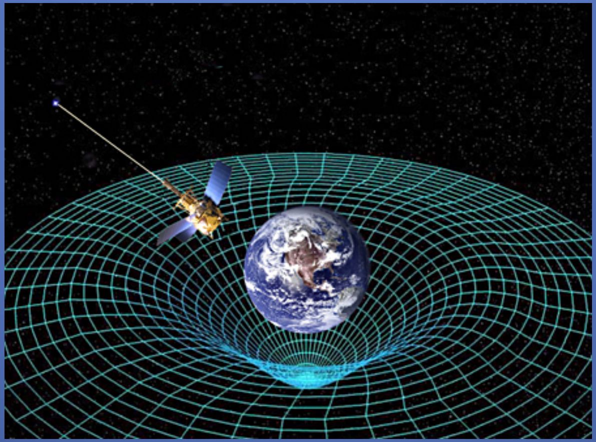
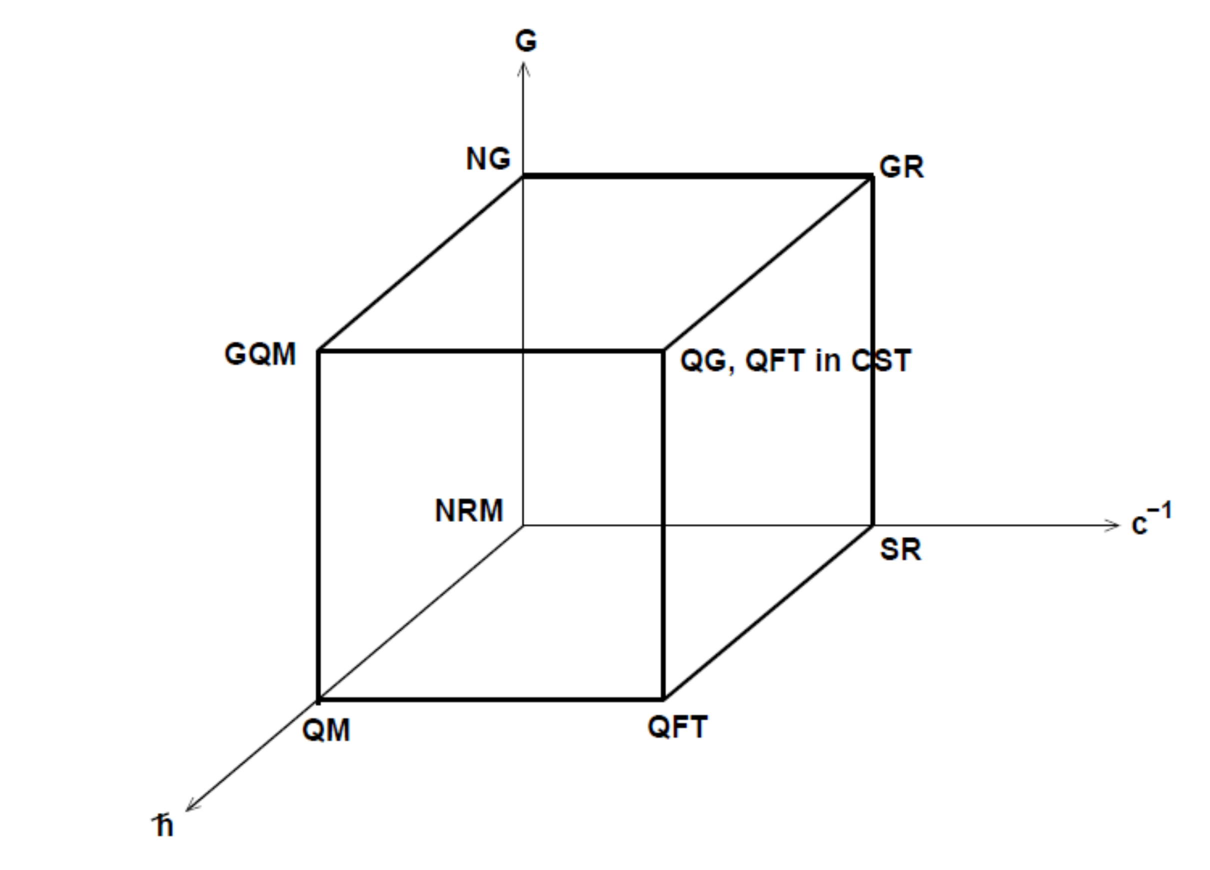
Relativistic Quantum Information and QFT on curved spacetimes



Marcos Sampaio

Setembro – 2018

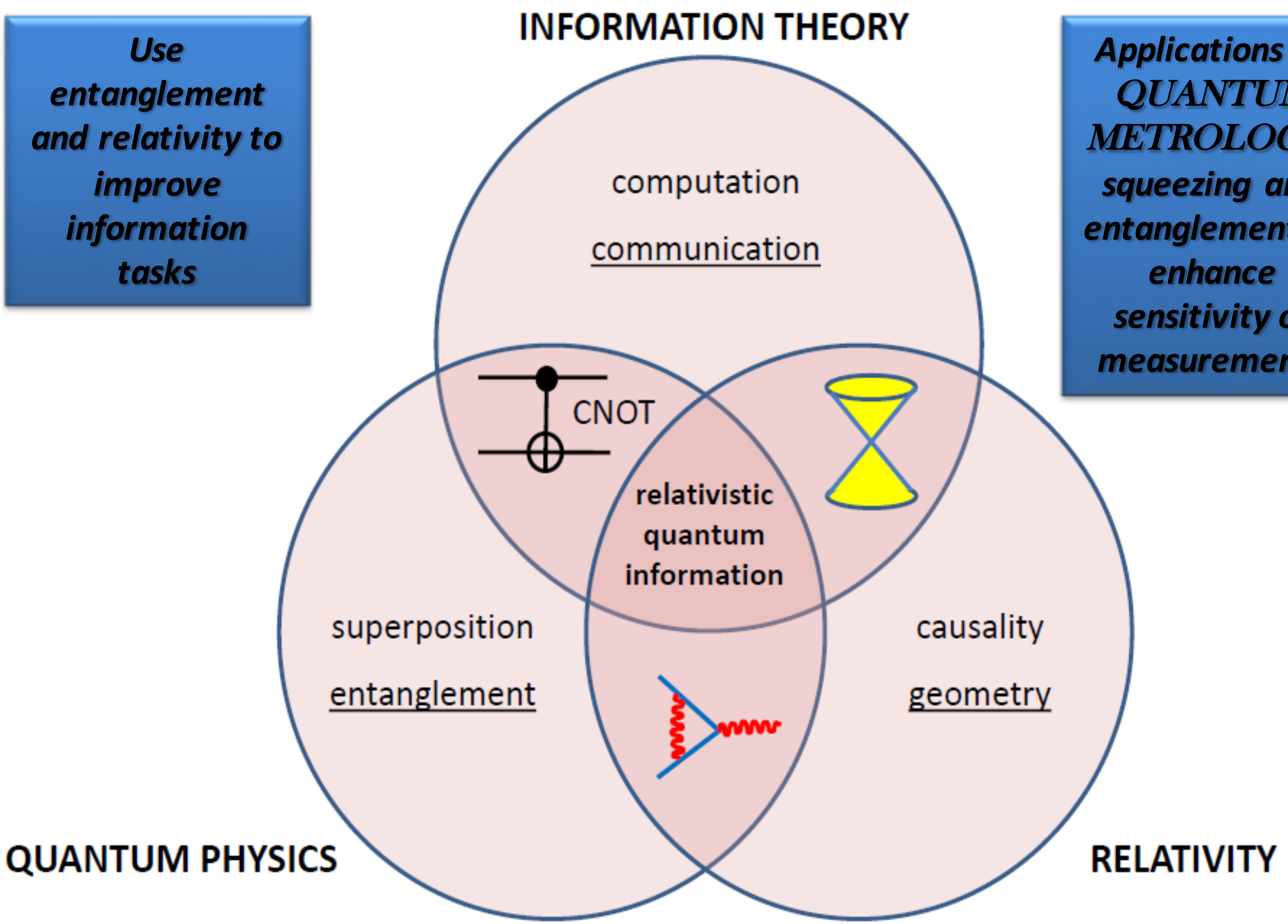
The cube of physical theories: $\hbar \quad G \quad c^{-1}$



