Deconfinement and chiral restoration within the SU(3) PNJL and EPNJL models in an external magnetic field

arXiv:1305.4751 [hep-ph]

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Coimbra, 10 July 2013

QCD phase structure

- **The very first "QCD" phase diagram taken from Cabibbo-Parisi (1975)**

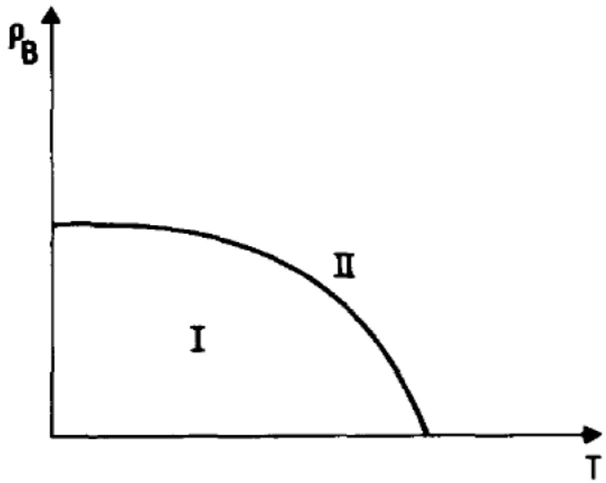

Fig. 1. Schematic phase diagram of hadronic matter. ρ_B is the density of baryonic number. Quarks are confined in phase I and unconfined in phase II.

– N. Cabibbo, G. Parisi, PLB 59 (1975) 67

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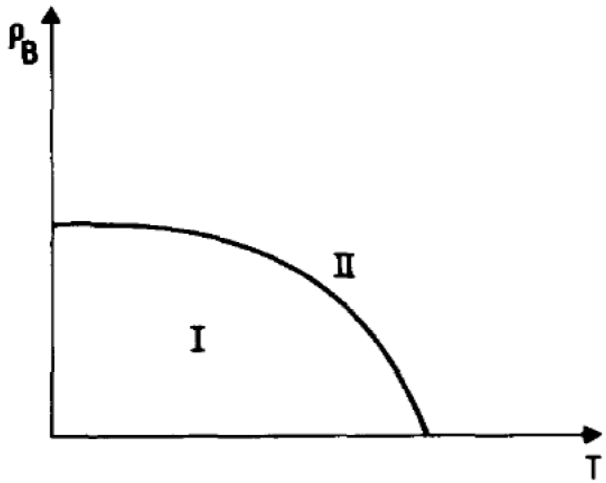

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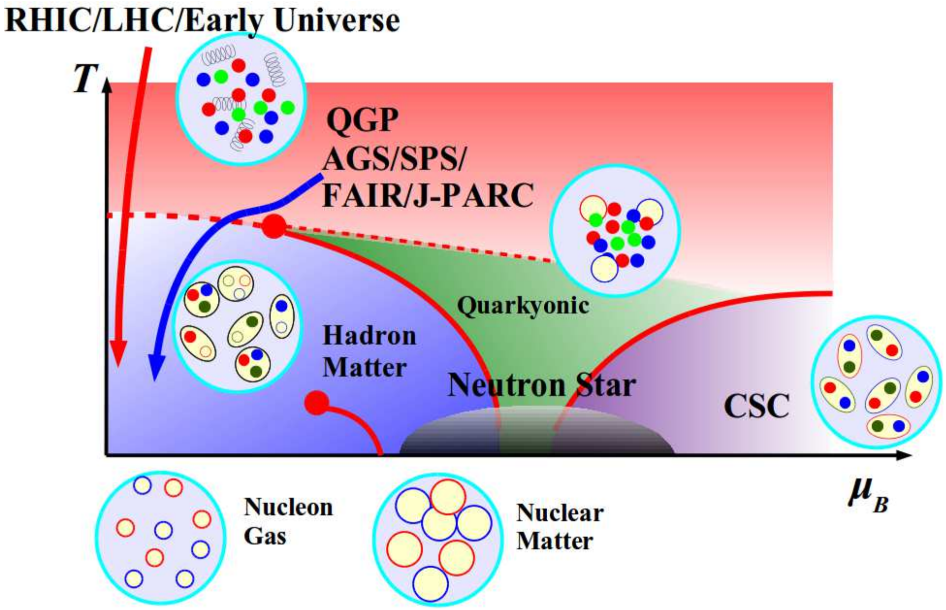
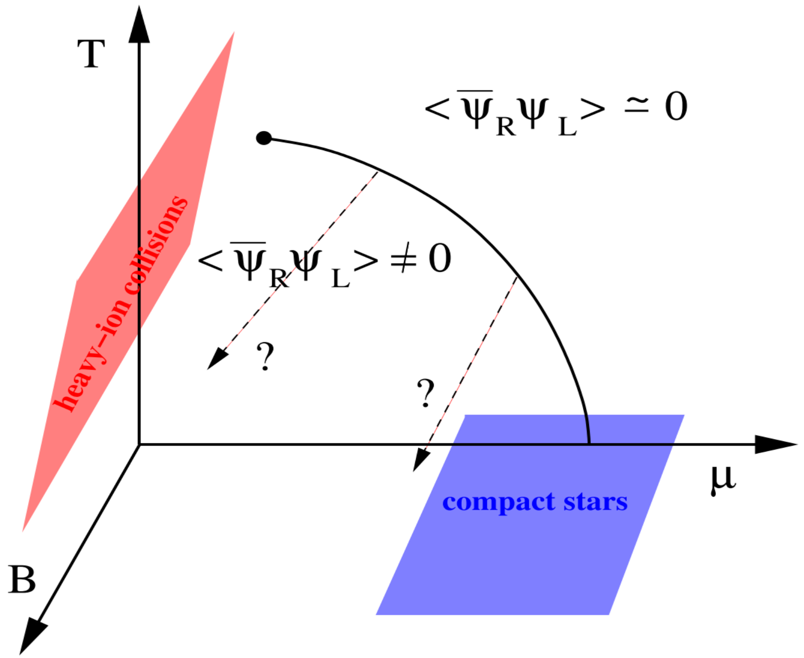
- **A schematic outline for the phase diagram of matter at ultrahigh density and temperature**



– A. Ohnishi, PTPS 193 (2012) 1-10

Deconfinement and chiral restoration in an external magnetic field:



Important for:

- physics of compact objects like magnetars;
- measurements in heavy ion collisions at very high energies;
  - RHIC energy scale: $eB_{max} \approx 5 \times 10^{18}$ G $(5 \times m_\pi^2)$
  - LHC energy scale: $eB_{max} \approx 5 \times 10^{19}$ G $(15 \times m_\pi^2)$
- first phases of the universe.

Concept of order parameter

Symmetries, order parameters, and phase transitions

- **Order parameter is a physical quantity that must**
  - vary under transformations of a symmetry
  - be nonzero in a state with spontaneously broken symmetry

⇓

It must be able to distinguish between two distinct phases!

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In QCD:

- it is helpful to use QCD symmetries to delineate the different phases
- to study transitions between phases, or thermodynamic states of the system, in which the symmetry is realized in one of two different ways:
  - as an exact symmetry
  - as a spontaneously broken symmetry

Properties of QCD I

- Chiral symmetry χ_S
  - Connection of left- and right-handed quarks
  - Exact symmetry if quarks are massless
  - Explicit breaking by quark masses
- Chiral symmetry spontaneously broken
  - Nucleon mass of ~ 1 GeV although bare quark masses ~ 5 MeV
  - "Constituent" or bound state quark masses larger by ~ 300 MeV

If the Lagrangian of a system has a certain symmetry, but the ground state (i.e., the vacuum) does not

⇓

Spontaneous symmetry breaking takes place

Properties of QCD II

- Confinement
  - Free quarks and gluons not observed
  - In general: No colored objects observed
- Not yet totally understood

Chiral symmetry χ_S

When $m_u = m_d = m_s = 0$ (chiral limit) χ_S is **spontaneously broken**

$$\Downarrow$$

- polarization of the vacuum turning it into a condensate of quark-antiquark pairs ($\langle \overline{\psi}_i \, \psi_i \rangle$);
- origin of 8 massless pseudoscalar mesons (Goldstone bosons);

At high values of T and $\mu_B \longmapsto$ restoration of χ_S

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order parameters: **quark condensates**

$$\langle\overline{\psi}_i\,\psi_i\rangle \begin{cases} \neq 0 & \Leftrightarrow \quad \text{symmetry broken}, T < T_c^{\chi} \\ = 0 & \Leftrightarrow \quad \text{symmetry restored}, T > T_c^{\chi} \end{cases}$$

