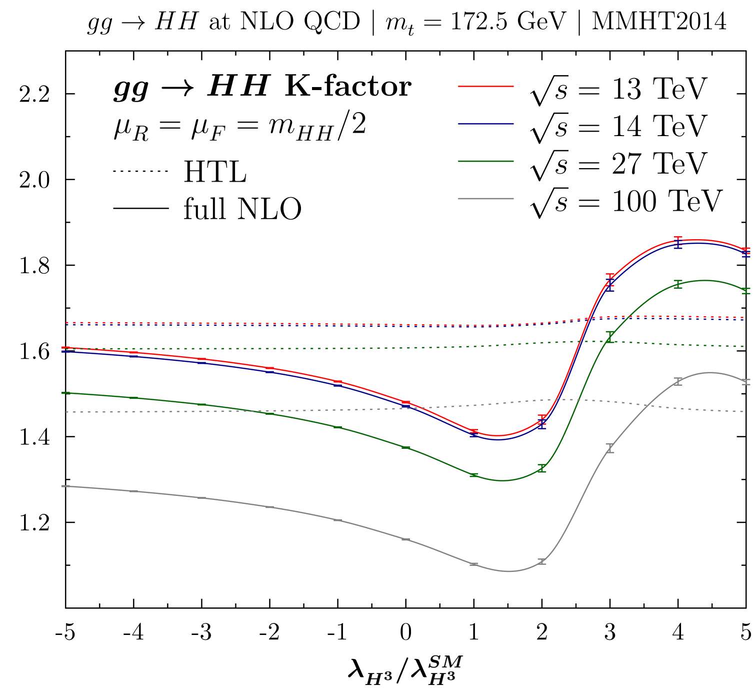
Total hadronic xs

Energy	m_t = 172.5 GeV
13 TeV	$27.73(7)^{+13.8\%}_{-12.8\%}$ fb
14 TeV	$32.81(7)^{+13.5\%}_{-12.5\%}$ fb
27 TeV	$127.0(2)^{+11.7\%}_{-10.7\%}$ fb
100 TeV	$1140(2)^{+10.7\%}_{-10.0\%}$ fb

(using PDF4LHC PDFs, central scale $\mu_R = \mu_F = m_{HH}/2$)

Variation of the triple Higgs coupling





- Minimum of the cross section shifted from $\lambda/\lambda_{\text{SM}} = 2.4$ to 2.3 due to mass effects in the real corrections
- *K*-factors vary a lot over the $\lambda/\lambda_{\text{SM}}$ range $\Rightarrow$ **mass effects have significant impact on the extraction of λ_{HHH}**

Top-mass scale-and-scheme uncertainties

- Top-quark mass can be renormalized in the on-shell (OS) scheme or in the $\overline{\text{MS}}$ scheme
- In the $\overline{\text{MS}}$ scheme: What scale choice for $\overline{m}_t(\mu_t)$?

$\neq$ choices $\Rightarrow$ $\neq$ results!

Envelop of the $\neq$ results $\equiv$
top-quark scale-and-scheme uncertainty

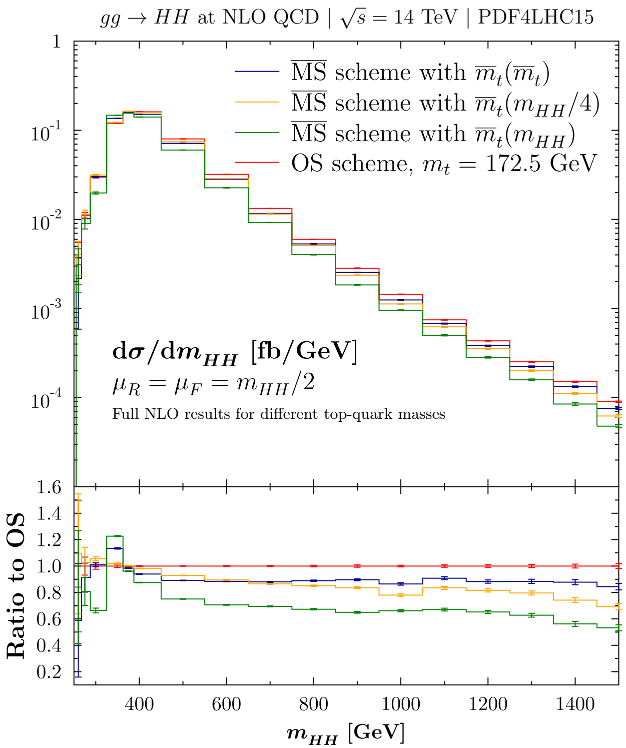
$\rightarrow$ At LO QCD: only parametric dependence on m_t

$\rightarrow$ At NLO QCD and beyond: logarithmic dependence on m_t in the virtual (and virtual-reals, etc) corrections

$\rightarrow$ How to cancel this dependence, and reduce the uncertainties?

