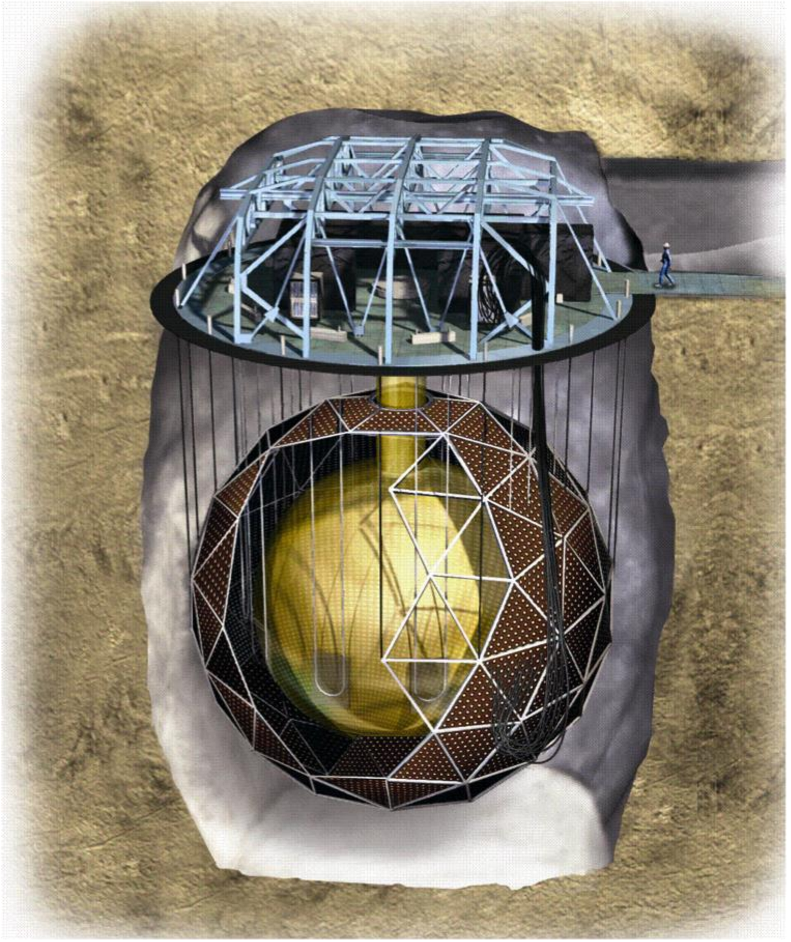


Cosmic ray muons reconstruction in the SNO+ experiment

Daniel Gonçalves

under the supervision of
Fernando Barão & Gersende Prior

LIP

LIP, Lisbon, September 5th 2018

Main Objective

Even if neutrinos are our main concern,

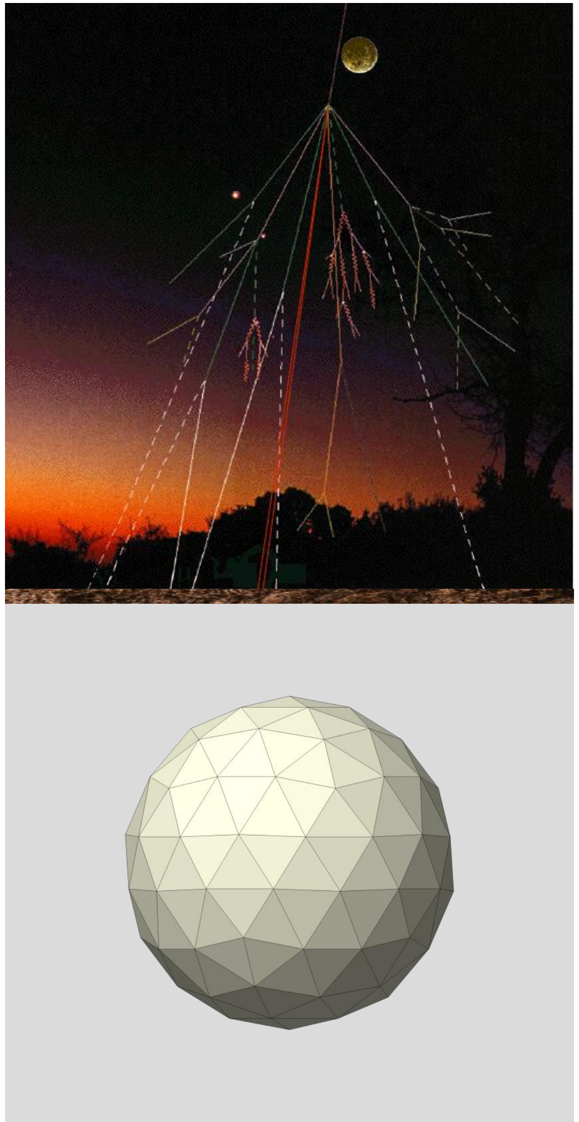
we must estimate the muon underground flux!

