ttH production and top-Higgs coupling properties

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Outline

- ttH production at the LHC
 - Search fot ttH production in ATLAS
 - ttH(bb) analysis
 - LIP contributions
 - Alternative boosted analysis for HL-LHC
- top-Higgs coupling CP structure
 - Phenomenology of CP-mixed top quark Yukawa coupling
 - ttH CP phenomenology @ LIP
 - Plan and current status
- > Summary

ttH production at the LHC

- Top quark is the most massive fermion, with a Yukawa coupling of order unity
- The dominant 125 GeV Higgs boson production mode (gluon fusion) allows only an indirect study of this coupling
- Associated production of a Higgs boson with a top quark pair (ttH) has direct dependence on the strength of the coupling







and new physics can be "conspiring" here



- \rightarrow ttH is rare compared to tt or to the other Higgs production modes
- CMS recently claimed observation of ttH production combining full Run 1 data with 35.9 fb⁻¹ of Run 2 data. Observed (expected) significance of 5.2 (4.2) standard deviations arXiv:1804.02610

Search for ttH production in ATLAS

- ▶ 4 analyses, defined by Higgs decay: multi-lepton (ML) (H \rightarrow ττ, H \rightarrow VV), bb, γγ, ZZ \rightarrow 4l
- > Multi-lepton
 - Multivariate methods to reject fake leptons
 - Equally limited by systematics and statistics
- > bb̄
 - Extensive use of b-tagging to enhance signal purity and constrain tt background
 - Limited by systematics, mainly modeling of tt produced with additional b-jets
- > γγ
 - Fitting $m_{\gamma\gamma}$ distribution in tt-enriched region
 - Limited by statistics
- > ZZ→4|
 - Very pure region with 0 events observed
- Evidence for ttH production with 36.1 fb⁻¹ of data. Observed (expected) significance of 4.2 (3.8) standard deviations

Phys. Rev. D 97, 072003 (2018)



ttH(bb) analysis

- ➤ Use multiple b-tagging working points (WP): loose, medium, tight, very tight → pseudo-continuous b-tagging
- Three categories: dilepton, single lepton boosted and single lepton resolved. Fully hadronic category to be included
- Regions for the fit are obtained taking advantage of the btagging score of all jets, attempting to:
 - Maximise signal-to-background ratio in signal regions
 - Get control regions as pure as possible in each of the tt+jets components (tt+light, tt+≥1c and tt+≥1b)
- > A simultaneous profile likelihood fit is performed
 - Classification multivariate analysis (MVA) output in signal regions, including boosted
 - > Event yields or H_T are used in control regions
- Observed (expected) signal significance with respect to the background-only hypothesis of 1.4 (1.6) standard deviations



Uncertainty source	$\Delta \mu$	
$t\bar{t} + \ge 1b$ modelling	+0.46	-0.46
Background model statistics	+0.29	-0.31
Jet flavour tagging	+0.16	-0.16
Jet energy scale and resolution	+0.14	-0.14
$t\bar{t}H$ modelling	+0.22	-0.05
$t\bar{t} + \geq 1c$ modelling	+0.09	-0.11
Total systematic uncertainty	+0.57	-0.54
$t\bar{t} + \ge 1b$ normalisation	+0.09	-0.10
$t\bar{t} + \geq 1c$ normalisation	+0.02	-0.03
Statistical uncertainty	+0.29	-0.29
Total uncertainty	+0.64	-0.61

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LIP contributions

- Modeling uncertainties of diboson backgrounds, by comparing kinematics and rates from different generator settings
- > Z+jets background: data-driven normalisation and uncertainties
 - Split control region in 3, based on b-tag, to get independent normalisations for 3 jet flavour components:
 - k(Z+0HF) = 1.0
 - k(Z+1HF) = k(Z+2HF)= 1.3
 - Stable with respect to changes in control region definition
 - Uncertainties from scale variations <10% in fit regions</p>
 - ➤ Mismodeling of no. of jets → 35% uncertainty on the Z+jets cross-section on the fit, decorrelated among jet multiplicities
- Earlier this year, data/MC comparisons with new software release and 2017 data
- Contributing to production of common ntuples to be used in the analysis



Data / Pred.

