Fourty years of the Coimbra pion cloud

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Outline

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Pion-nucleon resonances and the Peierls-Yoccoz projection

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(Received 6 July 1977)

An attempt is made to provide a microscopic interpretation of the pion-nucleon resonances in terms of the Peierls-Yoccoz projection applied to a coherent state of satellite mesons.

Forty years of collaboration with Joao, Manuel, Pedro, Ze, Luis

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Genuine quark state versus dynamically generated structure for the Roper resonance

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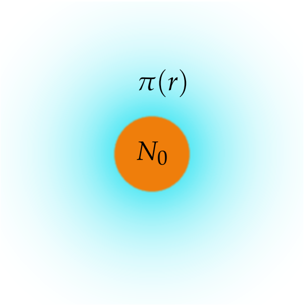
Pion cloud around a bare nucleon



Source $\rho_i = g \sum_k \boldsymbol{\sigma} \cdot \boldsymbol{k}\, \tau_i, \quad i = 1, 0, -1.$

Classical solution $\pi(r)$ interpreted as coherent state of pions:

$$\pi(\boldsymbol{r}) = \langle \Phi | \hat{\pi} | \Phi \rangle \,, \quad \hat{\pi} = \sum_{ik} a_{ik} + a^\dagger_{ik}$$

$$|\Phi\rangle = \exp\left\{ \sum_{ik} \xi_{ik} a^\dagger_{ik} \right\} |N_0\rangle$$

$$= |N_0\rangle + |\pi\, N_0\rangle + |\pi^2\, N_0\rangle + |\pi^3\, N_0\rangle + \cdots$$

Projection onto states with good angular momentum and isospin (Peierls-Yoccoz projection)

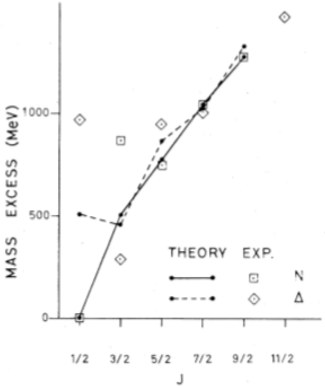
$$|\Phi\rangle = \sum_{IJ} |\Phi_{IJ}\rangle \,, \qquad |\Phi_{IJ}\rangle = P^{PY}_{IJ} |\Phi\rangle \qquad J = \tfrac{1}{2}, \tfrac{3}{2}, \tfrac{5}{2}, \ldots \quad I = \tfrac{1}{2}, \tfrac{3}{2}, \ldots$$

N^* and Δ^* excited states

Excited states with $I = \frac{1}{2}$ and $\frac{3}{2}$:

$$E_{IJ} = \frac{\langle \Phi_{IJ} | H | \Phi_{IJ} \rangle}{\langle \Phi_{IJ} | \Phi_{IJ} \rangle}$$

Free parameters:
coupling constant g, cut-off in k-space, Λ (assuming a finite size of the bare nucleon)



However: performing full minimization of E_{JT} (variation-after-projection, VAP), we end up with

$$E = m_{N_0} + m_\pi\,, \qquad J = \tfrac{3}{2},\ \ I = \tfrac{1}{2}\,,\tfrac{3}{2}$$
$$E = m_{N_0} + 2m_\pi\,, \qquad J = \tfrac{5}{2}$$
$$E = m_{N_0} + 3m_\pi\,, \qquad J = \tfrac{7}{2}$$

Scattering states

Excited baryons are resonances appearing in the pion-nucleon scattering with the energy corresponding to the energy where the phase shift passes through 90° (Breit-Wigner mass).

The proper ansatz:

free meson pion cloud

$$|\Psi_{JI}(W)\rangle = a^\dagger|N\rangle + \tan\delta(W)|\Phi_{\mathrm{res}}\rangle$$

W is the energy of the πN system in the c.m. frame.