Probabilistic Approach

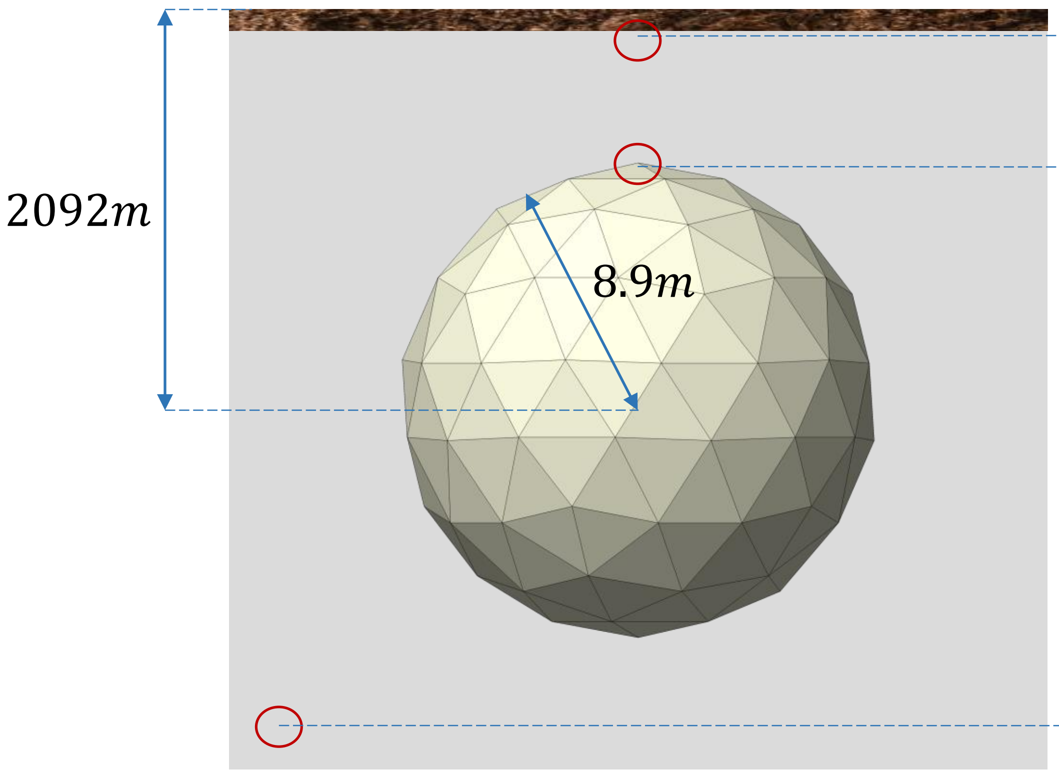
Collision-type Approach

Energetic Approach

Numerical Approach



The SNO+ detector and its surroundings



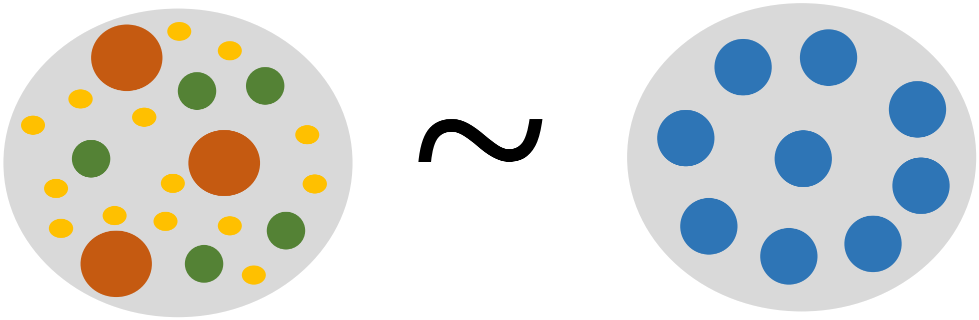
$$\rho_{2,s} = 2.8 g \cdot cm^{-3}$$

$$\rho_{2,v} = 2.9 g \cdot cm^{-3}$$

