BLASTs
Black hole Lasers powered by Axion Superradiant instabilities

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Lasers and stimulated emission

Kerr black hole having a BLAST:

- superradiant instability
- stimulated axion decay
Outline

1. Black hole superradiance
2. Axions
3. Lasing in superradiant axion clouds
4. Primordial black hole lasers
5. Fast Radio Bursts
6. Summary & Future prospects
Black hole superradiance

[Zeldovich (1966)]

Low frequency waves can be amplified by scattering off a Kerr black hole:
Black hole superradiance

Kerr metric \((G = c = \hbar = 1)\)

\[
ds^2 = - \left(1 - \frac{2Mr}{\Sigma}\right) dt^2 + \frac{\Sigma}{\Delta} dr^2 + \Sigma d\theta^2 \\
\quad + \frac{(r^2 + a^2)^2 - a^2 \sin^2 \theta \Delta}{\Sigma} \sin^2 \theta d\phi^2 - \frac{4Mra \sin^2 \theta}{\Sigma} dt d\phi
\]

\[J = aM \quad \Delta = r^2 + a^2 - 2Mr \quad \Sigma = r^2 + a^2 \cos^2 \theta\]

Event horizon and Cauchy (inner) horizon:

\[r_+ = M \pm \sqrt{M^2 - a^2} \quad 0 \leq a \leq M\]
Superradiance for scalar waves

Klein-Gordon equation in Kerr space-time:

\[ g^{\mu\nu} \nabla_\mu \nabla_\nu \Phi = 0 \]

Separation of variables:

\[ \Phi(t, r, \theta, \phi) = e^{-i\omega t} e^{im\phi} S(\theta) R(r) \]

stationary and axisymmetric  
spheroidal harmonics
Superradiance for scalar waves

Schrodinger-like equation

\[
\frac{d^2 \psi}{dr_*^2} + \left[ \omega^2 - V(\omega) \right] \psi = 0
\]

where \( \psi = \sqrt{r^2 + a^2 R} \), \( dr_* = \frac{(r^2 + a^2)}{\Delta} dr \)
Toy model for superradiance

\[ V(\omega) = \alpha \delta(x) + m\Omega(2\omega - m\Omega)(1 - \Theta(x)) \]

General solutions:
\[ \psi_I = e^{-i(\omega-m\Omega)x}, \quad \psi_{II} = Ae^{-i\omega x} + Be^{i\omega x} \]

Boundary conditions:
\[ \psi_I(0) = \psi_{II}(0), \quad \psi'_I(0) - \psi'_{II}(0) = \alpha\psi_I(0) \]

Reflection coefficient:
\[ R = \left| \frac{B}{A} \right|^2 = \frac{\alpha^2 + m^2\Omega^2}{\alpha^2 + (2\omega - m\Omega)^2} > 1 \quad \Rightarrow \quad \omega < m\Omega \]
Toy model for superradiance

In the superradiant regime:
• negative phase velocity: \( k_I = \omega - m\Omega < 0 \)
• positive group velocity: \( v_g = d\omega/dk_I = 1 \)

Waves carry negative energy into the BH

Energy and spin extraction from BH
Superradiant amplification

• Superradiant amplification for different waves:
  – Scalar: 0.3%
  – Electromagnetic: 4.4%
  – Gravitational: 138%

• No superradiance for fermions

[Press & Teukolsky (1974)]
Black hole bombs

[Press & Teukolsky (1972); Cardoso, Dias & Lemos (2004)]

• surround black hole with mirror
• multiple superradiant scatterings
• exponential amplification of signal
• extract large amount of energy and spin
• radiation pressure eventually destroys mirror
Massive black hole bombs

Massive fields can become bound to the black hole:

“gravitational atoms”

[Arvanitaki et al. (2009)]
Toy model for superradiant instabilities

Bound states satisfy: \[ \omega \cot(\omega L) + \alpha = i(\omega - m\Omega) \]

In the limit \( \alpha \gg \omega \): \[ \omega = \omega_R + i\Gamma \]

\[ \omega_R \simeq \frac{n\pi}{2L} \]

\[ \Gamma \simeq -\frac{\omega_R(\omega_R - m\Omega)}{\alpha} \]

\[ \Phi \propto e^{-i\omega_R t} e^{\Gamma t} \quad \rightarrow \quad \text{Superradiant instability} \]
Matching asymptotics method

