Neutral Meson Masses in Strong Magnetic Fields

Sidney dos Santos Avancini

Departamento de Física - Universidade de Coimbra
Departamento de Física - Universidade Federal de Santa Catarina
Florianópolis - SC - Brasil

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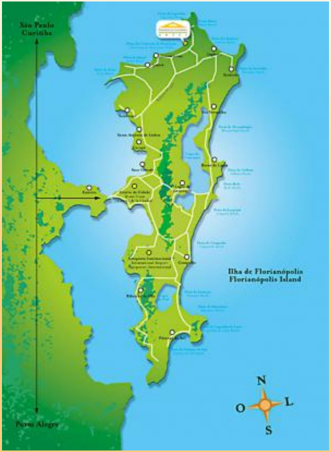
Florianópolis



Azorean Colonization - population: 350.000 (2010).
Latitude 27° - Subtropical (54 Km long x 18 Km wide)
Mean annual temperature (period 1923-1984) 20,4° C.
Means: February (hottest) 24,5°C and July (coldest) 16,4°C.

Introduction

Motivation for the study of strong magnetic fields:

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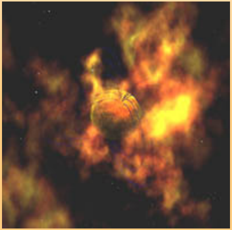
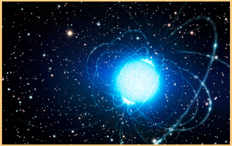
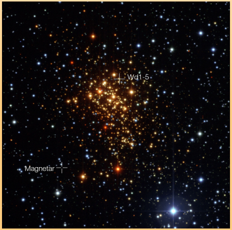


(a) artistic image



(b) artistic image of a magnetar in the star cluster Westerlund 1

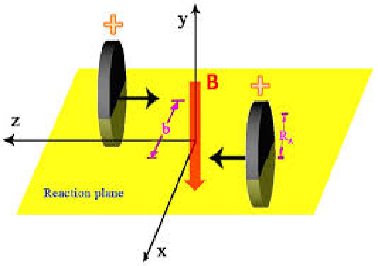


(c) magnetar and its probable former companion star in Westerlund 1

credits: NASA and ESO

Introduction

- Non-central heavy-ion collisions (B $\sim 10^{20}$ Gauss)



collision Au+Au, b=10fm, COM-Energy = $\sqrt{s}$= 200 GeV
(RHIC - Brookhaven National Lab)
But problably the duration of the field is short ($\sim$ 1fm/c)

figure from: *"Electromagnetic fields and anomalous transports in heavy-ion collisions - A pedagogical review"*, Xu-Guang Huang - arxiv: 1509.04073

Magnetic Field Scales

	B [Gauss]	eB [MeV^2]
Earth surface	0.5	$(0.05\text{x}10^{-6}\text{MeV})^2$
MRI - Tomography	$1.5\text{x}10^4$	$(8.6\text{x}10^{-6}\text{MeV})^2$
CERN - magnet	$8.4\text{x}10^4$	$(20.5\text{x}10^{-6}\text{MeV})^2$
Levitating frogs*	10^5	$(25\text{x}10^{-6}\text{MeV})^2$
Quantum electron critical field	$4.4 \text{ x } 10^{13}$	$(0.5 \text{ MeV})^2 = \mathbf{m}_e{}^2$
Magnetars (surface field)	$5.0\text{x}10^{15}$	$(5 \text{ MeV})^2 = (10\mathbf{m}_e)^2$
(Au+Au) RHI Collision	10^{19}	$(400 \text{ MeV})^2 = (3\mathbf{m}_\pi)^2$

(1 Tesla = 10^4 Gauss)

Andre Geim - Ig Nobel-2000 and Nobel-2010 (grapheno)