Padmanabhan
PHYS REV D , vol. 84, 2011

relativistic quantum information

Use entanglement and relativity to improve information tasks



Applications to QUANTUM METROLOGY: squeezing and entanglement to enhance sensitivity of measurements

RQI Conference Wien 2018

Jason Pie
Lorentz-Covariant Generalised Uncertainty Principles

Aida Ahmadzadegan
Exploring the boundaries of multipartite quantum communication

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Exploring the boundaries of multipartite quantum communication

Maria E. Papageorgiou
Impact of relativity on localizability and vacuum entanglement

R. Schützhold
Interaction of a Bose-Einstein condensate with a gravitational wave

R. Wald
Quantum Superposition of Massive Objects and the Quantization of Gravity

R. Ursin
Quantum optics experiment in a relativistic environment

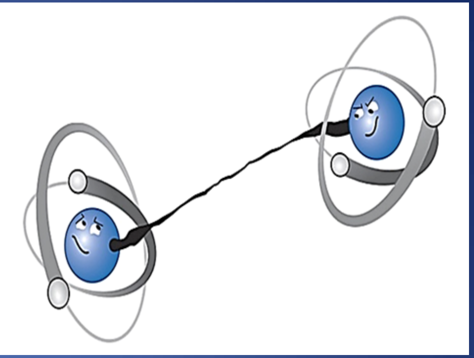
M. Zych
Relativity of quantum superpositions

Dennis Raetzel
Optical resonators in curved spacetime

QUANTUM ENTANGLEMENT

Alice can make unitary transformations and measurements only in *A*, Bob only in the complement *B*

$$|\psi\rangle = \cos\theta\,|\uparrow\rangle_A|\downarrow\rangle_B + \sin\theta\,|\downarrow\rangle_A|\uparrow\rangle_B$$



QUANTIFYING ENTANGLEMENT

PURE STATES:

Schmidt basis

$$|\Phi\rangle_{AB} = \sum_{ij}\omega_{ij}|i\rangle_A \otimes |j\rangle_B \Rightarrow |\Phi\rangle_{AB} = \sum_n \omega_n |n\rangle_A \otimes |n\rangle_B$$

Measure of entanglement: use density matrix $\rho_{AB} = |\Phi\rangle\langle\Phi|_{AB}$

DEFS: reduced density matrix (subsystem A) $\rho_A = \mathbf{Tr}_B(\rho_{AB})$

von Neumann entropy $S(\rho) = -\mathbf{Tr}(\rho\log_2(\rho))$

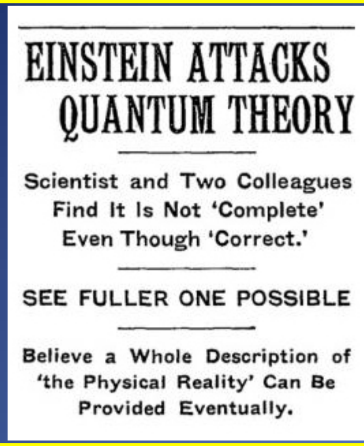
DEF: entanglement between A and B = $S(\rho_A) = S(\rho_B)$

MIXED STATES ➡ no analogue to Schmidt decomposition (entropy no longer quantifies entanglement)

but necessary condition for separability (no negative eigenvalues) suggest to use

negativity = sum of negative eigenvalues of ρ_{AB}^{PT}

1935


EINSTEIN ATTACKS QUANTUM THEORY

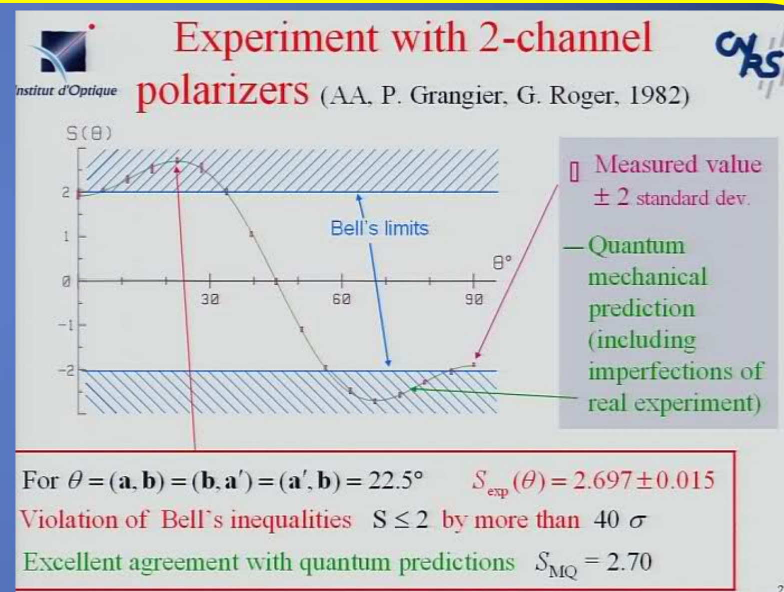
Scientist and Two Colleagues Find It Is Not 'Complete' Even Though 'Correct.'

SEE FULLER ONE POSSIBLE

Believe a Whole Description of 'the Physical Reality' Can Be Provided Eventually.

1982

A. Aspect



2015

Loophole-free Bell inequality violation using electron spins separated by 1.3 kilometres

B. Hensen et al.,

Nature 526, 682–686 (29 October 2015) doi:10.1038/nature15759

Special Relativistic Effects

Is Entanglement Lorentz Invariant?

Bertlmann et al. Phys.Rev.A81(2010)042114

Wigner Rotations

massive particles

$$|\, p_\Lambda,\, s'\,\rangle = U(\Lambda)\,|\, p,\, s\,\rangle$$

$$U(\Lambda)\,|\, p,\, s\,\rangle = U(\Lambda L(p))\,|\, k,\, s\,\rangle = U(L(p_\Lambda))\,U(L^{-1}(p_\Lambda)\Lambda L(p))\,|\, k,\, s\,\rangle$$

$$\mathrm{W}(\Lambda, p) = L^{-1}(\mathrm{p}_\Lambda)\Lambda L(\mathrm{p})$$

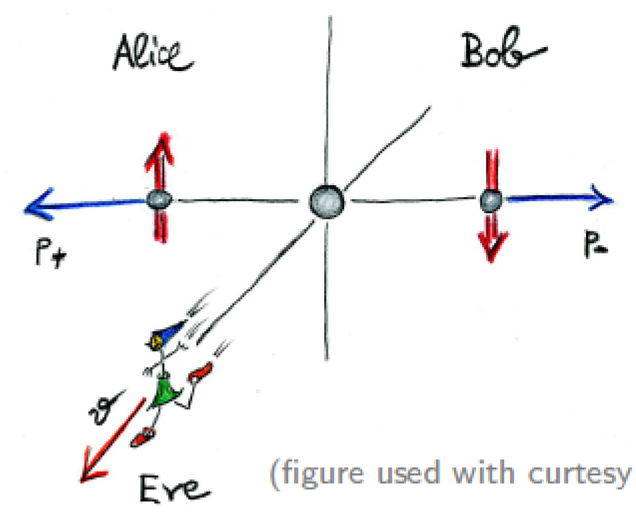
$$\text{spin}-\tfrac{1}{2} \quad SU(2)$$

$$|\,\psi\,\rangle_{\text{total}} = (\cos\alpha\,|\,p_+, p_-\,\rangle + \sin\alpha\,|\,p_-, p_+\,\rangle\,)\,(\cos\beta\,|\!\uparrow\downarrow\rangle + \sin\beta\,|\!\downarrow\uparrow\rangle\,)\,.$$

$$\rho = |\,\psi\,\rangle\langle\,\psi\,|$$

$$E(\rho) = \sum_i (1 - \mathrm{Tr}\,\rho_i^2)$$

ρ_i is obtained by tracing over all subsystems except the i-th.