- Solve radial equation in both regions
- Match asymptotic behaviour in overlap region
- Possible for small mass coupling:
  \[ \alpha_\mu = GM\mu/\hbar c \ll 1 \]
Gravitational "atoms"

Gravitational Hydrogen-like spectrum \((n, l, m)\)
[Detweiler (1980); Furuhashi & Nambu (2004)]

\[
\hbar \omega_n \simeq \mu c^2 \left(1 - \frac{\alpha_\mu^2}{2n^2}\right) \quad \alpha_\mu \ll 1
\]

\[
\Gamma = -C_{ln}(\omega_R - m\Omega)\alpha_\mu^{4l+5} \left(\frac{r_+ - r_-}{r_+ + r_-}\right)^{2l+1}
\]

superradiant instability

Note: matching not possible for extremal BH [JGR (2009)]
Superradiant growth rate

\[ n = 2, l = m = 1 \]

[Dolan (2007)]

[see review by Brito, Cardoso & Pani (2015)]
Non-relativistic scalar clouds

\[ n = 2, l = m = 1 \]

\[ \sqrt{\langle v^2 \rangle} \simeq (\alpha_\mu/2)c \]

Bohr radius:

\[ r_0 = \frac{\hbar}{\mu c \alpha_\mu} \simeq 66 \left( \frac{\alpha_\mu}{0.03} \right)^{-1} \left( \frac{\mu}{10^{-5} \text{ eV}} \right)^{-1} \text{ cm} \]

Event horizon:

\[ r_+ \simeq 0.1 \left( \frac{\alpha_\mu}{0.03} \right) \left( \frac{\mu}{10^{-5} \text{ eV}} \right)^{-1} \left( 1 + \sqrt{1 - \tilde{a}^2} \right) \text{ cm} \]

Superradiant growth rate:

\[ \Gamma_s \simeq \frac{\tilde{a}}{24} \alpha_\mu^9 \left( \frac{c^3}{GM} \right) \simeq 4 \times 10^{-4} \tilde{a} \left( \frac{\mu}{10^{-5} \text{ eV}} \right) \left( \frac{\alpha_\mu}{0.03} \right)^8 \text{ s}^{-1} \]
Astrophysical black hole bombs

Superradiant condition implies (for l=m=1):

\[ \omega < \Omega \quad \Rightarrow \quad \alpha_\mu < 1/2 \]

For stellar mass black holes:

\[ \alpha_\mu \simeq 0.75 \left( \frac{M}{M_\odot} \right) \left( \frac{\mu}{10^{-10} \text{eV}} \right) \]

Can use superradiance to probe the low-energy BSM particle spectrum
CP-violation in Quantum Chromodynamics (QCD):

\[ \mathcal{L} = -\frac{1}{4} F_{\mu \nu} F^{\mu \nu} - \frac{\theta}{32 \pi^2} F_{\mu \nu} \tilde{F}^{\mu \nu} \]

Neutron electric dipole moment:

\[ \theta \lesssim 10^{-10} \]

Introduce dynamical axion field [Peccei & Quinn (1977)]:

\[ \mathcal{L} = -\frac{1}{4} F_{\mu \nu} F^{\mu \nu} - \frac{1}{32 \pi^2} \frac{\phi}{F_\phi} F_{\mu \nu} \tilde{F}^{\mu \nu} - \frac{1}{2} \partial_\mu \phi \partial^\mu \phi - V(\phi) \]
Axions

Axion is a pseudo-NG boson of the U(1) Peccei-Quinn symmetry that gets potential from QCD instantons:

\[ V(\phi) \]

Axion can be very light:

\[ \mu \simeq 10^{-5} \left( \frac{F_\phi}{6 \times 10^{11} \text{ GeV}} \right)^{-1} \text{ eV} \]

[Weinberg; Wilczek (1978)]
Axion-photon coupling

The electromagnetic triangle anomaly leads to an axion-photon-photon interaction:

\[ \mathcal{L}_{\phi\gamma\gamma} = \frac{\alpha K}{8\pi F_\phi} \phi F_{\mu\nu} \tilde{F}^{\mu\nu} \]