Free Dirac Particle

Dirac equation $(\vec{B}=0) \Leftrightarrow$ Relativistic Quantum Mechanics

$$\left(\not{p} - M\right)\Psi(t,\vec{r}) \;=\; 0 \;\;,\;\; \not{p} \equiv \hat{p}^{\mu}\gamma_{\mu} \;\;,\;\; \beta = \gamma^0, \vec{\alpha} = \gamma^0\vec{\gamma}$$

$$i\partial_t\Psi(t,\vec{r}) \;=\; H\Psi(t,\vec{r}) \;=\; \left(\vec{\alpha}\cdot\hat{\vec{p}} + \beta M\right)\Psi(t,\vec{r})$$

ansatz for the solution:

$$\Psi(t,\vec{r}) \;=\; \Psi(\vec{p})e^{-ip^{\mu}x_{\mu}} \;=\; \begin{bmatrix} \phi \\ \chi \end{bmatrix} e^{-i(p_0 t - \vec{p}\cdot\vec{r})}$$

positive and negative energy solutions:

$$\Psi_s^{(+)}(t,\vec{r}) \;=\; N\begin{bmatrix} \chi_s \\ \frac{\vec{\sigma}\cdot\vec{p}}{E+M}\chi_s \end{bmatrix} e^{-ip^{\mu}x_{\mu}}$$

$$\Psi_s^{(-)}(t,\vec{r}) \;=\; N\begin{bmatrix} \frac{\vec{\sigma}\cdot\vec{p}}{E+M}\chi_s \\ \chi_s \end{bmatrix} e^{ip^{\mu}x_{\mu}}$$

"Free Particle" in a Magnetic Field

$\vec{B}$ is introduced via minimal coupling to the 4-vector potential (A^μ):

$$\not{p} \equiv \hat{p}^\mu \gamma_\mu \rightarrow (\hat{p}^\mu - qA^\mu)\gamma_\mu \quad , \quad \mathrm{q = particle\ electric\ charge}$$

$A^\mu = (0, 0, Bx, 0)$ (Landau gauge)

$$\vec{B} = \nabla \times \vec{A} \Rightarrow \vec{B} = B\hat{z} \quad , \quad \nabla \cdot \vec{A} = 0$$

$$\begin{aligned} \left(\not{p} - q\not{A} - M\right)\Psi(t,\vec{r}) &= 0 \ , \\ i\partial_t \Psi(t,\vec{r}) = H(A^\mu(\vec{r}))\Psi(t,\vec{r}) &= \left(\vec{\alpha}\cdot\left[\hat{\vec{p}} - q\vec{A}(x^\mu)\right] + \beta M\right)\Psi(t,\vec{r}) \end{aligned}$$

“Free Particle” in a Magnetic Field

electron (q=-e) , $A^\mu = (0, 0, Bx, 0)$ (Landau gauge)

- $$i\partial_t \Psi(t,\vec{r}) \;=\; H(x)\Psi(t,\vec{r}) \;=\; \left(\vec{\alpha}\cdot\left[\hat{\vec{p}} + eBx\hat{y}\right] + \beta M\right)\Psi(t,\vec{r})$$

  ansatz for the solution (positive enery)

  $$\Psi(t,\vec{r}) \;=\; f(x)e^{-iEt+p_y y+p_z z} \quad , \quad f(x) \rightarrow 4-\mathrm{spinor}$$

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- $$\begin{bmatrix} -E+M & 0 & p_z & \hat{O}_1 \\ 0 & -E+M & \hat{O}_2 & -p_z \\ p_z & \hat{O}_1 & -E-M & 0 \\ \hat{O}_2 & -p_z & 0 & -E-M \end{bmatrix} \begin{bmatrix} f_1(x) \\ f_2(x) \\ f_3(x) \\ f_4(x) \end{bmatrix} \; = \; 0$$

  where $\hat{O}_1 = -i(eB)^{1/2}(\frac{\partial}{\partial\xi} + \xi)$, $\hat{O}_2 = -i(eB)^{1/2}(\frac{\partial}{\partial\xi} - \xi)$

  $\xi = (eB)^{1/2}(x + \frac{p_y}{eB})$ (D. Melrose, LNP854 ,(2013).)