Variational principle (Kohn principle)

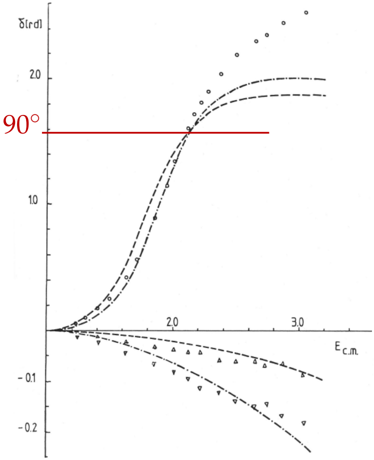
$$\tan\delta(W) = \text{stationary}$$

reducing to the usual form for the parameters of the scattering wave function:

$$\langle\delta\Psi_{JI}|\hat{H} - W|\Psi_{JI}\rangle = 0\,.$$

$\Delta(1232)$ resonance

Phase shift $\delta(W \equiv E_{\mathrm{c.m.}})$
($E_{\mathrm{c.m.}}$ in units m_π)
For sufficiently large Λ:

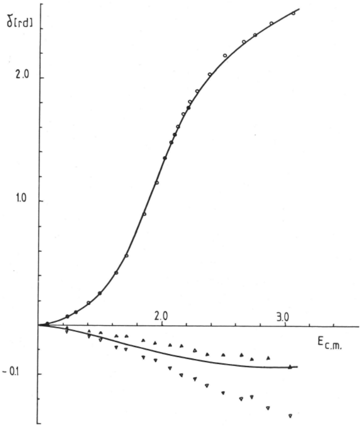


pion cloud around bare N

$$|\Psi_{\frac{3}{2}\frac{3}{2}}(W)\rangle = a^\dagger|N\rangle + \tan\delta(W)|\Phi_N\rangle$$

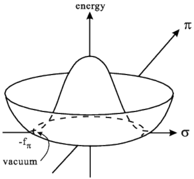
pion cloud around bare Δ

$$+ \quad |\Phi_\Delta\rangle$$



The linear σ-model with quarks

$$\mathcal{L}_{\mathrm{q-meson}} = g\bar{q}\,(\sigma + \mathrm{i}\vec{\tau}\cdot\vec{\pi}\,\gamma_5)\,q\,, \qquad \mathcal{L}_{\mathrm{meson\ self-int}} =$$



Meson self-interaction generates nonzero σ-field in the vacuum $\sigma_{\mathrm{vac}} = f_\pi = 93$ MeV, and provides a fine mass for the quark, $m_q = g\sigma_{\mathrm{vac}}$.
"Hedgehog" coherent states with pion and σ clouds around 3-quark core:

$$|\mathrm{Hh}\rangle = \mathrm{e}^{\left\{\int \mathrm{d}k\,\xi(k)a^\dagger_{m,i=-m}(k)\right\}}\,\mathrm{e}^{\left\{\int \mathrm{d}k\,\sigma(k)a^\dagger_{\sigma}(k)\right\}}\left(q^\dagger_{\mathrm{u}\downarrow} - q^\dagger_{\mathrm{d}\uparrow}\right)^3|0\rangle\,.$$

$$|\Psi_N\rangle = P^{\mathrm{PY}}_{I=\frac{1}{2},J=\frac{1}{2}}\,|\mathrm{Hh}\rangle\,, \qquad |\Psi_\Delta\rangle = P^{\mathrm{PY}}_{I=\frac{3}{2},J=\frac{3}{2}}\,|\mathrm{Hh}\rangle\,.$$

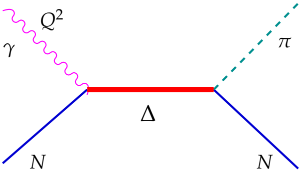
EM current and density:

quark   pion contribution

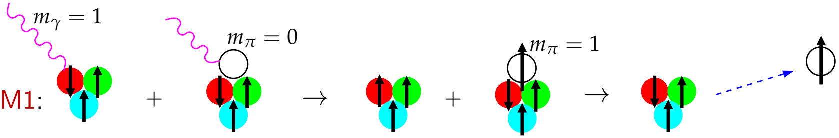
$$j = \bar{q}\gamma\left(\tfrac{1}{6} + \tfrac{1}{2}\tau_3\right)q + (\vec{\pi}\times\nabla\vec{\pi})_3\,, \qquad \rho = \bar{q}\gamma_0\left(\tfrac{1}{6} + \tfrac{1}{2}\tau_3\right)q + \left(\vec{\pi}\times\vec{P}_\pi\right)_3\,.$$

Electroproduction of π on Δ

The $\gamma N \Delta$ vertex is dominated by the dipole magnetic (M1) multipole; minor contribution comes from the quadrupole electric (E2) and Coulomb (C2) multipole.