NLO uncertainties in differential distributions

- Switch to $\overline{\text{MS}}$ scheme, with $\rightarrow$ modification of the mass counterterm
- Compare the predictions with OS m_t, $\overline{m}_t(\overline{m}_t)$, $\overline{m}_t(\mu_t)$ with $Q/4 \leq \mu_t \leq Q$, take the envelop $\rightarrow$ our uncertainty



$\sqrt{s} = 14$ TeV:

$$\left.\frac{d\sigma}{dQ}\right|_{Q=300\text{GeV}} = 0.02978(7)^{+6\%}_{-34\%}\ \text{fb/GeV}$$

$$\left.\frac{d\sigma}{dQ}\right|_{Q=400\text{GeV}} = 0.1609(4)^{+0\%}_{-13\%}\ \text{fb/GeV}$$

$$\left.\frac{d\sigma}{dQ}\right|_{Q=600\text{GeV}} = 0.03204(9)^{+0\%}_{-30\%}\ \text{fb/GeV}$$

$$\left.\frac{d\sigma}{dQ}\right|_{Q=1200\text{GeV}} = 0.000435(4)^{+0\%}_{-35\%}\ \text{fb/GeV}$$

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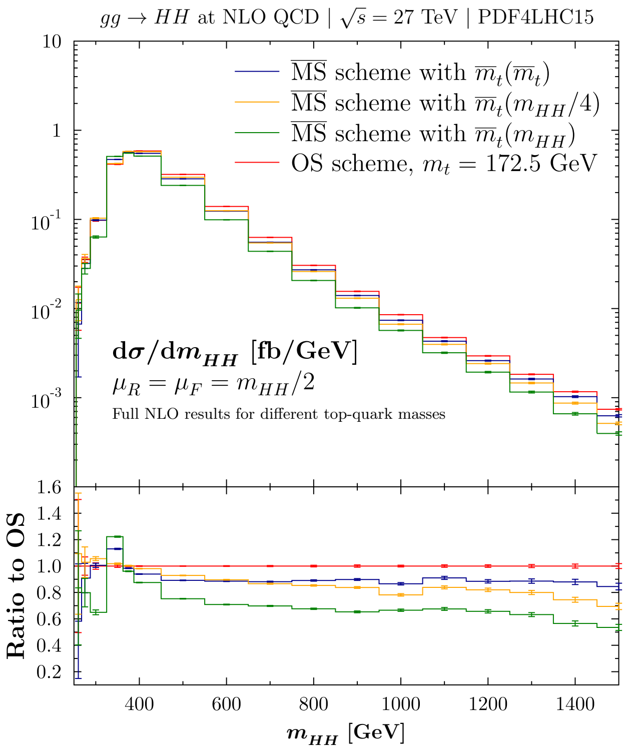

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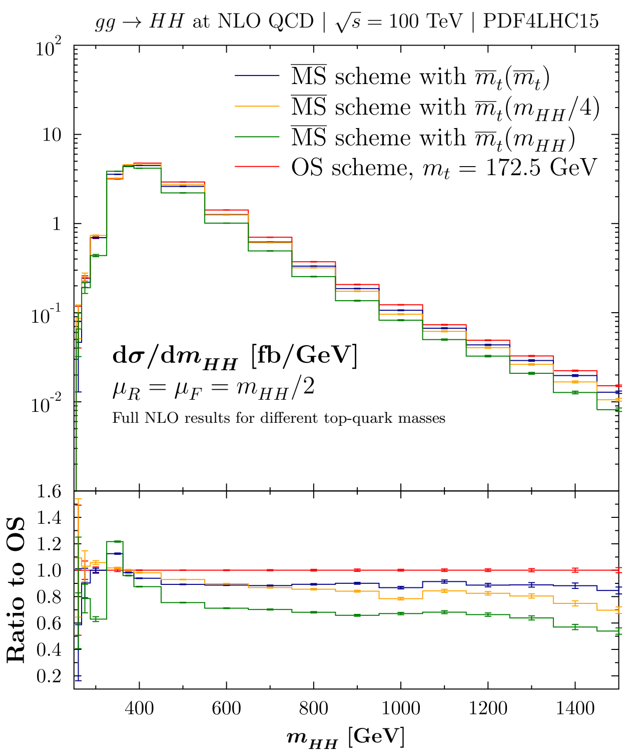

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Uncertainty on the total cross section

Take for individual Q values the maximum / minimum differential cross section and integrate

$$\sigma^{\rm NLO}_{13\ {\rm TeV}}(gg \to HH) = 27.73(7)^{+4\%}_{-18\%}\ {\rm fb}$$

$$\sigma^{\rm NLO}_{14\ {\rm TeV}}(gg \to HH) = 32.81(7)^{+4\%}_{-18\%}\ {\rm fb}$$

$$\sigma^{\rm NLO}_{27\ {\rm TeV}}(gg \to HH) = 127.0(2)^{+4\%}_{-18\%}\ {\rm fb}$$

$$\sigma^{\rm NLO}_{100\ {\rm TeV}}(gg \to HH) = 1140(2)^{+3\%}_{-18\%}\ {\rm fb}$$

Sizable uncertainty comparable to the usual factorization/renormalization scale uncertainty

Combination of uncertainties

How to combine usual scale uncertainty with m_t scale-and-scheme uncertainty?

