Mott Insulator to High-Tc Superconductor

Spin-Charge Split Pairing in Underdoped Cuprates

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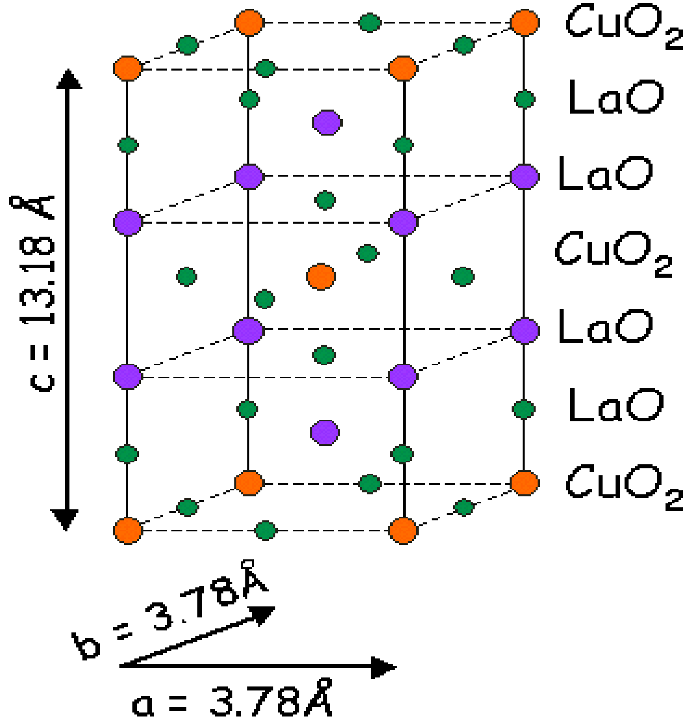
High Temperature Superconductivity

Discovered 1986 (Bednorz and Muller)

- Underlying model (Anderson 1987)

Hole-Doped Mott Insulator or Quantum Anti-ferromagnet in a layered 3D lattice

Described by some variant of t-J model (large-U Hubbard Model) On the Cu Lattice



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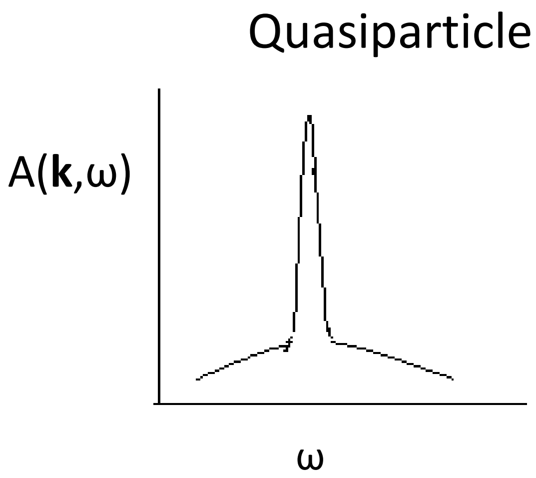
DEAD

Conventional Superconductors

- Metallic State: FERMI LIQUID, described by electron quasiparticles (carry spin and charge).

- quasiparticles form spatially overlapping (zero momentum) Cooper pairs

•> Landau-BCS Theory



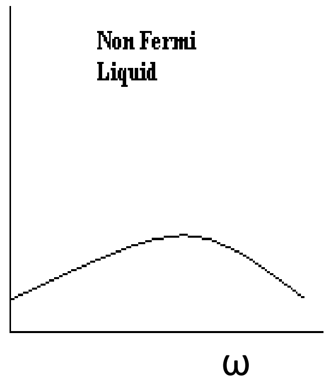
Cuprates (expt)

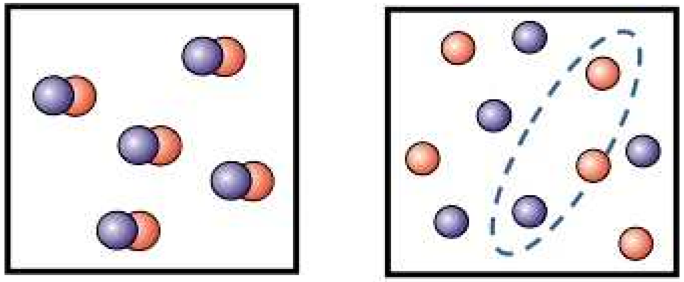
- Metallic state 2D, superconductivity 3D.

- Two metallic states - but no electron quasiparticles (Non-Fermi Liquid)

- Spin and charge are carried by separate excitations, spectrally hidden.

- Superconducting Mechanism is Unknown -- Not clear what pairs and how?





