Relativistic Quantum Information and QFT on curved spacetimes

Marcos Sampaio
Setembro – 2018
The cube of physical theories:

Padmanabhan

*Phys Rev D*, vol. 84, 2011
Use entanglement and relativity to improve information tasks.

Applications to QUANTUM METROLOGY: squeezing and entanglement to enhance sensitivity of measurements.
R. Schützhold
Interaction of a Bose-Einstein condensate with a gravitational wave

R. Wald
Quantum Superposition of Massive Objects and the Quantization of Gravity

R. Ursin
Quantum optics experiment in a relativistic environment

M. Zych
Relativity of quantum superpositions

Dennis Raetzel
Optical resonators in curved spacetime

Maria E. Papageorgiou
Impact of relativity on localizability and vacuum entanglement

Aida Ahmadzadegan
Exploring the boundaries of multipartite quantum communication

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Jason Pie
Lorentz-Covariant Generalised Uncertainty Principles
**QUANTUM ENTANGLEMENT**

Alice can make unitary transformations and measurements only in $A$, Bob only in the complement $B$

$$|\psi\rangle = \cos \theta |\uparrow\rangle_A |\downarrow\rangle_B + \sin \theta |\downarrow\rangle_A |\uparrow\rangle_B$$

**QUANTIFYING ENTANGLEMENT**

**PURE STATES:**

$$|\Phi\rangle_{AB} = \sum_{i,j} \omega_{ij} |i\rangle_A \otimes |j\rangle_B \quad \Rightarrow \quad |\Phi\rangle_{AB} = \sum_n \omega_n |n\rangle_A \otimes |n\rangle_B$$

Measure of entanglement: use density matrix

$$\rho_{AB} = |\Phi\rangle \langle \Phi|_{AB}$$

DEFS: reduced density matrix (subsystem $A$)

$$\rho_A = \text{Tr}_B(\rho_{AB})$$

von Neumann entropy

$$S(\rho) = -\text{Tr}(\rho \log_2(\rho))$$

DEF: entanglement between $A$ and $B$ =

$$S(\rho_A) = S(\rho_B)$$

**MIXED STATES**

no analogue to Schmidt decomposition

(entropy no longer quantifies entanglement)

but necessary condition for separability (no negative eigenvalues) suggest to use

negativity = sum of negative eigenvalues of $\rho_{AB}^{PT}$
Loophole-free Bell inequality violation using electron spins separated by 1.3 kilometres

B. Hensen et al.,

Special Relativistic Effects

Is Entanglement Lorentz Invariant?

Wigner Rotations

massive particles

$$|p_{\Lambda}, s'\rangle = U(\Lambda) |p, s\rangle$$

$$U(\Lambda) |p, s\rangle = U(\Lambda L(p)) |k, s\rangle = U(L(p_{\Lambda})) U(L^{-1}(p_{\Lambda})\Lambda L(p)) |k, s\rangle$$

$$W(\Lambda, p) = L^{-1}(p_{\Lambda})\Lambda L(p)$$

$$\text{spin}-\frac{1}{2} \quad SU(2)$$

$$|\psi\rangle_{\text{total}} = (\cos \alpha |p_+, p_-\rangle + \sin \alpha |p_-, p_+\rangle) (\cos \beta |\uparrow \downarrow\rangle + \sin \beta |\downarrow \uparrow\rangle)$$

$$\rho = |\psi\rangle \langle \psi |$$

$$E(\rho) = \sum_i \left( 1 - \text{Tr} \rho_i^2 \right)$$

$\rho_i$ is obtained by tracing over all subsystems except the $i$-th
Entanglement in different Partitions

1 vs 3 Qubit Partition

- Momentum $\Lambda p_+, \Lambda p_-$ or spin $\uparrow, \downarrow$

Entanglement-Egg-Tray:
- Difference between linear entropies of initial and $\delta = \pm \frac{\pi}{2}$ Wigner rotated Bell-type state
  - $\exists$ entanglement change
  - only for entangled momenta
  - change maximal for separable spins