This leads to axion decay into photon pairs:

\[ \tau_\phi \simeq 3 \times 10^{32} K^{-2} \left( \frac{\mu}{10^{-5} \text{ eV}} \right)^{-5} \text{ Gyr} \]

Axions can account for dark matter!
**Axion dark matter**

**Misalignment production:**
[Preskill, Wise & Wilczek; Abbott & Sikivie; Dine & Fischler (1983)]
- axion field initially displaced from origin
- oscillates about origin after QCD transition
- stable cold dark matter condensate

\[
\Omega_\phi h^2 \approx 0.11 \left( \frac{10^{-5} \text{ eV}}{\mu} \right)^{1.19} F(F_\phi, \Theta_i) \Theta_i^2
\]

Fine-tuned initial conditions and other production mechanisms yield axion DM range:

\[
10^{-12} \text{ eV} \lesssim \mu \lesssim 10^{-2} \text{ eV}
\]
Axion decay into photon pairs can be stimulated by the passage of a photon:

$$\Gamma^s_t = 2f_\gamma \Gamma_\phi$$

In dense axion clusters, this can lead to lasing:

- cross section:  $$\sigma_{st} = \Gamma^s_t / \Phi_\gamma$$
- photon mean free path:  $$\lambda_\gamma = 1 / (n_\phi \sigma_{st})$$
- lasing condition:

$$\frac{\lambda_\gamma}{2R} = \frac{\mu^4 \tau_\phi \beta R^2}{24\pi M} \lesssim 1$$
Lasing axion clusters

Lasing can occur only for sufficiently dense clusters:

\[
\frac{\rho_\phi}{\rho_\odot} \gtrsim \frac{6}{K^4} \left( \frac{\mu}{1 \text{ eV}} \right)^{-2}
\]

which translates into a large axion degeneracy:

\[
\frac{n_\phi}{\mu^3} \gtrsim \frac{2 \times 10^{19}}{K^4} \left( \frac{\mu}{1 \text{ eV}} \right)^{-6}
\]

Can lasing occur in superradiant axion clouds?
Boltzmann equation for axion decay/inverse decay:

\[
\frac{dn_\lambda(k)}{dt} = \int dX_{LIPS} \left[ f_\phi(p)(1 + f_\lambda(k))(1 + f_\lambda(k')) - f_\lambda(k)f_\lambda(k')(1 + f_\phi(p)) \right] |\mathcal{M}|^2
\]

where:

\[
n_i = \int \frac{d^3 k_i}{(2\pi)^3} f_i(k_i)
\]
Lasing equations

Simplified cloud model:

- “2p”-toroidal axion cloud (flat space, non-relativistic)
- homogeneous and isotropic phase space distributions

\[ p_\phi \lesssim \frac{\alpha_\mu}{2} \mu c, \quad p_\gamma \simeq \frac{\mu c}{2}, \quad \Delta p_\gamma \simeq \frac{\alpha_\mu}{2} \mu c \]

\[
\frac{dn_\gamma}{dt} = \Gamma_\phi \left[ 2n_\phi \left( 1 + \frac{8\pi^2}{\mu^3 \beta} n_\gamma \right) - \frac{16\pi^2}{3\mu^3} \left( \beta + \frac{3}{2} \right) n_\gamma^2 \right]
\]

“active” axions
“sterile” axions
spontaneous decay
stimulated decay
photon annihilation
Lasing equations

We also need to include:

• axion source from BH superradiant instability
• photon escape rate: $\Gamma_e = c/(\sqrt{5}r_0)$

Photon-axion dynamics: $\Gamma_\phi \ll \Gamma_s \ll \Gamma_e$

\[
\frac{dN_\phi}{dt} = \Gamma_s N_\phi - \Gamma_\phi \left[ N_\phi (1 + AN_\gamma) - B_1 N_\gamma^2 \right],
\]

\[
\frac{dN_\gamma}{dt} = -\Gamma_e N_\gamma + 2\Gamma_\phi \left[ N_\phi (1 + AN_\gamma) - B N_\gamma^2 \right],
\]