BEC

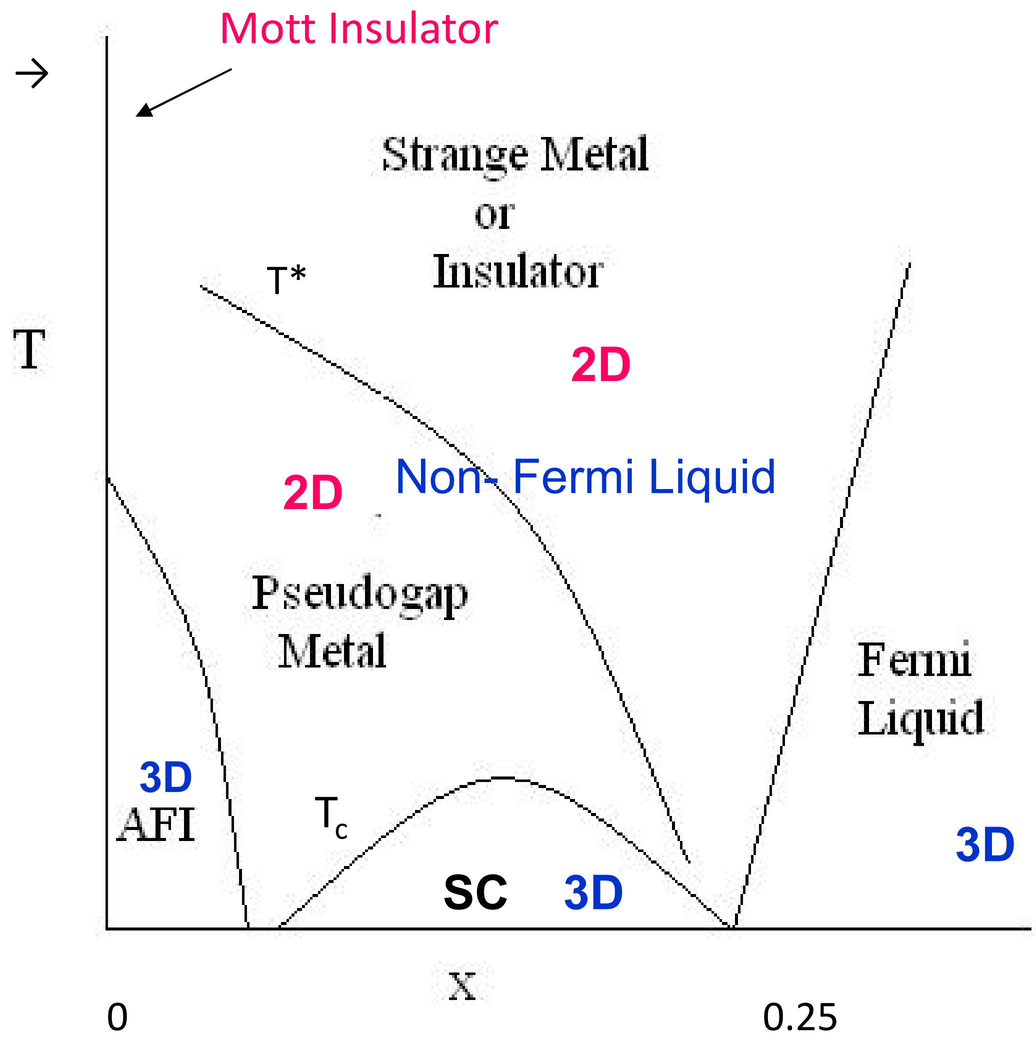
BCS

Phase Diagram (Hole Doped)

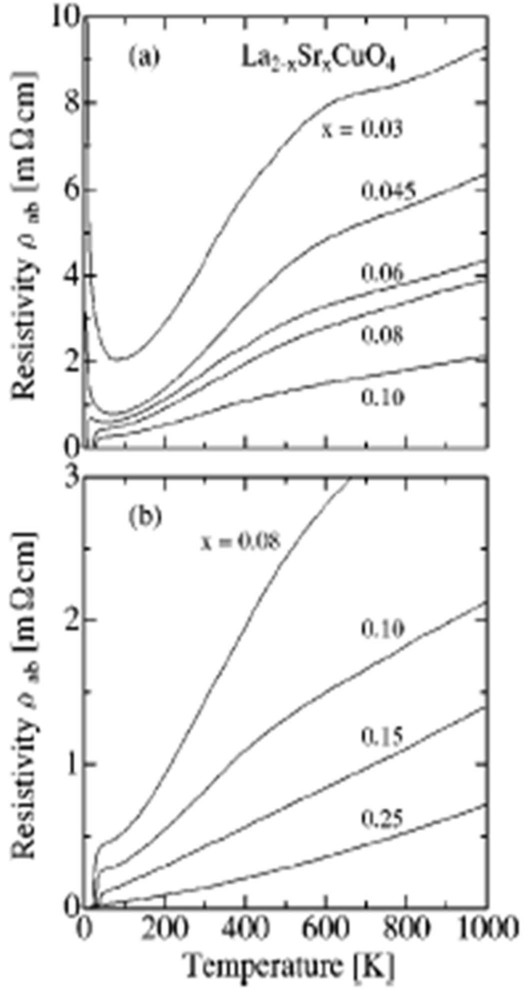
Temperature (T) vs Hole Density (x)   x = 1 – electron density (n)

Half-filled →

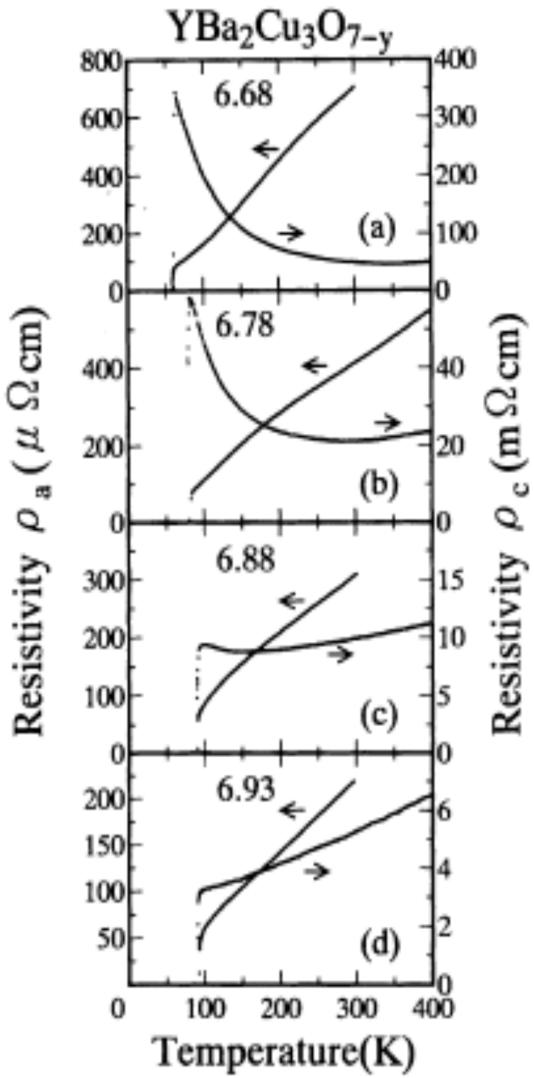
(x = 0)