Alternative boosted analysis for HL-LHC



- Select a boosted topology and then perform a fit to m_{bb} , analogously to the $H \rightarrow \gamma \gamma$ analysis
- Such a strategy has been proposed for the FCC (√s=100 TeV), with a promise of reduced systematics J. Phys. G, 43, 3 (2016)
- Would it be useful already at the HL-LHC (3ab⁻¹ @√s=14 TeV)? We are trying to answer these questions:
 - What is the luminosity needed for observation of ttH using this analysis?
 - What is the expected uncertainty on the top Yukawa coupling given the full HL-LHC luminosity?



top-Higgs coupling CP structure

- > Baryon asymmetry in the Universe motivates search for additional sources of CP violation
- In some SM extensions, CP violation is introduced by having the physical neutral Higgs boson be a mixture of CP-odd and CP-even. Impact on couplings to fermions

$$\mathbf{L}_{ht\bar{t}} = y_t \bar{t} (a + ib\gamma_5) th$$

- Searches for anomalous couplings of the 125 GeV Higgs boson to gauge bosons set upper limits to CP-odd fraction of production cross-section @95% CL: 0.41 from ATLAS and 0.25 from CMS Eur. Phys. J., C75(10):476, 2015. ; Phys. Lett. B, 759:672. 25 p, 2016.
- Should pursue equivalent searches in fermionic sector: top is the first obvious candidate, but also tau, bottom, then muon, and so on...
- ► Even if we are convinced the 125 GeV Higgs is purely CP-even, new physics can manifest through anomalous couplings → we must directly measure Yukawa couplings

Phenomenology of a CP-mixed top Yukawa coupling

- Fit of a and b to combined data from all Higgs searches at LHC Run I and Tevatron, assuming all other couplings to be SM-like
 - * SM; - best-fit values; = = 68%, 95% and 99.7% CL regions
- Production cross-section of ttH decreases while that of tH/tH increases as the CP-odd fraction of the coupling is increased
- Impact on kinematics of ttH production: m_{ttH} distribution shifts to higher mass values → Higgs with higher p_T
- Observables sensitive to tt spin correlations also sensitive to CP nature of the coupling
- > CP-odd observables are needed to be sensitive to relative sign of a and b







ttH CP phenomenology @ LIP Phys. Rev. D 96, 013004 (2017), arXiv:1711.05292

- ttH(→bb) and ttbb events at NLO, including alternative CP scenarios, and set of other SM backgrounds at LO. Fast detector simulation
- > Kinematical reconstruction routines for single lepton and dilepton
- New family of angular observables using boosted reference frames, to use for signal/background and CP-even/CP-odd discrimination
- Studied impact of reconstruction on the distributions of the new observables and of those previously proposed by phenomenologists
- Expected limits @95% CL for cross-section times H→bb branching ratio in different CP scenarios









ttH CP phenomenology @ LIP

Phys. Rev. D 96, 013004 (2017), arXiv:1711.05292

- Angular observables retain information about CP nature of the coupling, even after detector effects and reconstruction
- Sensitivity of a search for ttH has small dependence on CP-odd fraction
- Best observables require reconstruction of top quarks and Higgs boson
- Collaborated with theorists in computation of differential distributions at NLO+NLL for pure CP-odd scenario (Phys. Rev. D 96, 073005 (2017))





95% CL limit on σ×BR(h→bb)[pb]

Plans and current status

- Perform measurement of CP-odd coupling within ATLAS ttH(bb) analysis
 - Request signal samples with modified top quark Yukawa coupling
 - Select events from signal-rich regions defined by the search analysis, compromising between purity and statistics
 - > Compute angular observables
 - Template fit or asymmetries using best observables (or output from dedicated multivariate method) to simultaneously extract a and b parameters
- Currently doing template fit studies with ATLAS fully-simulated backgrounds and private fast-simulated signal samples to obtain approximate sensitivity estimates, including main systematic uncertainties



