Multiplicity fluctuations near the QCD critical point

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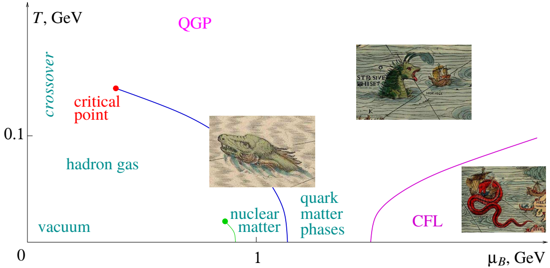
Outline

The phase diagram of the Strong Interactions

We would like to map/probe the phase diagram of strongly interacting matter.



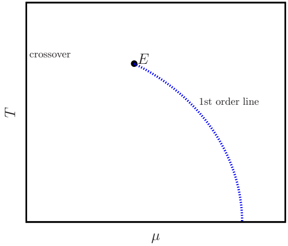
M. Stephanov J.Phys.Conf.Ser. 27 (2005).

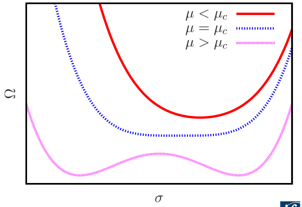
- Different regimes / tools.
- Very difficult task.
- Critical endpoint
  → marked by fluctuations
  → possible landmark?

The QCD critical point

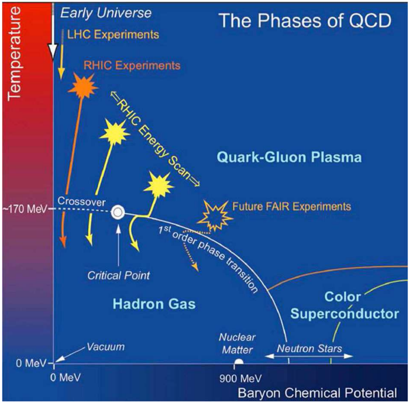
- Consequence of the 1$^{\text{st}}$ order line for high μ.
- Flat effective potential for order parameter $\sigma \sim \langle \bar{q} q \rangle$.
- Diverging correlation length $\xi \sim m_\sigma^{-1}$.
- Strong, long-range fluctuations and universal features.

Experimental search!





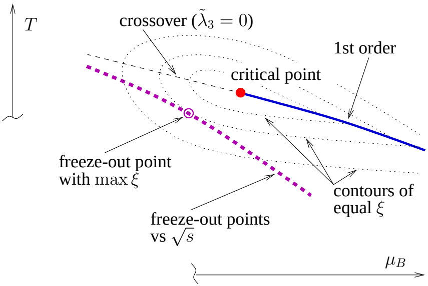
Searching for the critical point

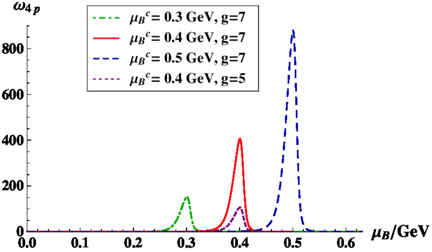


- Heavy-ion collisions: STAR, NA61, CBM, NICA...
- Freeze-out near the CEP. (Hope fluctuations survive!)
- Distinct signatures:
  → Non-monotonic behavior?
  → Scaling? (Dynamical/FSS)

Stephanov, Rajagopal, Shuryak, PRD 60 (1999),
https://drupal.star.bnl.gov/STAR/files/BES_WPII_ver6.9_Cover.pdf

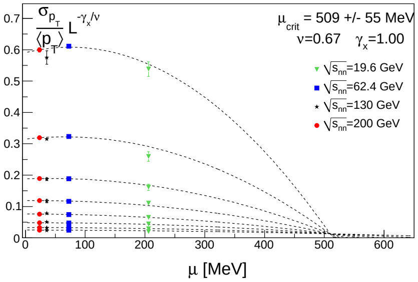
Proposals: non-monotonic behavior

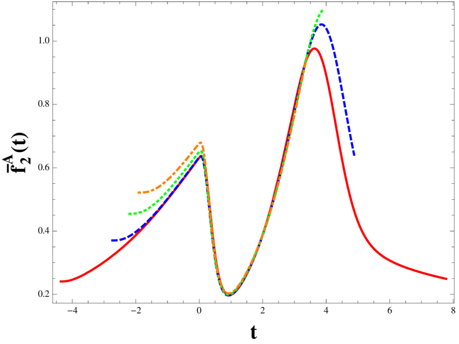




Stephanov, PRL 102 (2009)
Athanasiou, Rajagopal and Stephanov, PRD 82 (2010)

Proposals: universal scalings





Fraga, Palhares and Sorensen, PRC 84 (2011)
Mukherjee, Venugopalan and Y. Yin, PRL 117 (2016)

Calibrating expectations

Expectations must be adapted to experimental limitations...