$A = 8\alpha^2_\mu/25$, $B_1 = 2\alpha^4_\mu/75$, $B_2 = 2\alpha^3_\mu/25$, $B = B_1 + B_2$
Numerical solution

\[ \mu = 10^{-5} \text{ eV} \]
\[ K = 1 \]
\[ M_{BH} = 8 \times 10^{23} \text{ kg} \]
\[ \tilde{a} = 0.7 \]
\[ \alpha_{\mu} \approx 0.03 \]

\[ N_{\phi}^c = \frac{\Gamma_e}{2A\Gamma_{\phi}} \]
\[ N_{\gamma}^c = \frac{\Gamma_s}{A\Gamma_{\phi}} \]
Laser pulses

Neglecting annihilation, close to lasing threshold:

\[ X = N_\phi/N_\phi^C - 1 \ll 1 , \quad Y = N_\gamma/N_\phi^C \ll 1 \]

\[ \frac{d^2 \log Y}{du^2} = \eta - \frac{1}{2} Y \]

where:

\[ u = \Gamma_e t , \quad \eta = \Gamma_s/\Gamma_e \ll 1 \]

\[ Y = \begin{cases} Y_0 e^{\eta u^2/2} & , \ Y \ll 2\eta \\ C \cosh^{-2} \left( \sqrt{C}(u - u_{\text{max}})/2 \right) & , \ Y \gg 2\eta \end{cases} \]
BLAST properties

For the first laser pulses:

\[ L_B \sim 2 \times 10^{42} \frac{\tilde{a}}{K^2} \left( \frac{10^{-5} \text{ eV}}{\mu} \right)^2 \left( \frac{\alpha_\mu}{0.03} \right)^7 \left( \frac{\xi}{100} \right) \text{ erg/s} \]

\[ \tau_B \sim \frac{1}{\sqrt{\tilde{a}}} \left( \frac{10^{-5} \text{ eV}}{\mu} \right) \left( \frac{\alpha_\mu}{0.03} \right)^{-9/2} \left( \frac{\xi}{100} \right)^{-1/2} \text{ ms} \]

\[ \nu_B \sim 1.2 \left( \frac{\mu}{10^{-5} \text{ eV}} \right) \text{ GHz} \]

\[ \xi = \log \left( \frac{\Gamma_s}{\Gamma_\phi} \right) \]
\[ \sim 107 - 4 \log \left( \frac{\mu}{10^{-5} \text{ eV}} \right) + 8 \log \left( \frac{\alpha_\mu}{0.03} \right) + \log \left( \frac{\tilde{a}}{K^2} \right) \]
Plasma effects

BLASTs typically exceed BH Eddington luminosity

\[ L_{Edd} \sim 10^{38} \left( \frac{M}{M_\odot} \right) \text{ erg/s} \]

i.e. radiation pressure blows away surrounding plasma.

However, electric field within cloud can exceed the Schwinger threshold for e\(^+\)e\(^-\) production:

\[ |E| \sim E_c \left( \frac{\mu}{10^{-5} \text{ eV}} \right) \left( \frac{\alpha \mu}{0.03} \right) \left( \frac{L}{10^{43} \text{ erg/s}} \right)^{1/2} \]
Plasma effects

Rate of Schwinger electron-positron plasma production:

\[
\frac{dn_e}{dt} = \frac{\alpha \epsilon_0}{\pi^2 \hbar} |E|^2 e^{-\pi E_c/|E|} \simeq 10^{56} \frac{|E|^2}{E_c^2} e^{-\pi E_c/|E|} \text{ m}^3 \text{s}^{-1}
\]

Expect plasma to reach critical density in a single pulse:

\[
\nu_p = \sqrt{\frac{n_e e^2}{4\pi^2 \epsilon_0 m_e}} \simeq 1 \left( \frac{n_e}{10^{16} \text{ m}^{-3}} \right)^{1/2} \text{ GHz}
\]

and effective photon plasma mass blocks axion decay.

**BLASTs are short electromagnetic bursts!**
Plasma effects

Electrons and positrons eventually annihilate:

\[ \tau_{\text{ann}} \approx 4 \left( \frac{n_e}{10^{16} \text{ m}^{-3}} \right)^{-1} \text{ hours} \]

allowing lasing to restart.