“Free Particle” in a Magnetic Field

one can show that the system of eqs. is equivalent to:

$$\left[\frac{d^2}{d\xi^2} + \left[\frac{E^2 - M^2 - p_z^2}{eB} \mp 1\right] - \xi^2\right] \left[\begin{array}{c} f_{1,3} \\ f_{2,4} \end{array} \right] \;=\; 0$$

comparing with the 1-dimensional harmonic oscillator eq.

$$\left[\frac{d^2}{dq^2} + [2n+1] - q^2\right] v_n(q) \;=\; 0\,,\; v_n(q) = \frac{1}{(\pi^{1/2} 2^n n!)^{1/2}} H_n(q) e^{-\frac{1}{2}q^2}$$

$\Rightarrow$ plane wave solution ansatz (positive energy):

$$\Psi(t, \vec{r}) \;=\; \left(\begin{array}{c} C_1 v_{n-1}(\xi) \\ C_2 v_n(\xi) \\ C_3 v_{n-1}(\xi) \\ C_4 v_n(\xi) \end{array} \right) e^{-iEt + p_y y + p_z z}$$

Johnson-Lippmann Solution

particular choice of $C_1, C_2, C_3, C_4 \Rightarrow$ four independent solutions:

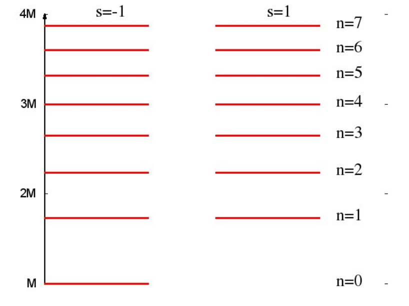
$$\Psi_s^\epsilon(\vec{r}) = \left[\frac{1+s}{2} \begin{bmatrix} (\epsilon E_n + M) v_{n-1}(\xi) \\ 0 \\ \epsilon p_z v_{n-1}(\xi) \\ i p_n v_n(\xi) \end{bmatrix} + \frac{1-s}{2} \begin{bmatrix} 0 \\ (\epsilon E_n + M) v_n(\xi) \\ -i p_n v_{n-1}(\xi) \\ -\epsilon p_z v_n(\xi) \end{bmatrix} \right]$$

$$\Psi_s^\epsilon(t,\vec{r}) \;=\; \frac{(eB)^{1/4}}{(2\pi)} \frac{1}{\sqrt{2\epsilon E_n(\epsilon E_n + M)}} \; \Psi_s^\epsilon(\vec{r}) \; e^{-i\epsilon(Et + p_y y + p_z z)}$$

$\epsilon = +1(-1) \rightarrow$ positive(negative) energy state
s=+1(-1) $\rightarrow$ spin up (down) states
$p_n = \sqrt{2eBn} \quad \xi = (eB)^{1/2}(x + \epsilon \frac{p_y}{eB})$

$$\sum_{\epsilon=\pm1} \sum_{s=\pm1} \sum_{n=0}^{\infty} \int dp_y \int dp_z \Psi_s^\epsilon(t,\vec{r}) \, \Psi_s^\epsilon(t,\vec{r\prime}) \;=\; \delta^3(\vec{r} - \vec{r\prime}) \,, \; \text{(Completeness)}$$

"Free Particle" in a Magnetic Field



Electron Landau Levels

$$E_n = \sqrt{p_z^2 + M^2 + 2eBn} \ ,$$

$$n = l + \frac{1}{2}(1+s) \ , \ \ s = \pm 1 \ ,$$

$$l = 0, 1, 2, ...$$

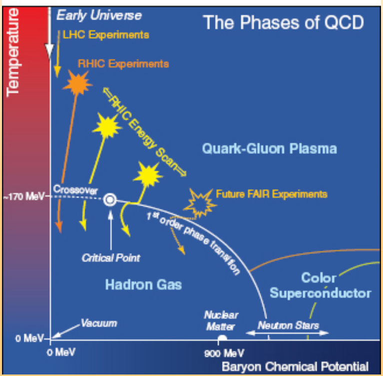
$$E_n^2 - M^2 - 2eBn = p_z^2 \geq 0 \Rightarrow$$

$$n \leq \left[\frac{E_n^2 - M^2}{2eB} \right] \ , \ 2eBn \to p_x^2 + p_y^2$$

- Landau levels with n=1,2,3... are doubly degenerate (spin $s = \pm 1$)
- Ground state, $n = 0$, is non-degenerate and has spin s=-1 (for the electron)

(In the figure, we set $p_z = 0$, $\frac{eB}{M} = 1$)

Quark Matter in Strong Magnetic Field



The whole phase diagram may be explored!