Chiral symmetry χ_S

When $m_u = m_d \neq 0$ and $m_s \neq 0$ (small values) χ_S is **explicitly broken**

⇓

- transformation of an initially pointlike quark with its small bare mass into a massive quark;
- mesonic spectrum for pseudoscalar mesons (pions, kaons, eta);

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Still used as order parameter for the chiral transition

De/confinement

Deconfinement:

- $\mathcal{L}_{QCD}$ has an (exact) $\mathrm{SU}_c(3)$ local color symmetry
- **low temperature**: $\mathbb{Z}_3$ symmetric, confined phase ($\mathbb{Z}_3$ center of $\mathrm{SU}_c(3)$ symmetry)
- **high temperature**: deconfined phase characterized by the spontaneous breaking of the $\mathbb{Z}_3$ symmetry

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Order parameter in pure gauge theory ($m_f \to \infty$):

Polyakov loop $\Phi = \frac{1}{N_c}\mathrm{Tr}_c \left\langle\left\langle \mathcal{P} \exp\left[i \int_0^\beta d\tau A_4(\vec{x},\tau)\right]\right\rangle\right\rangle$

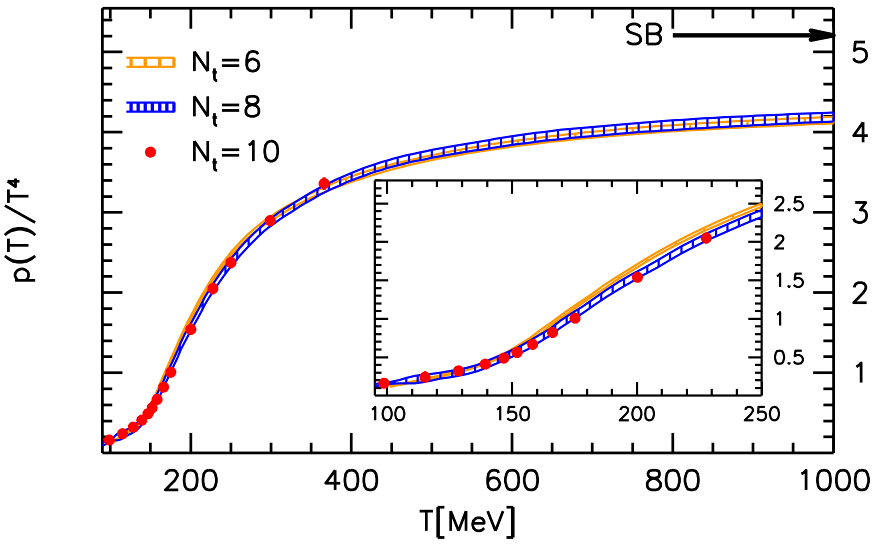
– Not well defined with quarks
– Used anyway (still possible to distinguish both phases)

$$\Phi \begin{cases} = 0 \;\Leftrightarrow\; T < T_c^\Phi, \text{ hadronic (confined) phase} \\ \neq 0 \;\Leftrightarrow\; T > T_c^\Phi, \text{ QGP (deconfined) phase} \end{cases}$$

Understand the QCD phase structure is one of the most important topics in the physics of strong interactions

Theoretical point of view:

- Effective model calculations
- Lattice calculations



– S. Borsanyi et al., JHEP 1011 (2010) 077

QCD phase structure

Understand the QCD phase structure is one of the most important topics in the physics of strong interactions

Theoretical point of view:

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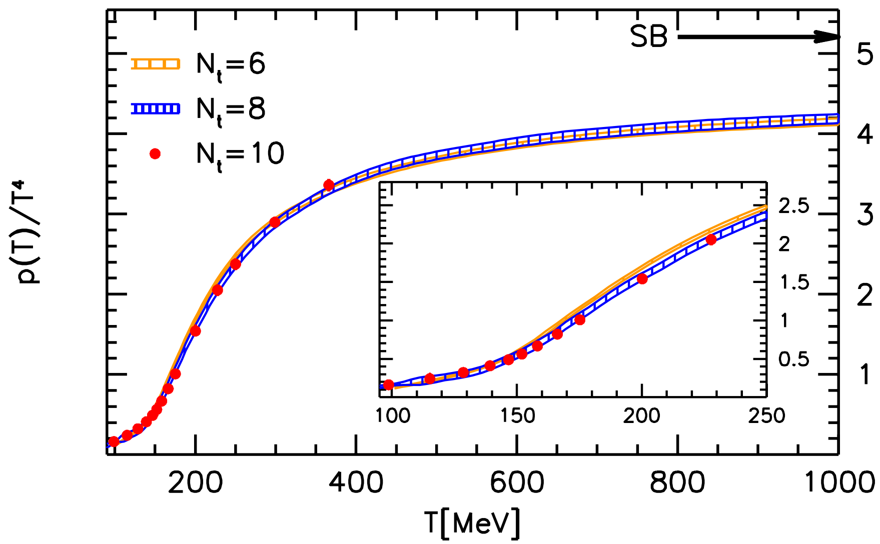
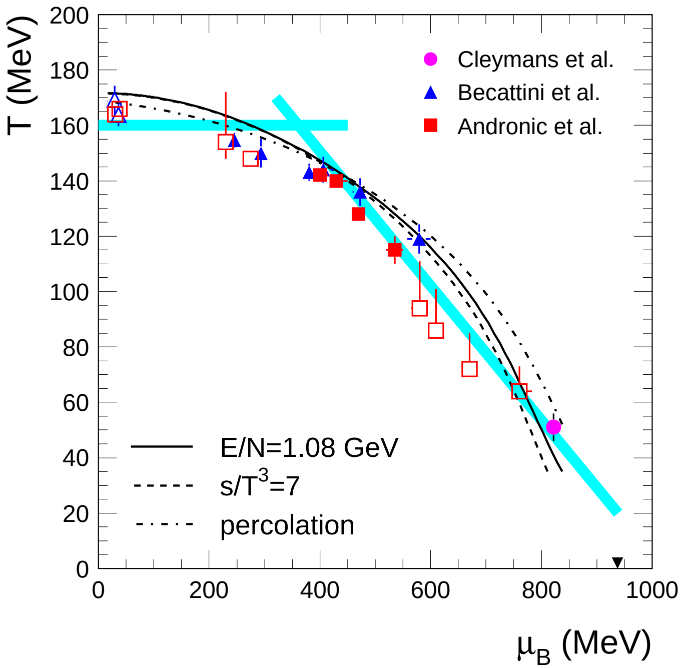
- Lattice calculations

Experimental point of view:

- Map the QCD phase boundary
- Localization of the CEP

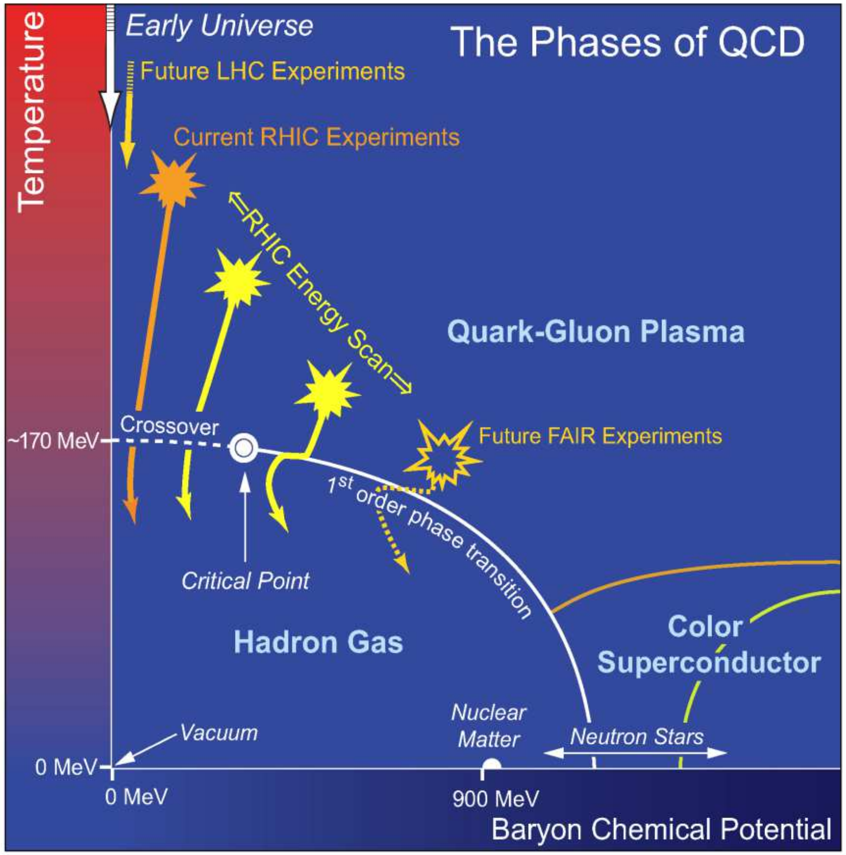




– S. Borsanyi et al., JHEP 1011 (2010) 077 – A. Andronic et al., NPA 837 (2010) 65

QCD phase structure

Experimental point of view:



– "The Frontiers of Nuclear Science, A Long Range Plan", arXiv:0809.3137 [nucl-ex]

QCD phase structure

Understand the QCD phase structure is one of the most important topics in the physics of strong interactions

Experimental point of view:

Beam Energy Scan (BES) program at RHIC[1]:

- map the QCD phase boundary;
- search for the QCD Critical End Point
  - "energy scan" of Au+Au collisions at energies from $\sqrt{s_{NN}} = 7.7 - 200$ GeV
    - higher moments of net-proton multiplicity distributions
    - particle ratio fluctuations (K/π, p/π and K/p)

CEP:
No clear evidence yet!