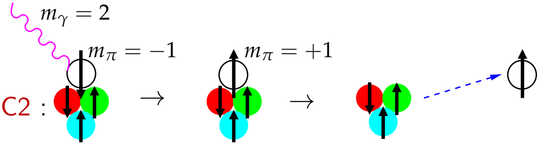


M1:



$M_N = \frac{1}{2}$ $\quad$ $M_\Delta = \frac{3}{2}$ $\quad$ $M_N = \frac{1}{2}, \quad m_\pi = 1$

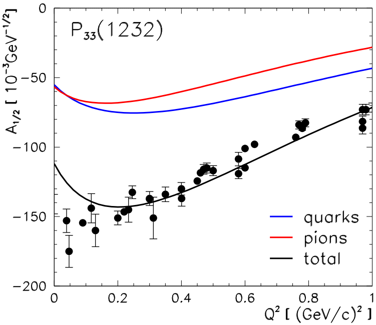
E2, C2 :

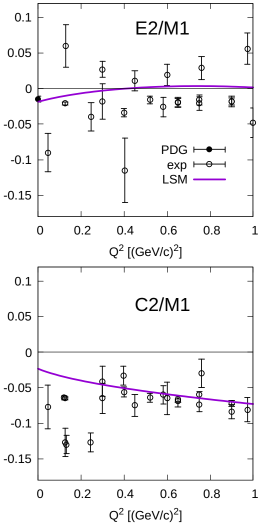


$M_N = -\frac{1}{2}$ $\quad$ $M_\Delta = \frac{3}{2}$ $\quad$ $M_N = \frac{1}{2}, \quad m_\pi = 1$

Δ(1232) helicity amplitude

P.Alberto, M.Fiolhais, S.Širca, B.G.,PLB **373** (1996) 229.





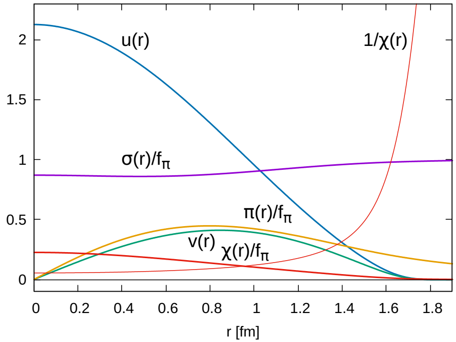
$A_{1/2} \approx M1$ amplitude
Beyond the $kR \ll 1$ approximation:
$C2 \neq E2$ ("Siegert theorem" not valid)

The chiral chromodielectric model

In the linear σ-model higher 3-quark states are not bound; a possible extension is to add a chromodielectric field via quark-boson coupling:

$$\mathcal{L}_{\text{int}} = \frac{g}{\chi}\bar{q}(\hat{\sigma} + \mathrm{i}\vec{\tau}\cdot\hat{\vec{\pi}}\gamma_5)q\,, \qquad r\to\infty : \chi(r)\to 0\,, \quad m_q = \frac{g\sigma(r)}{\chi(r)} \to \infty\,.$$

Self consistent solution for the quark, meson and χ fields:



P. Alberto, M. Fiolhais, B. G., and J. Marques, Phys. Lett. B **523**, 273 (2001)

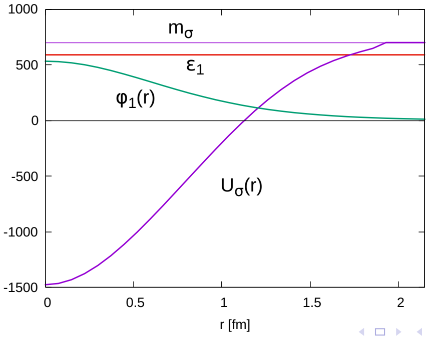
Vibrational modes for the sigma field

We expand the field operators of the bosons around their expectation value in the ground state $|\mathrm{N}\rangle$; for the σ we write

$$\hat{\sigma}(\boldsymbol{r}) = \sum_n \frac{1}{\sqrt{2\varepsilon_n}}\,\varphi_n(r)\,\frac{1}{\sqrt{4\pi}}\left[\tilde{a}_n + \tilde{a}_n^\dagger\right] + \sigma(r)\,, \qquad \tilde{a}_n|N\rangle = 0$$

The stability conditions implies a Klein-Gordon equation for the σ-meson modes

$$\left(-\nabla^2 + m_\sigma^2 + U_\sigma(r)\right)\varphi_n(r) = \varepsilon_n^2\varphi_n(r)\,, \qquad U_\sigma(r) = \frac{\mathrm{d}^2 V(\sigma(r))}{\mathrm{d}\sigma(r)^2}$$