Ideal/pure theory

- Thermodynamic limit.
- Equilibrium.
- Diverging ξ.
- Large fluctuations.

HIC experiments

- Tiny system.
- Rapid expansion and short time.
- Limited ξ.
- Noise, decays, acceptance...

Detectable after all? Smoking-gun signatures?

This work

Mission

Construct a simple yet somewhat general framework for studying different effects/contributions.

Focus: Fluctuations of particle multiplicities.

Not done yet

First steps... For now, no predictions/comparisons to data.

- Second-order moments of the pions.
- No real dynamics, transport or scatterings.
- Toy model for spurious fluctuations.

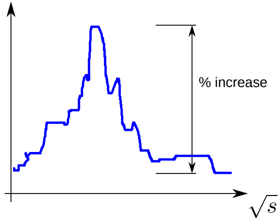
MH, Fraga, Santos, PRD 93 (2016),
MH, Fraga, PRD 96 (2017).

Outline

Methods and approach

In particular, we focus on effects of the critical point upon particle multiplicity fluctuations.

- Statistical moments of N.
- Non-monotonic dependence on collision conditions.
- How large is the resulting peak?



Fluctuations, statistics $\Rightarrow$ *probabilistic description.*

General idea

We use a grand canonical ensemble with fluctuating parameters.

Critical + spurious fluctuations

$$m^2 \to m^2 + \delta m^2 \,, \qquad V \to V + \delta V \,, \qquad T \to T + \delta T \,\ldots \tag{1}$$

Fluctuations of the order parameter σ taken into account in the squared masses:

$$\delta m^2 \approx (\partial m^2/\partial\sigma)\,\delta\sigma \,. \tag{2}$$

Assumption

- Homogeneous fluctuations for all parameters!

Monte Carlo simulations

Fluctuations can be simulated using Monte Carlo methods.

Algorithm:

1. Draw parameters from $\mathcal{P}(T)$, $\mathcal{P}(R)$ etc (spurious fluctuations).
2. Draw σ_0, m^2 from $\mathcal{P}(\sigma_0)$ (critical fluctuations).
3. Draw occupation numbers from Boltzmann factor $e^{-\beta\,(\omega_p - \mu)\,n_p}$ ("grand canonical" fluctuations).

- Discretization requires boundary conditions: $p_i^\ell = \alpha_i^\ell / R$.
- Statistics $\rightarrow \langle N \rangle$, $\langle \Delta N\, \Delta N \rangle$, ...

Background can be systematically added!

Analytic expressions

Calculations by series and averages over fluctuations are possible.

Effective energy level fluctuations

- Critical: $\omega_0 + \delta\omega_\sigma = \sqrt{p^2 + m^2 + \delta m^2(\sigma_0)}$.
- System size: $p_i^\ell + \delta p_i^\ell = \alpha_i^\ell/(R + \delta R)$.
- T and μ fluctuations: $\dfrac{\omega + \delta\omega_{T,\mu} - \mu}{T} = \dfrac{\omega - (\mu + \delta\mu)}{T + \delta T}$.

Taylor expanding in $\delta\omega_i^\ell$ and averaging → *general formulae*,

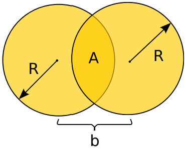
$$\overline{\langle \cdots \rangle} \approx \langle (\cdots)_0 \rangle + \sum_\omega \langle (\cdots)_\omega \rangle \, \overline{\delta\omega} + \sum_{\omega,\omega'} \langle (\cdots)_{\omega,\omega'} \rangle \, \overline{\delta\omega \delta\,\omega'} + \ldots . \quad (3)$$

Spurious fluctuations models

In HICs, thermodynamic parameters are not fully controlled. A model for spurious fluctuations is necessary:

1. Gaussian temperature fluctuations ($\sigma_T/T = 5\%$)
2. Geometrical fluctuations (below)

Geometric fluctuations



- Impact parameter distribution $\Rightarrow$ Overlap area.
- Assumption $V(b) = C(\sqrt{s})\ A(b)$.
- Fix $R_p = 6.8$ fm for $0 - 5\%$ centrality.
- Fluctuations of C missing!