[1] X. Luo, STAR Collaboration, APPB 5 No 2 (2012) 497

QCD phase structure

Understand the QCD phase structure is one of the most important topics in the physics of strong interactions

Much more to come in the future

- CERN (NA61)
- FAIR (CBM)
- NICA (MPD)

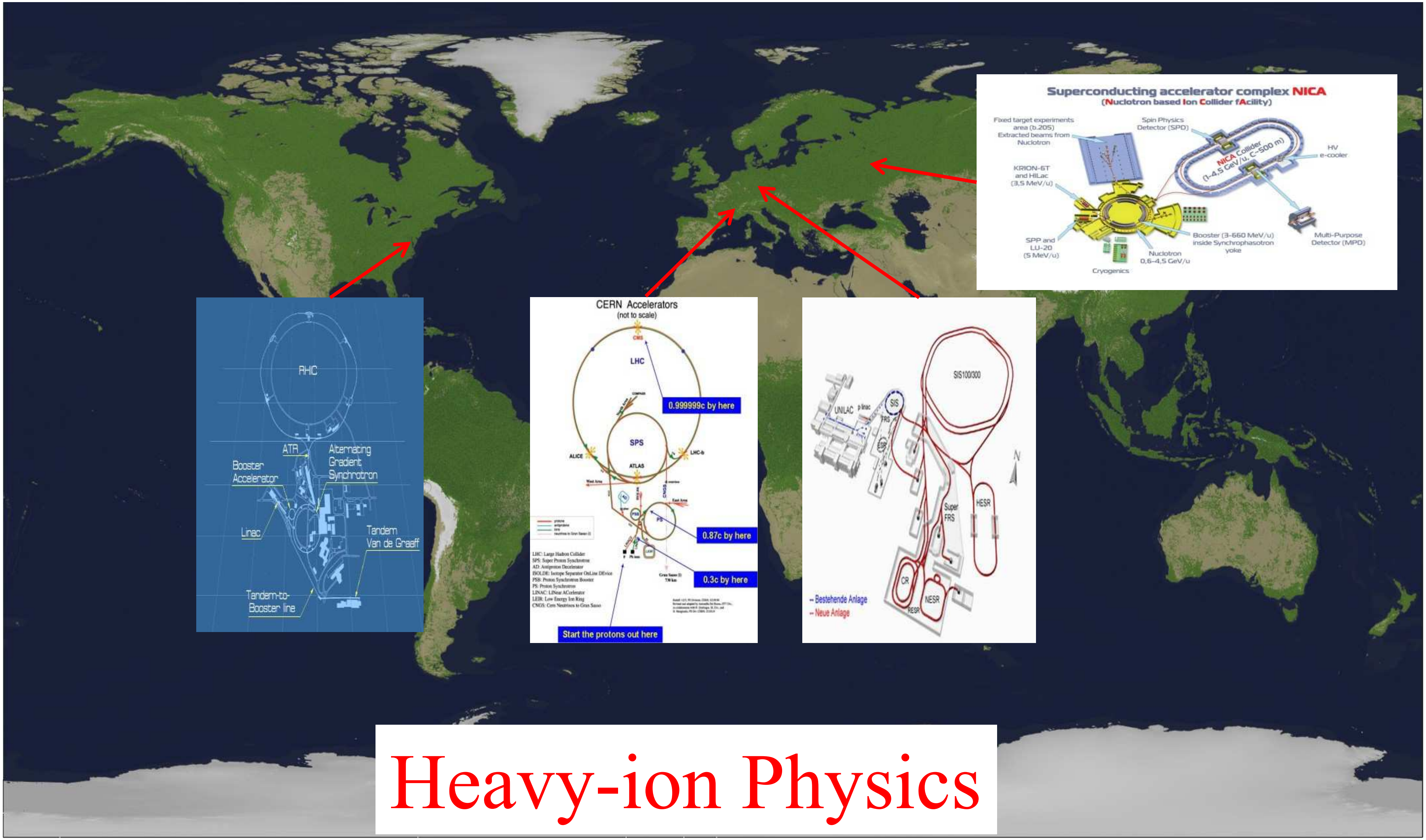
NICA: **N**uclotron based **I**on **C**ollider f**A**cility **C**ollider with $\sqrt{s_{NN}} = 3.5 - 11$ GeV (Begin 2015)

Program:

- Systems with highest baryon density
- Critical point
- Quarkyonic phase
- Chiral symmetry restoration

If the CEP is found, it would be the first clear indication for the chiral phase transition in the heavy-ion experiments

QCD phase structure



Heavy-ion Physics

Open questions:

Restoration of chiral symmetry (χ_S) and deconfinement:

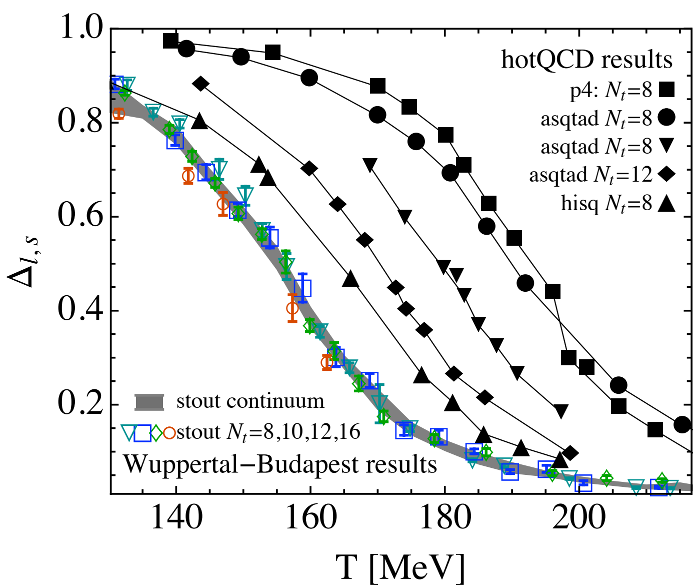
- connection between chiral symmetry and confinement?
- can both transitions occur simultaneously?
- $1^{st}-$ order chiral phase transition at high baryon density?
- where is the CEP?
- does an external magnetic field enhances the χ_S breaking? Magnetic catalysis. (The magnetic field has a strong tendency to enhance ("catalyze") spin-zero fermion-antifermion condensates);
- or, can the magnetic field suppress the quark condensate (inverse magnetic catalysis)?

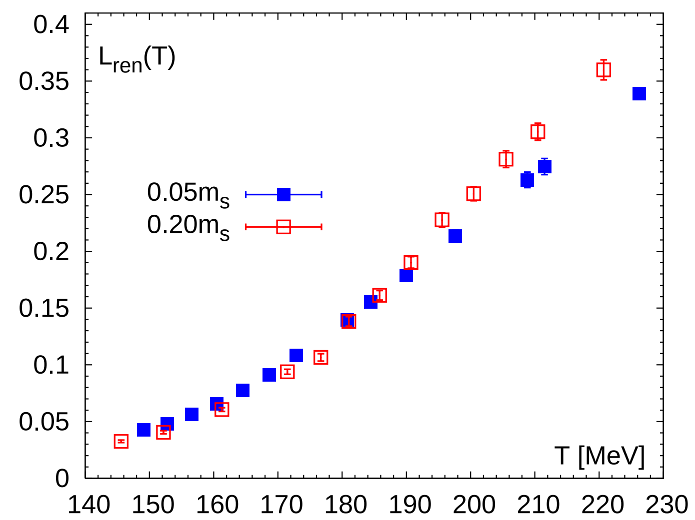
Lattice calculations at Finite Temperature:

Restoration of chiral symmetry and deconfinement:
both transitions can occur simultaneously?