Expect BLASTs to repeat!
Constraints

1. Critical cloud mass/spin for lasing:

\[
\frac{J^c_\phi}{J_{BH}} \approx \frac{0.06}{\tilde{a} \alpha_\mu^3 K^2} \left( \frac{\mu}{10^{-8} \text{ eV}} \right)^{-2} \lesssim 1
\]

BLASTs only possible for

\[
\mu \gtrsim 10^{-8} \text{ eV} \quad \Rightarrow \quad M_{BH} \lesssim 10^{-2} M_\odot
\]

Primordial Black Holes

2. Non-linear axion self-interactions quench superradiance ("bosenova"):

\[
\phi^c \lesssim F_\phi \quad \Rightarrow \quad \alpha_\mu \lesssim 0.03 K
\]

[Kodama & Yoshino (2012-2015)]
Primordial black holes

Large density fluctuations collapse directly into BHs once they re-enter the horizon in radiation era:

\[ \frac{\delta \rho}{\rho} \gtrsim 0.1 \]

Possible mechanisms:
(multi-field) inflation, curvaton, phase transitions

PBH mass comparable to mass within Hubble horizon:

\[ M = \frac{c^3 t}{G} \approx 10^{25} g_*^{-1/2} \left( \frac{T}{300 \text{ GeV}} \right)^{-2} \text{ kg} \]
Primordial black holes

PBHs heavier than $10^{12}$ kg survive until today and may contribute to dark matter!

[Carr, Kuhnel & Sandstad (2017)]
Primordial black holes

PBHs are born with no spin and accrete mostly in spherical configuration [Ali-Haimoud & Kamionkowski (2017)]

However, PBHs can merge into spinning PBHS!

e.g. two equal Schwarzschild BHs merge into BH with spin \( \tilde{a}_f \approx 0.7 \) [Scheel et al. (2009)]

PBH merger rate in clustered scenario:
[Clesse & Garcia-Bellido (2016)]

\[
\Gamma_{\text{capt}}^{\text{total}} \approx 3 \times 10^{-9} f_{\text{DM}} \delta_{\text{loc}} \text{ yr}^{-1} \text{Gpc}^{-3}
\]

Clusters, dwarf spheroidal galaxies: \( \delta_{\text{loc}} \approx 10^9 - 10^{10} \)
Fast Radio Bursts

If axions account for most of the dark matter:

\[ \mu \sim 10^{-5} \text{ eV} \]

They can produce repeating BLASTs for PBHs with mass

\[ M_{PBH} \sim (2 - 8) \times 10^{23} \text{ kg} \]

BLAST properties:

\[ \nu_B \sim 1 \text{ GHz} \quad L_B \sim 10^{39} - 10^{42} \text{ erg/s} \quad \tau_B \sim 100 - 1 \text{ ms} \]

- longer BLASTs are fainter and yield short-lived continuous lasers (1-100 years)
- shorter BLASTs are brighter and yield long-lived repeating fast radio bursts (10^4 years)
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Fast Radio Bursts

More than 30 FRBs with these properties have been observed in the past decade [Lorimer et al. (2007)]

FRB 121102 repeats and is localized within a faint dwarf galaxy [Chatterjee et al.; Marcote et al.; Tendulkar et al. (2017)]

Possible mechanisms:
compact object mergers; NS collapse; energetic pulsars/magnetars…

Could BLASTs account for some of these FRBs?

- QCD axion exists and is the dark matter
- Sub-terrestrial PBHs exist and merge
Predictions

• Up to $10^5$ active FRBs across the sky (PBHs <10% of DM)

• X-rays afterglows from $e^+e^-$ annihilation

• Gravitational waves from mergers

• Gravitational waves from bosenova between bursts

• DM axion-photon conversion in galactic B-field @ SKA
  [Kelley & Quinn (2017)]

• QCD axion with mass $\sim 10^{-5}$ eV and $g_{\phi\gamma\gamma} \sim 10^{-15}$ GeV$^{-1}$
  [ADMX, X3, CULTASK, MADMAX, ORPHEUS]
Summary

• **Superradiant instabilities** can produce dense axion clouds around spinning black holes

• **Stimulated axion decay** leads to extremely bright lasing bursts (BLASTs)

• Tantalizing agreement with observed **Fast Radio Bursts** for **cosmological axions** and **primordial black holes**

• Lasing may occur for other light **BSM bosons**?
Thank you!