



## Machine Learning



*Lorenzo Moneta CERN - EP-SFT* Lorenzo.Moneta@cern.ch

#### LIP Data Science School / 12-14 March 2018

## Outline

#### Lecture 1 (today)

- Introduction to Machine Learning
- Supervised Learning
- Linear Models
  - Regression
  - Classification
- Hypothesis Tests and ROC curve
- Overfittig and Regularization
- Cross-Validation
- Machine Learning Software
  - Introduction to ROOT/TMVA



## **Outline (2)**

- Lecture 2:
  - Support Vector Machine (SVM)
  - Decision Trees
- Lecture 3:
  - Introduction to Neural Networks
  - Deep Learning



Ο

Vaximum.

X<sub>1</sub>

margin

Г



### References

Lots of materials presented taken from these lectures:

- *M. Kagan*: <u>CERN Academic Training Lectures</u> (2017)
- S. Gleyzer: <u>TAE 2017 Lectures</u>
- *A. Rogozhnikov*: Lecture at Yandex summer school of Machine Learning in HEP (2016)

Books:

- Elements of Statistical Learning (*Friedman et al...*)
- Pattern Recognition and Machine learning (Bishop)

## What is Machine Learning?

- Field of study that gives computer the ability to learn without being explicitly programmed (Arthur Samuel, 1959)
- Better definition (T. Mitchell, 1998)
  - Study of algorithms that improve their performance **P** for a given task **T** with more experience **E**

• Example:

• Task: Identify Higgs boson, faces in pictures, etc...

### Where is Used ?

- Natural Language Processing
- Speech and handwriting recognition
- Object and image recognition
- Fraud detection
- Financial marker analysis
- Search engines
- Spam and virus detection
- Medical diagnosis
- Robotics control
- Automation: e.g. self-driving cars
- Advertising (recommender systems)

### Growing very fast !





Facial recognition



Autonomous ("selfdriving") vehicles



# Examples





## Machine Learning in HEP

- In analysis and reconstruction
  - Classifying signal from background events
  - Reconstructing particles and improving energy/mass resolution
  - Particle identification
  - Energy calibration
- In the trigger and Data Acquisition
  - Quickly identifying complex final states
  - Data quality monitoring
- In computing
  - Estimate dataset popularity
  - Optimisation of resources

# Machine Learning in HEP

#### Classification

- Particle Identification
- Pattern Recognition (tracks)
- Searches for new Physics

#### Regression

- Function Estimation
  - e.g. estimate better particle energy





# **Example: Higgs Discovery**





- Identification of particles
- Identification of interactions
- Energy regression
- Event selection

#### **Improvement in analysis from all 4 areas**

# Mathematical Modeling

- Key element in machine learning is a mathematical model
  - mathematical characterisation of system(s) of interest, typically via random variable
  - Chosen model depends on the task and on the available data





# Mathematical Modeling

Key element is a mathematical model

### Learning

- Estimate statistical model from the data
- Prediction and Inference

 use the statistical model to make predictions on new data points and infer properties of system(s)



## **Mathematical Models**

- Parametric Models
  - described with a fixed set of parameters



- independent of data set sizes
- Non-parametric models
  - do not have a fixed set of parameters
  - complexity grows with data size

Binary kNN Classification (k=1)



### **Generative and Discriminative Models**

#### Generative model

- Estimate probability density functions p(x,y)
  - estimate p(x | y) and prior p(y) and then using Bayesian theorem
  - $p(y | x) \propto p(x | y)p(y)$
- Discriminative model
  - Model directly the p(y | x)
- Majority of methods (e.g. logistic regression, neural networks) are discriminative models

# Machine Learning Tasks

- Classification
- Regression
- Clustering



Dimensionality Reduction







# Supervised Learning

- Given N examples (events in HEP) with features (Training Data)
  - {  $\mathbf{x_i} \in \boldsymbol{\mathcal{X}}$  } and targets {  $y_i \in \boldsymbol{\mathcal{Y}}$  }
- Learn function mapping y = f(x)
  - **x**<sub>i</sub> is typically a n-dimensional vector (number of features)
    - X is a matrix (number of events × number of features)
  - y<sub>i</sub> is instead a scalar