Spin & Momentum Partition

- 2 momentum or 2 spin qubits

Entanglement change between momentum and spin of $\delta = \pm \frac{\pi}{2}$ Wigner rotated spin-Bell-type state
  - entanglement change for entangled momenta
  - identical to 1 vs 3 partition for $\delta \to \frac{\pi}{2}$
  - maximal change also for entangled spins

Generally (e.g. for $\psi^-$) there is **entanglement - tradeoff** between 

**spin - spin**, **momentum - momentum** & **spin - momentum entanglement**

<table>
<thead>
<tr>
<th>Alice</th>
<th>Bob</th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="image1" alt="Spin-Spin" /></td>
<td><img src="image2" alt="Spin-Spin" /></td>
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<tr>
<td><img src="image3" alt="Momentum-Momentum" /></td>
<td><img src="image4" alt="Momentum-Momentum" /></td>
</tr>
<tr>
<td><img src="image5" alt="Spin-Momentum" /></td>
<td><img src="image6" alt="Spin-Momentum" /></td>
</tr>
</tbody>
</table>

**but entanglement between particles is **genuinely invariant****

**Alice - Bob partition**

- **NO entanglement change**

- **Which physical partitions are accessible?**
- **Separation of spin and momentum possible?**

**Vedral, Saldanha, PRA87(2013)042102**
1. Uniformly Accelerated Observer ROB

• An accelerating observer in Minkowski spacetime detects a thermal spectrum of particles in the vacuum state.

• Analogous to the Hawking radiation in black hole spacetime.
Vacuum is a relative concept, observer dependent

Rindler Coordinates \((\eta, \chi)\)

There is the physical situation of two observers, Alice and (Anti)Rob, one of which, Alice, is inertial, while the other one, (Anti)Rob, is uniformly accelerated, see Fig.1.

\[
x = -\frac{e^{a\chi}}{a} \cosh(a\eta)
\]
\[
t = -\frac{e^{a\chi}}{a} \sinh(a\eta)
\]
\[
x^2 - t^2 = \frac{e^{a\chi}}{a}
\]

The uniformly accelerated observers Rob and AntiRob are confined to the Rindler wedges I (\(|t| < x\)) and II (\(|t| < -x\)) respectively, which are causally disconnected from each other. Their worldlines are hyperbolas, which correspond to lines of constant \(\chi = c^2/a\), where a is their proper acceleration.
KLEIN GORDON EQUATION (2D)

**FLAT SPACE**

\[ \Box \phi = 0 \text{ in 2-dim are plane waves of the form} \]

\[ u_k = \frac{1}{\sqrt{2\pi \omega}} e^{i(kx - \omega t)} \]

\[ i\partial_t u_k = -i\omega u_k \]

\[ i\partial_t u_k^* = i\omega u_k^* \]

\[ u_k \rightarrow \text{positive frequency solutions} \]

\[ u_k^* \rightarrow \text{negative frequency solutions} \]

\[ \phi = \int (u_k a_k + u_k^* a_k^\dagger) dk. \]

\[ |n_1, ..., n_k\rangle = a_1^{\dagger n_1} ... a_k^{\dagger n_k} |0\rangle \]

**RINDLER SPACE**

\[ \Box \phi := \frac{1}{\sqrt{-g}} \partial_\mu (\sqrt{-g} g^{\mu \nu} \partial_\nu \phi) \]

\[ \hat{\phi} = \int (u_k a_k^I + u_k^* a_k^I + h.c.) dk \]

\[ |0\rangle_R = |0\rangle_I \otimes |0\rangle_{II} \]

\[ a_k^I |0\rangle^I = 0 \]

\[ a_k^I |0\rangle^I = 0 \]

**Global Killing Vector Field**

**Bogoliubov Transformation**

\[ a_k = \int ((u_k, u_k') a_k + (u_k, u_k') a_k^\dagger + (u_k, u_k') a_k^I + (u_k, u_k') a_k^I + (u_k, u_k') a_k^I) dk \]

\[ u_k^U = \cosh ru_k^I + \sinh ru_k^{I*}, \quad \text{sech}^2(r) = 1 - e^{\frac{2\pi \omega}{\alpha}}. \]