- **Phase transition:**

$N_f = 2{+}1$:





$T_c^\chi = 154 \pm 9$ MeV $\qquad$ $T_c^\Phi = 170(4)(3)$ MeV

$\Delta_T \sim 16$ MeV

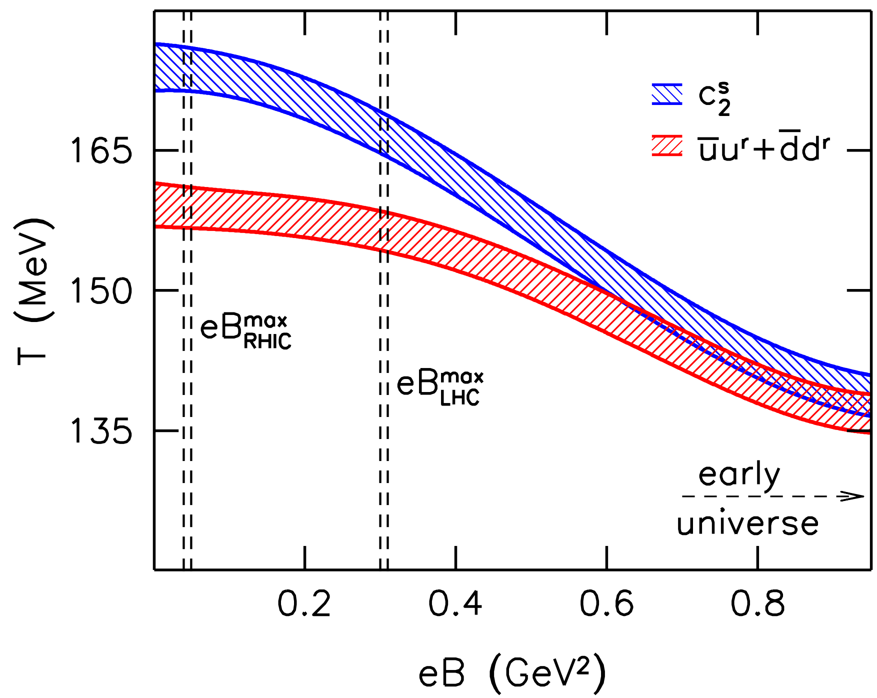
– A. Bazavov, et al., PRD 85 (2012) 054503

– Y. Aoki, et al., JHEP 0906 (2009) 088

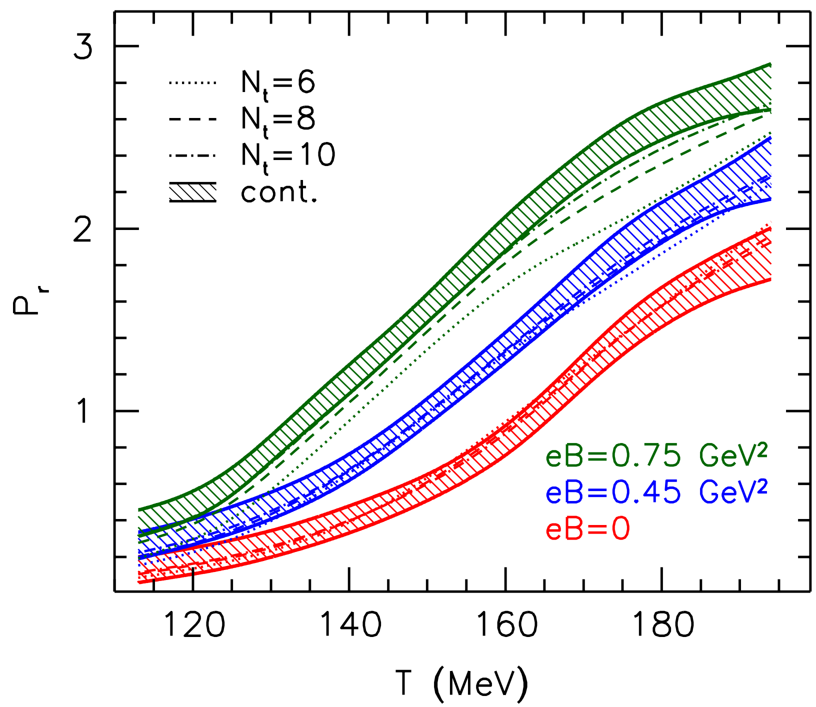
Lattice calculations at Finite Temperature:

Effect of an external magnetic field on the finite temperature transition of QCD

- **Phase transition deduced from peaks in susceptibilities**



- **Polyakov Loop**



– G.S. Bali, et al., JHEP 1202 (2012) 044 – F. Bruckmann, et al., JHEP 1304 (2013) 112

Polyakov loop extended Nambu–Jona-Lasinio model with strange quarks

Symmetries of QCD

- **QCD**
  - Chiral (χ_S) symmetry $\Leftrightarrow \begin{cases} - \text{ hadron masses} \\ - \text{ dynamics of hadrons at low energy} \end{cases}$
  - Center ($\mathbb{Z}_3$) symmetry $\Leftrightarrow$ de/confinement

$\Downarrow$

- **explicitly** broken (softly) by the presence of dynamical quarks

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- **QCD inspired models**
  - NJL model: only chiral symmetry aspects

    $\Downarrow$

    - chiral symmetry is **explicitly** and **spontaneously** broken
    - $U_A(1)$ symmetry breaking is implemented by ´t Hooft interaction
  - PNJL model: **synthesis** between chiral and de/confinement aspects

Model and formalism

PNJL model in the presence of an external magnetic field

$$\mathcal{L}_{PNJL} \quad = \quad \bar{q}\left(i\gamma_\mu D^\mu - \hat{m}\right)q + \frac{g_S}{2}\sum_{a=0}^{8}\left[\left(\bar{q}\lambda^a q\right)^2 + \left(\bar{q}(i\gamma_5)\lambda^a q\right)^2\right]$$

scalar-pseudoscalar four qua

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$$+ \; g_D\left[\det\left[\bar{q}(1+\gamma_5)q\right] + \det\left[\bar{q}(1-\gamma_5)q\right]\right] + \mathcal{U}\left(\Phi,\bar{\Phi};T\right) - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}$$

't Hooft determinant which breaks explicitly the axial UA(1)

effective potential fixed by comparison with pure-gauge Lattice QCD: mimic the gluonic degrees of freedom of QCD

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where $\hat{m} = \mathrm{diag}(m_u, m_d, m_s)$ is the current quark mass matrix

- $D^\mu = \partial^\mu - iq_f A^\mu_{EM} - igA^\mu; \quad A^\mu = \delta^\mu_0 A^0$ (Polyakov gauge)
- $F_{\mu\nu} = \partial_\mu A^{EM}_\nu - \partial_\nu A^{EM}_\mu$
  - $A^{EM}_\mu = \delta_{\mu 2} x_1 B$ static and constant magnetic field in the z direction

Coupling between Polyakov loop and quarks uniquely determined by covariant derivative D^μ

$$\Phi(\vec{x}) = \frac{1}{N_c}\mathrm{Tr}_c\left\langle\left\langle \mathcal{P}\exp\left[i\int_0^\beta d\tau A_4(\vec{x},\tau)\right]\right\rangle\right\rangle$$

– C. Ratti, et al., PRD 73 (2006) 014019

Polyakov loop extended NJL model

- The model includes features of both **chiral** and $\mathbb{Z}_3$ symmetry breaking
- The coupling is fundamental for reproducing lattice results concerning QCD thermodynamics: it originates a **suppression** of the **unconfined quarks** in the hadronic phase [1] (low temperature)

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- A non-zero **Polyakov loop** reflects the **spontaneously broken** $\mathbb{Z}_3$ symmetry characteristic of deconfinement (high temperature)
    - $\mathbb{Z}_3$ is **broken** in the **deconfined phase** ($\Phi \to 1$)
    - $\mathbb{Z}_3$ is **restored** in the **confined one** ($\Phi \to 0$)

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  - $\mathbb{Z}_3$ is **restored** in the **confined one** ($\Phi \to 0$)
- At $T = 0$: $\Phi = \bar{\Phi} = 0 \longmapsto$ both sectors decouple