### **Supervised Learning: Classification**

- {  $\mathbf{x}_i \in \mathcal{X}$  } and targets {  $y_i \in \mathcal{Y}$  }
- Learn function mapping y = f(x)



- **Classification** : *Y* are a finite set of labels
  - binary classification *Y* = {0,1}
    e.g. Higgs event vs Background events
    multi-class classification *Y* = {c<sub>1</sub>, c<sub>2</sub>,...,c<sub>n</sub>}









### **Supervised Learning: Regression**

- {  $\mathbf{x}_i \in \mathcal{X}$  } and targets {  $y_i \in \mathcal{Y}$  }
- Learn function mapping y = f(x)
  Regression: *Y* are Real Numbers





# Unsupervised Learning

- Given some data  $D = \{x_i\}$ , but no labels
- Find possible structure in the data
  - Clustering:
    - partition the data into groups  $\mathbf{D} = \{ \mathbf{D}_1 \cup \mathbf{D}_2 \cup \mathbf{D}_3 \dots \cup \mathbf{D}_n \}$
  - Dimensionality reduction:
    - find a low dimensional
       representation of the data
       with a mapping Z = h(X)





# **Example: Clustering**

- Astronomical analysis:
  - grouping of galaxies
- Genetic analysis in biology
- Market Segmentation
- Organization of computing clusters
- Social network analysis
- Grouping of information (e.g. Google news)





Andrew Ng

### **Classification and Regression Tasks**

Classification - How to find the best decision boundary ?



Regression - How to determine the correct model ?



# Supervised Learning

#### How does it works?

- Choose a function with parameters
  - $\mathcal{F} = \{ f(\mathbf{x}; \mathbf{w}) \}$
  - with optional constraint  $\Omega(\mathbf{w})$
- Design a Loss function measuring the cost of choosing badly

$$L(\mathbf{w}, \mathbf{x}) = \frac{1}{N} \sum_{i=1}^{N} L(y_i, f(\mathbf{x}_i, \mathbf{w}))$$

• Find best values of the parameters **w** that minimize the loss function L(**w**, **x**)



• Estimate final performance on an independent data set

### **Loss Function Minimization**

Minimization of Loss Function



- Use a labeled training-set to compute loss function
- Iterative optimisation procedure (e.g. gradient descent) to find parameter values, i.e. *f*(*x*; *w*), which gives the minimum of the Loss function

$$\arg\min_{\mathbf{w}} L(\mathbf{w}, \mathbf{x}) = \arg\min_{\mathbf{w}} \frac{1}{N} \sum_{i=1}^{N} L(y_i, f(\mathbf{x}_i, \mathbf{w})) + \lambda \Omega(\mathbf{w})$$

λΩ(w) is a constraint term on the parameters w
regularisation, penalising certain values of w

## **Example: Linear Regression**

- Loss Function often used for regression
  - Square Error Loss

$$L(f(\mathbf{x};\mathbf{w}),y) = (f(\mathbf{x};\mathbf{w}) - y)^2$$

Measurement XYZ

Linear Regression :
assume a linear model

$$f(\mathbf{x}; \mathbf{w}) = \mathbf{w}^T \mathbf{x}$$

• Find **w**<sup>\*</sup> minimum of L(**w**)



### $L(\mathbf{w}) = \frac{1}{2} \sum (y_i - f(\mathbf{x}_i; \mathbf{w}))^2$

Arb.Units

60

50

40

**Least Square Regression** 

• Find minimum of L(w)

Least Square Loss function

Used also for parameter estimation
 i.e. Least square fit (X<sup>2</sup> fit)

i=1



10

Lab. Lesson 1



6

Maximum

Deviation

10 lenght [cm]

8

L. Moneta

Measurement XYZ

### **Parameter Estimation**

Model process using likelihood function of the observed data

$$\mathcal{L}(\mathbf{w}) = P(\mathbf{y}|\mathbf{x}; \mathbf{w}) = \prod_{i} p(y_i|\mathbf{x}_i; \mathbf{w})$$