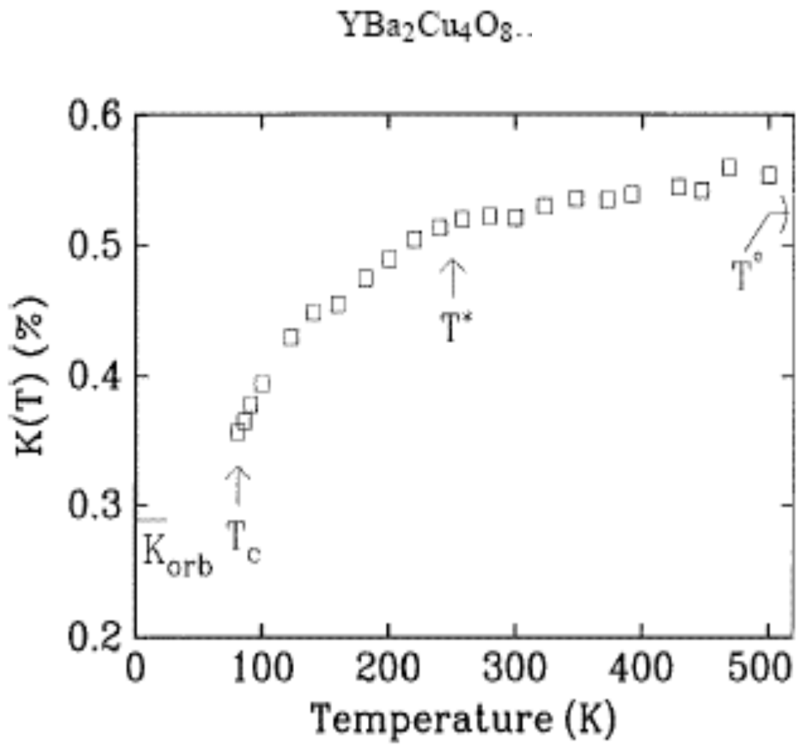
A. Normal State is Two Dimensional

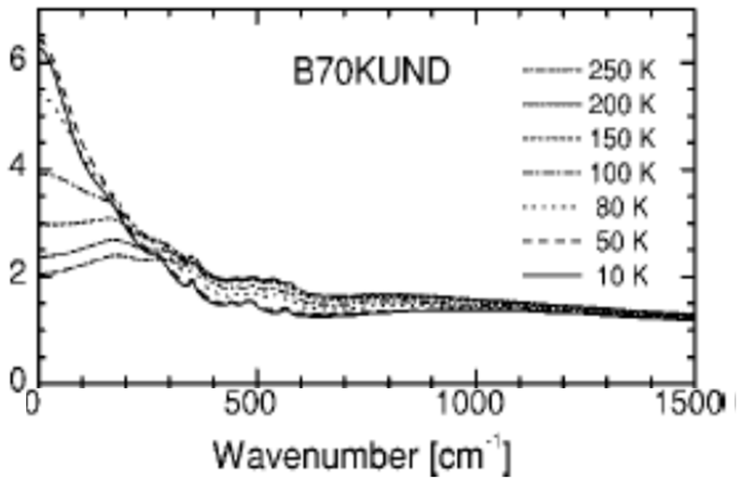


ω



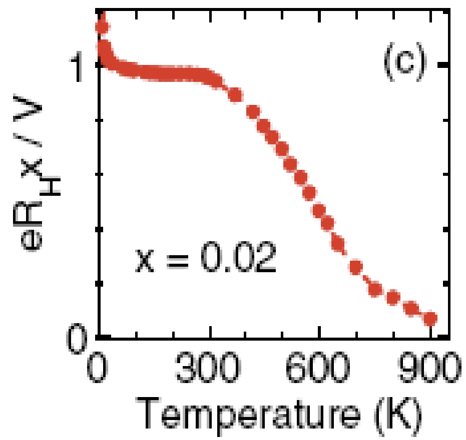
Spin-Charge Separation





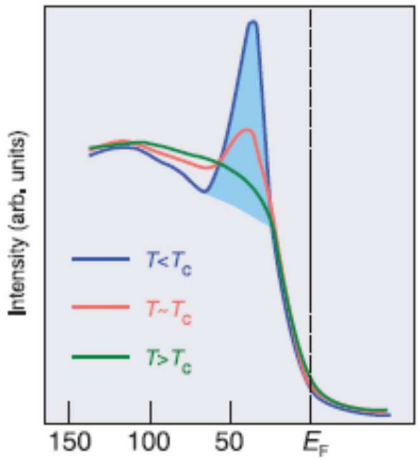
Hall Coeff: $R_H^{-1} \sim x$

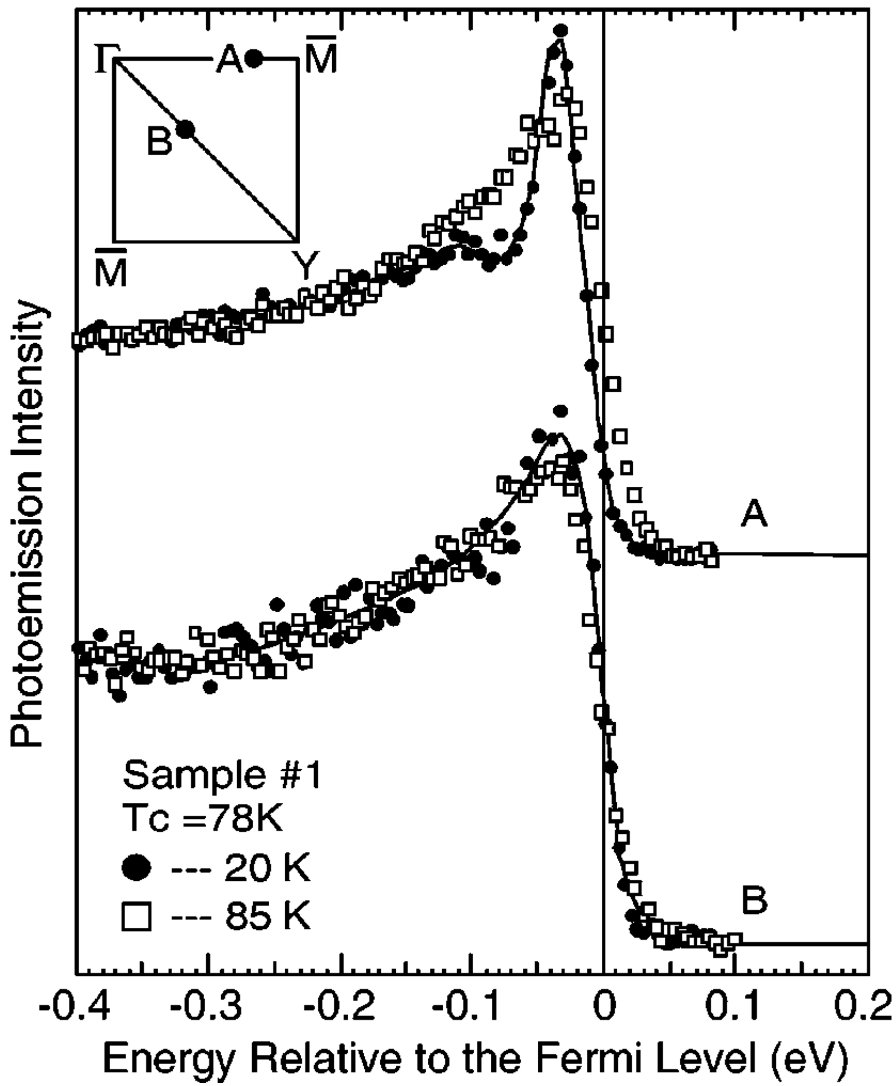
Independent of T



BELIEF

- Superconducting State is Conventional d-wave BCS state, electrons paired.





Electron Peak below Tc in ARPES

D-wave spectrum. Nodal Quasiparticles?

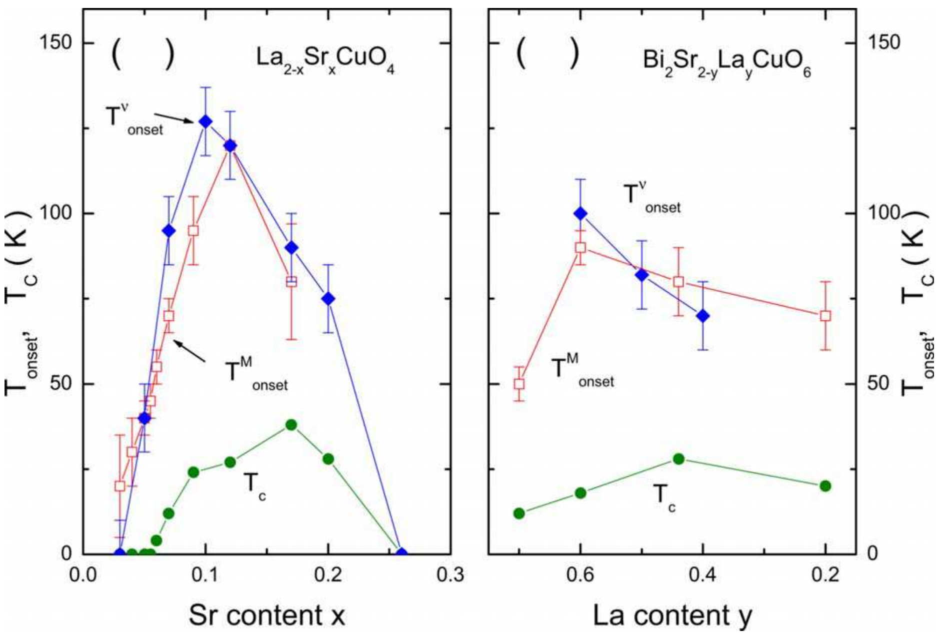
Electron Pairing -- Inconsistent

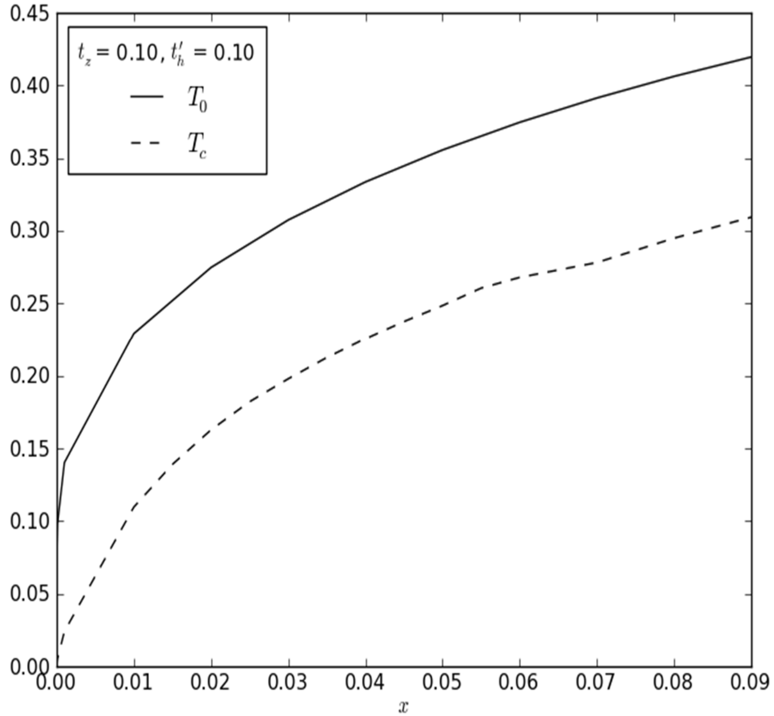
- Large Nernst Signal, Diamagnetism
  >> Pairs above Tc,
  But cannot see electron peaks above Tc

- T_c order x, decreases toward insulator

- Superfluid Density ≈ x (not 1 – x)
- No mechanism connecting electron pairing to non-FL metal

T*





OUTLINE

PROPOSITION: The theory must account for all the phases and main features without contradictions. Because the phenomena are connected.

- Central principle – spin-charge separation in a doped Mott insulator. (Anderson). Spin states shared with insulator

- Renormalized t-J model

- Spin-charge split pairing mechanism.