- Envelope of all uncertainties: vary at the same time μ_R, μ_F, μ_t. **Found equivalent to a *linear* addition of the relative errors!**

- Relative errors are scaling with universal factors
$\Rightarrow$ we can combine the scale-and-scheme uncertainty (at NLO) with the scale uncertainty (at NNLO): universal scaling

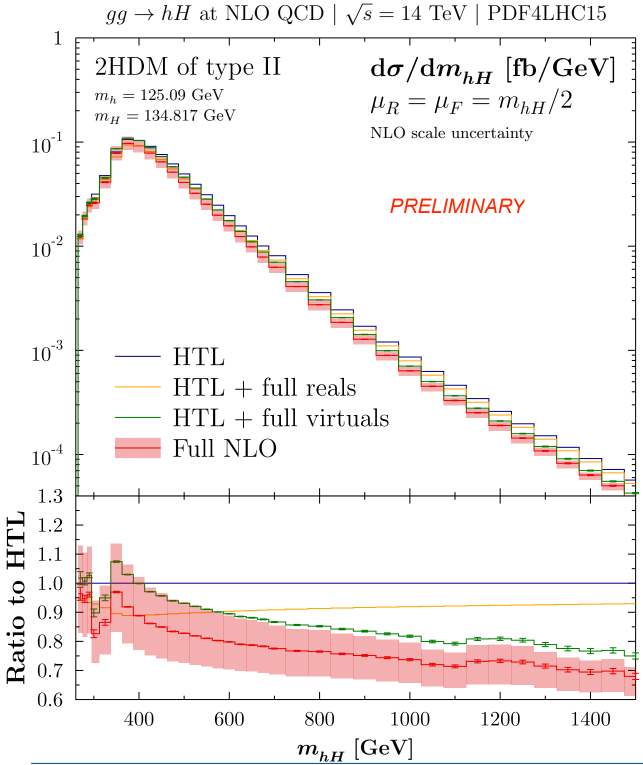
State-of-the-art recommended prediction

$$\sigma^{\rm NNLO}_{\rm 14\ TeV}(gg \to HH) = 36.69\ ^{+6\%}_{-23\%}\ \rm fb$$

Preliminary results in 2HDM of type II

Two-Higgs-Doublet Model (2HDM): 5 physical Higgs bosons with **2 CP-even** *h* and *H*, **1 CP-odd** *A* and **2 charged Higgs bosons** $H^{\pm}$

- 2HDM of type II: H_u couples to up-type fermions, H_d to down-type fermions
- Benchmark point compatible with experimental constraints: $h \equiv H_{obs}$ with $m_h = 125.09$ GeV; H with $m_H = 134.817$ GeV; $\tan\beta = v_{H_u}/v_{H_d} = 3.23375$



- Ratios of 2HDM couplings to SM couplings: $g_{htt} = 0.64939$, $g_{Htt} = -0.34069$, $g_{hhH} = -12.511$, $g_{hHH} = 79.528$
- Total cross section: $\sigma(hH) = 18.53(2)^{+14.0\%}_{-12.7\%}$ fb; 2HDM results comparable to SM results

Conclusions and outlook

- **Calculation of the two-loop integrals of $gg \to HH$ with three mass scales without reduction to master integrals**

  $\to$ Results obtained in the OS scheme and in the $\overline{\text{MS}}$ scheme, scale uncertainty $\sim \pm 10 - 15\%$

  $\to$ Large NLO top-quark mass effects, $\sim -15\%$ in the total cross section

  $\to$ Extraction of λ_{HHH} sizably impacted by the top-quark mass effects

- **Sizable top-quark scale-and-scheme uncertainty:**
  $\sim 30\%$ at large Q, $\sim 21\%$ on the total cross section

$$\sigma^{\text{NLO}}_{14\text{ TeV}}(gg \to HH) = 32.81(7)\ ^{+13.5\%}_{-12.5\%}\ (\mu_R, \mu_F)\ ^{+4\%}_{-18\%}\ (\mu_t)\ \text{fb}$$

$$\sigma^{\text{NNLO}}_{14\text{ TeV}}(gg \to HH) = 36.69\ ^{+6\%}_{-23\%}\ (\mu_R, \mu_F, \mu_t)\ \text{fb}$$

- Top-quark scale-and-scheme uncertainty sizable in a variety of processes

  $\to$ Issue not only for *HH* production

  $\to$ **Full NNLO calculation required to decrease the uncertainty: Tough!**

- **Outlook: Release of 2HDM results for *hH* and *AA* production; study of *hA* production and bottom-quark loop**

Backup slides

2-loop virtual box corrections

- **Extraction of ultraviolet (UV) divergences:**
  Endpoint subtraction of the Feynman integrals

$$\int_0^1 dx\, \frac{f(x)}{(1-x)^{1-\epsilon}} = \frac{f(1)}{\epsilon} + \int_0^1 dx\, \frac{f(x)-f(1)}{1-x} + \mathcal{O}(\epsilon)$$

- **Infrared (IR) divergences in the middle of the range**
  $\Rightarrow$ Subtraction of the integrand and analytical integration

  - Generic denominator $N = ar^2 + br + c$, $N_0 = br + c$
  - Singular infrared behavior in the limit $r \to 0$
  - $a, c = \mathcal{O}\left(1/m_t^2\right), \quad b = 1 + \mathcal{O}\left(1/m_t^2\right)$

$$\int_0^1 dxdr\, \frac{rH(x,r)}{N^{3+2\epsilon}} = \int_0^1 dxdr \left[\left(\frac{rH(x,r)}{N^{3+2\epsilon}} - \frac{rH(x,0)}{N_0^{3+2\epsilon}}\right) + \frac{rH(x,0)}{N_0^{3+2\epsilon}}\right]$$

Handling threshold instabilities

- **Threshold at $\hat{s} = m_{HH}^2 = 4m_t^2$:**
  $\Rightarrow$ Analytical continuation in the complex plane with

$$m_t^2 \to m_t^2\,(1 - i\tilde{\epsilon})\,, \quad \tilde{\epsilon} \ll 1$$

- **Enhance stability above threshold with integration by parts** Example with $N = a + bx$:

$$\int_0^1 dx\,\frac{2b\,f(x)}{N^3} = \frac{f(0)}{a^2} - \frac{f(1)}{(a+b)^2} + \int_0^1 dx\,\frac{f'(x)}{N^2}$$