**Unruh basis**

2-mode squeezed state

\[ |0_k\rangle^M = \frac{1}{\cosh(r)} \sum_n \tanh^n(r) |n_k\rangle^I |n_k\rangle^{II} \]
Observer region I has no access to information region II. Trace over II

\[ \rho_I^k = \text{Tr}_{II}(|0_k\rangle^\mathcal{M}\langle 0_k|) = \frac{1}{\cosh^2(r)} \sum_n \tanh^{2n}(r)|n_k\rangle^I\langle n_k| = \left(e^{-\frac{2\pi \omega}{a}} - 1\right) \sum_n \left(e^{-\frac{2\pi \omega}{a}}\right)^n |n_k\rangle^I\langle n_k| \]

Canonical thermal state with temperature \( T_U = \frac{a}{2\pi} \), a observer acceleration (\( \hbar = c = k_B = 1 \))

Let \( v_k^I \) be the mode functions of region I \( (b_k, b_k^\dagger) \) and \( u_k \) those of inertial system \( (a_k, a_k^\dagger) \).

Related by Bogoliubov transformation

\[ v_k^I = \int_{k'} \alpha_{kk'} u_{k'} + \beta_{kk'} u_{k'}^\ast \]

\[ \hat{b}_k = \int_{k'} \alpha_{kk'}^\ast \hat{a}_{k'} - \beta_{kk'}^\ast \hat{a}_{k'}^\ast \]

Occupation number: Accelerated observer will perceive a thermal bath of particles

\[ \mathcal{M} \langle 0|\hat{b}_k^\dagger \hat{b}_k|0\rangle^\mathcal{M} = \int_{k'} |\beta_{k'k}|^2 = \frac{1}{e^{2\pi|k|/a} - 1} \]

Bose Black Body Radiation with \( \bar{T} = \frac{2\pi}{a} \)
What are the effects of gravity and motion on quantum properties?

Fuentes-Schuller, Mann PRL 2005
Adesso, Fuentes-S, Ericsson PRA 2007

Entanglement • observer-dependent
• degrades with acceleration, vanishes for \( \infty \) acceleration

\[ |\text{Bell}\rangle \sim |0\rangle \otimes |0\rangle + |1\rangle \otimes |1\rangle \]

more realistic states:

\[ |\text{squeezed}\rangle \sim \sum_{n \geq 0} \tanh^n s \ |n\rangle \otimes |n\rangle \]
The Hawking radiation

“Black holes ain’t so black”
Hawking v.s. Unruh

A stationary observer outside the black hole would see the thermal Hawking radiation.

An accelerating observer in vacuum would see a similar Hawking-like radiation called Unruh radiation.

### Special and General Relativistic Effects

<table>
<thead>
<tr>
<th>Effect</th>
<th>Frame of Reference</th>
<th>Timeline</th>
<th>Level</th>
</tr>
</thead>
<tbody>
<tr>
<td>Lorentz transformed polarization</td>
<td>LEO and beyond</td>
<td>mid-term</td>
<td>QM + SR</td>
</tr>
<tr>
<td>Relativistic frame dragging</td>
<td>TBD</td>
<td>TBD</td>
<td>QM + GR</td>
</tr>
<tr>
<td>Entanglement with curvature</td>
<td>TBD</td>
<td>visionary</td>
<td>QM + GR</td>
</tr>
<tr>
<td>Fermi problem</td>
<td>Sunshielded satellites</td>
<td>long-term</td>
<td>QFT</td>
</tr>
<tr>
<td>Optical Colella-Overhauser-Werner experiment</td>
<td>LEO and beyond</td>
<td>near-term</td>
<td>QM + GR</td>
</tr>
</tbody>
</table>