- Origin of the phases and their properties

- Experiments, Connecting the dots

SPIN-CHARGE Separation Anderson 1987

Physics in large U Hubbard Model

$$H = -\sum_{ij} t_{ij} \; a^{+}_{i\sigma} a_{j\sigma} \; + \; U \sum_{i} n_{i\uparrow} n_{i\downarrow}$$ ← On-Site Repulsion

Half Filling (x = 0) > Electrons Localized > MOMENTS --

S = ½ SPINS > **MOTT Insulator**

Project out doubly occupied sites > *t – J* Model

> Quantum Antiferromagnet with Vacancies (holons)

$$H = \sum_{ij} J_{ij} S_i \cdot S_j - \sum_{ij\sigma} t_{ij} C^{+}_{i\sigma} C_{j\sigma} \qquad J_{ij} = 4t_{ij}^2 / U$$

Spin Exchange projected hopping x > 0

In plane: nn t/J ~ 3,4 nnn t' ~ t/4, J'/J = (t'/t)^{2}, small out-of-plane t_z

Metal in doped region $x > 0$, proposition:

Electron behaves like a composite object,

spin behaves like local moment (rep: spin ½ 'spinons')

Charge carried by spinless vacancies (holons) of charge +e and concentration x.

Both terms represent strong interactions

- $S_1 . S_2$ describes EXCHANGE of two electrons, two spin ½ “SPINONS”.
- Hopping (t) describes exchange of spinon and a holon

□ □ □ □ □ □ □ □

$$C^{+}_{i\uparrow} = b^{+}_{i\uparrow} h_{i}$$

$$S^{+}_{i} = b^{+}_{i\uparrow} b_{i\downarrow}, \qquad S_{iz} = (b^{+}_{i\uparrow} b_{i\uparrow} - b^{+}_{i\downarrow} b_{i\downarrow}) / 2$$

Gauge invariance

$$b_{i\sigma} \rightarrow b_{i\sigma}\, e^{i\theta i} \qquad h_{i} \rightarrow h_{i}\, e^{i\vartheta i}$$

Spin RVB State (Anderson)

AF interaction J, causes neighboring spins to PAIR as singlets

(Ground State of 2 site problem)

• Forms a BCS like state of Paired spinons. (No Charge)

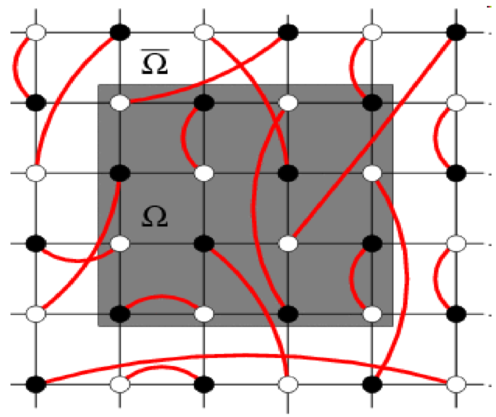
• Bosonic spinons ---- (Arovas-Auerbach)

•RVB mixed with NEEL at x = 0

Very Accurate: in 2D at T = 0 (Confirmed by Monte Carlo)

Reproduces Quantum Spin-Wave Theory Results 3D

Singlet $(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle)$



$$H = -2\sum_{ij} J_{ij} A^{+}_{ij} A_{ij} \qquad 2A_{ij} = b_{i\uparrow} b_{j\downarrow} - b_{i\downarrow} b_{j\uparrow}$$

↑ ↓ ↑ ↓ ↑ ↓
↓ ↑ ↓ ↑ ↓ ↑
↑ ↓ ↑ ↓ ↑ ↓
↓ ↑ ↓ ↑ ↓ ↑

Neel State

Singlets Condense $\langle A_{im} \rangle = A \exp[iQ_0 \cdot (r_i - r_m)/2]$

Q_0 = zone corner (two sublattice quantum order)

Proposition

- For x > 0, hopping will convert spin RVB singlets into Cooper pairs.
- Will create spin-charge separated metal (spinons and holons quasiparticles.

BUT:

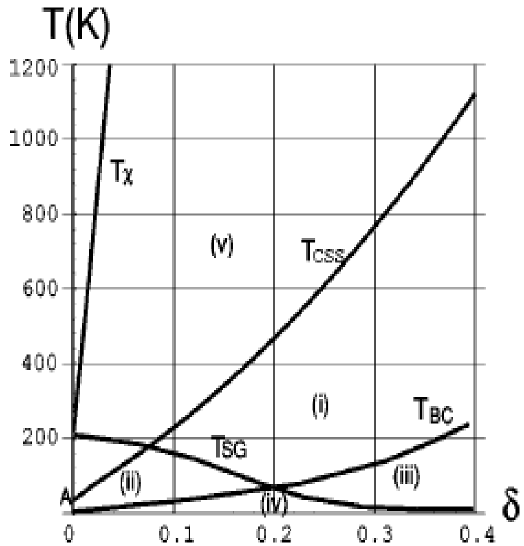
$$\sum_{jm} t b^{+}_{m\sigma} b_{j\sigma} h^{+}_{j} h_{m}$$

a) Bare t-J model has no charge pairing mechanism

b) Attempts to create useful metallic state based on bare model failed

$$C^{+}_{i\uparrow} C_{j\uparrow} = b^{+}_{i\uparrow} b_{j\uparrow} h^{+}_{j} h_{i} \rightarrow \langle b^{+}_{i\uparrow} b_{j\uparrow} \rangle h^{+}_{j} h_{i}$$

Mess -→



Activity died 1990

Revival: From Mott Insulator to Small x

ASSUMPTION

- Moving Holes Destroy Magnetic Order beyond small x_c at T = 0.

Renormalizes the model from t >>J to t_eff << J_eff. Creates a spinon gap.