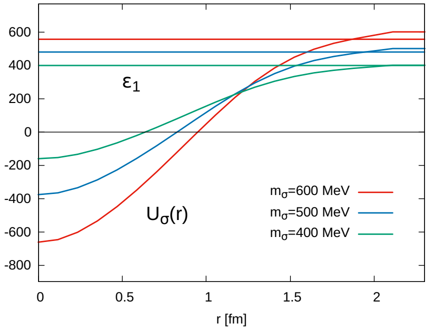


Results

Roper: cohabitation of the quark core excitation ($1s \to 2s$) and a molecular state σN:

m_σ	E_N	2s–1s	ε_1	c_2	$\tilde{E}_N$	$\Delta\tilde{E}_R$
1200	1269	446	1090	0.05	1256	380
700	1249	477	590	0.12	1235	396

A bound state exists also for smaller (more realistic) m_σ:

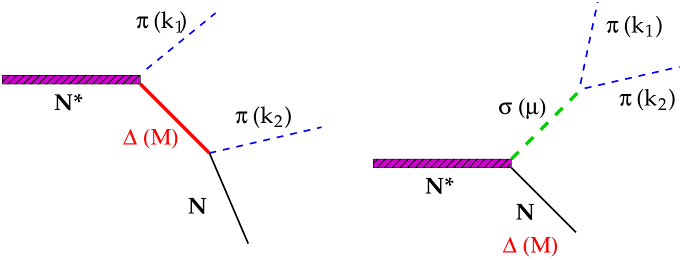


Coupled channel approach for the Roper resonance

πN, $\pi\Delta$, σN channels:
S. Širca, B.G., Eur. Phys. J. A **38**, 271 (2008),
S. Širca, and M. Fiolhais, B.G., Eur. Phys. J. A **42**, 185 (2009),

Assumption about the two-pion channels:

$$\pi N \to N^* \to \pi\Delta \to \pi\pi N \qquad \pi\mathrm{N} \to N^* \to \sigma N \to (2\pi)^{l=0}_{I=0} N$$



Incorporating quark states and meson clouds in the coupled channel framework

Channel state

$$|\Psi_\alpha\rangle = \mathcal{N}_\alpha\Big\{\Big[a^\dagger_\alpha(k_\alpha)|\Phi_\alpha\rangle\Big] + c_{\alpha N}|\Phi_N\rangle + c_{\alpha R}|\Phi^0_R\rangle + \sum_\beta \int \frac{\mathrm{d}k\ \chi_{\alpha\beta}(k_\alpha,k)}{\omega(k)+E_\beta(k)-W}\,[a^\dagger_\beta(k)|\Phi_\beta\rangle]\Big\},$$

$\alpha,\beta = \pi N, \pi\Delta, \sigma N$

The meson amplitude $\chi_{\alpha\beta}$ is proportional to the scattering K matrix:

$$K_{\alpha\beta}(k_\alpha,k_\beta) = K_{\beta\alpha}(k,k_\alpha) = \pi\,\mathcal{N}_\alpha\mathcal{N}_\beta\,\chi_{\alpha\beta}(k_\alpha,k_\beta)\,, \quad \mathcal{N}_\alpha = \sqrt{\frac{\omega_\alpha E_\alpha}{k_\alpha W}}$$

The **S** matrix

$$\mathbf{S} = \frac{1+\mathrm{i}\mathbf{K}}{1-\mathrm{i}\mathbf{K}}, \qquad \mathbf{S} = 1 + 2\mathrm{i}\mathbf{T}\,, \qquad \mathbf{T} = \mathbf{K} + \mathrm{i}\mathbf{K}\mathbf{T}$$

Pion-baryon vertices

p-wave quark-pion vertex, determined in the Cloudy Bag Model:

$$V^{\pi}_{\alpha\beta} = r_q \frac{1}{2f} \frac{k^2}{\sqrt{12\pi^2\omega(k)}} \frac{\omega_{\mathrm{MIT}}}{\omega_{\mathrm{MIT}}-1} \frac{j_1(kR_{\mathrm{bag}})}{kR_{\mathrm{bag}}} \langle\Phi_\alpha||\sum_{i=1}^{3}\sigma^i_m\tau^i_t||\Phi_\beta\rangle$$

$$r_q = \begin{cases} 1 & \text{for } (1s)^3 \text{ configuration} \\ r_\omega = \left[\frac{\omega^1_{\mathrm{MIT}}(\omega^0_{\mathrm{MIT}}-1)}{\omega^0_{\mathrm{MIT}}(\omega^1_{\mathrm{MIT}}-1)}\right]^{1/2} = 0.457 & \text{for } B = (1s)^3\text{—}(1s)^2(2s)^1 \\ \frac{2}{3}+r_\omega^2 & \text{for } (1s)^2(2s)^1 \end{cases}$$