- Parameter Estimation: find parameters that maximise likelihood function
  - Equivalent: minimise log-likelihood

$$\mathbf{w}^* = \arg\max_w \mathcal{L}(\mathbf{w}) = \arg\min_w -\log \mathcal{L}$$

### **Parameter Estimation: Linear model**

- Assume:  $y_i = f(x_i) + e_i$  with  $f(x) = wx_i$
- With Normal random error e<sub>i</sub> ~ N(0, σ) p(e<sub>i</sub>) ∝ exp(-e<sub>i</sub><sup>2</sup>/(2σ<sup>2</sup>)
  the model for y<sub>i</sub> is described by a p(y<sub>i</sub>|x<sub>i</sub>,w) ∝ exp((wx<sub>i</sub>-y<sub>i</sub>)<sup>2</sup>/(2σ<sup>2</sup>)
- The likelihood function is then

$$\begin{aligned} \mathcal{L}(w) &= p(\mathbf{y}|\mathbf{x}, w) = \prod_{i} p(y_i|x_i; w) \\ \rightarrow -\log \mathcal{L}(w) &= \sum_{i}^{i} (y_i - w_i x_i)^2 \end{aligned}$$

- The negative log-likelihood function is equivalent to the least square loss
- Probabilistic interpretation for our simple regression machine learning model

Measurement XY7

### Classification

- Want to learn a function to separate different class of events.
  - Problem is to find best decision boundary



## **Linear Decision Boundary**



Predict class 0 if h(x) < 0 otherwise class 1</li>
 if h(x) > 0

• distance from boundary =  $h(\mathbf{x}) / ||\mathbf{w}||$ 

# Logistic Regression

• Linear discriminant

$$h(\mathbf{x}; \mathbf{w}) = \mathbf{w}^T \mathbf{x}$$

• Use probability that example **x** is in a given class using the sigmoid function

$$p(y = 1 | \mathbf{x}) \equiv p_i = \frac{1}{1 + e^{-h(\mathbf{x};\mathbf{w})}} = \frac{1}{1 + e^{-\mathbf{w}^T \mathbf{x}}}$$



further from the boundary, more certain about class

# Logistic Regression

- Probabilistic interpretation
- 2 classes classification: Bernoulli process

$$p(y_i|x_i) = (p_i)^{y_i} (1 - p_i)^{1 - y_i} \quad y_i = \{0, 1\}$$

• Negative Log-Likelihood is then

$$-\ln \mathcal{L} = -\sum_{i} (y_i \ln p_i + (1 - y_i) \ln(1 - p_i))$$

#### **Binary Cross Entropy Loss function**

### **Binary Cross Entropy Function**

$$L(\mathbf{w}) = -\sum_{i} (y_i \ln p_i + (1 - y_i) \ln(1 - p_i))$$



# Logistic Regression

- Find the values of **w** minimising the crossentropy loss function
  - $\mathbf{w}^* = \arg\min_{\mathbf{w}} -\ln L(\mathbf{w})$
- Use iterative algorithm to find optimal value
   w\* (e.g. gradient descent )





### **Gradient Descent**

- Minimize Loss function by repeated gradient steps
  - Compute gradient w.r.t. parameters:

- Update parameters 
$$\mathbf{w}' \leftarrow \mathbf{w} - \eta \frac{\partial L(\mathbf{w})}{\partial \mathbf{w}}$$

 $-\eta$  is called the learning rate, controls how big of a gradient step to take

Many variants exists especially in case of deep neural network training

 $\frac{\partial L(\mathbf{w})}{\partial \mathbf{w}}$ 

# Why Sigmoid function?

• If we use probabilistic theory and Bayes statistics, the posterior p(y=1 | x) :

$$p(y = 1|x) = \frac{p(x|y = 1)p(y = 1)}{p(x|y = 1)p(y = 1) + p(x|y = 0)p(y = 0)}$$

• Then by using 
$$z = \ln \frac{p(x|y=1)p(y=1)}{p(x|y=0)p(y=0)}$$

$$p(y = 1|x) = \frac{1}{1 + e^{-z}}$$


Which hypothesis is the most consistent with the data we have observed ?