$$\rho(890 \leq r \leq 209200, \phi, \theta) = -\frac{1}{2083100} r + \frac{302094}{104155} \quad g \cdot cm^{-3}$$

$$\rho_1 = 2.83 g \cdot cm^{-3}$$

Major simplifications



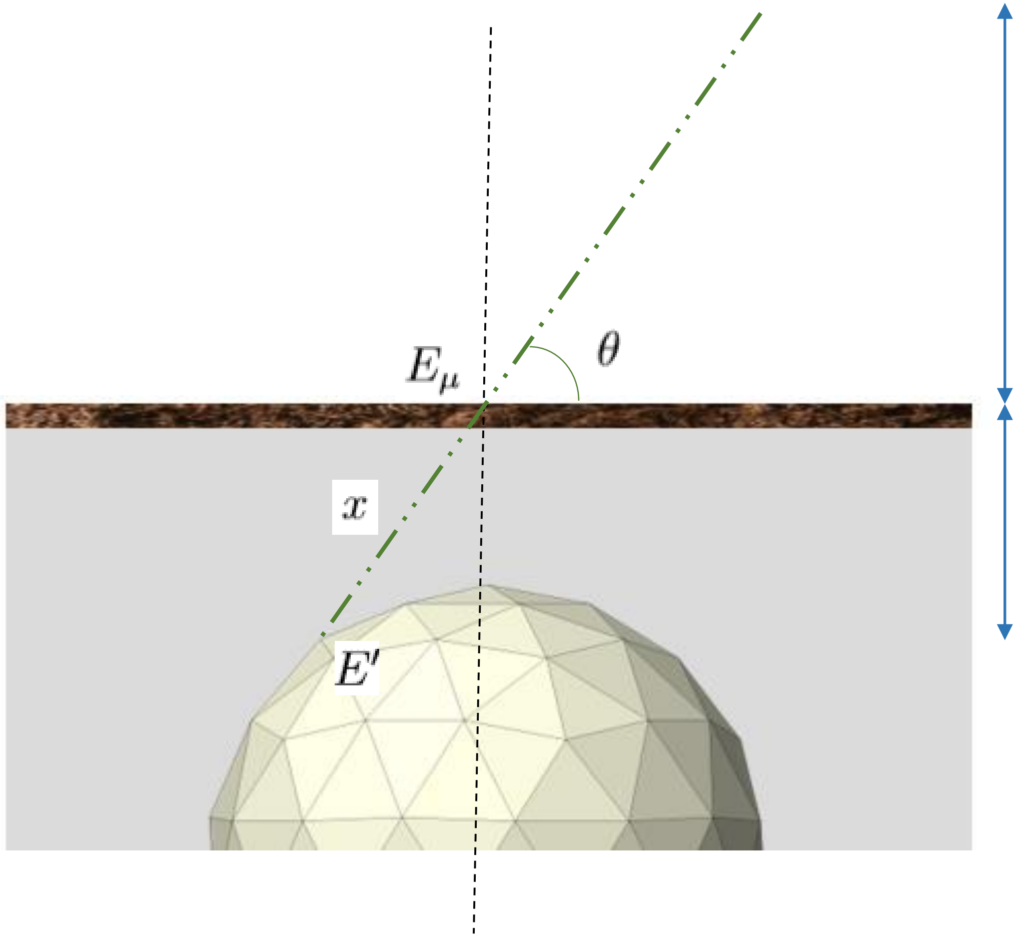
$\rho = \rho(r, \phi, \theta)$ → $\rho = 2.85 g \cdot cm^{-3}$