Supported by Rigorous One-hole Results (Dagotto -- RMP)

- Decouple spinons perturbatively and obtain a reduced Hamiltonian in which t is renormalized away

New Hamiltonian has hopping of spinon pairs (RVB singlets) exchanged with (i) one holon and (ii) two holons

One holon term gives a spin-charge separated Metal

Two holon terms charge pairing (superconductivity)

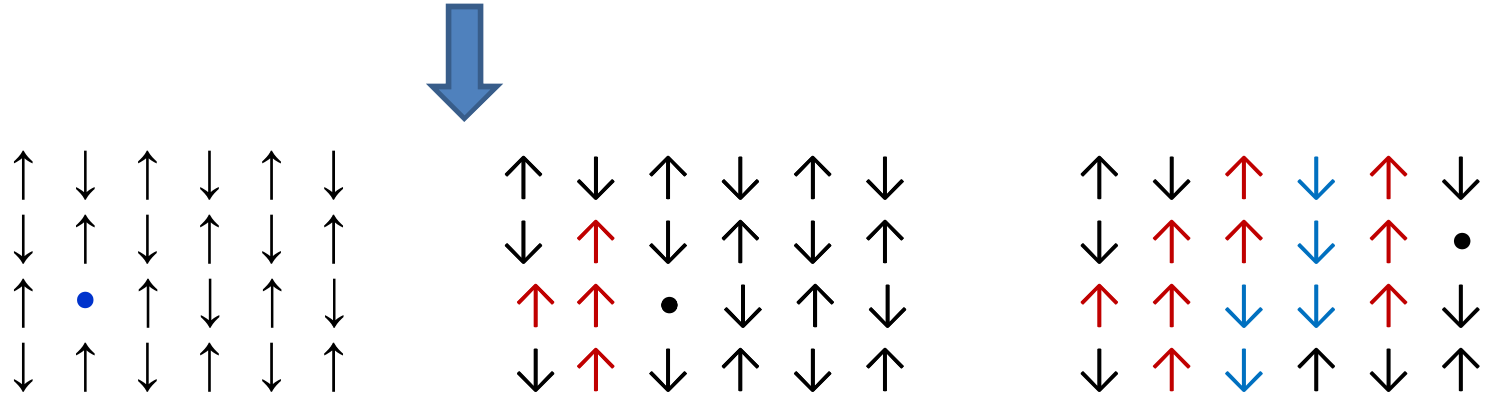
Sarker, Phys. Rev. B 77, 052505 (2008);
Sarker & Lovorn, PRB 82, 014504 (2010)).
Sarker & Lovorn, PRB 85, 144502 (2012).

RENORMALIZATION

Hole Motion Faces Two Impediments 2D

1. Antiferromagnetism: High Energy or Fast

2. RVB Singlets: Slower



Energy ~ 2JL L= Length Must Retrace Path > Localization

- Hole Forms a Spin Bag: Renormalized Hole
- Inside bag hole hops around fast (t) → Broad Spectral background (t)

Localization (MF) →

Phase Separation Visscher (73)

Holes Localized in Domain Walls Schultz (89)

Energetically Better for $x < 0.3$ $E_{localized} < E_{spiral} < E_{slave\text{-}boson}$

Hu & Sarker (94), Sarker (93)

Case: No broken Symmetry - (Use Continuity)
Renormalized Hole: Bigger and Slower

Small x: Renormalized Hamiltonian

$$H = -2J_{eff} \sum_{ij} A^{+}_{ij} A_{ij} - t_{eff} \sum_{ij} c^{+}_{i\sigma} c_{j\sigma}$$

- Prevents Magnetic Order
- Opens a Spinon Gap ~ Ω/2

RG Parameters: Estimate from accurate one- hole calculations →

Bare parameters t/J = 3 - 4

$\Omega \approx$ singlet breaking scale ~ J

J_{eff} scales with J
t_{eff} also scale with J

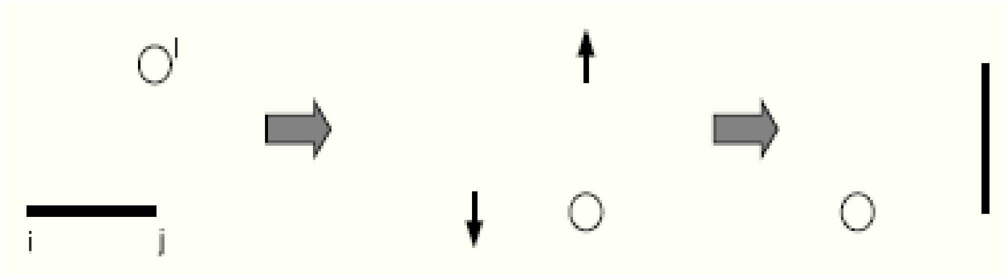
Renormalized Hamiltonian

Gap > Low-energy Configurations:
Holes + Singlets

Renormalized Hole Hops > Breaks a Singlet >
Creates Two Spinons ---- Energy Ω

One-Hole Process

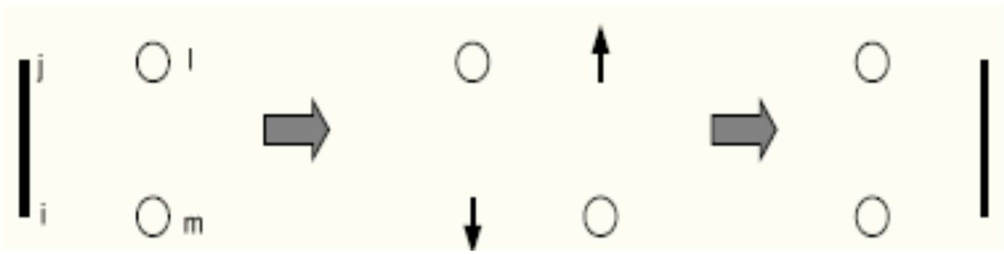
The Hole Hops to Another Site, Singlet is Reconstructed



$$-\frac{t_s}{2}(1-x)\sum_{ijl} \mathrm{A}^\dagger_{jl}\mathrm{A}_{ij}h^\dagger_i h_l,$$