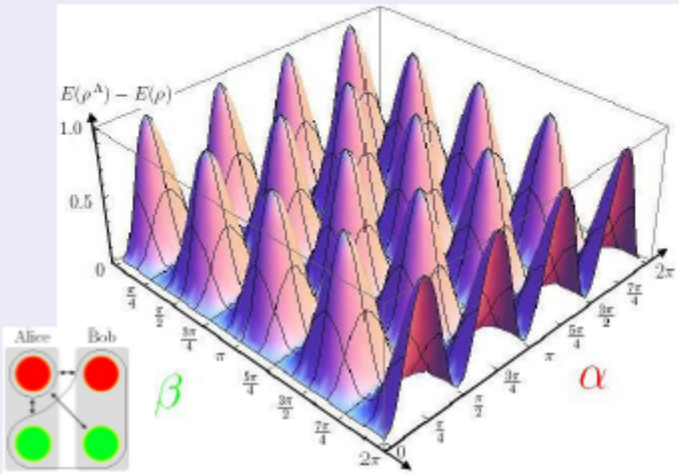


(figure used with curtesy of R. Bertlmann)

Entanglement in different Partitions

1 vs 3 Qubit Partition

momentum Λp_+, Λp_- or spin $\uparrow$, $\downarrow$



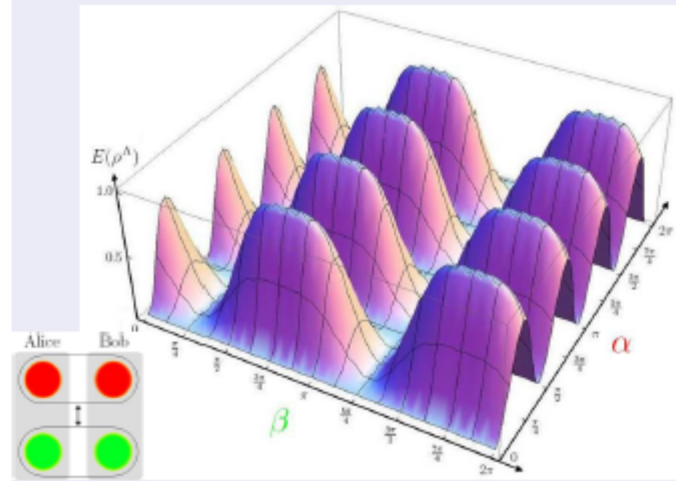
Entanglement-Egg-Tray:

Difference between linear entropies of initial and $\delta = \pm\frac{\pi}{2}$ Wigner rotated Bell-type state

- $\exists$ entanglement change
- only for entangled momenta
- change maximal for separable spins

Spin & Momentum Partition

2 momentum or 2 spin qubits



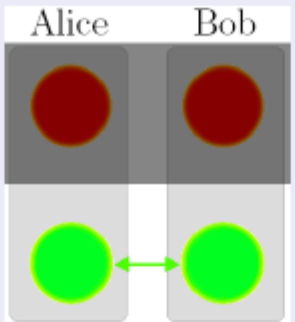
Entanglement change between momentum and spin of $\delta = \pm\frac{\pi}{4}$ Wigner rotated spin-Bell-type state

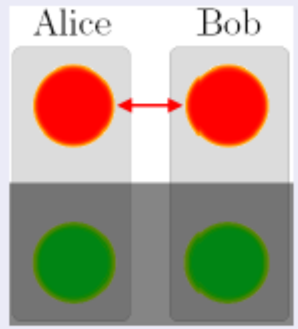
- entanglement change for entangled momenta
- identical to 1 vs 3 partition for $\delta \to \frac{\pi}{2}$
- maximal change also for entangled spins

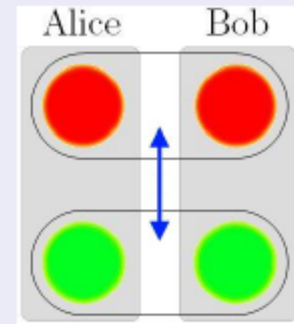
N. F., R. Bertlmann, M. Huber, & B. C. Hiesmayr, Phys. Rev. A **81** (2010) 042114

Generally (e.g. for ψ^-) there is entanglement - tradeoff between

spin - spin, momentum - momentum & spin - momentum entanglement



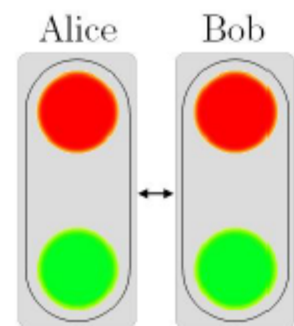




but entanglement between particles is **genuinely invariant**

Alice - Bob partition

- **NO** entanglement change



- **Entanglement not generally invariant**
- **Particle entanglement is invariant**
- **Bell inequality violation invariant**

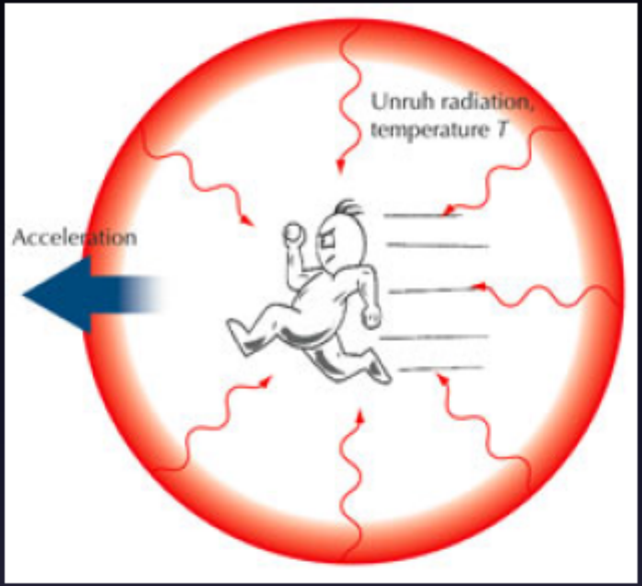
- **Which physical partitions are acessible?**
- **Separation of spin and momentum possible?**

Vedral, Saldanha, PRA87(2013)042102

General Relativistic Effects

1. Uniformly Accelerated Observer ROB

- An accelerating observer in Minkowski spacetime detects a thermal spectrum of particles in the vacuum state.



- Analogous to the Hawking radiation in black hole spacetime.