- ttH production at the LHC provides direct access to top quark Yukawa coupling
- Observation of ttH means a shift of focus towards measuring properties, with increasing precision, as happened in the dominant Higgs production modes
- ► H→bb is not the most sensitive channel to observation, but allows reconstruction of top quarks and Higgs boson, which makes it preferred to measuring coupling properties
- First step towards generalizing top quark Yukawa coupling is adding a CP-odd component, also motivated by SM extensions
- Such a measurement would differentiate ATLAS ttH(bb) analysis → LIP in privileged position to lead the effort
- In parallel, phenomenology work about prospects for ttH in HL-LHC and future colliders, in close contact with theorists

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Phenomenology of a CP-mixed top Yukawa coupling

Indirect constraints

- Fit of a and b to combined data from all Higgs searches at LHC Run I and Tevatron, assuming all other couplings to be SM-like
 - * SM; - best-fit values; = = 68%, 95% and 99.7% CL regions

tH/tH and ttH cross-sections

- Production cross-sections expected to deviate from SM expectation with increasing CP-odd component, in opposing directions
- > α is the CP-mixing angle, given by atan(b/a)
- > 20% accuracy in ttH cross-section at 14TeV LHC $\rightarrow \pi/6$ accuracy in α





Experimental Status

CP properties of the 125 GeV Higgs Boson

- Measurements focused on couplings to gauge bosons
- > ATLAS spin-parity measurements on $H \rightarrow \gamma\gamma$, $H \rightarrow ZZ^* \rightarrow 4I$, $H \rightarrow WW^* \rightarrow ev\mu v$ with full Run I dataset
 - Excluded spin 2, spin 0 CP-odd and BSM spin 0 CP-even scenarios at >99.9% CL
 - Upper limit to effective CP-odd cross-section fraction f<0.41 at 95% CL</p>
- ► ATLAS CP invariance test on vector boson fusion (VBF), H→ττ with full 8TeV dataset
 - Dimensionless CP-violating parameter constrained to [-0.11,0.05] at 68% CL
- > CMS search for CP-odd anomalous couplings in $H \rightarrow ZZ^* \rightarrow 4I$, $H \rightarrow WW^* \rightarrow ev\mu v$ combined with VH $(H \rightarrow b\overline{b})$
 - f<0.25 at 95% CL and f<0.0034 assuming SM-like ratio between the top and bottom Yukawa couplings</p>
 - CP-odd Higgs boson wouldn't couple at tree level to gauge bosons, contribution to these channels loopsuppressed compared to CP-even
- Coupling to photons more "democratic", since it is also loop-induced in the SM. CMS 95% CL intervals for Hγγ couplings
 - CP-even: [-0.011,0.054] (SM predicts ~0.004); CP-odd: [-0.039,0.037]

Large Hadron Collider (LHC)

- > Circular accelerator at CERN, 27km in circumference
- Collides proton-proton, proton-lead, lead-lead (recently also xenon)
- > pp collisions at 13TeV centre-of-mass energy in the present run

ATLAS detector

- Multi-purpose detector, partially motivated to search for unknown phenomena
- Solenoidal magnetic field for inner detector and toroidal field for muon tracking
- Several layers covering nearly the whole solid 25m angle: tracking, calorimetry and muon spectrometer
- Trigger system: reduce collection rate to a rate susceptible of storage and analysis



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Phenomenology of a CP-mixed top Yukawa coupling Spin correlations

> Three out of **many** observables in ttH, to illustrate variety of methods



- CP-odd factor is sensitive to sign of b
- > Only available in tt→dilepton
- Need to reconstruct Higgs
- Boost enhances sensitivity to spin correlations
- Also for dilepton only and requires Higgs reconstruction
- Boosts "open" the angles increasing sensitivity
- Combines production with decay
- In principle accessible in all tt decays
- Need t, t and H reconstruction

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Phenomenology of a CP-mixed top Yukawa coupling

Spin correlations

Comparison of distribution predicted by theory with that after detector simulation, cuts and reconstruction in a tt→dilepton channel analysis



Angle between leptons' momenta projected on the plane perpendicular to the Higgs direction in the lab frame