Luo, Xu, Mohanty and Xu, JPG **40** (2013)
Skokov, Friman and Redlich, PRC **88** (2013)

Outline

Critical fluctuations

Our main ingredient is a fluctuating order parameter σ:

$$\mathcal{P}[\sigma] \sim e^{-\Omega[\sigma]/T}\,, \tag{4}$$

where

$$\Omega[\sigma] = \int d^3x \,\left\{ \frac{(\nabla\sigma)^2}{2} + \frac{m_\sigma^2}{2}\sigma^2 + \frac{\lambda_3}{3}\sigma^3 + \frac{\lambda_4}{4}\sigma^4 + \cdots \right\}. \tag{5}$$

The correlation length determines $\mathcal{P}$, with

$$m_\sigma = \xi^{-1}\,, \qquad \text{Ising: } \lambda_3 = \tilde{\lambda}_3\, T\,(T\,\xi)^{-3/2}\,,\ \ \lambda_4 = \tilde{\lambda}_4\,(T\,\xi)^{-1}\,. \tag{6}$$

Tsypin, PRB 55 (1996), Stephanov, PRL 102 (2009).

Homogeneous approximation

Focusing on homogeneous fluctuations, Gaussian approximation for $\sigma_0 = \int d^3x\, \sigma(x)/V$:

$$\mathcal{P}(\sigma_0) \propto \exp\left(\frac{-V}{T\,\xi^2}\,\sigma_0^2\right)\,, \tag{7}$$

- Increase in ξ leads to broader distribution!
- Only second-order moments to this approximation.
- Interactions $\rightarrow$ influence on particle production.

Interactions

Influence through mass corrections on particle-production:

$$\mathcal{L} \approx -G\,\sigma_0\,\vec{\pi}\cdot\vec{\pi} - g\,\sigma_0\,\bar{\psi}_p\psi_p\,\ldots \qquad (G \approx 300\text{MeV},\ g \approx 10?). \qquad (8)$$

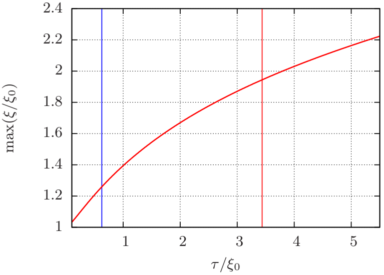
- Fluctuations of observables from $\mathcal{P}(\sigma_0) \to \delta m_\pi, \delta m_p, \ldots$
- Pion-sigma interaction found from Ginzbourg-Landau argument.
- Proton-sigma interaction a lot more uncertain...

Stephanov, Rajagopal, Shuryak, PRD 60 (1999).

Limiting ξ

Critical Slowing Down

- Non-equilibrium effects $\rightarrow \xi \nrightarrow \infty$.
- Evolution inspired by dynamical universality class.
- Growth limited by cooling timescale, initial value and causality.
- Optimistically, cooling over critical point.

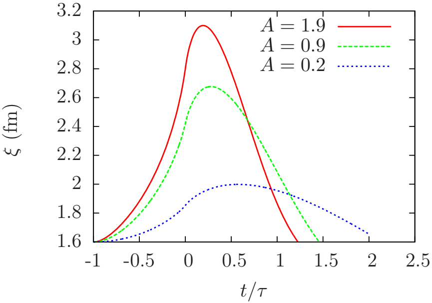


$\xi_0 = 1.6\,\text{fm}$, $\tau = 1\,\text{fm}$, $\tau = 5.5\,\text{fm}$

In more detail...

$$\frac{d\xi}{dt} = A \left(\frac{\xi}{\xi_0}\right)^{2-z} \left(\frac{\xi_0}{\xi} - \frac{\xi_0}{\xi_{eq}(t)}\right) .$$

$$\xi_{eq}(t) = \xi_0 \left|\frac{t}{\tau}\right|^{-\nu/\beta\delta}$$



Berdnikov, Rajagopal, PRD 61 (2000),
Hohenberg and Halperin, RMP **49** (1977)
MH, Fraga, Santos, PRD 93 (2016).