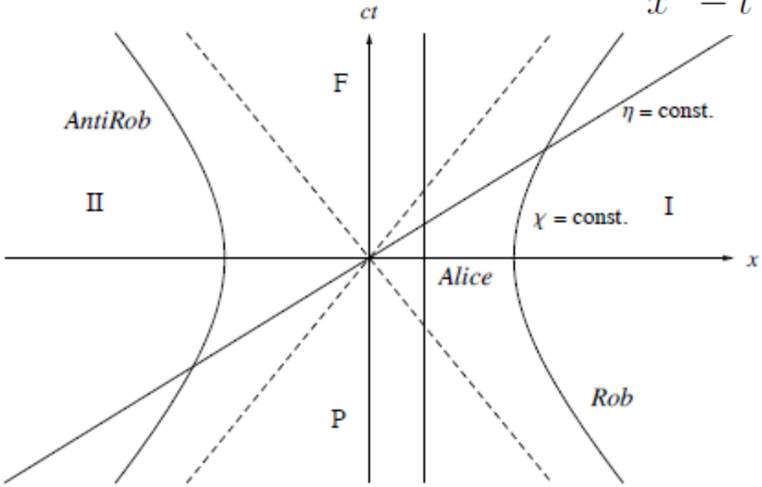
Vacuum is a relative concept, observer dependent

Rindler Coordinates (η, χ)

There is the physical situation of two observers, Alice and (Anti)Rob, one of which, Alice, is inertial, while the other one, (Anti)Rob, is uniformly accelerated, see Fig.1.

$$x^2 - t^2 = \frac{e^{a\chi}}{a}.$$

$$x = \frac{-e^{a\chi}}{a}\cosh(a\eta) \qquad t = \frac{-e^{a\chi}}{a}\sinh(a\eta)$$



$$x = \frac{e^{a\chi}}{a}\cosh(a\eta) \qquad t = \frac{e^{a\chi}}{a}\sinh(a\eta)$$

$$ds^2 = e^{2a\chi}(d\eta^2 - d\chi^2)$$

The uniformly accelerated observers Rob and AntiRob are confined to the Rindler wedges I ($|t| < x$) and II ($|t| < -x$) respectively, which are causally disconnected from each other. Their worldlines are hyperbolas, which correspond to lines of constant $\chi = c^2/a$, where a is their proper acceleration.

WEDGE I IS CAUSALLY DISCONNECTED OF WEDGE II

KLEIN GORDON EQUATION (2D)

FLAT SPACE

$\Box\phi = 0$ *in 2-dim are plane waves of the form*

$$u_k = \frac{1}{\sqrt{2\pi\omega}} e^{i(kx - wt)}$$

$$i\partial_t u_k = -i\omega u_k$$
$$i\partial_t u_k^* = i\omega u_k^*$$

Global Killing Vector Field

$u_k \rightarrow$ positive frequence solutions

$u_k^* \rightarrow$ negative frequence solutions

$$\hat{\phi} = \int (u_k a_k + u_k^* a_k^\dagger) dk.$$

$$|n_1, ..., n_k'\rangle = a_1^{\dagger n} ... a_k^{\dagger n'} |0\rangle$$

RINDLER SPACE

$$\Box\phi := \frac{1}{\sqrt{-g}} \partial_\mu (\sqrt{-g} g^{\mu\nu} \partial_\nu \phi)$$

$$\hat{\phi} = \int (u_k^I a_k^I + u_k^{II} a_k^{II} + h.c.) dk$$

$$|0\rangle_R = |0\rangle_I \otimes |0\rangle_{II}$$

$$a_k^I |0\rangle^I = 0 \quad a_k^{II} |0\rangle^{II} = 0.$$

Bogoliubov Transformation $a_k = \int ((u_k, u_{k'}^I) a_k^I + (u_k, u_{k'}^{*I}) a_k^{\dagger I} + (u_k, u_{k'}^{II}) a_k^{II} + (u_k, u_{k'}^{*II}) a_k^{\dagger II}) dk$

Unruh basis $u_k^U = \cosh r u_k^I + \sinh r u_k^{II*},$ $\quad \mathrm{sech}^2(r) = 1 - e^{\frac{2\pi\omega}{a}}.$

2-mode squeezed state $|0_k\rangle^{\mathcal{M}} = \frac{1}{\cosh(r)} \sum_n \tanh^n(r) |n_k\rangle^I |n_k\rangle^{II}$

Unruh Temperature and Entanglement

Observer region I has no access to information region II. Trace over II

$$\rho_I^k = \mathrm{Tr}_{II}(|0_k\rangle^{\mathcal{M}}\langle 0_k|) = \frac{1}{\cosh^2(r)}\sum_n \tanh^{2n}(r)|n_k\rangle^I\langle n_k| = (e^{\frac{-2\pi\omega}{a}} - 1)\sum_n (e^{\frac{-2\pi\omega}{a}})^n |n_k\rangle^I\langle n_k|$$

Canonical thermal state with temperature $T_U = \frac{a}{2\pi}$, a observer acceleration ($\hbar = c = k_B = 1$)

Let v_k^I be the mode functions of region I $(b_k, b_k^\dagger)$and u_k those of inertial system$(a_k, a_k^\dagger)$.

Related by Bogoliubov transformation

$$v_k^I = \int_{k'} \alpha_{kk'} u_{k'} + \beta_{kk'} u_{k'}^*$$

$$\hat{b}_k = \int_{k'} \alpha_{kk'}^* \hat{a}_{k'} - \beta_{kk'}^* \hat{a}_{k'}^*$$

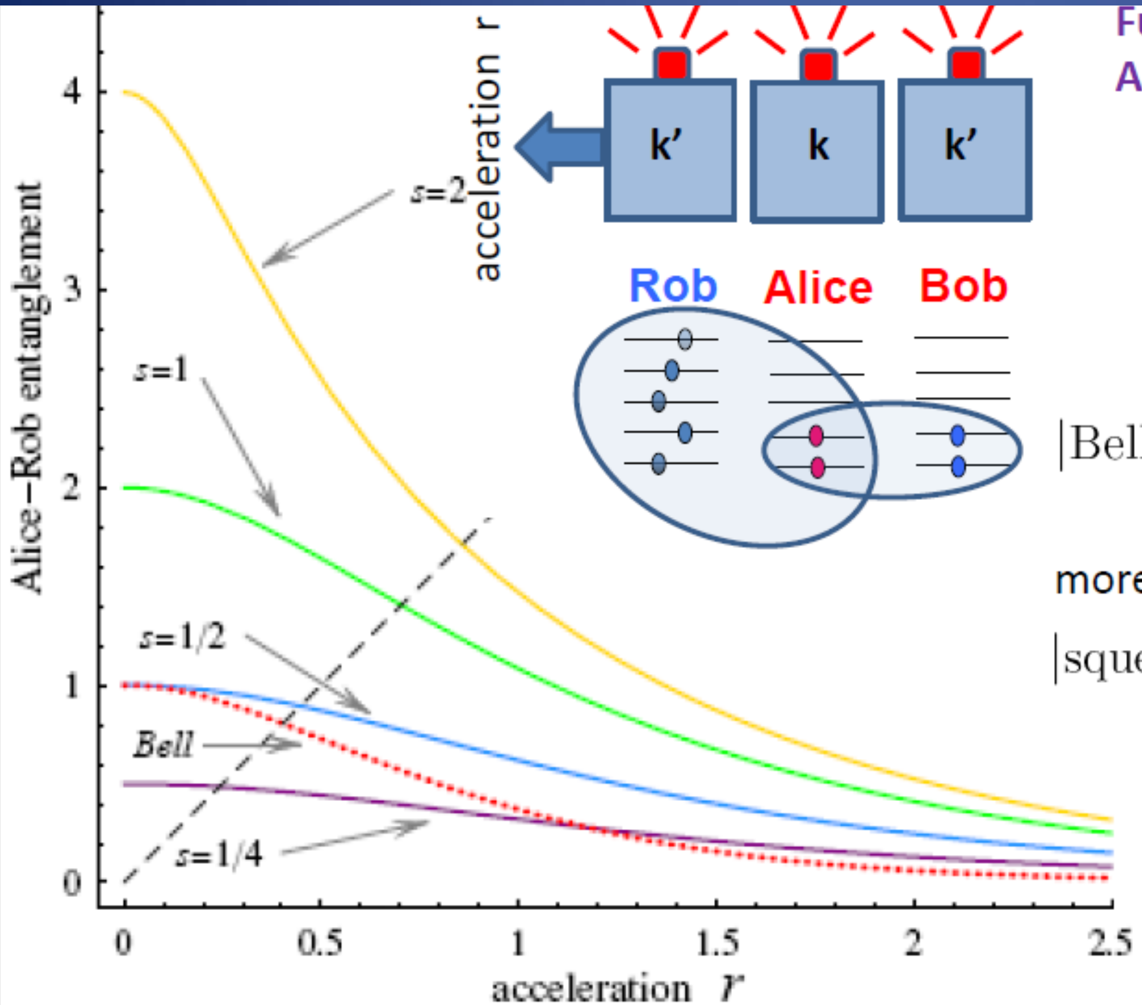
Occupation number: Accelerated observer will perceive a thermal bath of particles