- For b-quark loop, same game but with more integration by parts (b-quark loop left for future work)

Details for the renormalization

- **UV renormalization:** δ_{α_s}, δ_g, δ_{m_t}

  → $\overline{\text{MS}}$ renormalization for α_s with 5 active flavors $N_F = 5$

$$\frac{\delta\alpha_s}{\alpha_s} = \frac{\alpha_s}{\pi}\Gamma(1+\epsilon)\,(4\pi)^\epsilon\left[-\frac{33-2(N_F+1)}{12\epsilon} + \frac{1}{6}\log\left(\frac{\mu_R^2}{m_t^2}\right)\right], \qquad \delta_{\alpha_s} = \frac{\delta\alpha_s}{\alpha_s}\mathcal{M}_{\rm LO}$$

  → Top-quark contribution to the external gluons self-energies

$$\delta_g = \frac{\alpha_s}{\pi}\Gamma(1+\epsilon)\left(\frac{4\pi\mu_R^2}{m_t^2}\right)^\epsilon\left(-\frac{1}{6\epsilon}\right)\mathcal{M}_{\rm LO}$$

  → On-shell renormalization for m_t

$$\frac{\delta m_t}{m_t} = \frac{\alpha_s}{\pi}\Gamma(1+\epsilon)\left(\frac{4\pi\mu_R^2}{m_t^2}\right)^\epsilon\left(\frac{1}{\epsilon}+\frac{4}{3}\right), \qquad \delta_{m_t} = -2\frac{\delta m_t}{m_t}m_t^2\frac{\partial\mathcal{M}_{\rm LO}}{\partial m_t^2}$$

- **IR subtraction:**

$$\delta_{\rm IR} = \frac{\alpha_s}{\pi}\frac{\Gamma(1-\epsilon)}{\Gamma(1-2\epsilon)}\left(\frac{4\pi\mu_R^2}{-m_{HH}^2}\right)^\epsilon\left[\frac{3}{2\epsilon^2} + \frac{33-2N_F}{12\epsilon}\left(\frac{\mu_R^2}{-m_{HH}^2}\right)^{-\epsilon} - \frac{11}{4} + \frac{\pi^2}{4}\right]\mathcal{M}_{\rm LO}$$

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$$\delta_g = \frac{\alpha_s}{\pi}\Gamma(1+\epsilon)\left(\frac{4\pi\mu_R^2}{m_t^2}\right)^\epsilon\left(-\frac{1}{6\epsilon}\right)\mathcal{M}_{\rm LO}$$

  → $\overline{\text{MS}}$ renormalization for m_t

$$\frac{\delta m_t}{m_t} = \frac{\alpha_s}{\pi}\Gamma(1+\epsilon)\left(\frac{4\pi\mu_R^2}{m_t^2}\right)^\epsilon\left(\frac{1}{\epsilon} - \log\left(\frac{\mu_{R,t}^2}{m_t^2}\right)\right), \quad \delta_{m_t} = -2\frac{\delta m_t}{m_t}m_t^2\frac{\partial\mathcal{M}_{\rm LO}}{\partial m_t^2}$$

- **IR subtraction:**

$$\delta_{\rm IR} = \frac{\alpha_s}{\pi}\frac{\Gamma(1-\epsilon)}{\Gamma(1-2\epsilon)}\left(\frac{4\pi\mu_R^2}{-m_{HH}^2}\right)^\epsilon\left[\frac{3}{2\epsilon^2} + \frac{33-2N_F}{12\epsilon}\left(\frac{\mu_R^2}{-m_{HH}^2}\right)^{-\epsilon} - \frac{11}{4} + \frac{\pi^2}{4}\right]\mathcal{M}_{\rm LO}$$

Analytical integration of the infrared regulator

In topology 6:
Start from the form subtracted form factors F_i,

$$\Delta F_i = \frac{\alpha_s}{\pi}\,\Gamma(1+2\epsilon)\left(\frac{4\pi\mu_0^2}{m_t^2}\right)^{2\epsilon}(G_1+G_2)\,,$$

$$G_1 = \int_0^1 d^6x\; x^{1+\epsilon}(1-x)^\epsilon r^{1+\epsilon} s^{-\epsilon}\left\{\frac{H_i(\vec{x})}{N^{3+2\epsilon}(\vec{x})} - \frac{H_i(\vec{x})|_{r=0}}{N_0^{3+2\epsilon}(\vec{x})}\right\}\,,$$

$$G_2 = \int_0^1 d^6x\; x^{1+\epsilon}(1-x)^\epsilon r^{1+\epsilon} s^{-\epsilon}\frac{H_i(\vec{x})|_{r=0}}{N_0^{3+2\epsilon}(\vec{x})}$$

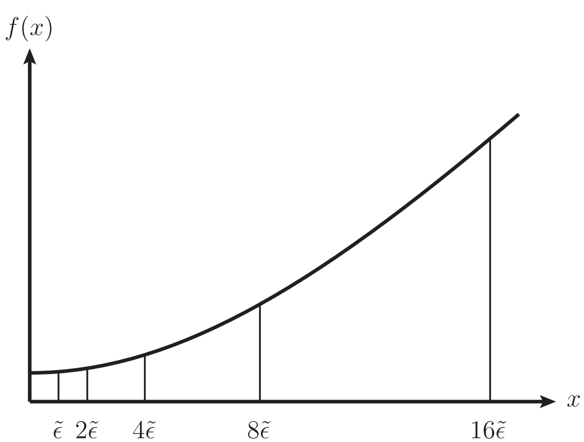
with $N(\vec{x}) = ar^2 + br + c$, $N_0(\vec{x}) = br + c$, $a, c = \mathcal{O}\left(1/m_t^2\right)$, $b = 1 + \mathcal{O}\left(1/m_t^2\right)$.