credit: FAIR - CBM - GSI

Quark Matter in Strong B

Very active area of research nowadays:

- QCD phase diagram is being tested in RHIC (T , $\mu = 0$)
- FAIR in near future is going to prove the matter at finite T and μ
- QCD phase diagram in strong B is not completely understood
- Some lattice calculations are available
- Several theoretical calculations of the equations of state (EOS) have been done: mostly using effective models such as: NJL, PNJL, sigma model, etc
- Calculations using Chiral Perturbation Theory
- Hadrons are observed not quarks $\rightarrow$ to understand hadronization process is very important

Very few calculations of hadron properties under strong B $\Rightarrow$ a lot to be done!

ref: (J. O. Andersen and W. R. Naylor, “Phase diagram of QCD in a magnetic field : A review”, arXiv:1411.7176)

su(2)-NJL Model in a B-Field

Two-flavor NJL model Lagrangian:

$$\mathcal{L} \;=\; \overline{\psi}\left(i\not{D} - \tilde{m}\right)\psi \;+\; G\left[(\overline{\psi}\psi)^2 + (\overline{\psi}i\gamma_5\vec{\tau}\psi)^2\right] \;-\; \frac{1}{4}F^{\mu\nu}F_{\mu\nu}$$

interaction terms: scalar-isoscalar + pseudoscalar-isovector

$F^{\mu\nu} = \partial^\mu A^\nu - \partial^\nu A^\mu$ - eletromagnetic tensor field

$D^\mu = (i\partial^\mu - QA^\mu)$ - covariant derivative

$\vec{\tau}$ are isospin Pauli matrices

ψ is the quark fermion field,

$$\psi = \begin{pmatrix} \psi_u \\ \psi_d \end{pmatrix}, \; \tilde{m} = \begin{pmatrix} m_u & 0 \\ 0 & m_d \end{pmatrix}, \; Q = \begin{pmatrix} q_u = \frac{2}{3}e & 0 \\ 0 & q_d = -\frac{1}{3}e \end{pmatrix}.$$

We take m_u=m_d=m and use the Landau gauge $\rightarrow \vec{B} = B\hat{z}$

su(2)-NJL Model in a B-Field

NJL Lagrangian as an effective model for the QCD:
$\rightarrow$ has to reflect the symmetries of the strong interaction!
Positive points:

- Invariant under global phase transformation $\rightarrow$ baryon number conservation
- chiral symmetric Lagrangian(in the limit m_u=m_d=0)
- spontaneous symmetry breaking mechanism (dynamical mass generation)
- The whole QCD phase diagram can be described using just one effective model

Negative points:

- Model is non-renormalizable (needs regularization, i. e., Λ-cutoff)
- Interaction is not confining (no gluons or color charge)

NJL in the Mean Field Approximation

$$\mathcal{L} \; = \; \overline{\psi}\left(i\not{D} - \tilde{m}\right)\psi \, + \, G\left[(\overline{\psi}\psi)^2 + (\overline{\psi}i\gamma_5\vec{\tau}\psi)^2\right] \, - \, \frac{1}{4}F^{\mu\nu}F_{\mu\nu}$$

MFA $\rightarrow$ linearization of the $\mathcal{L}$ interaction terms disregarding quadratic fluctuations:

$$\hat{O} \equiv \langle\hat{O}\rangle + (\hat{O} - \langle\hat{O}\rangle) = \langle\hat{O}\rangle + \Delta\hat{O} \;\;,\;\; \hat{O} = (\overline{\psi}\psi) \text{ or } (\overline{\psi}\mathrm{i}\gamma_5\tilde{\tau}\psi)$$

MFA $\rightarrow (\Delta\hat{O})^2 \cong 0$; $\langle\overline{\psi}i\gamma_5\vec{\tau}\psi\rangle$ = 0 (symmetry)

$$\mathcal{L}_{MFA} = \overline{\psi}\left(i\not{D} - M\right)\psi + G\left\langle\overline{\psi}\psi\right\rangle^2 - \frac{1}{4}F^{\mu\nu}F_{\mu\nu} \;,$$

constituent quark mass

$$M = m - 2G\left\langle\overline{\psi}\psi\right\rangle$$

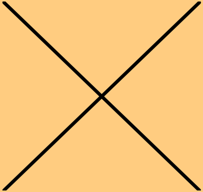
NJL Model in a Strong B

Feynman Rules from Standard Quantum Field Theory

Propagator

$$iS_q(x,x') \equiv \langle 0|T[\psi_q(x)\bar{\psi}_q(x')]|0\rangle \,,\ q = u,d$$