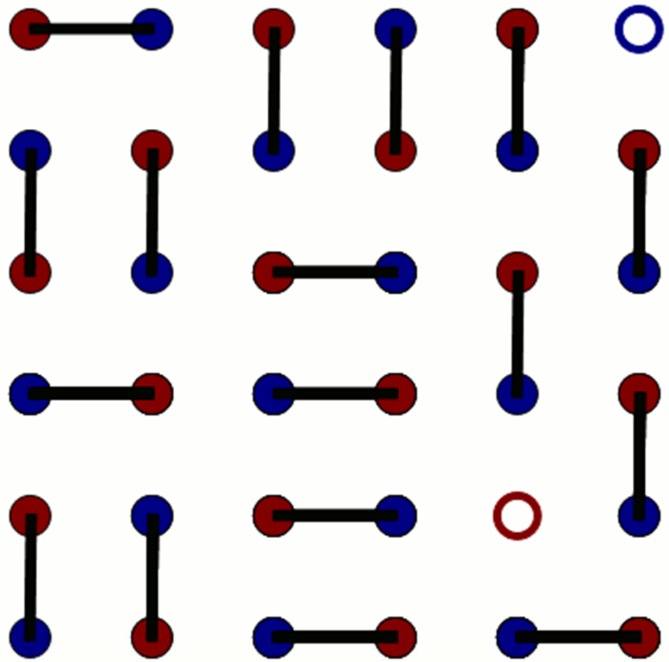
Two-Hole Process

A Second Hole Follows the First, Singlet is Reconstructed

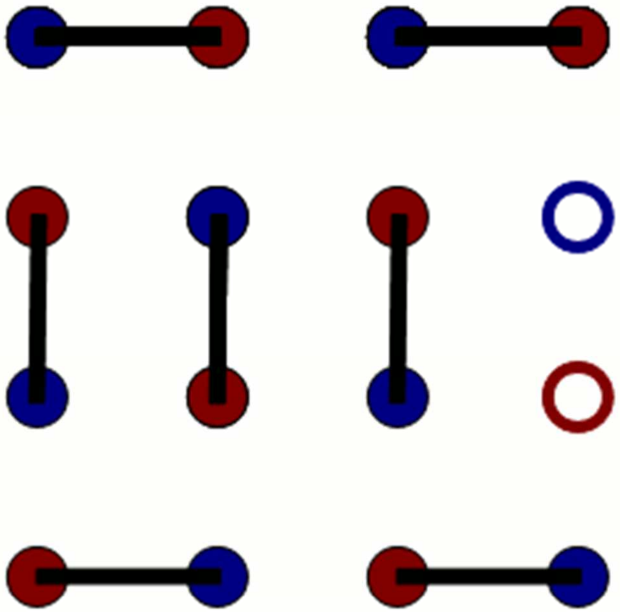


$$-t_s\sum_{ij;lm} \mathrm{A}^\dagger_{ml}\mathrm{A}_{ij}h^\dagger_i h^\dagger_j h_l h_m$$

One-Hole Process



Two-Hole Process



Hopping Hamiltonian (small x – in-plane)

$$H_{hopp} = -\frac{t_s}{2}(1-x)\sum_{ijl} A_{jl}^{+} A_{ij} h_i^{+} h_l \quad - t_s \sum_{ijlm} A_{ml}^{+} A_{ij} h_i^{+} h_j^{+} h_l h_m$$

Same energy scale $t_s = 4t_{eff}^{\ 2}/\Omega$ scales with J

> Related to Short-Range RVB Model Kivelson, Rokhsar and Sethna ('88,'89)

- Symmetry of the original model PLUS
  Total number holons (and spinons) are separately
  CONSERVED in each sublattice. (revealed at low energies)

Valid for small x (UNDERDOPED) and T and ω
< Ω (Spin Gap),

Expt Consequences →

- 1. No magnetic order
- 2. No Conventional Fermi Liquid

Consequences (3): TWO DIMENSIONALITY of METAL EMERGES

Two dimensionality: UNIQUE to cuprates, >> central importance

Take layered system and turn on small t_c

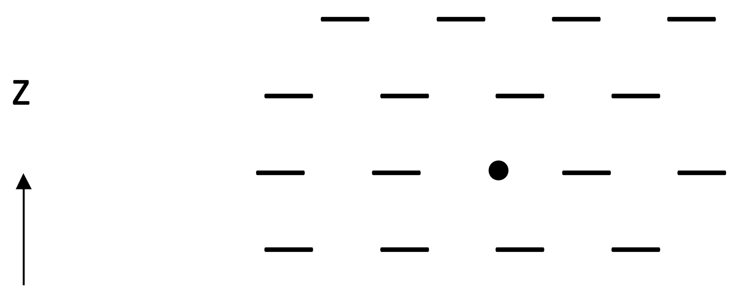
OUR CASE: Spin Gap BLOCKS coherent single holon hopping
in the c direction, but not of pairs

Origin of Two Dimensionality

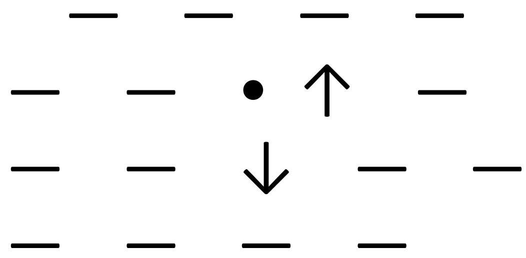
Consider Out of Plane Hopping $t_z << t$

$J_z / J = (t_z / t)^2$ So Singlets form in the xy plane

Holon hops, Creates Two Spinons in Adjacent Planes: High Energy (Ω)



(1)



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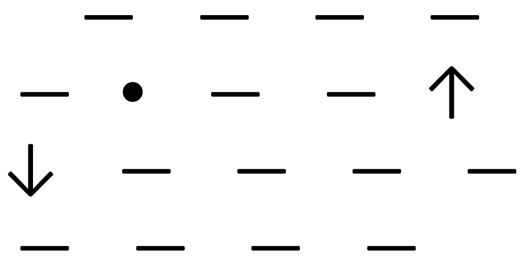

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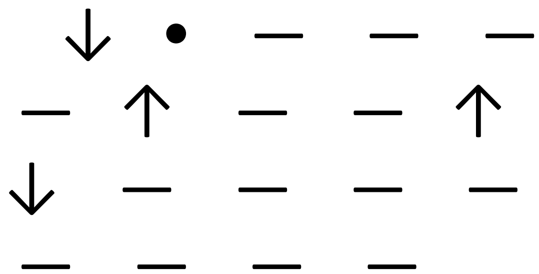