$$^{\mathcal{M}}\langle 0|\hat{b}_k^\dagger \hat{b}_k|0\rangle^{\mathcal{M}} = \int_{k'} |\beta_{k'k}|^2 = \frac{1}{e^{2\pi|k|/a} - 1}$$

Bose Black Body Radiation with $T = 2\pi/a$

What are the effects of gravity and motion on quantum properties?

Fuentes-Schuller, Mann PRL 2005
Adesso, Fuentes-S, Ericsson PRA 2007



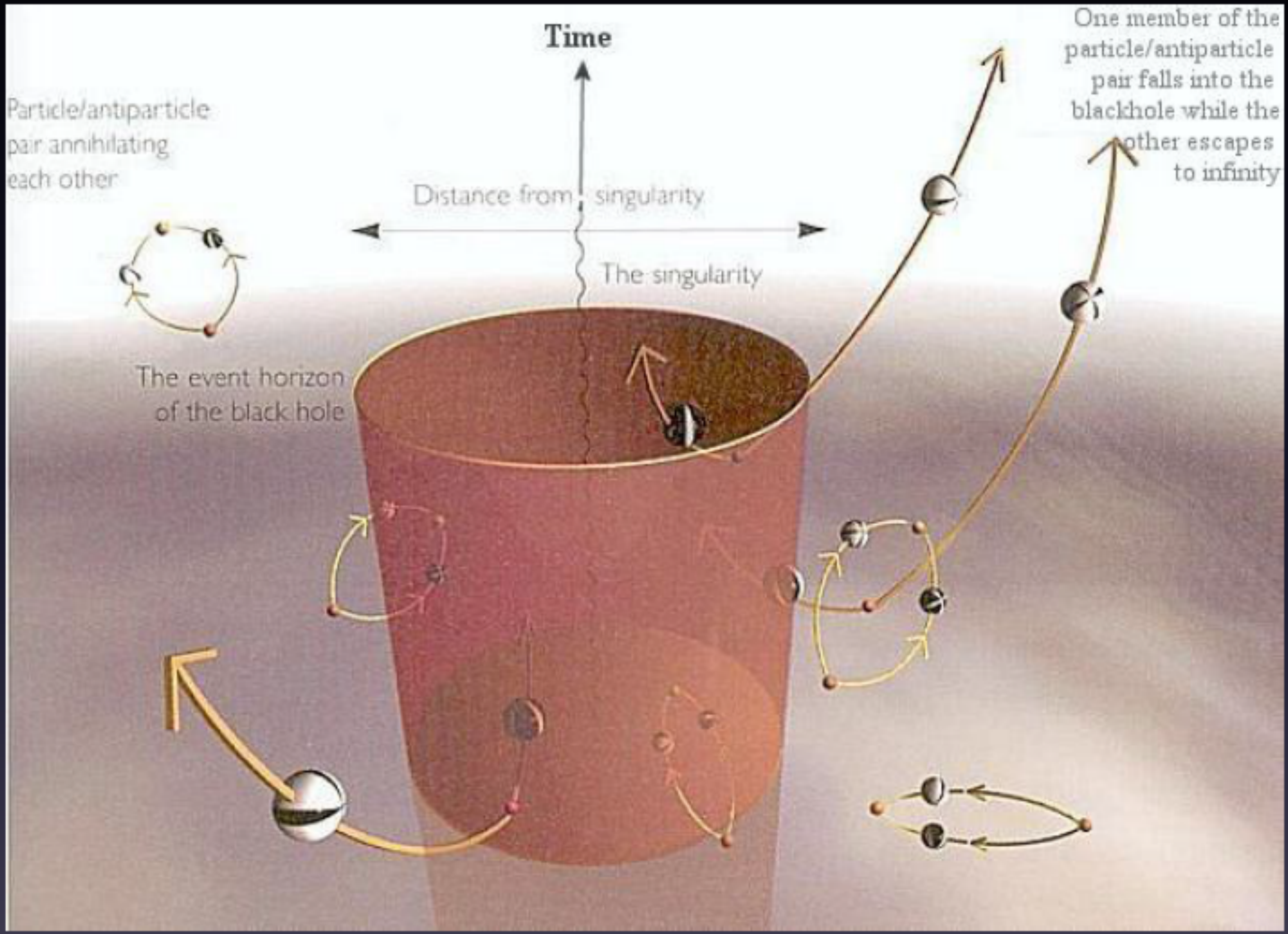
$$|\text{Bell}\rangle \sim |0\rangle \otimes |0\rangle + |1\rangle \otimes |1\rangle$$

more realistic states:

$$|\text{squeezed}\rangle \sim \sum_{n\geq 0} \tanh^n s\ |n\rangle \otimes |n\rangle$$