# **Hypothesis Test**

- H<sub>0</sub>: null hypothesis
  - the hypothesis we want to reject
  - e.g. the data contains only background
- **H**<sub>1</sub>: alternative hypothesis
  - e.g. the data consists of signal + background events
- Critical region:
  - regions of the test statistics defining the hypothesis rejection
- $\alpha$  : significance level (Type 1 error)
  - probability to reject H<sub>0</sub> when is true (false positive)
- *β*: Type 2 error
  - probability to accept H<sub>0</sub> when is false (false negative)
  - 1- $\beta$ : power of the test

#### **Classification as Hypothesis Test**



## Hypothesis Test

	State of Nature		
Decision we make	H <sub>o</sub> is true	H <sub>o</sub> is false	
Accept H <sub>o</sub>	ok	Type II error	
		probability $\beta$	
Reject H <sub>o</sub>	Type I error	ok	
	probability $\alpha$		

### **ROC Curve**



### **ROC Curve**

Receiver Operating Characteristic (ROC) Curve classifying quarks vs. gluons



### **ROC Curve**



### Sensitivity and Specificity

The Truth

Fes Sca	st (	Has the disease	Does not have the disea	• _	
	pre: Positive	True Positives (TP) a	False Positives (FP) b	PPV = TP TP + FP	
	Negative	c False Negatives (FN)	d True Negatives (TN)	<b>NPV = </b> TN + FN	
		Sensitivity TP TP + FN	Specificity Sen TN TN + FP	nsitivity: • Signal efficiency • True Positive rate	
	0	r, — a+c	dSpo d+b	ecificity: • 1 Background efficienc • True Negative rate	

## Purity

- Purity = Number of signal Events passing selection / Total number of events passing the selection
  - Purity = True Positive / (True Positive + False Positive)
  - important value but dependent on total number of Signal/ Background events)
- Optimize selection depending on analysis
  - e.g.  $S/\sqrt{(S+B)}$  or expected Asimov significance for discovery



Significance

20

15

10

5

## Neyman-Pearson Lemma

The likelihood ratio λ(x) used as selection criteria, gives for each selection efficiency α the best possible rejection of H<sub>0</sub> in favour of H<sub>1</sub> (background rejection)

$$\lambda(x) = \frac{L(x|H_0)}{L(x|H_1)} \le c$$

where  $P(\lambda(X) \le c | H_0) = \alpha$ 

The cut value c defines the rejection region of the null hypothesis H<sub>0</sub>

## **Polyonial Regression**

TMultiGraph of 3 TGraphErrors



### What is the correct model?



$$f(x|\mathbf{w}) = w_0 + w_1 x \qquad f(x|\mathbf{w}) = w_0 + w_1 x + w_2 x^2 + w_3 x^3 \qquad f(x|\mathbf{w}) = w_0 + w_1 x + \dots + w_9 x^9$$

#### **Under fitting Large Bias**

model does not reproduce well the data

#### **Over fitting** Large Variance

model reproduce the training data too well

# Overfitting

- Model reproduce too well training data
  - In the extreme limit it will follow exactly the data  $(L(w) \approx 0)$
- It might fail miserably on an independent data set (a validation/test data set)



# Overfitting

 Same happens also for classification (e.g. logistic regression)



In case of overfitting decision boundary follows the data

## **Bias-Variance Trade Off**

- Simple model under-fit: it will deviate from data (high bias) but not influenced by its peculiarity (low variance)
- Complex model over-fit: it will not deviate from data (low bias) but it will be very sensitive to the data (high variance)
  - **Bias** : systematic error of the model
  - Variance: sensitivity of prediction
- If model is more complex
  - will capture more data points → lower bias
  - will move more to capture the data → higher variance

### **Bias - Variance Trade Off**

#### **Generalization Error**



# Regularization

- Method to find optimal model is to add a parameter constraint in the loss function
  - aim to trade some bias to reduce variance
- Modify loss function (e.g. for linear regression):

$$L(\mathbf{w}) = \frac{1}{2} \sum_{i=1}^{N} (y_i - f(\mathbf{x}_i; \mathbf{w}))^2 + \lambda \Omega(\mathbf{w})$$

L2 norm Ω(w) = ||w||<sup>2</sup> = ∑ w<sub>i</sub><sup>2</sup>
equivalent to Gaussian prior on the weights
L1 norm Ω(w) = ||w|| = ∑ |w<sub>i</sub>|
equivalent to Laplace prior on the weights shore 12-14 March 2018

53



• L2 keeps weights small, L1 keeps weights sparse!