Analytical integration of G_2 gives rise to hypergeometric functions

$$G_2 = \frac{1}{2+\epsilon}\int_0^1 d^5x\,\frac{x^{1+\epsilon}(1-x)^\epsilon s^{-\epsilon}}{c^{3+2\epsilon}}\;{}_2F_1\left(3+2\epsilon, 2+\epsilon; 3+\epsilon; -\frac{b}{c}\right) H_i(\vec{x})\Big|_{r=0}$$

Richardson extrapolation

- **Goal: From $m_t^2(1 - i\tilde{\epsilon})$, obtain the limit $\tilde{\epsilon} \to 0$**
- **Solution: Richardson extrapolation of the result!**
  Assuming $f(\tilde{\epsilon}) - f(0)$ polynomial for small $\tilde{\epsilon}$, method to accelerate the convergence of $f(\tilde{\epsilon})$ to $f(0)$



$$\text{RiEx}_{2,\tilde{\epsilon}} = 2f(\tilde{\epsilon}) - f(2\tilde{\epsilon}) = f(0) + \mathcal{O}(\tilde{\epsilon}^2)$$

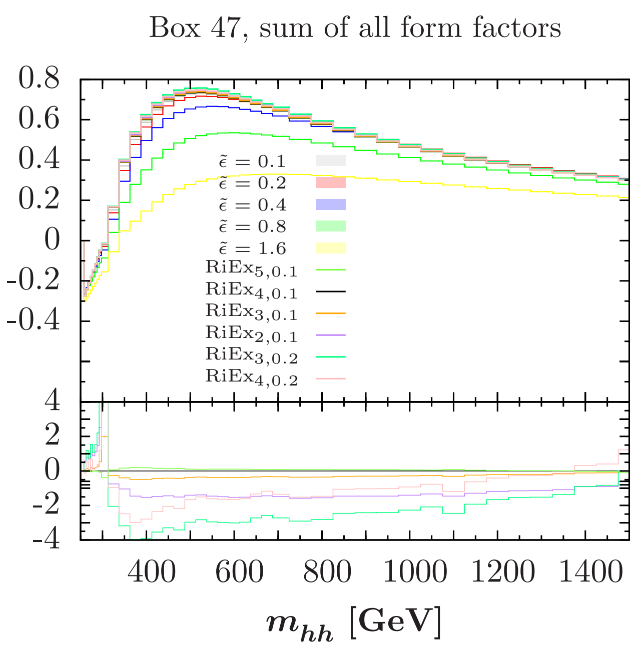
$$\text{RiEx}_{3,\tilde{\epsilon}} = \frac{1}{3}\Big[8f(\tilde{\epsilon}) - 6f(2\tilde{\epsilon}) + f(4\tilde{\epsilon})\Big] = f(0) + \mathcal{O}(\tilde{\epsilon}^3)$$

$$\text{RiEx}_{4,\tilde{\epsilon}} = \frac{1}{21}\Big[64f(\tilde{\epsilon}) - 56f(2\tilde{\epsilon}) + 14f(4\tilde{\epsilon}) - f(8\tilde{\epsilon})\Big] = f(0) + \mathcal{O}(\tilde{\epsilon}^4)$$

$$\text{RiEx}_{5,\tilde{\epsilon}} = \frac{1}{315}\Big[1024f(\tilde{\epsilon}) - 960f(2\tilde{\epsilon}) + 280f(4\tilde{\epsilon}) - 30f(8\tilde{\epsilon}) + f(16\tilde{\epsilon})\Big] = f(0) + \mathcal{O}(\tilde{\epsilon}^5)$$

Richardson extrapolation

- **Goal: From $m_t^2(1 - i\tilde{\epsilon})$, obtain the limit $\tilde{\epsilon} \to 0$**
- **Solution: Richardson extrapolation of the result!**
  Assuming $f(\tilde{\epsilon}) - f(0)$ polynomial for small $\tilde{\epsilon}$, method to accelerate the convergence of $f(\tilde{\epsilon})$ to $f(0)$



$$\mathrm{RiEx}_{2,\tilde{\epsilon}} = 2f(\tilde{\epsilon}) - f(2\tilde{\epsilon}) = f(0) + \mathcal{O}(\tilde{\epsilon}^2)$$

$$\mathrm{RiEx}_{3,\tilde{\epsilon}} = \frac{1}{3}\Big[8f(\tilde{\epsilon}) - 6f(2\tilde{\epsilon}) + f(4\tilde{\epsilon})\Big] = f(0) + \mathcal{O}(\tilde{\epsilon}^3)$$

$$\mathrm{RiEx}_{4,\tilde{\epsilon}} = \frac{1}{21}\Big[64f(\tilde{\epsilon}) - 56f(2\tilde{\epsilon}) + 14f(4\tilde{\epsilon}) - f(8\tilde{\epsilon})\Big] = f(0) + \mathcal{O}(\tilde{\epsilon}^4)$$