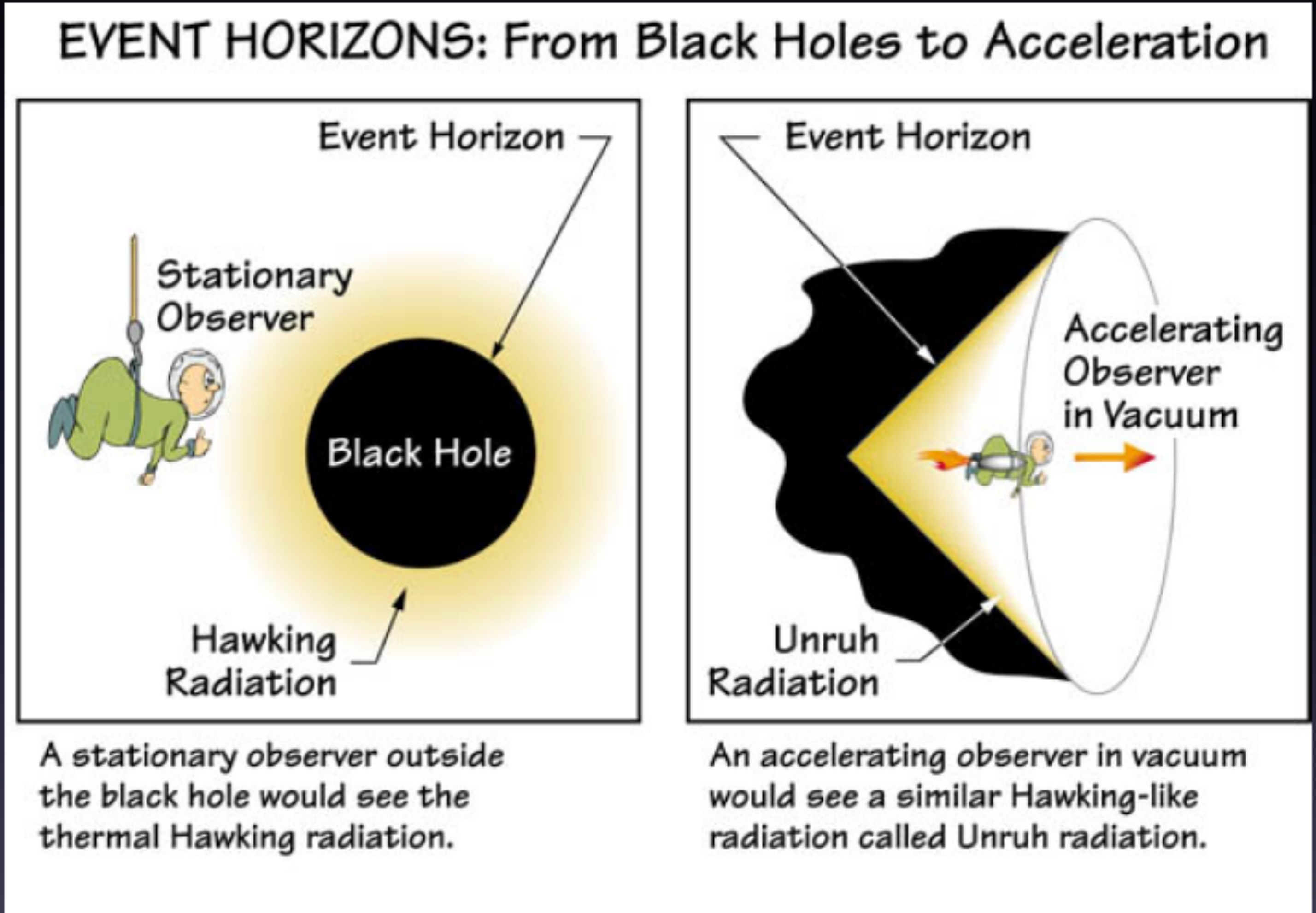
Entanglement
- observer-dependent
- degrades with acceleration , vanishes for ∞ acceleration

The Hawking radiation



“Black holes ain’t so black”

Hawking v.s. Unruh



Alice falls into a Black Hole, *Fuentes et al. Phys.Rev.Lett.95(2005)120404*

Fundamental Quantum Optics Experiments Conceivable with Satellites

Special and General Relativistic Effects			
Lorentz transformed polarization	LEO and beyond	mid-term	QM + SR
Relativistic frame dragging	TBD	TBD	QM + GR
Entanglement with curvature	TBD	visionary	QM + GR
Fermi problem	Sunshielded satellites	long-term	QFT
Optical Colella-Overhauser-Werner experiment	LEO and beyond	near-term	QM + GR
Accelerating Detectors in Quantum Field Theory			
Acceleration induced fidelity loss	TBD	visionary	QFT + GR
Berry phase interferometry	LEO	mid-term	QFT + GR
Gravitationally induced entanglement decorrelation	LEO and beyond	near-term	Non-standard QFT + GR
Spacelike entanglement extraction	TBD	visionary	QFT + GR

Table 1. Summary of possible experiments. LEO refers to Low Earth Orbit, an elliptical orbit about the Earth with altitude up to 2000 km. The timeframes are mentioned in Section 1. Roughly, 'near-term' experiments (∼ 5 years) can be accomplished with a single satellite in LEO, 'mid-term' experiments (25 years) require multiple satellites or higher orbits, 'long-term' experiments involve Earth-Moon distances, and 'visionary' experiments extend to solar orbits and beyond. Under "Regime" (and throughout the paper) QM refers to ordinary quantum mechanics, QFT to quantum field theory, SR to special relativity, GR to general relativity, and QG to quantum gravity. The "Level" classifications are explained in Section 1.1.

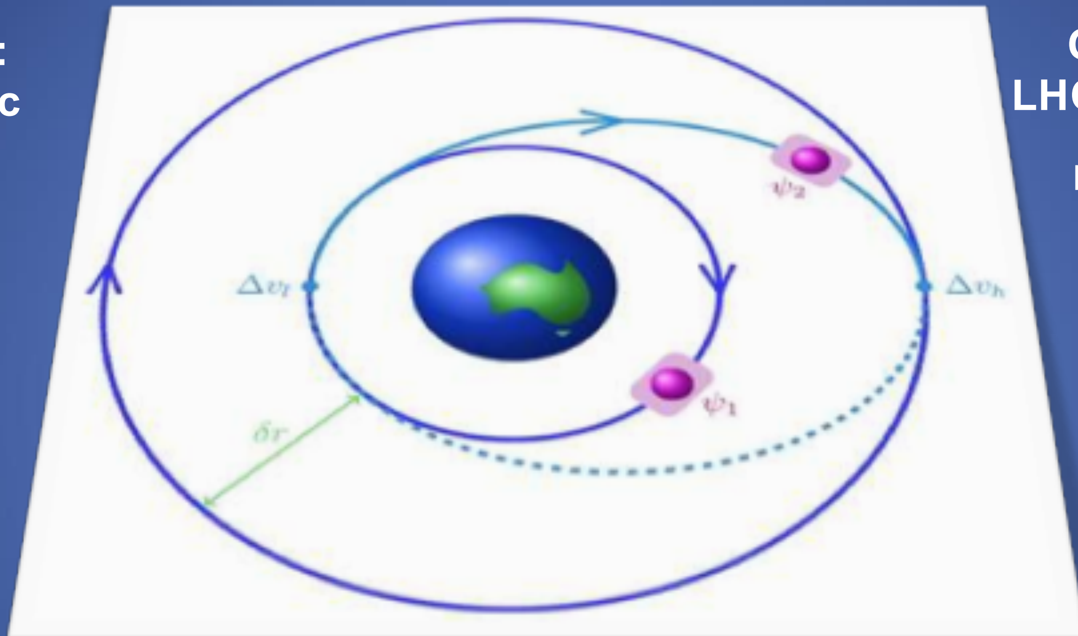
D. Rideout et al. Class.Quantum Grav.29(2012)224011

Gravitationally induced quantum decorrelation

Space-based experiment could test gravity's effects on quantum entanglement

General Relativity: tested from Cosmic scales to 10 m

PRD78(2008)042003



Quantum Theory: LHC 10^{-20} m to ~100 km

Nature 3-7 (2007) 481

The change in gravity is predicted to cause a degradation of the entanglement between the BECs

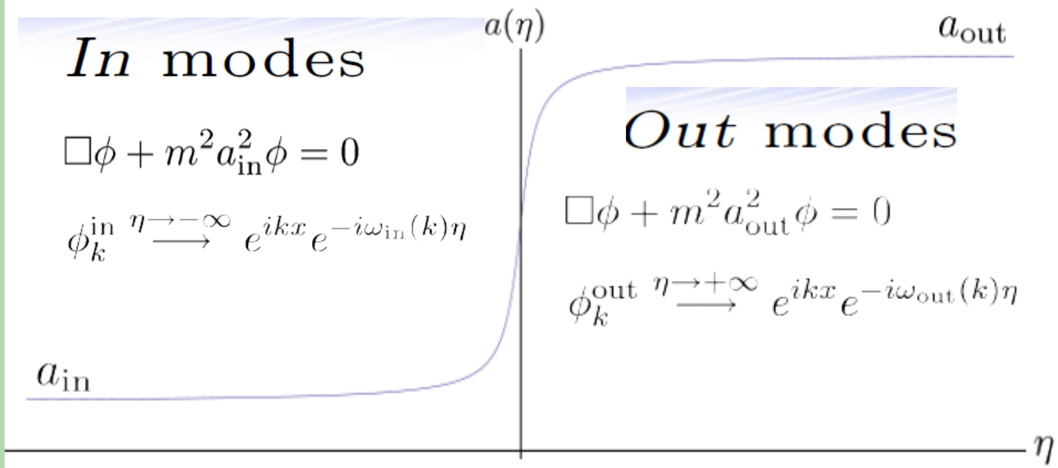
David Edward Bruschi, et al, *New Journal of Physics*. 2014 DOI: 10.1088/1367-2630/16/5/053041