 $\blacktriangleright \Delta \phi^H(\ell^+,\ell^-) \longrightarrow$

zimuthal angle difference tween leptons' momenta in the Higgs rest frame

 $\sin(\theta_t^{t\bar{t}H})\sin(\theta_{W^+}^H)$

Angle between t momentum in ttH rest-frame and ttH momentum in lab frame

Angle between H momentum in tH rest-frame and W+ in H rest-frame



Phenomenology of a CP-mixed top Yukawa coupling **Spin correlations**

Comparison of distribution predicted by theory with that after detector simulation, cuts and reconstruction in a tt \rightarrow dilepton channel analysis

 $\operatorname{sgn}\left(\left(\vec{p}_{b}-\vec{p}_{\bar{b}}\right)\cdot\left(\vec{p}_{\ell^{-}}\times\vec{p}_{\ell^{+}}\right)\right)\Delta\theta^{\ell h}(\ell^{+},\ell^{-})$



ttbb

tth (h=H) m_{H} =125 GeV tth (h=A) m_{A} =125 GeV

 $\Delta \phi^{h}(I^{+},I^{-})$ [rads]

륑충 Azimuthal angle difference LHC, √s = 13 TeV MadGraph5_aMC@NLO $\Delta \phi^H(\ell^+,\ell^-)$ Ξ between leptons' momenta in dilepton channel (e+u) 0.2 the Higgs rest frame NLO+Pythia6 0.15 0.1 $\sin(\theta_t^{t\bar{t}H})\sin(\theta_{W^+}^H)$ 0.05 -2 10

Exp x = $\Delta \phi^{h}(I+,I-)$

Phenomenology of a CP-mixed top Yukawa coupling

Spin correlations LHC, √s = 13 TeV MadGraph5_aMC@NLO dilepton channel (e+µ) 0.3 > Comparison of distribution at generator+parton-shower level with that after NLO+Pythia6 detector simulation, cuts and reconstruction in a tt-dilepton channel analysis 0.2 $\operatorname{sgn}\left((\vec{p}_b - \vec{p}_{\bar{b}}) \cdot (\vec{p}_{\ell^-} \times \vec{p}_{\ell^+})\right) \Delta \theta^{\ell h}(\ell^+, \ell^-)$ 0.1 0.2 원 장 $\Delta \phi^H(\ell^+,\ell^-)$ -IZ 0.12 dilepton channel (e+µ) NLO+Pvthia6 0.1 0.08 0.06 $\sin(\theta_t^{t\bar{t}H})\sin(\theta_{W^+}^H)$ 0.04 Anale between H > momentum in tH rest-frame 0.02 Angle between t momentum in and W+ in H rest-frame ttH rest-frame and ttH momentum in lab frame 0.2 10



Phenomenology of a CP-mixed top Yukawa coupling

Spin correlations in the boosted Higgs regime

- By requiring a Higgs boson with p_T > 200 GeV in ttH, regions with signal-to-background ratios of order one can be attained in the tt→ dilepton channel
- **>** Bonus: high- p_T Higgs enhances $t\bar{t}$ spin correlations
- Solution Use lab-frame $\Delta \Phi(I^+,I^-)$, not requiring reconstruction of t, \overline{t} or H

Expected exclusion of |α|>π/3 at 95% CL with 3ab⁻¹ of 13TeV LHC data



Phenomenology of a CP-mixed top Yukawa coupling Angular observables sensitive to production

Authors suggested it should allow measurement of α =0.3 π

with 3 significance with 300 fb⁻¹ of 13 TeV LHC data

10 Δθ^{tīH}(ī,H) Δθ^{tīH}(t,H) Angle between the Higgs boson and t/ī in the tīH rest-frame