1− C. Ratti et al., PRD 73 (2006) 014019

Effective potential $\mathcal{U}\left(\Phi,\bar{\Phi};T\right)$

- Effective potential for the (complex) Φ field: is conveniently chosen to reproduce results obtained in lattice calculations

$$\frac{\mathcal{U}\left(\Phi,\bar{\Phi};T\right)}{T^4}=-\frac{a\left(T\right)}{2}\bar{\Phi}\Phi+b(T)\ln[1-6\bar{\Phi}\Phi+4(\bar{\Phi}^3+\Phi^3)-3(\bar{\Phi}\Phi)^2]$$

with

$$a\left(T\right)=a_0+a_1\left(\frac{T_0}{T}\right)+a_2\left(\frac{T_0}{T}\right)^2,\quad b(T)=b_3\left(\frac{T_0}{T}\right)^3$$

a_0	a_1	a_2	b_3
3.51	-2.47	15.2	-1.75

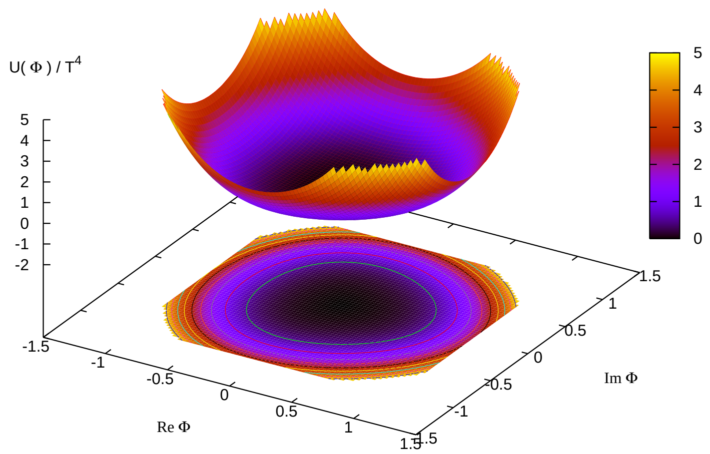
and $T_0=270$ MeV

– S. Roessner, C. Ratti, W. Weise, PRD 75 (2007) 034007

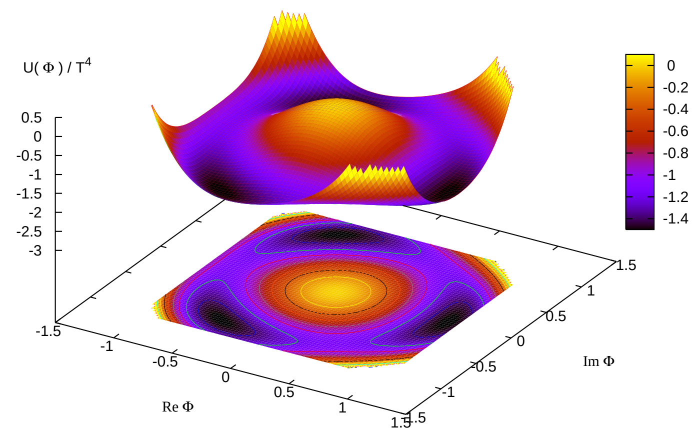
Model and formalism

Effective potential $U(\Phi, \bar{\Phi}, T)$:

$\Phi \to 0$: **confined phase**



$\Phi \to 1$: **deconfined phase**



- **low temperature**: $\mathbb{Z}_3$ symmetric, confined phase ($\mathbb{Z}_3$ center of $SU_c(3)$ symmetry)
- **high temperature**: deconfined phase characterized by the spontaneous breaking of the $\mathbb{Z}_3$ symmetry

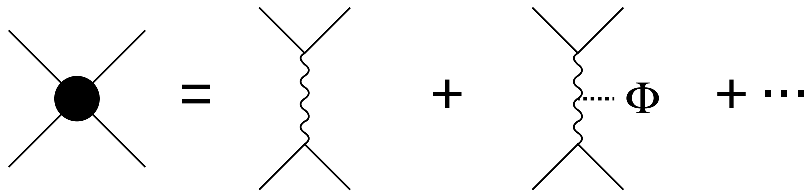
– H. Hansen, et al., PRD 75, (2007) 065004

Model and formalism

Entangled PNJL (EPNJL)

$g_S = const. \longmapsto g_S(\Phi)$

The diagrammatic description of the effective vertex $g_S(\Phi)$:



$$\boxed{g_S(\Phi,\bar{\Phi}) = g_S[1-\alpha_1\Phi\bar{\Phi} - \alpha_2(\Phi^3+\bar{\Phi}^3)]}$$

- entanglement interactions between $\langle \bar{q}_i q_i \rangle$ and Φ in addition to the covariant derivative in the original PNJL model;
- the entanglement vertex $g_S(\Phi)$ makes the correlation between the chiral restoration and the deconfinement transition stronger;

The parameter set $(\alpha_1 = 0.25, \alpha_2 = 0.10)$ must satisfy the triangle region $\{-1.5\alpha_1 + 0.3 < \alpha_2 < -0.86\alpha_1 + 0.32\alpha_2, \alpha_2 > 0\}$.

–Y. Sakai, et al., PRD 82 (2010) 076003

Methodology:

Minimization of $\Omega(T,\mu_f)$ with respect to M_f $(f = u, d, s)$

$\Downarrow$

"Gap" equations:

$$\boxed{M_f = m_f - 2\, g_S \langle \bar{q}_f\, q_f \rangle + 2\, g_D \langle \bar{q}_j\, q_j \rangle \langle \bar{q}_k\, q_k \rangle}$$

Effective action for the scalar and pseudoscalar mesons

$\Downarrow$

Meson propagators, $g_{M\bar{q}q}$, $f_{M\bar{q}q}$,...

PNJL *Pseudo-critical* temperatures

Pseudo-critical temperatures:

⇓

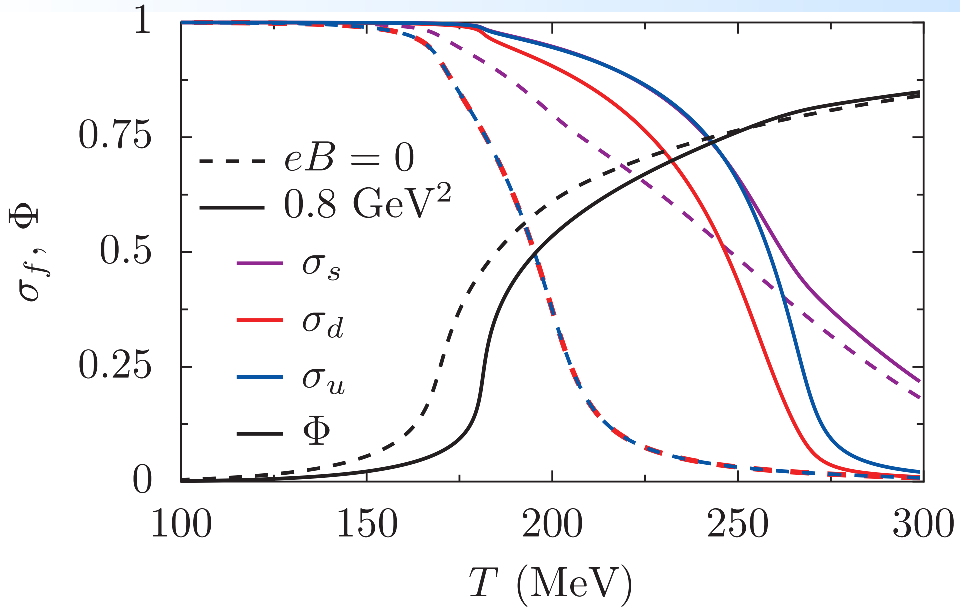
Separate the different phases in PNJL model

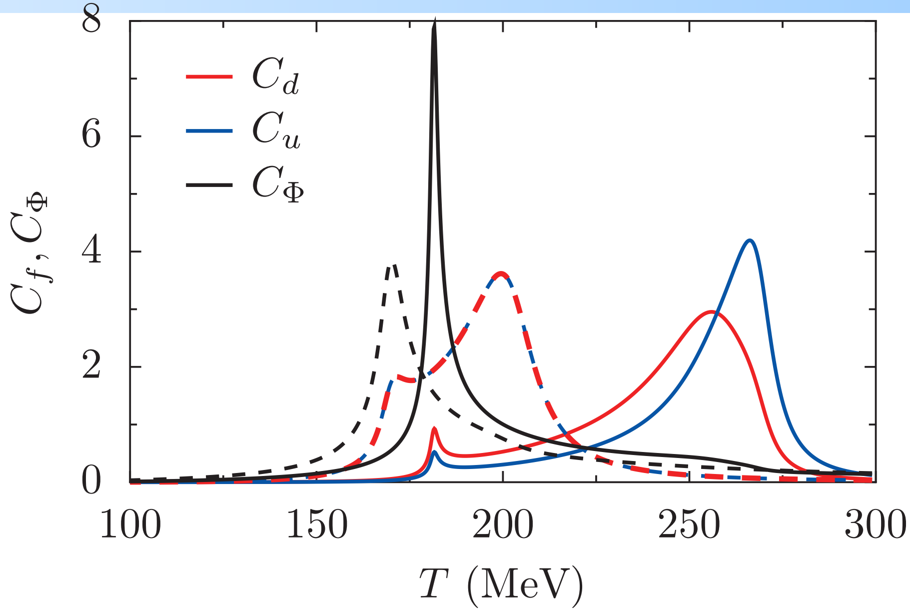
Criterion to identify ***partial*** and ***effective*** restoration of symmetries:

⇓

- Partial restoration of symmetries: $\frac{\partial^2 \langle \bar{u}u \rangle}{\partial T^2} = \frac{\partial^2 \langle \bar{s}s \rangle}{\partial T^2} = \frac{\partial^2 \Phi}{\partial T^2} = 0$
- Effective restoration of symmetries: the symmetry violating observables vanish (ex.: degenerescence of chiral partners)
- In the chiral limit both concepts coincide.