Quantum Fields in Expanding Spacetimes

1+1 FRW universe

$$ds^2 = a(\eta)^2(d\eta^2 - dx^2)$$

$$\sqrt{g}\mathcal{L} = \frac{1}{2}\eta^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - \frac{m^2a^2}{2}\phi^2$$



In modes

$$\Box\phi + m^2 a_{\text{in}}^2\phi = 0$$

$$\phi_k^{\text{in}} \xrightarrow{\eta\to-\infty} e^{ikx}e^{-i\omega_{\text{in}}(k)\eta}$$

Out modes

$$\Box\phi + m^2 a_{\text{out}}^2\phi = 0$$

$$\phi_k^{\text{out}} \xrightarrow{\eta\to+\infty} e^{ikx}e^{-i\omega_{\text{out}}(k)\eta}$$

$$\hat{\phi} = \sum_k \hat{a}_k^{\text{in}}\,\phi_k^{\text{in}} + \hat{a}_k^{\text{in}\dagger}\,\phi_k^{\text{in}*}$$

$$\hat{\phi} = \sum_k \hat{a}_k^{\text{out}}\,\phi_k^{\text{out}} + \hat{a}_k^{\text{out}\dagger}\,\phi_k^{\text{out}*}$$

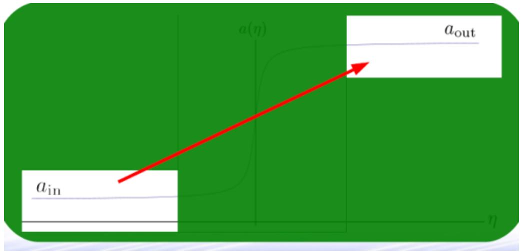
Relate IN and OUT states via Bogoliubov transformation (squeezing)

barred quantities (IN REGION) and unbarred quantities (OUT REGION)

$$\bar{a}_s = \alpha_s^* a_s - \beta_s^* a_{-s}^\dagger,$$

so that mixing occurs only between states labelled by s and $-s$.

Just like SHO with time dependent frequency



$$\langle\bar{0}|\hat{a}_s^\dagger \hat{a}_s|\bar{0}\rangle = |\beta_s|^2$$

$$ds^2 = a(\eta)(d\eta^2 - dx^2)$$

$$a(\eta) = 1 + \epsilon(1 + \tanh \sigma\eta)$$

$\epsilon, \sigma > 0$ volume and rapidity of expansion

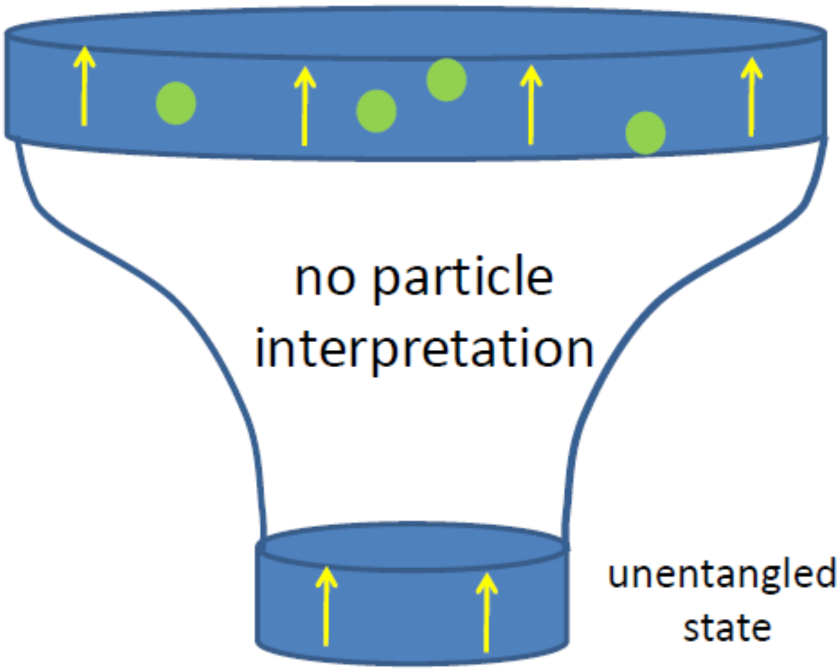
$\partial/\partial\eta$ Killing vector field in both asymptotic regions (particle content well defined)

. As a pure state of a bi-partite system, this can be written as a Schmidt decomposition

$$|\bar{0}\rangle_k|\bar{0}\rangle_{-k} = \sum_{n=0}^{\infty} c_n|n\rangle_k|n\rangle_{-k}, \qquad c_n = \left(\frac{\beta_k^*}{\alpha_k^*}\right)^n \sqrt{1 - \left|\frac{\beta_k}{\alpha_k}\right|^2}.$$

where n labels the number of excitations in the field mode k (as seen by an inertial observer in the out-region) and the coefficients c_n are real.

Ball, Fuentes-S, Schuller PLA 2006



"History of the universe encoded in entanglement"

<u>toy model</u>

expansion rate σ

expansion factor ϵ

- calculate entanglement

  asymptotic past $S = 0$

  asymptotic future $S = S(\sigma, \epsilon)$

- excitingly, can solve for

$\sigma = \sigma(S)$ $\epsilon = \epsilon(S)$

$$\varrho = |\bar{0}\rangle_{-k}|\bar{0}\rangle_{k}\,{}_{k}\langle\bar{0}|_{-k}\langle\bar{0}|$$

Density matrix

$$\varrho_k = \sum_{m=0}^{\infty} {}_{-k}\langle m|\varrho|m\rangle_{-k}$$

Reduced density matrix – observe only modes +k

$$S = -\mathrm{Tr}(\varrho_k \log_2 \varrho_k) = \log_2 \frac{\gamma^{\gamma/(\gamma-1)}}{1-\gamma},$$

Entropy quantifies entanglement

$$\gamma = \left|\frac{\beta_k}{\alpha_k}\right|^2$$

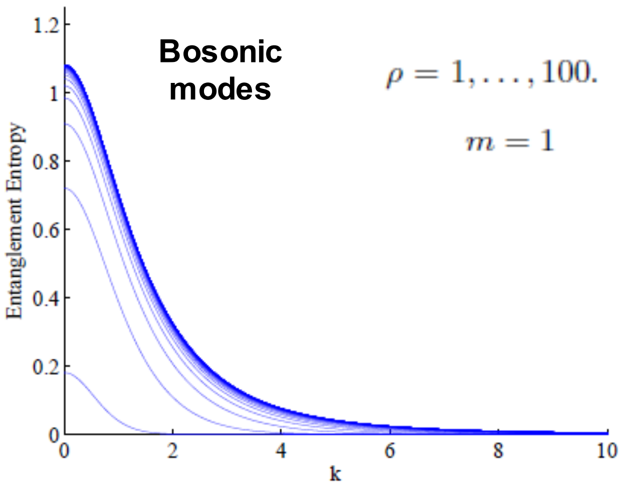
$$E_p = \sqrt{p^2+m^2} \text{ such that } m\sqrt{\epsilon} \ll E_p \ll 2\sigma,$$