45% oxygen
26% silicon
9% aluminum
4% iron

$$Z = \frac{0.45Z_0 + 0.26Z_{Si} + 0.09Z_{Al} + 0.04Z_{Fe}}{0.84} \approx 11.25$$

$$A = \frac{0.45A_0 + 0.26A_{Si} + 0.09A_{Al} + 0.04A_{Fe}}{0.84} \approx 3.788 \times 10^{-23} g$$

Method



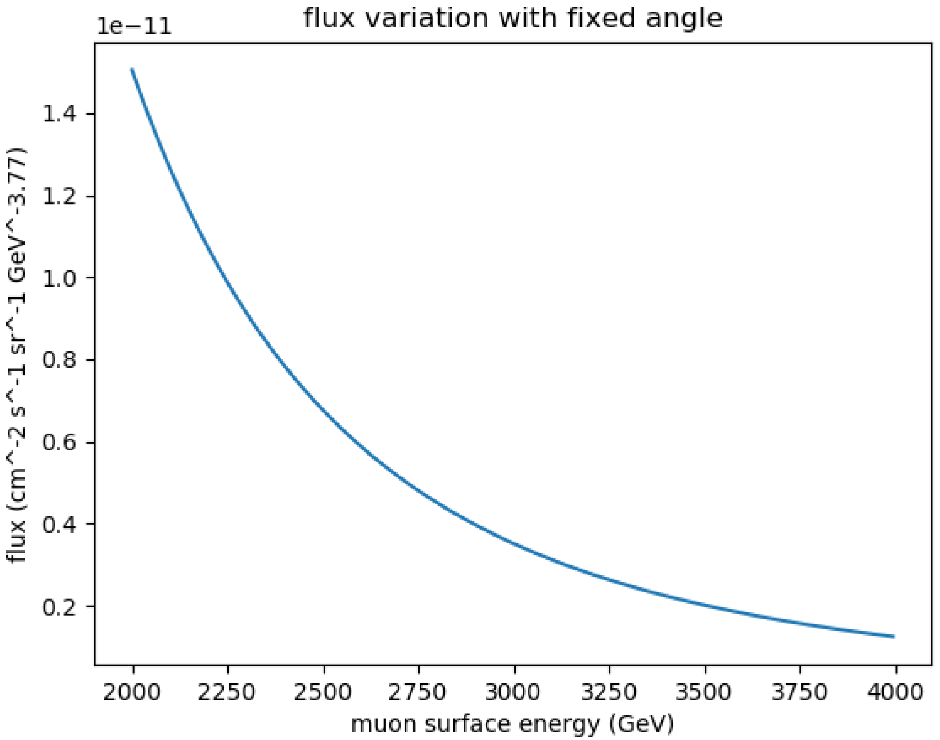
$$\Phi := \frac{dN}{dE} = \frac{0.14(E_\mu/GeV)^{-\gamma}}{cm^2 ssr GeV}\left(\frac{1}{1+\frac{1.1E_\mu cos\theta}{115GeV}} + \frac{0.054}{1+\frac{1.1E_\mu cos\theta}{850GeV}}\right)$$

$$= E_\mu^{-2.7}\left(\frac{16.1}{115+1.1E_\mu cos\theta} + \frac{6.426}{850+1.1E_\mu cos\theta}\right)$$