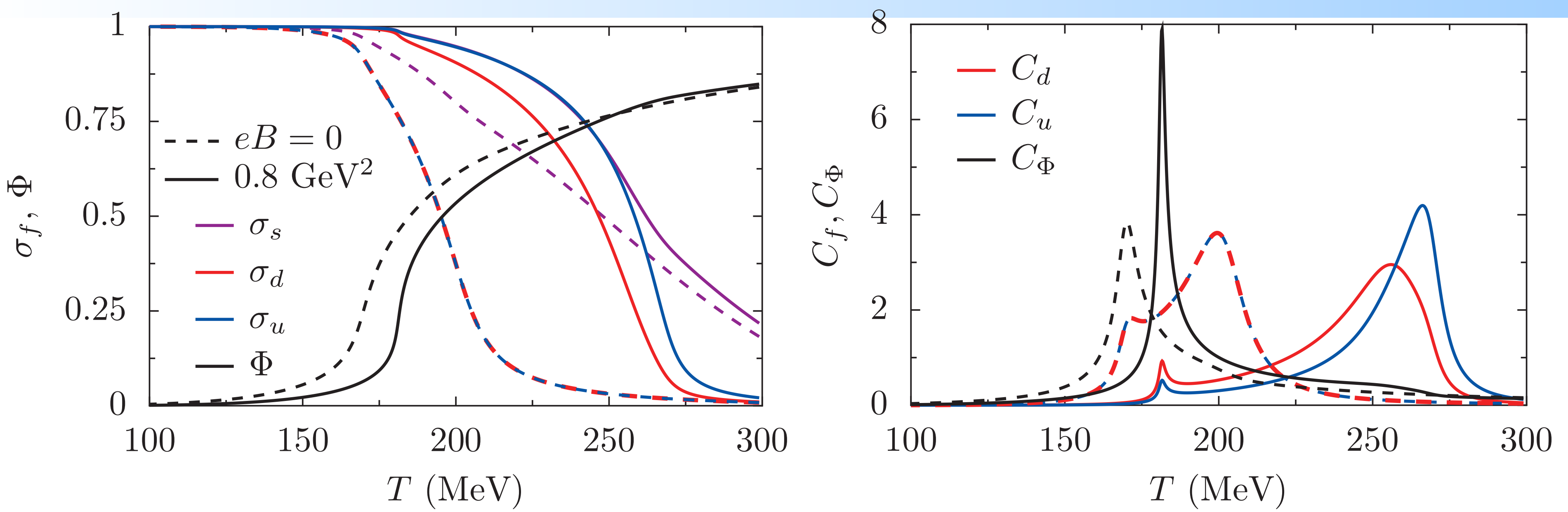
PNJL *Pseudo-critical* temperatures





$C_f = -m_\pi \partial\sigma_f/\partial T$ (where $\sigma_f = <\bar{q}_f q_f> (B,T)/ <\bar{q}_f q_f> (B,0)$); $C_\Phi = m_\pi \partial\Phi/\partial T$

PNJL *Pseudo-critical* temperatures



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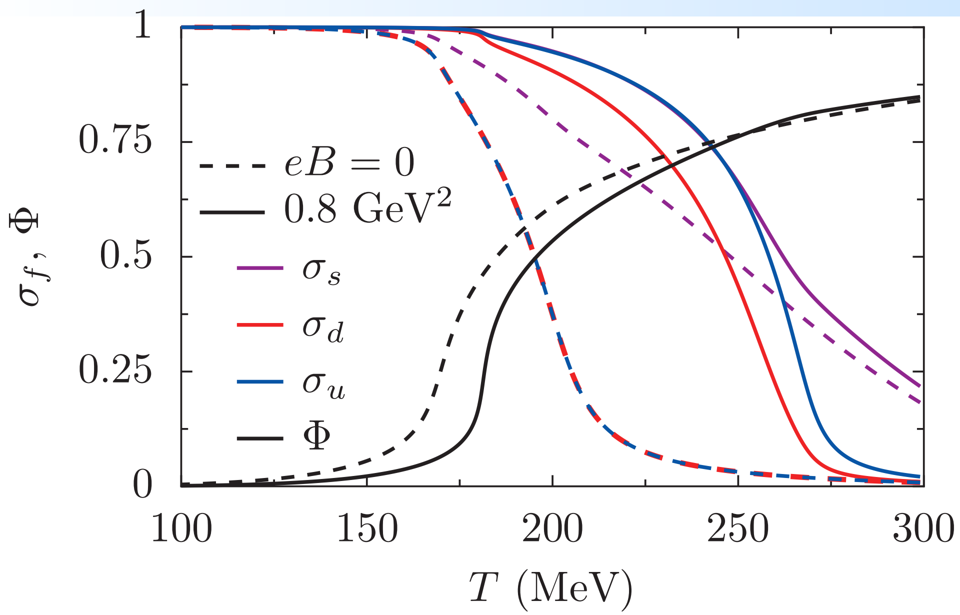
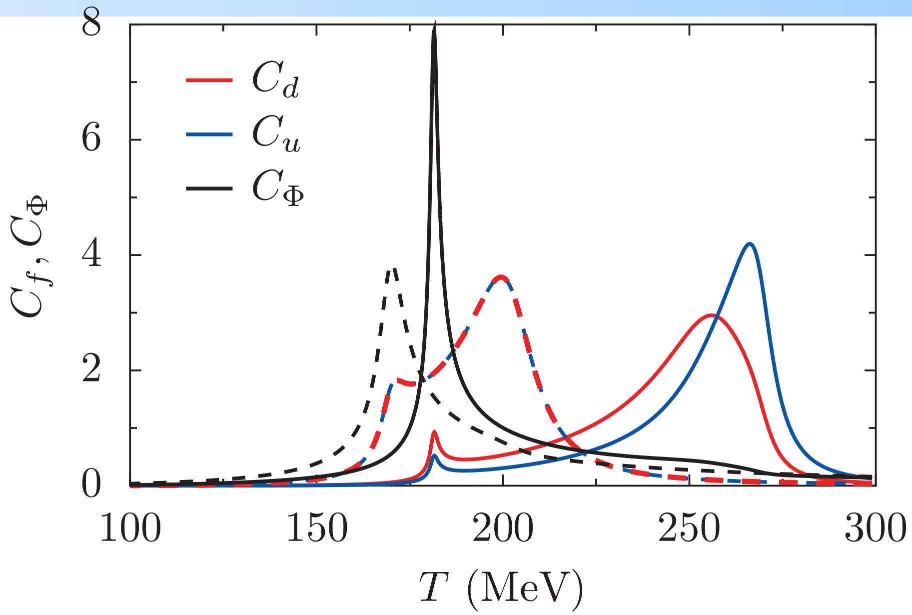
- Smooth **crossover** from the chirally broken to the chirally symmetric phase: ***partial* restoration of** χ_S
  - T^χ for u and d quark transitions become different as eB increases;
  - $q_u = 2e/3$; $q_d = -e/3 \longmapsto M_u$ becomes larger and the restoration of χ_S in the u sector is delayed:
    - T_u^χ is higher than T_d^χ

PNJL *Pseudo-critical* temperatures





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    - T^χ_u is higher than T^χ_d
- C_Φ becomes narrower as eB increases $\longmapsto$ eventually for sufficient strong eB a $1^{st}-$order phase transition takes place