(4)

Fig. 3. Spinons and Holons Move Away in the Plane

Fig (4): Broken Singlets Proliferate

> Single Holon Hopping is Blocked, so Metal is 2D

> Interplane hopping by a singlet does not create spinons, superconductivity is 3D

PREDICTION Strange-Metal Is (Gauge) Insulator

High T phase has full SYMMETRY (High Entropy)

$$H_{hopp} = -\frac{t_s}{2}(1-x)\sum_{ijl} A_{jl}^{+} A_{ij} h_i^{+} h_l \quad -t_s \sum_{ijlm} A_{ml}^{+} A_{ij} h_i^{+} h_j^{+} h_l h_m$$

No propagating ($h^{+}_{i}h_{j}$) holon hopping term

>> $\langle h^{+}_{i}h_{j}\rangle = 0$ No coherent holon propagation in Symmetric Phase

Fluctuating singlet backflow → Gauge Fluctuations

→ Holons Confined (Dynamically Localized)

Effective Holon Hopping :

$$t_h A_{ij}^{+} A_{jl}$$

6. NO ORDER > GAUGE INSULATOR! Underdoped

>> (6a) No Drude Peak

>> (6b) High Resistivity -- much above Mott Limit

NEXT: (small x): Phases

Spinon order same as in from Insulator in symmetry

- No new order parameters. Theory is constrained.

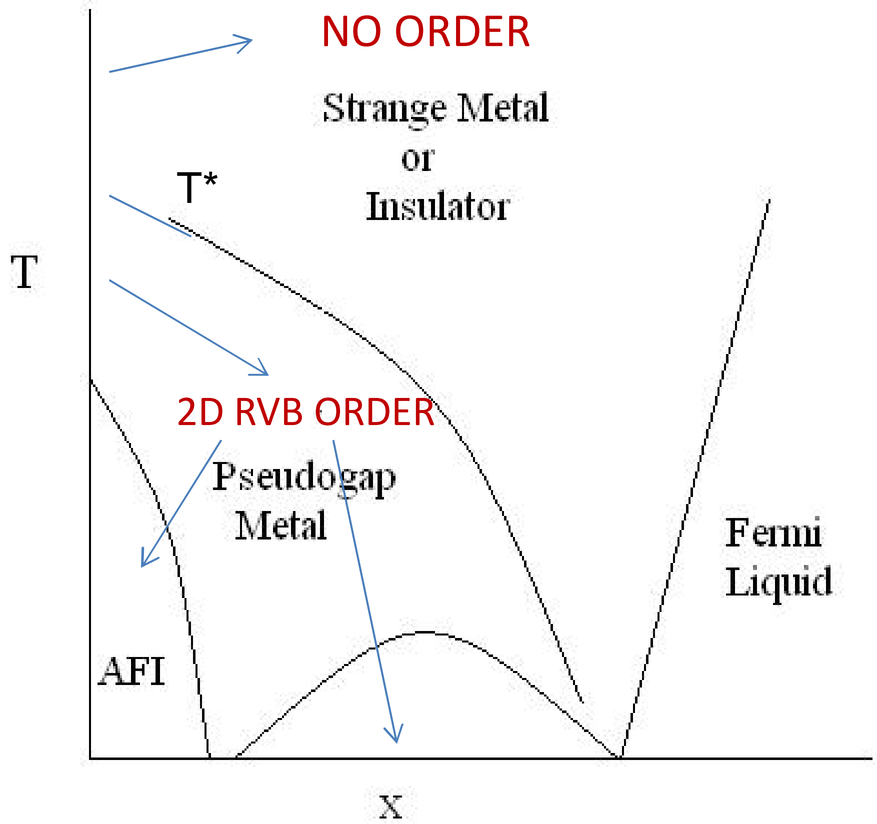
Consequence (4): Exactly 3 phases in (other than superconductivity).

- High T Strange Metal (NO Order) no quasiparticles of any type

- Below T* 2D RVB order

Singlets Condensed A_{ij} = nonzero

Paramagnetic susceptibility suppressed



Pseudogap Phase

RVB order > Coherent Holon Motion Within Same Sublattice

- 2D Spinless Fermi Liquid of concentration x,

Consistent wi2th observed transport, including quantum oscillations in a magnetic field

- Two holon hopping term charge pairing

  Combined with the spinon RVB order leads to

  d-wave superconductivity for electron pair

Real-Space Pairing of holons -- BEC

One-hole and pair hopping → same scale t_s → strong coupling
In the presence of RVB order

- Leads to real-space pair binding of holons below Tp and 3D Bose-Einstein Condensation (BEC) below Tc.

Consistent with observed superfluid density order x, and rise of Tc with x, existence of pairs above Tc.

As the holon pair of momentum k propagates
its phase is Locked to the phase of RVB spin singlets,

Physical Pair > is spin-charge split with a dual character

Charge is mobile holon pair (charge 2e) and the spin part is singlet (0 momentum) part of the condensate (concentration 1- x).

Further Test: Specific heat below T_c

Important > probes excitations, bulk property

Experimental low T $c(T) = dU/dT = A T + B T^3$

Conventional d-wave BCS □ T^2 (does not work)

Interacting Bosons on a lattice

Bogoliubov phonons > T^3

However, the zero-point energy is T-dependent in the Presence of condensate density which also depends on T (thermodynamics), gives a linear T

The crossover scale is 10^{-2} relative to T_c.

Split pairing works

Linear T term have consequences for other systems

SUMMARY

- Spin-Charge separation produces correct theory if t-J model is renormalized.
- Gives rise to split pairing - spin Cooper pair (BCS) shared with insulator, real space charge pair (BEC)
- Continuity from insulator reproduces Phase Diagram with main features.
- spin RVB order as common to pseudogap, superconducting and insulating phases is the Glue.
- Explains origin of 2D metal.
- Uncovers anomalous T linear term in specific heat