### Accelerating Detectors in Quantum Field Theory

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</tr>
</thead>
<tbody>
<tr>
<td>Acceleration induced fidelity loss</td>
<td>TBD</td>
<td>visionary</td>
<td>QFT + GR</td>
</tr>
<tr>
<td>Berry phase interferometry</td>
<td>LEO</td>
<td>mid-term</td>
<td>QFT + GR</td>
</tr>
<tr>
<td>Gravitationally induced entanglement decorrelation</td>
<td>LEO and beyond</td>
<td>near-term</td>
<td>Non-standard</td>
</tr>
<tr>
<td>Spacelike entanglement extraction</td>
<td>TBD</td>
<td>visionary</td>
<td>QFT + GR</td>
</tr>
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Table 1. Summary of possible experiments. LEO refers to Low Earth Orbit, an elliptical orbit about the Earth with altitude up to 2000 km. The timeframes are mentioned in Section 1. Roughly, ‘near-term’ experiments (~ 5 years) can be accomplished with a single satellite in LEO, ‘mid-term’ experiments (25 years) require multiple satellites or higher orbits, ‘long-term’ experiments involve Earth-Moon distances, and ‘visionary’ experiments extend to solar orbits and beyond. Under “Regime” (and throughout the paper) QM refers to ordinary quantum mechanics, QFT to quantum field theory, SR to special relativity, GR to general relativity, and QG to quantum gravity. The “Level” classifications are explained in Section 1.1.
The change in gravity is predicted to cause a degradation of the entanglement between the BECs

Quantum Fields in Expanding Spacetimes

1+1 FRW universe

\[ ds^2 = a(\eta)^2 (d\eta^2 - dx^2) \]

\[ \sqrt{g} \mathcal{L} = \frac{1}{2} \eta^{\mu \nu} \partial_\mu \phi \partial_\nu \phi - \frac{m^2 a^2}{2} \phi^2 \]

**In modes**

\[ \Box \phi + m^2 a_{in}^2 \phi = 0 \]

\[ \phi_k^{in} \xrightarrow{\eta \to -\infty} e^{ikx} e^{-i\omega_{in}(k)\eta} \]

\[ a_{in} \]

\[ \hat{\phi} = \sum_k \hat{a}_{k}^{in} \phi_k^{in} + \hat{a}_{k}^{in*} \phi_k^{in*} \]

**Out modes**

\[ \Box \phi + m^2 a_{out}^2 \phi = 0 \]

\[ \phi_k^{out} \xrightarrow{\eta \to +\infty} e^{ikx} e^{-i\omega_{out}(k)\eta} \]

\[ a_{out} \]

\[ \hat{\phi} = \sum_k \hat{a}_{k}^{out} \phi_k^{out} + \hat{a}_{k}^{out*} \phi_k^{out*} \]
Relate IN and OUT states via Bogoliubov transformation (squeezing)

barred quantities (IN REGION) and unbarred quantities (OUT REGION)

\[
\bar{a}_s = \alpha^*_s a_s - \beta^*_s \bar{a}^\dagger_{-s},
\]

so that mixing occurs only between states labelled by \(s\) and \(-s\).

\[
\langle \bar{0} | \hat{a}^\dagger_s \hat{a}_s | \bar{0} \rangle = |\beta_s|^2
\]

\[
ds^2 = a(\eta)(d\eta^2 - dx^2)
\]

\[a(\eta) = 1 + \epsilon(1 + \tanh \sigma \eta)\]

\(\epsilon, \sigma > 0\) volume and rapidity of expansion

\(\partial/\partial \eta\) Killing vector field in both asymptotic regions (particle content well defined)

As a pure state of a bi-partite system, this can be written as a Schmidt decomposition

\[
|\bar{0}\rangle_k |\bar{0}\rangle_{-k} = \sum_{n=0}^\infty c_n |n\rangle_k |n\rangle_{-k},
\]

\[
c_n = \left(\frac{\beta_k^*}{\alpha_k^*}\right)^n \sqrt{1 - \left|\frac{\beta_k}{\alpha_k}\right|^2}.
\]

where \(n\) labels the number of excitations in the field mode \(k\) (as seen by an inertial observer in the out-region) and the coefficients \(c_n\) are real.
“History of the universe encoded in entanglement”

**Toy model**

- expansion rate $\sigma$
- expansion factor $\epsilon$

- calculate entanglement
  - asymptotic past $S = 0$
  - asymptotic future $S = S(\sigma, \epsilon)$