$R_{\mathrm{bag}} = 0.83$ fm $f_\pi = 76$ MeV (reproducing $g_{\pi NN}$ and g.s. properties)

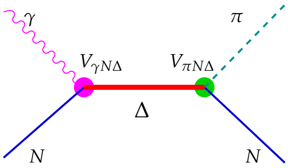
2-π decays:

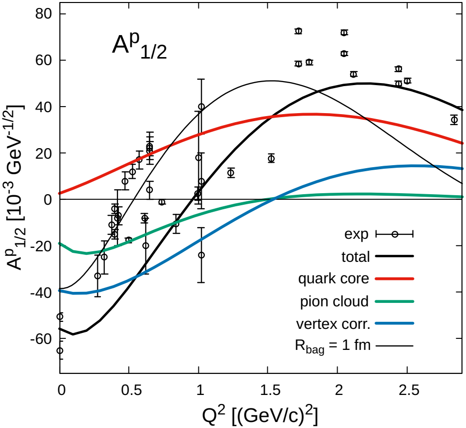
$$V^\sigma_0(k,\mu) = V^\sigma_0(k)\, w_{\mathrm{BW}}(\mu)\,, \qquad V^\sigma_0(k) = g_\sigma \frac{k}{\sqrt{2\omega(k)}}\,.$$

Two sets of BW parameters: $m_\sigma = \Gamma_\sigma = 500$ MeV and $m_\sigma = \Gamma_\sigma = 600$ MeV

Free parameters: baryon bare masses

Pion electroproduction on $N(1440)$





Note

$$\mathrm{sign}(A_{1/2}) = \frac{\mathrm{sign}(M1(\gamma N \to \pi N))}{\mathrm{sign}(V_{\pi N\Delta})}$$

Pion-cloud effects dominate at small Q^2, i.e. at large distances.

Molecular states revisited

Motivation:

Recent QCD lattice calculation by the Graz-Ljubljana group, searching for pion scattering states on top of the three-quark configuration: no indication for a "bare Roper" below ~ 2 GeV; but a strong $[\pi\pi]^{S=0}_{I=0}$ component.

Bethe Salpeter equation for the meson amplitude $\chi_{\alpha\gamma}$

$$\chi_{\alpha\gamma}(k_\alpha, k_\gamma) = \mathcal{K}_{\alpha\gamma}(k_\alpha, k_\gamma) + \sum_\beta \int \mathrm{d}k \, \frac{\mathcal{K}_{\alpha\beta}(k_\alpha, k)\chi_{\beta\gamma}(k, k_\gamma)}{\omega(k) + E_\beta(k) - W}$$

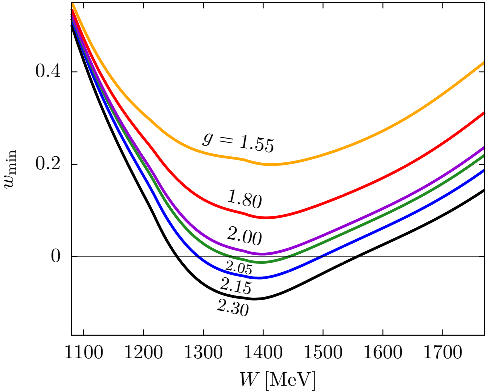
Approximating the kernel $\mathcal{K}$ by a separable form, the integral equation reduces to an algebraic equation which can be solved exactly.

For sufficiently strong coupling the kernel $\mathcal{K}$ may become singular and a quasi-bound state arises.

Dynamically generated resonances

Eq. for the expansion coefficients x: $x_{\alpha\beta} + \sum_\gamma M_{\alpha\beta,\beta\gamma}\, x_{\beta\gamma} = b_{\alpha\beta}$

The singularity arises when the lowest eigenvalue of M crosses 0:



Poles in the complex W-plane:

g	ReW_p	-2ImW_p
PDG	1370	180
1.80	1397	157
1.95	1383	112
2.00	1358	111
2.05	1331	44
	1438	147