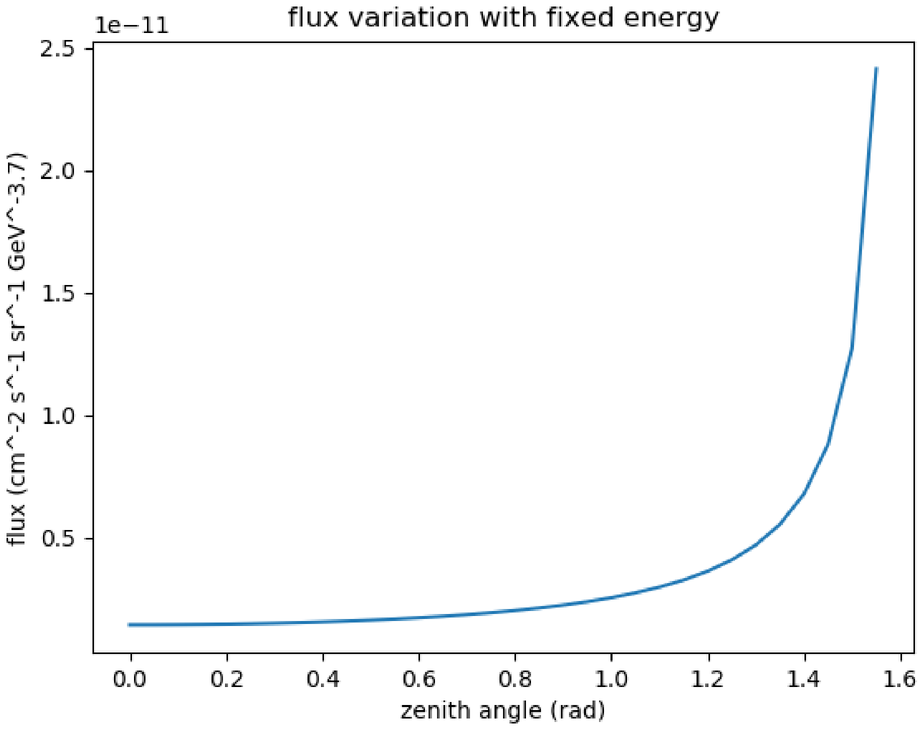
$$-\frac{\overline{dE}}{dx} = \frac{4\pi n z^2}{m_e c^2 \beta^2}\left(\frac{e^2}{4\pi\epsilon_0}\right)^2\left[\ln\left(\frac{2m_e c^2\beta^2}{I(1-\beta^2)}\right) - \beta^2\right]$$

Surface flux

$$\Phi = E_\mu^{-2.7}\left(\frac{16.1}{115+1.1E_\mu cos\theta}+\frac{6.426}{850+1.1E_\mu cos\theta}\right)\ cm^{-2}s^{-1}sr^{-1}GeV^{-1.7}$$



Angle fixed: $\pi/4$



Energy fixed: $3.5TeV$

Energy is lost: muon interaction with the rock

$$-\frac{\overline{dE}}{dx} = \frac{4\pi n z^2}{m_e c^2 \beta^2}\left(\frac{e^2}{4\pi\epsilon_0}\right)^2\left[\ln\left(\frac{2m_e c^2\beta^2}{I(1-\beta^2)}\right)-\beta^2\right]$$ where $n = \frac{N_A Z\rho}{A}$ and $I \approx (10eV)Z$