- excitingly, can solve for
  $\sigma = \sigma(S')$
  $\epsilon = \epsilon(S')$
Entropy quantifies entanglement

\[ \rho = |\bar{0}\rangle_{-k} |\bar{0}\rangle_k \langle 0|_{-k} \langle 0| \]

Density matrix

\[ \rho_k = \sum_{m=0}^{\infty} -k \langle m|\rho|m \rangle_{-k} \]

Reduced density matrix – observe only modes +k

\[ S = -\text{Tr}(\rho_k \log_2 \rho_k) = \log_2 \frac{\gamma^{\gamma/(\gamma-1)}}{1-\gamma} \]

\[ \gamma = \left| \frac{\beta_k}{\alpha_k} \right|^2 \]

Entropy quantifies entanglement

\[ E_p = \sqrt{p^2 + m^2} \text{ such that } m \sqrt{\epsilon} \ll E_p \ll 2\sigma. \]

\[ \epsilon \approx \frac{2E_p^2}{m^2} \sqrt{\gamma(S)} \]

\[ \sigma \approx \frac{\pi}{2} \left( \frac{1 + \gamma(S)}{E \frac{d}{dE} \ln \gamma(S) - 1} \right)^{\frac{1}{2}} E. \]

Ex: small mass limit...

... one can read ST evolution from entanglement!!!!!

Bosonic modes

\[ \rho = 1, \ldots, 100. \]

\[ m = 1 \]

Fermionic modes

\[ \rho = 1, \ldots, 100. \]

\[ m = 1 \]

Fuentes et al, PRD82(2010)045030
the fidelity of teleportation is effected by motion it is possible to correct by local rotations and trip planning
Alice and Bob in an expanding spacetime

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PACS 03.67.Mn – Entanglement measures, witnesses, and other characterizations
PACS 03.65.Ud – Entanglement and quantum nonlocality (\textit{e.g.} EPR paradox, Bell’s inequalities, GHZ states, etc.)
PACS 04.62.+v – Quantum fields in curved spacetime

Abstract – We investigate the teleportation of a qubit between two observers Alice and Bob in an asymptotically flat Robertson-Walker expanding spacetime. We use scalar or fermionic field modes inside Alice’s and Bob’s ideal cavities and show the degradation of the teleportation quality, as measured by the fidelity, through a mechanism governed by spacetime expansion. This reduction is demonstrated to increase with the rapidity of the expansion and to be highly sensitive to the coupling of the field to spacetime curvature, becoming considerably stronger as it reduces from conformal to minimal. We explore a perturbative approach in the cosmological parameters to compute the Bogoliubov coefficients in order to evaluate and compare the fidelity degradation of fermionic and scalar fields.

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Fig. 2: Fidelity as a function of the expansion parameter $\rho$ for the bosonic field case with minimal coupling (solid line) and conformal coupling (dotted line). The dashed line shows the fidelity for the fermionic case. We have fixed $|k| = m = 1$ and $\epsilon = 0.7$. The logarithmic negativity has a similar behavior.
REFERENCES

3. Lecture Series on Relativistic Quantum Information – Ivette Fuentes

LEADING RESEARCH GROUPS

Prof. Ivette Fuentes
- Relativistic Quantum Information

Prof. A. Zeilinger
- Quantum Information and Foundations of Physics

Prof. Thorsten Schumm
- Quantum Metrology

Prof. Reinhold Bertlmann
- Quantum Mechanics of Particle Physics
Perspectives: A growing field of research

- Holographic computation of entanglement using AdS/CFT – Ryu & Takayanagi
- Testing effects of gravity and motion on entanglement in space based experiments – Bruschi, grupo de Nottingham
- Event horizon and entropy in high energy hadron production – Castorini et al.
- Application of Unruh effect to neutrino physics
- Quantum Metrology
- Study of entanglement produced by scattering in a purely quantum field theoretical framework
- Effects of interaction on entanglement of fields in expanding spacetimes

Collaborators
- Paul Mansfield – Durham UK
- Alex Blin and Brigitte Hiller – Coimbra
- Gustavo Souza – UFOP
- J. Geraldo – UESC
- Irismar Paz – UFPI
- Manoel Messias – UFMA
- Karen Fonseca – Colômbia
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- Carolina Arias – UFABC