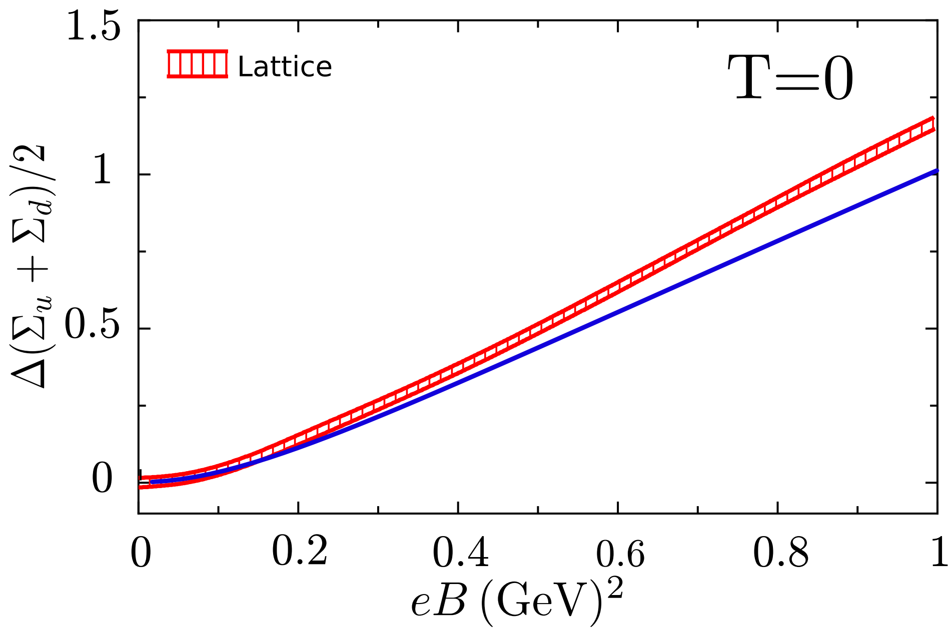
(E)PNJL *Pseudo-critical* temperatures

Pseudo-critical temperatures for the chiral transition $\left(T_c^\chi = (T_u^\chi + T_d^\chi)/2\right)$ and for the deconfinement (T_c^Φ) with $\mathbf{T_0 = 210}$ MeV.

	PNJL				EPNJL	
eB [GeV^2]	T_u^χ [MeV]	T_d^χ [MeV]	T_c^χ [MeV]	T_c^Φ [MeV]	T_c^χ [MeV]	T_c^Φ [MeV]
0	199	199	199	**170**	186	184
0.2	208	207	208	171	191	186
0.4	226	224	225	174	203	192
0.6	245	241	243	177	221	204
0.75	261	253	257	181	233	210
0.8	266	256	261	182	238	214
1	287	270	279	186	254	224

- as eB becomes stronger, the separation between T_c^χ and T_c^Φ increases;
- eB has a smaller impact in the location of the deconfinement crossover: T_c^Φ has just a weaker increase

(E)PNJL vs. lattice calculations



Change of the light condensate due to eB:

$$\Delta\Sigma_f(B,T) = \Sigma_f(B,T) - \Sigma_f(0,T),$$

with

$$\Sigma_f(B,T) = \frac{2m_f}{m_\pi^2 f_\pi^2}\left[\langle \bar{q}_f q_f \rangle (B,T) - \langle \bar{q}_f q_f \rangle (0,0)\right] + 1$$

- For $T = 0$ three models NJL, PNJL and EPNJL coincide;
- Results quantitatively agree with lattice** and even at $eB = 1$ GeV^2 there is a discrepancy of the order of $\sim 15\,\%$.

** G. S. Bali, et al., PRD 86 (2012) 071502

(E)PNJL vs. lattice calculations

Results obtained for PNJL (EPNJL) model together with the continuum extrapolated lattice results for the light condensates at $T = 0$ **

$T = 0$	$eB = 0$		$eB = 0.2$ GeV^2		$eB = 0.4$ GeV^2	
	$+/2$	$-$	$+/2$	$-$	$+/2$	$-$
(E)PNJL	1	0	1.11	0.08	1.32	0.23
Latt. **	1	0	1.14(2)	0.09(2)	1.37(2)	0.28(2)

$T = 0$	$eB = 0.6$ GeV^2		$eB = 0.8$ GeV^2		$eB = 1.0$ GeV^2	
	$+/2$	$-$	$+/2$	$-$	$+/2$	$-$
(E)PNJL	1.55	0.40	1.79	0.58	2.02	0.76
Latt. **	1.63(3)	0.47(3)	1.90(3)	0.67(3)	2.16(3)	0.87(3)

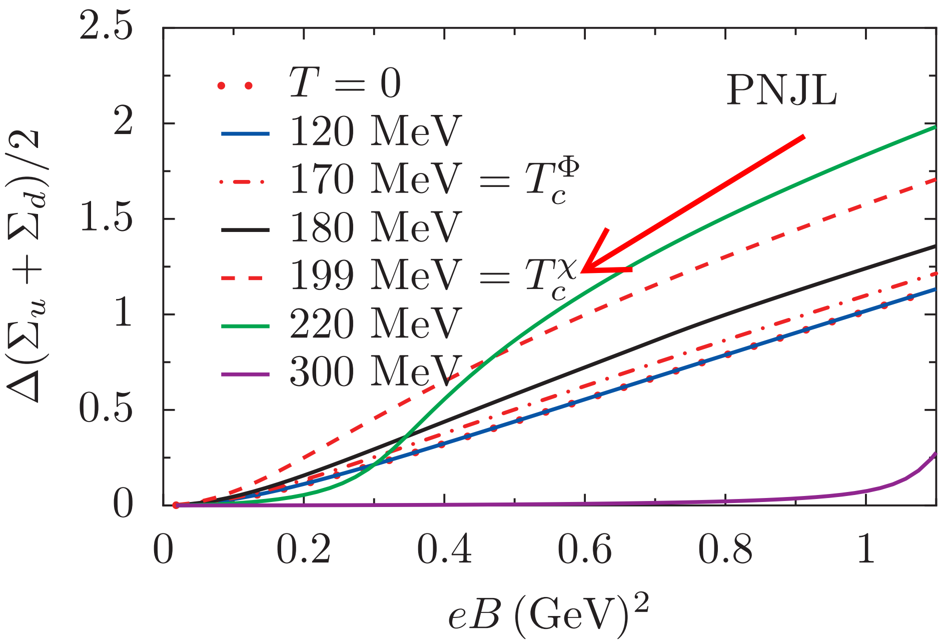
"$+/2$": average of the light condensates $((\Sigma_u + \Sigma_d)/2)$;
"$-$": difference of the light condensates $(\Sigma_u - \Sigma_d)$.

- The average of the light condensates is in good agreement with lattice results, specially at low magnetic fields;
- $eB = 1$ GeV^2: the average of the light condensates do not differ more than $\sim 10\%$.

** G. S. Bali, et al., PRD 86 (2012) 071502

PNJL magnetic catalysis

Change of the renormalized condensates as a function of eB for several temperatures:



$T < T_c^\chi(eB = 0)$:

- condensates average increase with eB

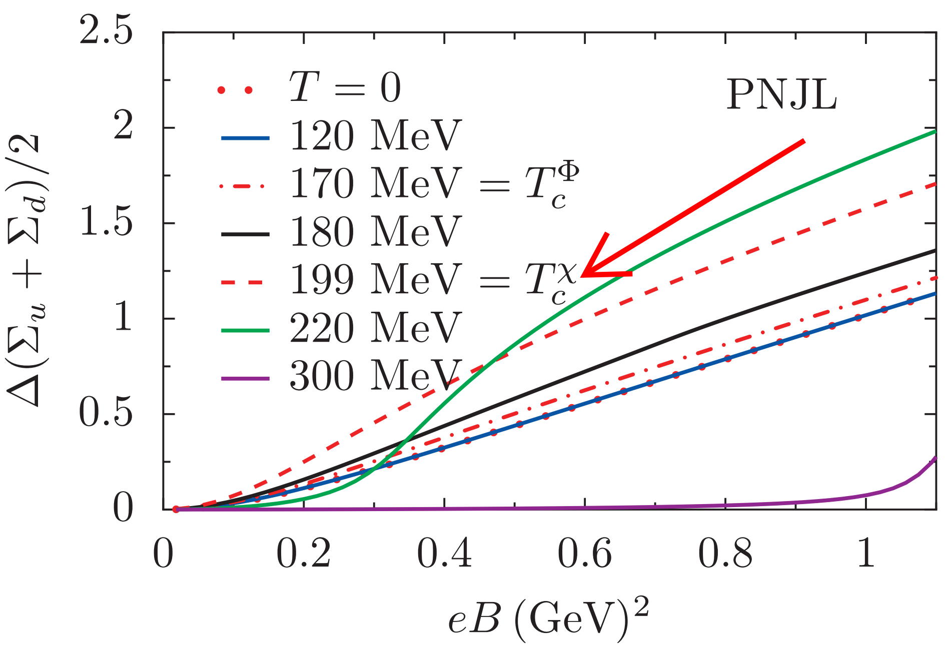
⬇

its value is greater the higher the temperature

(due to the magnetic catalysis effect)