[M. Kagan]

#### Data Science School in (Astro) Particle Physics, Lisbon, 12-14 March 2018 55

## Hyper-parameter Optimisation

- How to find optimal regularisation parameter ?
- We need to perform an hyper-parameter optimisation to find the best total error
- Split the data in 3 samples:
  - Training sample
    - used to train and fit the model
    - Find best parameter values
  - Validation sample
    - used to check the model and measure error as function of hyper-parameter
    - find best hyper-parameter values
  - Test Sample
    - Final check of the model when all parameters have been fixed
    - Need to be independent than validation since we have tune the model on the validation sample

Training Set	Validation Set	Test Set
--------------	-------------------	----------

### Hyper-Parameter Optimization



[M. Kagan]

### CrossValidation



- Divide data randomly in k-folds
  - Use (k-1) folds for training and 1 fold for validation
  - Repeat changing the validation set
- Use average estimate performances on the k-folds
- Can estimate variance on the performance
- Especially useful when data set is small

#### Machine Learning Software ROOT/TMVA

### ROOT

ROOT is a software toolkit which provides building blocks for:

- Data processing
- Data analysis
- Data visualisation
- Data storage



ROOT is written mainly in C++ (C++11 standard)

Bindings for Python are provided.

Adopted in High Energy Physics and other sciences (but also industry)

- ~250 PetaBytes of data in ROOT format on the LHC Computing Grid
- Fits and parameters' estimations for discoveries (e.g. the Higgs)
- Thousands of ROOT plots in scientific publications



TMVA



- ROOT Machine Learning tools are provided in the package TMVA (Toolkit for MultiVariate Analysis)
- Provides a set of algorithms for standard HEP usage
- Used in LHC experiment production and in several analysis (e.g. Higgs studies)
- Easy interface for beginners, powerful for experts
- Several active contributors and several features added recently (e.g. deep learning)







- TMVA is not only a collection of multi-variate methods. It is a
  - common interface to different methods
    - common interface for classification and regression
  - easy training and testing of different methods on the same dataset
    - consistent evaluation and comparison
    - same data pre-processing
  - several tools provided for pre-processing
  - embedded in ROOT
  - complete and understandable users guide

### **TMVA Methods**

The available methods are:

- Rectangular cut optimisation
- Projective likelihood estimation (PDE approach)
- Multidimensional probability density estimation (PDE rangesearch approach)
- Multidimensional k-nearest neighbour classifier
- Linear discriminant analysis (H-Matrix and Fisher discriminants)
- Function discriminant analysis (FDA)
- Artificial neural networks (various implementations)
- Boosted/Bagged decision trees
- Predictive learning via rule ensembles (RuleFit)
- Support Vector Machine (SVM)

### **New Features**

New features added since 2016:

• Deep Learning



- support for parallel training on CPU and GPU (with CUDA and OpenCL)
- Cross Validation and Hyper-parameter optimisation
- Improved loss functions for regression
- Interactive training and visualization for Jupyter notebooks
- new pre-processing features (variance threshold)

# Using TMVA



## Workflow in TMVA

- Reading input data
- Select input features and preprocessing
- Training
  - find optimal classification or regression parameters using data with known labels (e.g. signal and background MC events)

#### Testing

- evaluate performance of the classifier in an independent test sample
- compare different methods

#### Application

• apply classifier / regressor to real data where labels are not known



#### **TMVA Custumizations and Features**

TMVA supports:

- ROOT Tree input data (or ASCII, e.g. csv)
  - HSF support might come soon
- pre-selection cuts on input data
- event weights (negative weights for some methods)
- various method for splitting training/test samples
- k-fold cross-validation
- support variable importance
- hyper-parameter optimisations