Outline

Particle production

Limitations can have a strong effect on particle production. Not all of the produced are detected and not all of them are thermal!

- Ideally, Boltzmann factor $e^{-\beta\,(\omega_p-\mu)\,n_p}$.
- Limited acceptance window: acceptance probability $F(p)$!
- Resonance decay will dilute the signal!

Kinematic cuts

Acceptance factor

Kinematic cuts limit the solid-angle coverage and the acceptance probability. Assuming isotropy,

$$F(p) = \mathbf{e_0(p)} \cdot \frac{\Omega_{\rm acc}(p)}{4\pi}\,, \tag{9}$$

which depends on momentum p and detection efficiency e_0!

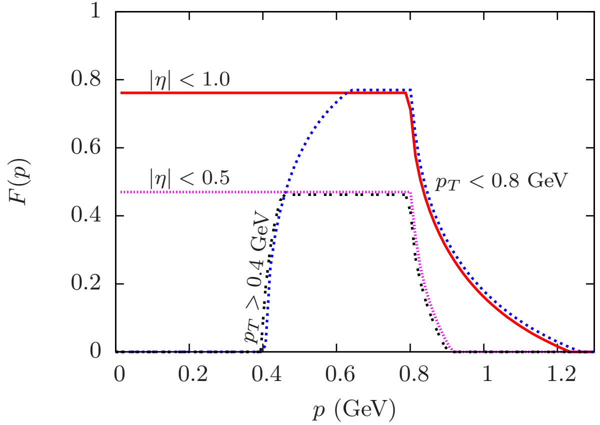
Impact on fluctuations from binomial-like expression:

$$\langle(\Delta n_p)^2\rangle_{acc} = F(p)^2\,\langle(\Delta n_p)^2\rangle + F(p)\big(1 - F(p)\big)\,\langle n_p\rangle\,. \tag{10}$$

Ling and Stephanov, PRC **93** (2016)
A. Bzdak and V. Koch, PRC **86** (2012)

Acceptance factor

The probability of accepting a particle with momentum p, assuming isotropy:



Resonance decay contributions

Decay products might also be detected, depending on their total momentum.

Decay into two particles

- One, both or none of the particles accepted.
- Independent decays and thermal resonance distribution.
- For each decay,

$$\langle n_l^m \rangle = P_2 + P_l \,, \tag{11}$$

$$\langle n_1 \, n_2 \rangle = P_2 \,. \tag{12}$$

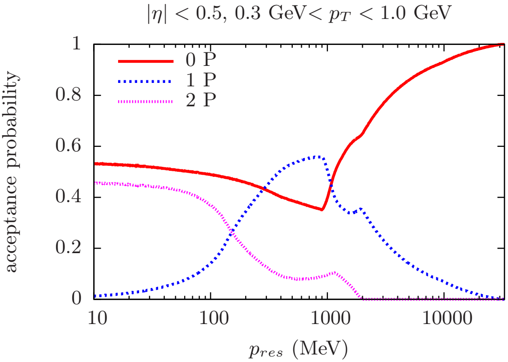
- Branching ratio < 1: $P_{n \neq 0} \to r_b \, P_{n \neq 0}$.

Nahrgang, Bluhm, Alba, Bellwied and Ratti, EPJC **75** (2015)

Resonance decay within acceptance window

Acceptance probability as function of the original momentum p_{res}:

- $p_{\text{res}} \to p_1 + p_2$.
- Probability from phase space volume.
- Isotropy + energy-momentum conservation.



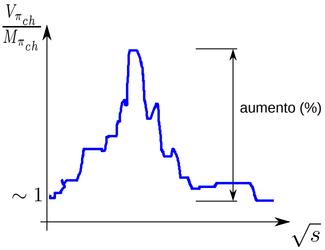
"$\rho \to \pi\ \pi$" decays (BR: 100%).

Outline

Results

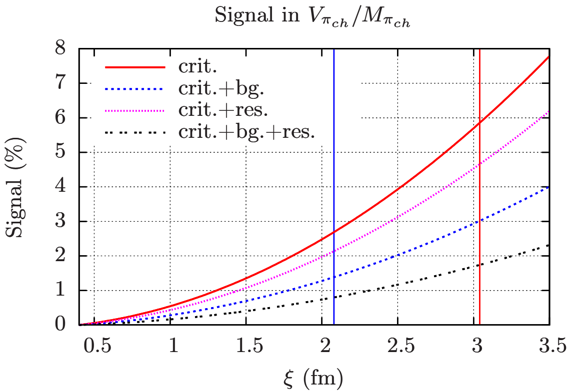
We now show some results for $V_{\pi_{ch}}/M_{\pi_{ch}}$, where $M_{\pi_{ch}} = \overline{\langle N_{\pi_{ch}} \rangle}$ and $V_{\pi_{ch}} = \overline{\langle (\Delta N_{\pi_{ch}})^2 \rangle}$.