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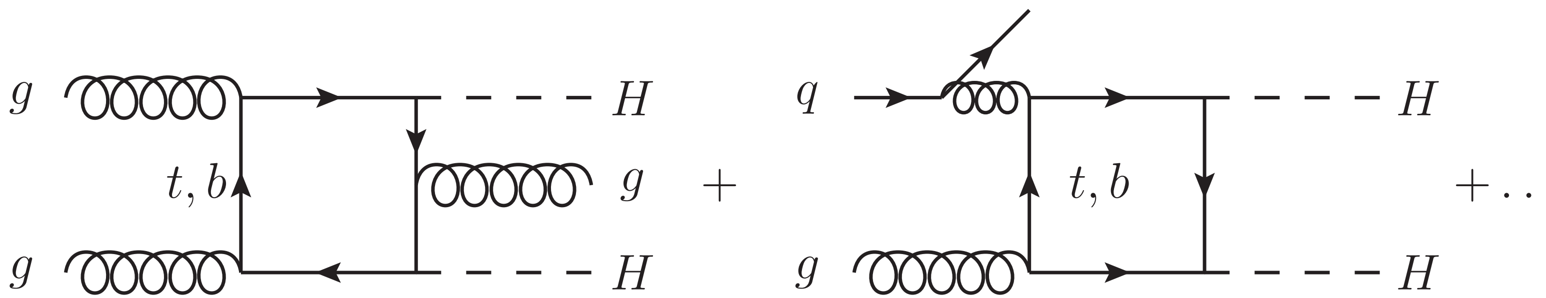
Calculation of the real corrections

Partonic sub-processes $gg \to HHg$, $gq/\bar{q} \to HHq/\bar{q}$, $q\bar{q} \to HHg$



- Full matrix elements generated with `FeynArts/FormCalc` [see Hahn, PoS ACAT2010 (2010) 078], evaluated with 1-loop library `COLLIER` [Denner, Dittmaier, Hofer, CPC 212 (2017) 220]
- Then subtracted with Born-improved HTL matrix-element squared calculated analytically
  $\Rightarrow$ **IR safe mass effects in the reals**

Calculation of the real corrections

Building the local IR counterterm:

$$d\Delta\hat{\sigma}_{ij}^{\text{mass}} = d\Delta\hat{\sigma}_{ij} - d\hat{\sigma}_{\text{LO}} \frac{d\Delta\hat{\sigma}_{ij}^{\text{HTL}}}{d\hat{\sigma}_{\text{LO}}^{\text{HTL}}}$$

Local IR counterterm with a projected on-shell LO $2 \to 2$ kinematics to rescale the $2 \to 3$ HTL

$2 \to 2$ OS LO from [Catani, Seymour, NPB 485 (1997) 291] with initial-state emitter, initial-state spectator

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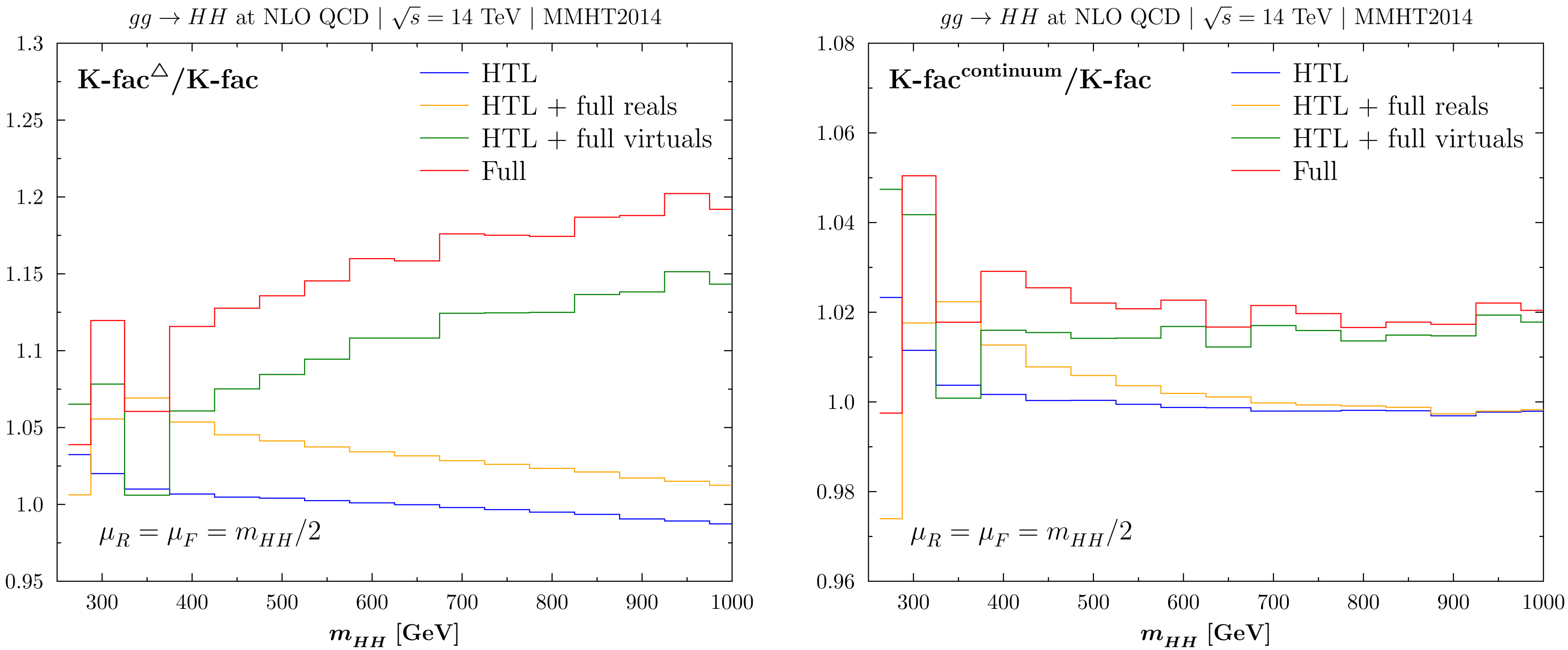
Local IR counterterm with a projected on-shell LO $2 \to 2$ kinematics to rescale the $2 \to 3$ HTL