12 > $b_4 = \frac{p_t^z p_{\overline{t}}^z}{p_t p_{\overline{t}}}$

- In principle accessible in all tt decays
 - Need t, t and H reconstruction

Experimental Status

Evidence for ttH production – Multilepton channel

- Most sensitive channel
- Seven categories:
 - 2 same-charge (SS) light leptons (e or μ) l and 0 τ leptons: 2ISS
 - > 3I with charges summing ± 1 , and 0τ
 - 4I with charges summing 0, further divided into Z-enriched and Z-depleted
 - > 2ISS+1τ
 - 2 opposite-charge (OS) I+1τ
 - > $3I+1\tau$ with charge summing 0
 - > 1|+2
- Besides, require 2 jets, of which at least one is b-tagged
- > Multivariate Analysis (MVA) dedicated to fake lepton rejection
- Simultaneous fit to classification MVA in some categories and just total number of events in others
- > Observed (expected) excess with respect to the background-only hypothesis of 4.1 (2.8) standard deviations
- > Uncertainty evenly coming from statistics and systematics, which are mainly jet energy scale and fake lepton estimate



Experimental Status Evidence for ttH production – yy channel

- Require diphoton system, 1b-tagged jet and either:
 - > 1 lepton and one additional jet
 - 0 leptons and two additional central jets. MVA used to discriminate from gluon fusion Higgs production
- Weight data in each category with ln(1+s/b)
- Fit two curves to data
- Very small systematics → sensitivity expected to improve with statistics
 Evidence for ttll production - 77 → 41 obcome

Evidence for ttH production – ZZ→4l channel

- Require 2 OS same-flavour lepton pairs, one b-tagged jet and either:
 - 1 additional lepton and two additional jets
 - 4 additional jets



CP violation in the Standard Model

$$\mathcal{L}_{Yukawa} = -\left(\overline{Q_L}\Gamma\phi d_R + \overline{Q_L}\Delta\tilde{\phi}u_R + \overline{L_L}\Pi\phi\ell_R\right) + \text{h.c.}$$

- > Yukawa matrices Γ and Δ mix different quark generations and are in principle arbitrary
- > They are bi-diagonalized by the U matrices which transform flavour states into mass states $\Gamma' = U_d^{L^{\dagger}} \Gamma U_d^R$, $\Delta' = U_u^{L^{\dagger}} \Delta U_u^R$
- > The weak interaction in terms of the mass states q' is:

$$\mathcal{L}_W = \frac{g}{2} (W^+_\mu \overline{u'_L} \gamma^\mu V d'_L + W^-_\mu \overline{d'_L} \gamma^\mu V^\dagger u'_L), V \equiv U^L_u^\dagger U^L_d$$

- CP symmetry of this interaction requires the CKM matrix V=V⁺
- Rephasing the quark fields reduces the physical parameters of the CKM matrix to 4:
 - 3 "rotation" angles
 - > 1 CP-violating phase

CP violation in a two-Higgs-doublets model



- After electroweak symmetry breaking, Goldstone bosons "absorbed" by W[±] and Z longitudinal components: two charged and one neutral (imaginary part)
- Physical states left: two neutral from real parts (η_1, η_2) , one neutral from an > imaginary part (η_3) and two charged
- CP violation can be introduced by mixing η_1 and η_2 with η_3 to form the physical Higgs bosons
- The Yukawa interaction of the top guark with the h₁ boson becomes

Higgs bosons
The Yukawa interaction of the top quark with the h₁ boson becomes
$$\begin{pmatrix} n_1 \\ h_2 \\ h_3 \end{pmatrix} = R \begin{pmatrix} \eta_1 \\ \eta_2 \\ \eta_3 \end{pmatrix}$$

$$\mathcal{L}_{ht\bar{t}} = y_t \bar{t} (a + (ib\gamma_5))th$$
where b is proportional to R₁₃

$$\mathbf{Flips sign under P}$$

$$\mathbf{a=1 and b=0 in the SM}$$

$$\mathbf{b} \neq 0 is evidence of new source of CP violation$$

Phenomenology of a CP-mixed top Yukawa coupling Matrix element likelihood method

- > Event kinematics fully characterized in terms of N invariant masses and boosted angles
- Probability distributions of the different hypotheses in the N-dimensional space of these variables are computed using LO matrix elements
- > Likelihood ratio between hypotheses is the optimal discriminant
- > For a continuum of hypotheses (α values), three ratios are sufficient
- NLO events depart from optimal case, but good discrimination remains. Real events with detector conditions will depart even further
- α=0, α=π/2, α=0.18π. Solid: LO; Hatched: difference between LO and NLO