- Signal in % compared to $\xi_r = 0.4$ fm.
- ξ_{max} depends on the cooling timescale.
- Signal S_5 for $\xi = 5\,\xi_r = 2$ fm ($\tau \sim 1$ fm).
- $T = 130$ MeV, $R_p = 6.8$ fm.
- $|\eta| < 0.5$, $0.3\,\text{GeV} < p_T < 1\,\text{GeV}$.



Results

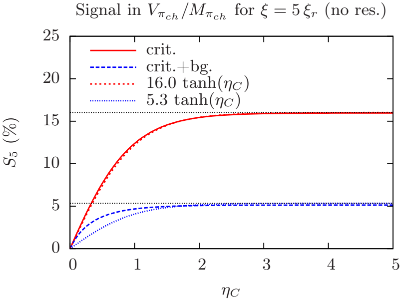
Peak height with relation to reference value:

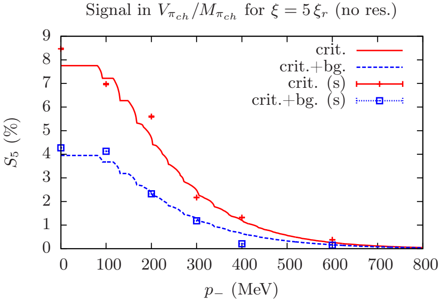


$\tau = 1\,\mathrm{fm}\,, \qquad \tau = 5.5\,\mathrm{fm}\,.$

Acceptance cuts

Results for $\xi \to 2.0$ fm.



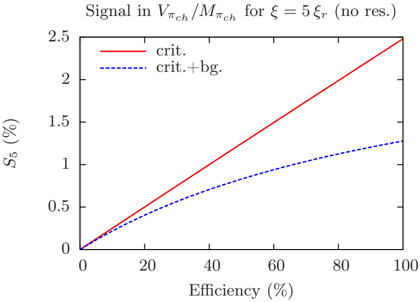


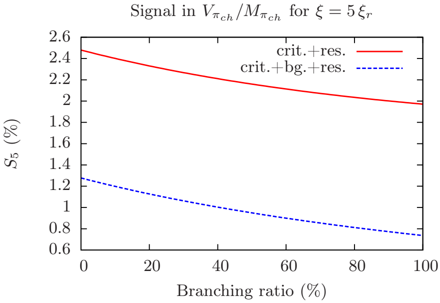
- Matches geometric argument!
- $0 < p_T < 2$ GeV.

- Superior cut ∼ irrelevant.
- Good agreement!

Efficiency and branching ratio

Results for $\xi \to 2.0$ fm.





- Binomial detection.
- Good agreement with Poissonian production.

Outline

Summary and outlook

- Initial approach in its first steps, but largely enhanceable.
- Both simulations and analytical expressions (to be extended).
- Possible extension for non-Gaussian fluctuations and protons.
- New sources/models of fluctuations can be incorporated.
- Finite-efficiency effects can also be introduced.
- Boosting of acceptance window?

Disclaimers

Caveats/Limitations

- Perfect equilibrium, no real dynamics → trend to overestimate signal, unreliable for p_T.
- Isotropy Assumption → effects of acceptance window should be taken with care!
- Homogeneous fluctuations → not realistic in relevant timescales.
- Background models still crude/incomplete → extra information and insight needed.
- Lack of control over some of the relevant parameters (protons and higher-order moments).

To keep in mind: still not exactly what we want!

But getting closer...

Acknowledgements

Thanks!

Thanks also to **FAPERJ** and **CNPq** for financial support!