- $n = n(r,\phi,\theta) \approx 5.097277933 \times 10^{47} cm^{-3}$
- $\frac{4\pi n z^2}{(4\pi)^2\epsilon_0^2 m_e c^2} = \frac{n}{4\pi\epsilon_0^2 m_e c^2} = 1.012578065 \times 10^{63}\frac{cm^{-3}}{eVC^4N^{-2}m^{-4}} = 1.012578065 \times 10^{69}\frac{1}{m^{-1}eVC^4N^{-2}}$
- $\frac{2m_e c^2}{I} = 9084.\overline{4}\frac{eV}{eV} = 9084.\overline{4}$

$$d\overline{E_p} := -\frac{\overline{dE}}{dx} \approx \frac{6.672605127 \times 10^{-7}}{\beta^2}\left[\ln\left(9084.44\frac{\beta^2}{1-\beta^2}\right)-\beta^2\right]\frac{1}{m^{-1}eVN^{-2}}$$

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Energy is lost: caution needed

- $\beta^2 = \beta^2(x)$
- $E = K + E_0 \to K$ where $E_0 = m_\mu c^2$
- $K = m_\mu c^2 \left(\frac{1}{\sqrt{1-\beta^2}} - 1 \right) \Longrightarrow \beta^2 = \beta^2(K) = 1 - \left(\frac{m_\mu c^2}{K + m_\mu c^2} \right)^2$

The Bethe Formula can thus be expressed as

$$-\frac{dK}{dx} = \xi(K) \quad \text{being}$$

$$\xi(K) = \frac{6.672605127 \times 10^{-7}}{1 - \left(\frac{m_\mu c^2}{K+m_\mu c^2}\right)^2} \left\{ \ln \left[9084.44 \frac{1 - \left(\frac{m_\mu c^2}{K+m_\mu c^2}\right)^2}{\left(\frac{m_\mu c^2}{K+m_\mu c^2}\right)^2} \right] + \left(\frac{m_\mu c^2}{K + m_\mu c^2} \right)^2 - 1 \right\}$$

Energy is lost: what we ‘just’ need to solve

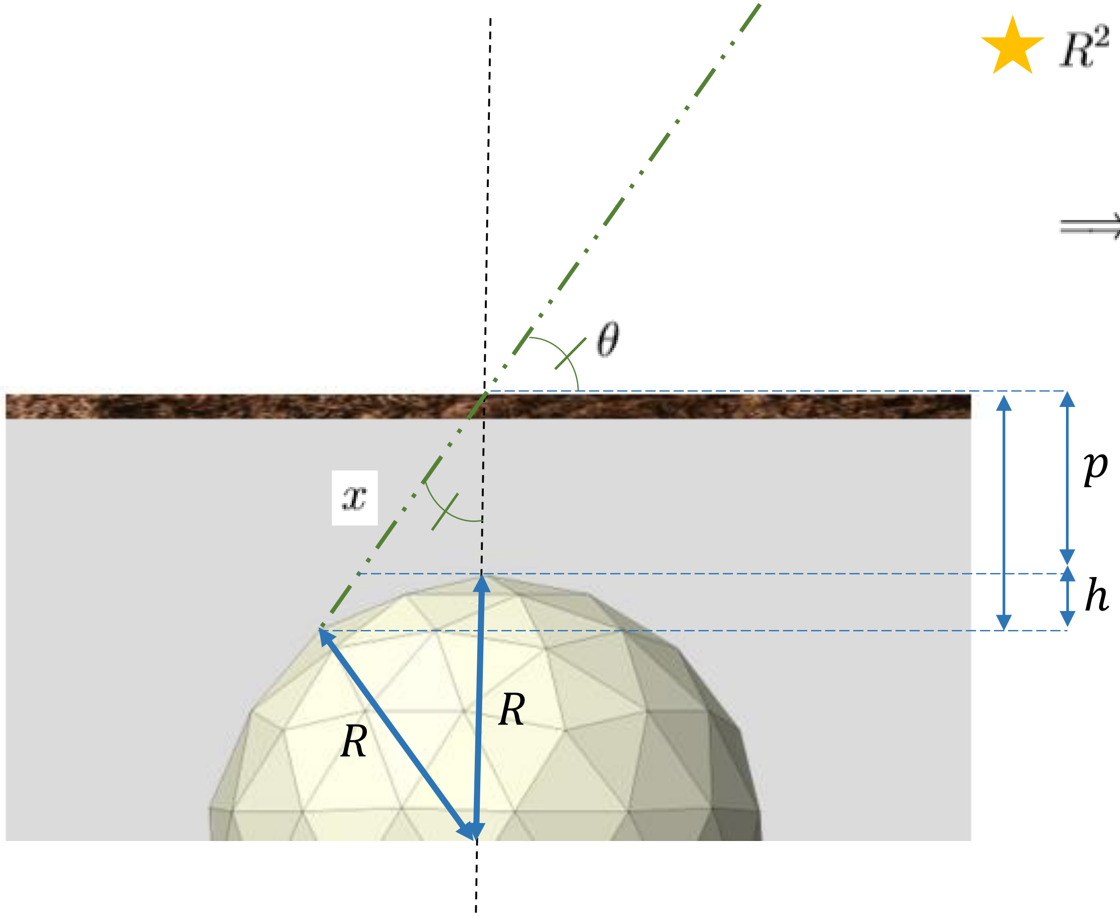
$$-\frac{dK}{dx} = \frac{2.6\times 10^{25}}{1-\frac{11172.49}{(K+105.7)^2}} \ln\left[9084.44\frac{1-\frac{11172.49}{(K+105.7)^2}}{\frac{11172.49}{(K+105.7)^2}}\right] - 2.6\times 10^{25} \Longrightarrow$$