Vertex



$$iK_M \,,\ (iK_\sigma = 2iG \,,\ iK_\pi = 2iG\, i\gamma_5\tau^a \otimes i\gamma_5\tau^a)$$

(Dressed) Fermion Propagator in B

$$S_q(x,x') = e^{i\Phi_q(x,x')} \sum_{n=0}^{\infty} S_{q,n}(x-x') \ , \ \Phi_q(x,x') = Q_q \int_x^{x'} dy_\mu A^\mu(y)$$

$\Phi_q(x,x')$ is the Schwinger phase, $q = u, d$, the integral is along a straight line connecting x and x'

$$S_{q,n}(Z) = \frac{\beta_q}{2\pi} exp\left(-\frac{\beta_q}{4} Z_\perp^2\right) \int \frac{d^2 p_\parallel}{(2\pi)^2} \frac{e^{(ip\cdot Z)_\parallel}}{p_\parallel^2 - M^2 - 2\beta_q n}$$
$$\times \left\{ \left[(p\gamma)_\parallel + M\right] \left[\Pi_- L_n\left(\frac{\beta_q}{2} Z_\perp^2\right) + \Pi_+ L_{n-1}\left(\frac{\beta_q}{2} Z_\perp^2\right)\right] + 2in\frac{(Z\cdot\gamma_\perp)}{Z_\perp^2}\left[L_n\left(\frac{\beta_q}{2} Z_\perp^2\right) - L_{n-1}\left(\frac{\beta_q}{2} Z_\perp^2\right)\right]\right\}$$

$Z = x - x'$, $Z_\parallel^2 = (Z_0^2 - Z_3^2)$, $Z_\perp^2 = (Z_1^2 + Z_2^2)$,
$\Pi_\pm = \frac{1}{2}(\mathbb{I} \pm i\gamma^1\gamma^2)$, $\beta_q = |Q_q| B$,

(ref. V. P. Gusynin et al, Nucl. Phys. B 462 (1996) 249)

NJL - GAP equation

$$M = m - 2G\langle\bar{\psi}\psi\rangle$$
$$= m + 2G \lim_{t'\to t^+} \lim_{\vec{x}'\to\vec{x}} \sum_{q=u,d} Tr[iS_q(x,x')] \, .$$

using the quark propagator and noticing that $\Phi_q(x,x) = 0$

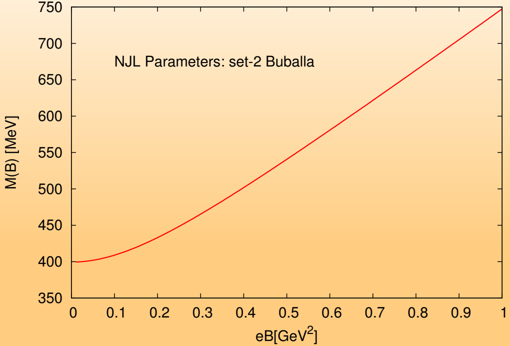
$$\frac{M-m}{2G} = \sum_{q=u,d} \sum_{n=0}^{\infty} \frac{i\beta_q}{2\pi} N_c \int \frac{d^2p_\parallel}{(2\pi)^2}$$
$$\times \frac{Tr\left[((p\cdot\gamma)_\parallel + M)(\Pi_- L_n(0) + \Pi_+ L_{n-1}(0))\right]}{p_\parallel^2 - M^2 - 2\beta_q n}$$

Calculating the trace:

$$\frac{M-m}{2MG} = \sum_{q=u,d} \sum_{n=0}^{\infty} 2g_n\beta_q N_c \int \frac{d^2p_\parallel}{(2\pi)^2} \frac{1}{p_\parallel^3 - M^2 - 2\beta_q n}$$

N_c=3, $g_n = 2 - \delta_{n0}$, $p_\parallel = (p_0, p_3)$.