### **TMVA Session**

void TMVAnalysis()	
TFile* outputFile = TFile::Open( "TMVA.root", "RECREATE" );	
TMVA::Factory *factory = new TMVA::Factory( "MVAnalysis", outputFile,"!V");	Create Factory
TFile *input = TFile::Open("tmva_example.root");	
factory->AddVariable("var1+var2", 'F'); factory->AddVariable("var1-var2", 'F'); //factory->AddTarget("tarval", 'F');	Add variables/ targets
factory->AddSignalTree ( (TTree*)input->Get("TreeS"), 1.0 ); factory->AddBackgroundTree ( (TTree*)input->Get("TreeB"), 1.0 ); //factory->AddRegressionTree ( (TTree*)input->Get("regTree"), 1.0 ); factory->PrepareTrainingAndTestTree( "", "", "nTrain_Signal=200:nTrain_Background=200:nTest_Signal=200:nTest_Background=200:!V"	Initialize Trees
factory->BookMethod( TMVA::Types::kLikelihood, "Likelihood", "!V:!TransformOutput:Spline=2:NSmooth=5:NAvEvtPerBin=50" ); factory->BookMethod( TMVA::Types::kMLP, "MLP", "!V:NCycles=200:HiddenLayers=N+1,N:TestRate=5" );	Book MVA methods
factory->TrainAllMethods(); // factory->TrainAllMethodsForRegression(); factory->TestAllMethods(); factory->EvaluateAllMethods();	n, test and evaluate
outputFile->Close(); delete factory; } We will see better with a real example	e
(e.g. TMVAClassification.C.tutorial)	[E. v. Toerne] Particle Physics, Lisbon, 12-14 March 2018

L. Moneta

## TMVA Toy Example

#### 4 Gaussian variable with linear correlations $\{x_1 = v_1 + v_2, x_2 = v_1 - v_2, x_3 = v_3, x_4 = v_4\}$ where $\{v_1, ..., v_4\}$ are normal variables



#### **Pre-processing of the Input Variables**

• Example: decorrelation of variable before training can be useful



#### Several others pre-processing available (see Users Guide)

# **Available Preprocessing**

This is the list of available pre-processing in TMVA

- Normalization
- Decorrelation (using Cholesky decomposition)
- Principal Component Analysis
- Uniformization
- Gaussianization

#### TMVA GUI

At the end of training + test phase TMVA produces an output file that can be examined with a special GUI (TMVAGui)



## **ROC Curve in TMVA**

For example from GUI one can obtain a ROC curve for each method trained and tested on an independent data set



→ Comparison of several methods
#### **TMVA Regression GUI** A dedicated GUI exists for regression (TMVARegGui)







73

## **Regression in TMVA**

- New Regression Features:
  - Loss function
    - Huber (default)
    - Least Squares
    - Absolute Deviation
    - Custom Function



#### Important for regression performance

## **Cross Validation in TMVA**

TMVA supports k-fold cross-validation



- Hyper-parameter tuning
  - find optimised parameters (BDT-SVM)
- Providing support for parallel execution
  - multi-process/multi-threads and on a cluster using Spark or MPI

Signal Efficiency

### **TMVA Interfaces**

External tools are available as additional methods in TMVA and they can be trained and evaluated as any other internal ones.

- **RMVA**: Interface to Machine Learning methods in R
  - c50, xgboost, RSNNS, e1071
    - see <u>http://oproject.org/RMVA</u>
- **PYMVA**: Python Interface
  - **skikit-learn** with RandomForest, Gradiend Tree Boost, Ada Boost)
    - see <u>http://oproject.org/PYMVA</u>
  - Keras (Theano + Tensorflow)
    - support model definition in Python



- See <a href="https://indico.cern.ch/event/565647/contributions/2308668/attachments/1345527/2028480/29Sep2016\_IML\_keras.pdf">https://indico.cern.ch/event/565647/contributions/2308668/attachments/1345527/2028480/29Sep2016\_IML\_keras.pdf</a>
- Input data are copied internally from TMVA to Numpy array

# Jupyter Integration

New Python package for using TMVA in Jupyter notebook (jsmva)

- Improved Python API for TMVA functions
- Visualisation of BDT and DNN
- Enhanced output and plots (e.g. ROC plots)
- Improved interactivity (e.g. pause/resume/stop of training)
- see example in SWAN gallery https://swan.web.cern.ch/content/machine-learning