$2 \to 2$ OS LO from [Catani, Seymour, NPB 485 (1997) 291] with initial-state emitter, initial-state spectator

$\Rightarrow$ **Mass effects IR safe in the real corrections**

Structure of the corrections



- Continuum diagrams ($\equiv$ all but with λ_{HHH}) play a dominant role at large m_{HH}
- No universal NLO top-mass effects (common in the triangle and box diagrams)
  $\Rightarrow$ not possible to approximate full NLO by single Higgs K-factors

More on the scale-and-scheme uncertainty

- Electroweak symmetry sum rule $y_t - \sqrt{2} m_t / v = 0$
  $\Rightarrow$ no rationale behind separating the treatment of the top-quark in Yukawa couplings from the top-quark propagator masses

- Conversion from OS pole mass to $\overline{\text{MS}}$ mass at N^{3}LO [Gray, Broadhurst, Grafe, Schilcher, ZPC 48 (1990) 673; Tarasov, JINR-P2-82-900; Chetyrkin, PLB 404 (1997) 161]

$$\overline{m}_t(m_t) = \frac{m_t}{1 + 4/3 a_s(m_t) + 10.9 a_s(m_t)^2 + 107.11 a_s(m_t)^3}, \quad a_s(\mu) = \frac{\alpha_s(\mu)}{\pi}$$

$$\overline{m}_t(\mu_t) = \overline{m}_t(m_t) \frac{c[a_s(\mu_t)]}{c[a_s(m_t)]}, \quad c(x) = \left(\frac{7}{2} x\right)^{\frac{4}{7}} \left(1 + 1.398 x + 1.793 x^2 - 0.6834 x^3\right)$$

With $m_t = 172.5$ GeV, $\overline{m}_t(\overline{m}_t) = 163.01516...$ GeV

High-Q expansion and proper scale choice

Scale-and-scheme uncertainty from logs of μ_t $\Rightarrow$ What scale to minimize these logs?

- Low Q-values: Peak of Q-distribution around the $t\bar{t}$-threshold
  $\Rightarrow$ Natural choice is OS m_t, or $\overline{m}_t(\overline{m}_t)$
- High Q-values: Analytical results in the $\overline{\text{MS}}$ scheme [see also Davies, Mishima, Steinhauser; Wellmann, JHEP 01 (2019) 176]

$$F_{i,\text{LO}} \to \frac{\overline{m}_t^2(\mu_t)}{Q^2} G_i^{\text{LO}}(Q^2, \hat{t})$$

$$\Delta F_{i,mass} \to \frac{\alpha_s}{\pi} \left\{ 2F_{i,\text{LO}} \left[\log \frac{\mu_t^2}{Q^2} + \frac{4}{3} \right] + \frac{\overline{m}_t^2(\mu_t)}{Q^2} G_i(Q^2, \hat{t}) \right\}$$

G_i and G_i^{LO} do not depend on $\overline{m}_t$

$\Rightarrow$ **Natural choice at high Q is $\mu_t \propto Q$**

m_t and off-shell Higgs production

Is the m_t-uncertainty seen in other processes?

Take a look at $\sigma(gg \to H^*)$ [Graudenz, Spira, Zerwas, PRL 70 (1993) 1372; Spira, Djouadi, Graudenz, Zerwas, NPB 453 (1995) 17; Harlander, Kant, JHEP 12 (2005) 015; Aglietti, Bonciani, Degrassi, Vicini, JHEP 01 (2007) 021; Anastasiou, Bucherer, Kunszt, JHEP 10 (2009) 068]

$$\sigma^{\rm LO}\Big|_{Q=125{\rm GeV}} = 18.43^{+0.8\%}_{-1.1\%}\ {\rm pb}$$
$$\sigma^{\rm LO}\Big|_{Q=300{\rm GeV}} = 4.88^{+23.1\%}_{-1.1\%}\ {\rm pb}$$
$$\sigma^{\rm LO}\Big|_{Q=600{\rm GeV}} = 1.13^{+0\%}_{-26.2\%}\ {\rm pb}$$
$$\sigma^{\rm LO}\Big|_{Q=1200{\rm GeV}} = 0.0249^{+0\%}_{-41.1\%}\ {\rm pb}$$

$$\sigma^{\rm NLO}\Big|_{Q=125{\rm GeV}} = 42.17^{+0.4\%}_{-0.5\%}\ {\rm pb}$$
$$\sigma^{\rm NLO}\Big|_{Q=300{\rm GeV}} = 9.85^{+7.5\%}_{-0.3\%}\ {\rm pb}$$
$$\sigma^{\rm NLO}\Big|_{Q=600{\rm GeV}} = 1.97^{+0\%}_{-15.9\%}\ {\rm pb}$$
$$\sigma^{\rm NLO}\Big|_{Q=1200{\rm GeV}} = 0.0402^{+0\%}_{-26.0\%}\ {\rm pb}$$

Similar uncertainties showing up at large *Q*! [Jones, Spira, in arXiv:2003.01700]

Is the m_t-uncertainty seen in other processes?