NJL - GAP equation



Effective mass increases with B
$\rightarrow$ Magnetic catalysis effect

refs: NJL parameters: M. Buballa, Physics Reports 407 (2005)205

su(2)-NJL EOS: D. P. Menezes, M. Benghi Pinto, S. S. Avancini, A. Pérez Martinez and C. Providência, Phys. Rev. C 79, 035807 (2009).

π^0 pole mass in strong magnetic field

T-matrix for the scattering of pairs of quarks, $(q_1 q_2) \to (q_1' q_2')$, can be calculated by solving the Bethe-Salpeter equation:

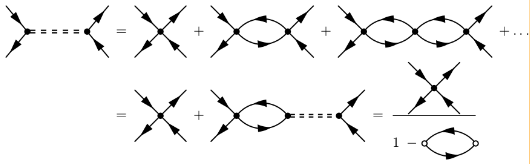


Figure: Diagrammatic representation of the RPA approximation.

Selecting the quantum numbers associated with the π^0
Left side: calculated using a quark-pion interaction

$$\mathcal{L}_{\pi qq} = i g_{\pi qq} \overline{\psi} \gamma_5 \vec{\tau} \cdot \vec{\pi} \psi \ ,$$

Right side: calculated in the NJL model

π^0 pole mass in strong magnetic field

using the Feynman rules

$$(ig_{\pi^0 qq})^2 \; iD_{\pi^0}(k^2) = \frac{2iG}{1 - 2G\Pi_{ps}(k^2)} \; ,$$

where the (π^0) pseudo-scalar polarization loop reads:

$$\frac{1}{i}\Pi_{ps}(k^2) = -\sum_{q=u,d}\int \frac{d^4p}{(2\pi)^4} Tr[i\gamma_5 iS_q(p+\frac{k}{2}) i\gamma_5 iS_q(p-\frac{k}{2})].$$

$D_{\pi^0}(k^2)$ represents the usual π^0-meson propagator:

$$D_{\pi^0}(k^2) = \frac{1}{k^2 - m_{\pi^0}^2} \; .$$

pole-mass of the π^0-meson $\rightarrow$ root of the equation:

$$1 - 2G\Pi_{ps}(k^2)|_{k^2 = m_{\pi^0}^2} = 0 \; .$$

π^0 polarization loop calculation

$$\frac{1}{i}\Pi_{ps}(k^2) = -\sum_{q=u,d}\sum_{n,m=0}^{\infty}\int d^4(x - x')e^{-ik\cdot(x-x')}$$
$$\times Tr[i\gamma_5 iS_{q,n}(x-x')i\gamma_5 iS_{q,m}(x'-x)]e^{i\Phi_q(x,x')}e^{i\Phi_q(x',x)} \; .$$

the Schwinger phases cancel out: $e^{i\Phi_q(x,x')}e^{i\Phi_q(x',x)} = 1$. After Fourier transforming:

$$\frac{1}{i}\Pi_{ps}(k^2) = -\sum_{q=u,d}\sum_{n,m=0}^{\infty}\int \frac{d^4p}{(2\pi)^4}$$
$$\times Tr[\gamma_5 S_{q,n}(p+\frac{k}{2})\gamma_5 S_{q,m}(p-\frac{k}{2})] \; .$$

Details: S. S. Avancini, W. R. Tavares and M. B. Pinto, “Properties of magnetized neutral mesons within a full RPA evaluation”, Phys. Rev. D 93, 014010 (2016)

π^0 polarization loop calculation

$$\frac{1}{i}\Pi_{ps}(k^2) = \sum_{q=u,d}\sum_{n=0}^{\infty} 2g_n\beta_q N_c \times \left(\int \frac{d^2p_\parallel}{(2\pi)^3}\frac{1}{p_\parallel^2 - M^2 - 2\beta_q n} - \int \frac{d^2p_\parallel}{(2\pi)^3}\frac{(k_\parallel^2/2)}{(p_\parallel^2 - M^2 - 2\beta_q n)((p+k)_\parallel^2 - M^2 - 2\beta_q n)}\right)$$

$$\frac{1}{i}\Pi_{ps}(k^2) \;=\; -i\left(\frac{M-m}{2MG}\right) - k_\parallel^2 \sum_{q=u,d}\beta_q N_c \sum_{n=0}^{\infty} g_n I_{q,n}(k^2) \;\;.$$

$$I_{q,n}(k^2) \equiv \int \frac{d^2p_\parallel}{(2\pi)^3}\frac{1}{[p_\parallel^2 - M^2 - 2\beta_q n][(p+k)_\parallel^2 - M^2 - 2\beta_q n]} \,.$$

and $k_\parallel$=(k_0,k_3) , $k_\parallel^2$=(k_0^2-k_3^2).