Correlation length and universality

Correlation length ξ

Roughly:

$$\langle \Delta\phi(x)\, \Delta\phi(x') \rangle \sim \exp\left(-\frac{|x - x'|}{\xi} \right)$$

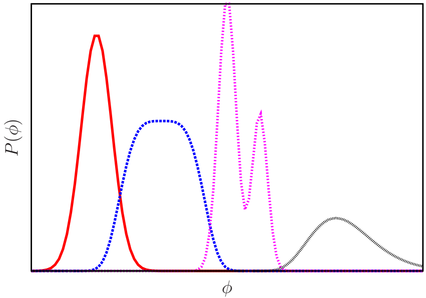
Universality

Roughly:

- Near second-order phase transition.
- Long range fluctuations (large ξ).
- Microscopic details become irrelevant.
- Relevant length-scale: ξ

Probability distributions

Some shapes:



- Poisson (rate λ):

$$P(k) = \frac{e^{-\lambda}\lambda^k}{k!},$$

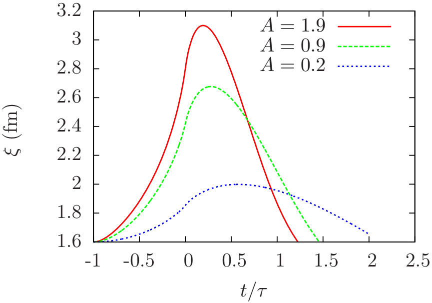
- Binomial (n tries):

$$P(k) = \frac{n!}{k!(n-k)!}\, p^k (1-p)^{n-k}$$

Critical slowing down

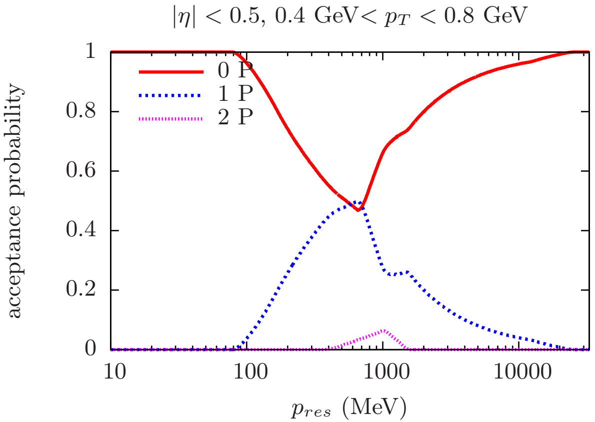
$$\frac{d\xi}{dt} = A \left(\frac{\xi}{\xi_0}\right)^{2-z} \left(\frac{\xi_0}{\xi} - \frac{\xi_0}{\xi_{eq}(t)}\right) .$$

$$\xi_{eq}(t) = \xi_0 \left|\frac{t}{\tau}\right|^{-\nu/\beta\delta}$$



Berdnikov, Rajagopal, PRD 61 (2000),
Hohenberg and Halperin, RMP **49** (1977)
MH, Fraga, Santos, PRD 93 (2016).

Resonance decay: other window



Boosting of acceptance window

Acceptance probability:

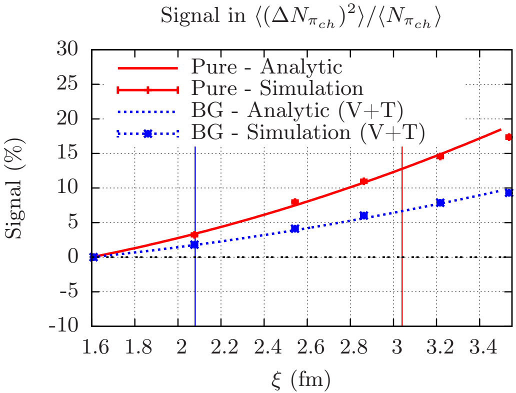
$$F(p) = \int_{\Omega_{\text{acc}}(p)} \frac{d\Omega}{4\,\pi} = \max[u_{\max}(p) - u_{\min}(p), 0]\,, \tag{13}$$

$$\Rightarrow \quad F(p) = \int \frac{d^3x}{V} \int_{\tilde{\Omega}_{\text{acc}}(p,\mathbf{x})} \frac{d\Omega}{4\,\pi}\,, \tag{14}$$

where $\tilde{\Omega}_{\text{acc}}(p, \mathbf{x})$ is the solid angle coverage of the acceptance window when boosted.

Simulation results

Peak height with relation to reference value:



$$T = 130\,\mathrm{MeV}\,, \qquad R_p = 6.8\,\mathrm{fm}. \tag{15}$$

$\tau = 1\,\mathrm{fm}\,,$ $\tau = 5.5\,\mathrm{fm}$