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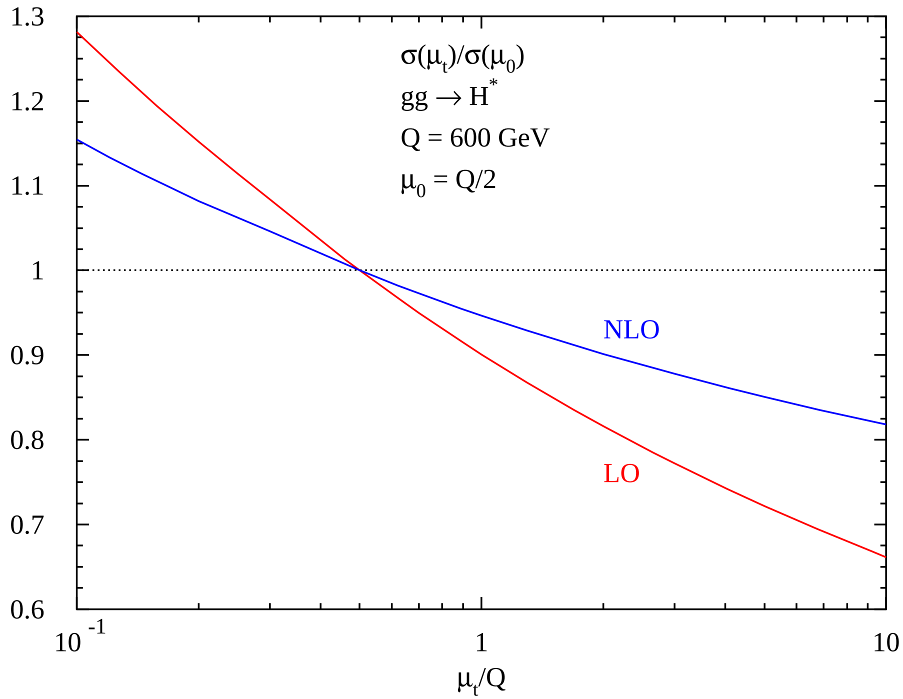
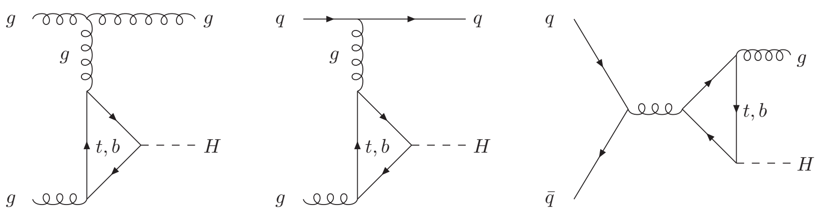
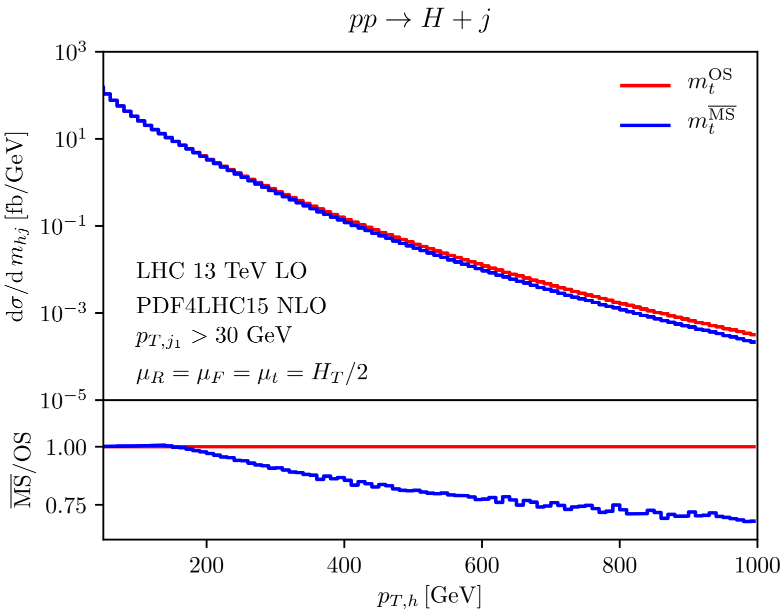


Similar uncertainties showing up at large *Q*! [Jones, Spira, in arXiv:2003.01700]

Is the m_t-uncertainty seen in other processes?

Take a look at $\sigma(gg \to Hj)$, relevant for high-p_T Higgs studies [Baur, Glover, NPB 339 (1990) 38; Schmidt, PLB 413 (1997) 391; De Florian, Grazzini, Kunszt, PRL 82 (1999) 5209; Glosser, Schmidt, JHEP 12 (2002) 016; Ravindran, Smith, Van Neerven, NPB 634 (2002) 247; Jones, Kerner, Luisoni, PRL 120 (2018) 162001]





Again sizable uncertainties, showing up at large $p_{T,h}$

[Jones, Spira, in arXiv:2003.01700]