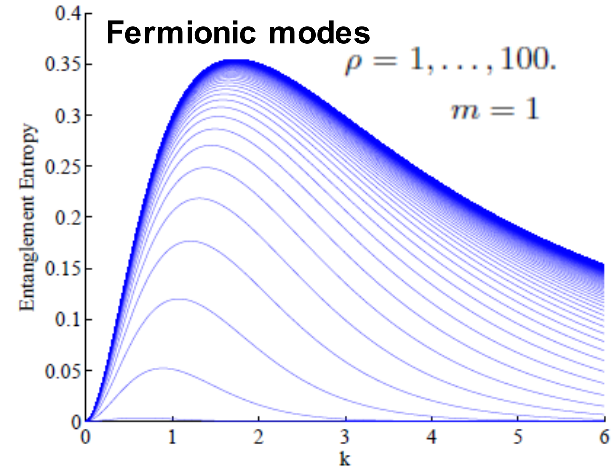
Ex: small mass limit…

$$\epsilon \approx \frac{2E_p^2}{m^2}\sqrt{\gamma(S)}.$$

$$\sigma \approx \frac{\pi}{2}\left(\frac{1+\gamma(S)}{-\frac{E}{4}\frac{d}{dE}\ln\gamma(S)-1}\right)^{\frac{1}{2}} E,$$

… one can read ST evolution from entanglement!!!!!

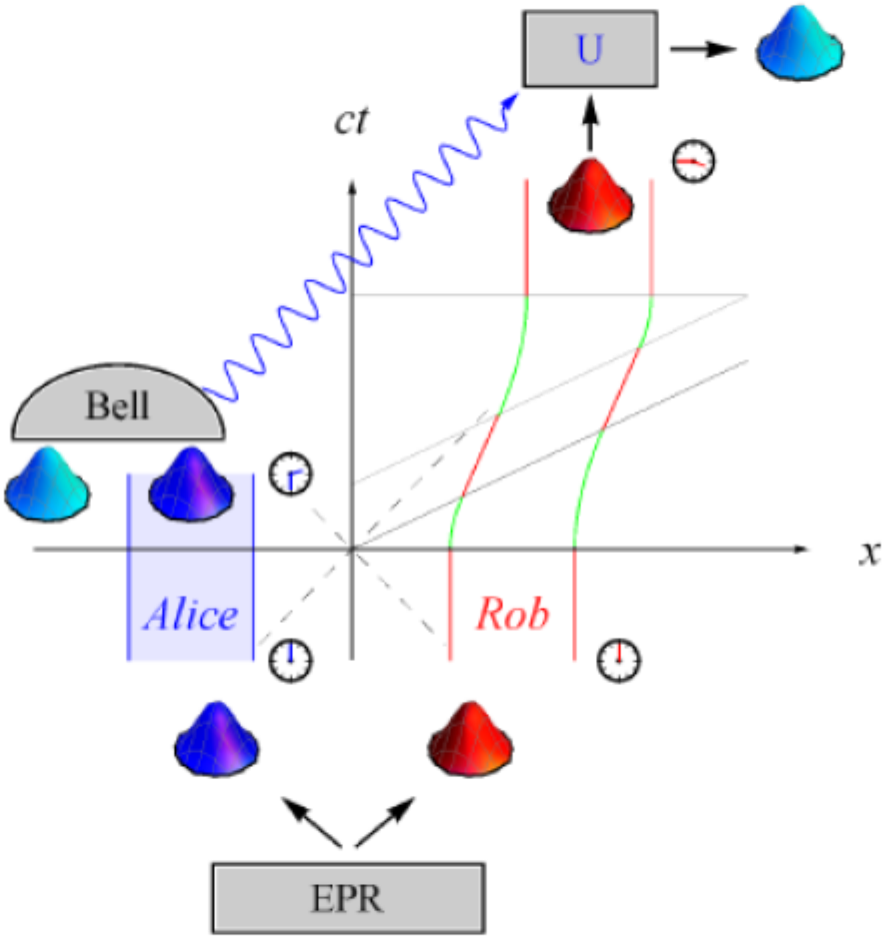




Fuentes et al, PRD82(2010)045030

teleporation with an accelerated partner

Friis, Lee, Truong, Sabin, Solano, Johansson & Fuentes PRL 2003



the fidelity of teleportation is effected by motion
it is possible to correct by local rotations and trip planning

EPL, **111** (2015) 60001
doi: 10.1209/0295-5075/111/60001
www.epljournal.o

Alice and Bob in an expanding spacetime

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[2] *Universidade Federal de Ouro Preto, Departamento de Matemática, ICEB - Campus Morro do Cruzeiro, s/n, 35400-000, Ouro Preto, MG, Brazil*
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published online 25 September 2015

PACS 03.67.Mn – Entanglement measures, witnesses, and other characterizations
PACS 03.65.Ud – Entanglement and quantum nonlocality (*e.g.* EPR paradox, Bell's inequalities, GHZ states, etc.)
PACS 04.62.+v – Quantum fields in curved spacetime

Abstract – We investigate the teleportation of a qubit between two observers Alice and Bob in an asymptotically flat Robertson-Walker expanding spacetime. We use scalar or fermionic field modes inside Alice's and Bob's ideal cavities and show the degradation of the teleportation quality, as measured by the fidelity, through a mechanism governed by spacetime expansion. This reduction is demonstrated to increase with the rapidity of the expansion and to be highly sensitive to the coupling of the field to spacetime curvature, becoming considerably stronger as it reduces from conformal to minimal. We explore a perturbative approach in the cosmological parameters to compute the Bogoliubov coefficients in order to evaluate and compare the fidelity degradation of fermionic and scalar fields.

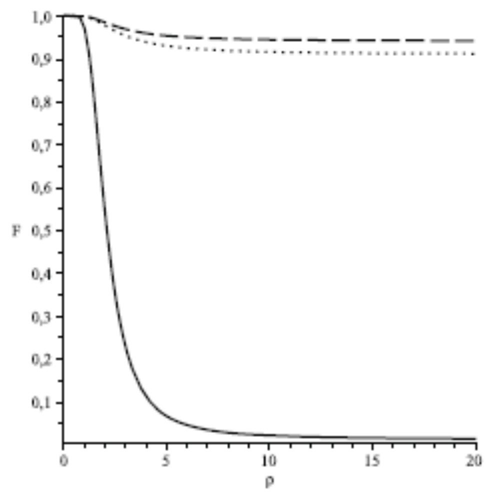

Fig. 2: Fidelity as a function of the expansion parameter ρ for the bosonic field case with minimal coupling (solid line) and conformal coupling (dotted line). The dashed line shows the fidelity for the fermionic case. We have fixed $|k| = m = 1$ and $\epsilon = 0.7$. The logarithmic negativity has a similar behavior.

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4. Crispino, Iguchi, Matsas REVIEWS OF MODERN PHYSICS, VOLUME 80, 2008

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Prof. Reinhold Bertlmann

Perspectives: A growing field of research

- Holographic computation of entanglement using AdS/CFT – Ryu & Takayanagi
- Testing effects of gravity and motion on entanglement in space based experiments – Bruschi, grupo de Nottingham
- Event horizon and entropy in high energy hadron production – Castorini et al.
- Application of Unruh effect to neutrino physics
- Quantum Metrology
- Study of entanglement produced by scattering in a purely quantum field theoretical framework
- Effects of interaction on entanglement of fields in expanding spacetimes

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