π^0 polarization loop calculation

$$1 - 2G\Pi_{ps}(k^2)|_{k^2=m^2_{\pi^0}} = 0$$

$$m^2_{\pi^0}(B) = -\frac{m}{M(B)} \frac{1}{i2GN_c \sum_{q=u,d} \beta_q \sum_{n=0}^{\infty} g_n I_{q,n}(m^2_{\pi^0})} \, .$$

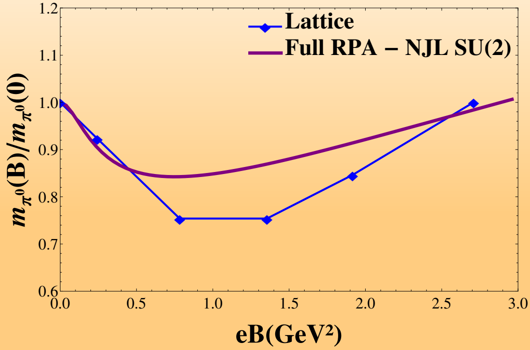
σ-meson is associated to the scalar polarization loop:

$$\frac{1}{i}\Pi_s(k^2) = -\sum_{q=u,d} \int \frac{d^4p}{(2\pi)^4} Tr[iS_q(p+\frac{k}{2})iS_q(p-\frac{k}{2})] \, ,$$

One can show that:

$$m^2_\sigma(B) = 4M^2(B) + m^2_{\pi^0}(B) \, .$$

π^0 mass results



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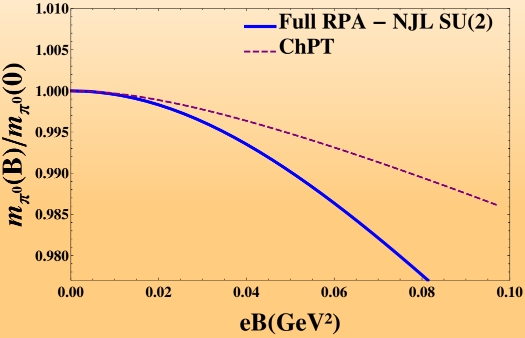

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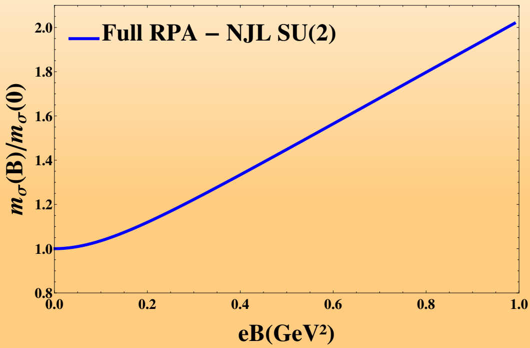
σ mass results



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Conclusions

- The π_0 and σ masses in a strong magnetic field were obtained in a full RPA calculation
- Our formalism performs the sum over the Landau levels analytically, hence, avoiding spurious solutions which can be found in the literature (for example, Tachyonic)
- Our calculation shows the same trend as the lattice and ChPT results
- It is a starting point for several generalizations

Future Perpectives

The properties of the magnetized hadronic matter are very important for a complete understanding of the QCD phase diagram.

This is currently a new and promising area of research in theoretical physics.

- It is still necessary to study mesons at finite T and μ=0 (RHIC-CERN and Lattice QCD)
- It is still necessary to undestand mesons at finite T and μ (RHIC-Fair - physics)
- To include strange quarks (su(3)-NJL)
- confinement - PNJL

Colaborations: Marcus Benghi Pinto, Ricardo Sonego - William Tavares (PHD - Student)- Brasil
Pedro Costa, Pedro Vieira Alberto and Constança Providência and (?) - Coimbra - Portugal

Thank you - Obrigado!