$$\Longrightarrow |dK| = \int_0^{x(\theta)} \left\{\frac{2.6\times 10^{25}}{1-\frac{11172.49}{(K+105.7)^2}} \ln\left[9084.44\frac{1-\frac{11172.49}{(K+105.7)^2}}{\frac{11172.49}{(K+105.7)^2}}\right] - 2.6\times 10^{25}\right\} dx$$

where $E_0 = 105.7 MeV$

and $6.672605127\times 10^{-7} mN^2/eV \approx 2.599409161\times 10^{25} MeV/m \approx 2.6\times 10^{25} MeV/m$.

Energy is lost: distance traveled by the muon



★ $R^2 = x^2\sin^2\theta + (R-h)^2 \Leftrightarrow h^2 - 2Rh + x^2\sin^2\theta = 0 \Longrightarrow$

$$\Longrightarrow h = \frac{2R \pm \sqrt{4R^2 - 4x^2\sin^2\theta}}{2} \Longrightarrow h = R - \sqrt{R^2 - x^2\sin^2\theta}$$

★ $x\cos\theta = p + R - \sqrt{R^2 - x^2\sin^2\theta} \Longrightarrow$

$$x = x(\theta) = (p+R)\cos\theta \pm \sqrt{(p+R)^2\cos^2\theta - p(p+2R)}$$

which only gives

$$x = x(\theta) = 2092\cos\theta - \sqrt{4376464\cos^2\theta - 4376384.79}$$

that **cannot** be correct!

Final remarks: a numerical job

Let $a := 2.6 \times 10^{25} MeV/m, b := 11172.49(MeV)^2, c := 105.7 MeV, d := 9084.44$

★ $$-\frac{dK}{dx} = \xi(K) \implies \frac{-dK}{\xi(K)} = dx \implies \int \frac{-dK}{\xi(K)} = \int dx \quad \text{for} \quad \xi(K) \neq 0$$

but $$\frac{1}{\xi(K)} = \frac{(K+c)^2 - b}{a\left\{(K+c)^2\left[\ln\left(\frac{d((K+c)^2-b)}{b}\right) - 1\right] + b\right\}}$$

★ Taylor expansion? **Better not**

★ So we are left with $$|dK| = \int_0^{2092\cos\theta - \sqrt{4376464\cos^2\theta - 4376384.79}} \left[\frac{a}{1-\frac{b}{(K+c)^2}} \ln\left(d\frac{1-\frac{b}{(K+c)^2}}{\frac{b}{(K+c)^2}}\right) - a\right] dx$$

and a *python* program eager to be run!

Thank You