S. Širca, H. Osmanović (Tuzla), A. Švarc (Zagreb), B.G. Phys. Rev. C **97**, 035204 (2018)

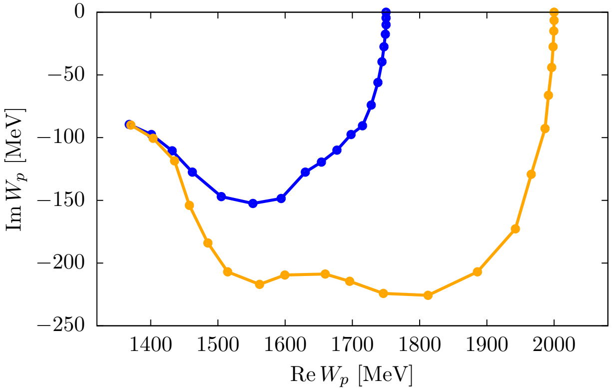
Including an excited three-quark state

Adding a three-quark configuration $(1s)^2 2s$ with mass m_R.

m_R [MeV]	m_σ [MeV]	g_σ	$\mathrm{Re}W_p$	$-2\mathrm{Im}W_p$	$\lvert r \rvert$	ϑ
PDG			1370	180	46	$-90°$
2000	600	1.55	1368	180	48.0	$-87°$
2000	600	1.70	1361	156	41.9	$-77°$
1530	600	1.55	1367	180	47.5	$-86°$
2400	600	1.68	1370	177	42.6	$-87°$
2000	500	1.43	1369	172	40.2	$-82°$
1530	500	1.36	1365	174	43.6	$-82°$

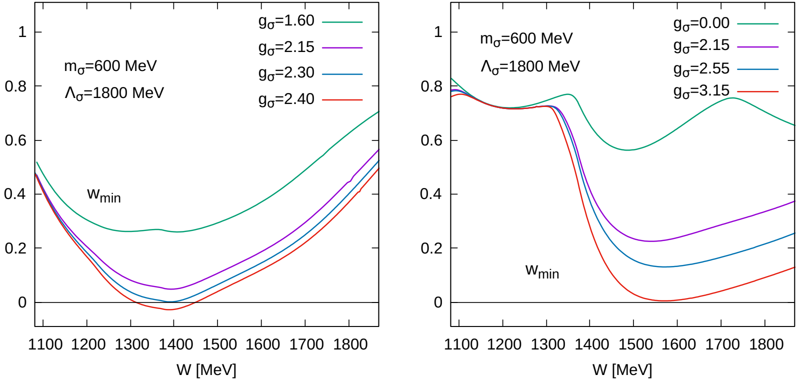
Evolution of the resonance pole in the complex W-plane

Evolution of the pole in the complex-energy plane as the interaction strength is being switched-off for $m_R = 1750$ MeV and $m_R = 2000$ MeV



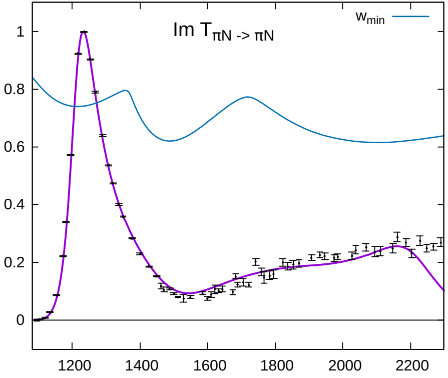
Dynamically generated Δ resonances

Coupled channel approach involving the p-wave πN and $\pi\Delta$ channels, and the s-wave $\sigma\Delta$ channel; check the behaviour of the lowest eigenvalue of the kernel:



N^* Δ^*

Is there a resonance around $W \approx 1500$ MeV?



Three-quark configuration $(1s)^2 2s$ at 2150 MeV.

ReW_p	-2ImW_p	$\|r\|$	ϑ
PDG			
1470	314	38	173°
Model no σ			
1491	408	49	170°
Model with σ			
1494	365	43	178°

Manifestation of the meson cloud:

- Clear and "unmistakable" signal
  - E2/M1 and S2/M1 ratio of helicity amplitudes for $\Delta(1232)$
- Strong signal, commonly agreed upon
  - strong enhancement of the $\pi N\Delta(1232)$ vertex
  - strong enhancement $M1$ electro-production amplitude on the Δ
- Strong signal but still disputable (model dependent)
  - zero-crossing of the helicity amplitude for $N^*(1440)$
  - strong enhancement of the the $\pi NN^*(1440)$ vertex
- Possible explanation – work in progress
  - N(1440): molecular state strongly coupled to $(1s)^2 2s$ configuration.
  - Dynamically generated $\Delta(1600)$ resonance