PNJL magnetic catalysis

Change of the renormalized condensates as a function of eB for several temperatures:



$T < T_c^{\chi}(eB = 0)$:

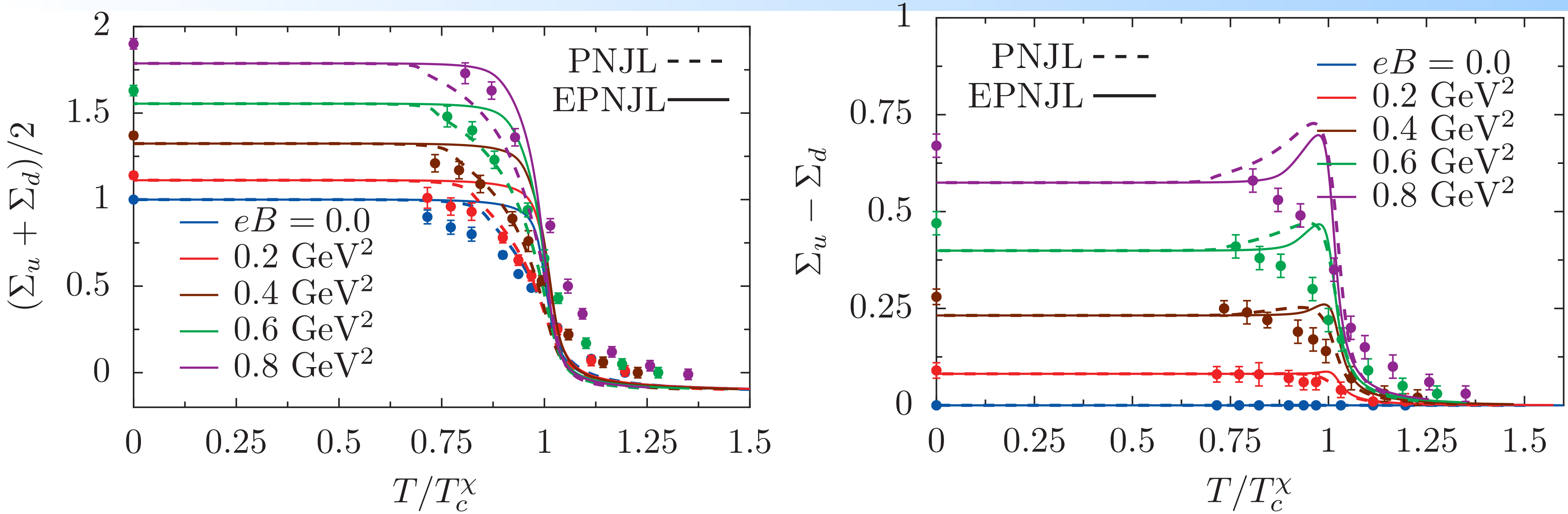
- condensates average increase with eB

  $\Downarrow$

  its value is greater the higher the temperature
  (due to the magnetic catalysis effect)

- $T > T_c^{\chi}(eB = 0)$:
  - Two competitive effects: partial restoration of χ_S and magnetic catalysis.
    - Partial restoration of χ_S prevails at lower values of eB:
      the change of the renormalized condensates is approximately zero;
    - The magnetic catalysis becomes dominant as eB increases:
      the change of the renormalized condensates condensate becomes nonzero.

PNJL vs. lattice calculations



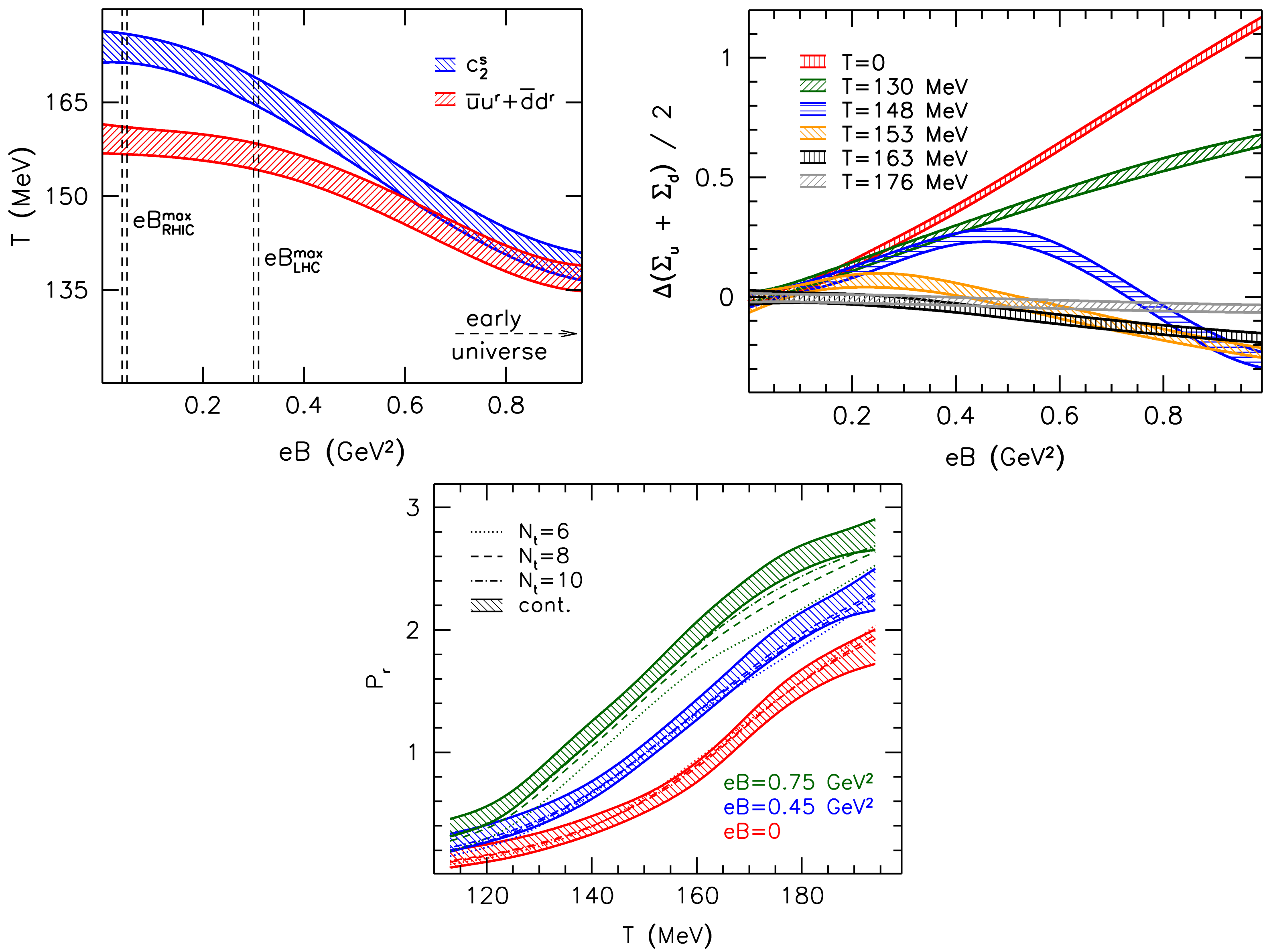
- (E)PNJL have the same behavior as the lattice results** except for a too fast drop above the respective transition temperatures.
- stronger magnetic catalysis for the u quark, due to its larger electric charge
  - higher $eB \longmapsto$ larger difference between u and d condensates;
  - higher $eB \longmapsto$ larger difference between T_c^u and T_c^d;
  - After T_c^χ: partial restoration of χ_S prevails over the magnetic catalysis

⇓

smaller $\Sigma_u - \Sigma_d$

** G. S. Bali, et al., PRD 86 (2012) 071502

Lattice calculations



– G.S. Bali, et al., JHEP 1202 (2012) 044

– F. Bruckmann, et al., JHEP 1304 (2013) 112

EPNJL *Inverse* magnetic catalysis

$T_0(eB)$ allows to describe the back reaction on the Polyakov loop due to eB

$$\boxed{T_0(eB) = T_0(eB=0) - 282.5(eB)^2 + 38.1(eB)^4, \quad eB < 0.75 \text{ GeV}^2}$$

- $T_0(eB=0) = 270$ MeV
- Pseudo-critical temperatures in EPNJL model with $T_0(eB)$:
  - Reproduces the shifted $T_c^{\chi}(eB)$ for eB up to 0.75 GeV^2.

eB [GeV^2]	0	0.2	0.4	0.6	0.75
T_c^{χ} [MeV]	215	214	211	205	200
T_c^{Φ} [MeV]	214	211	200	198	200

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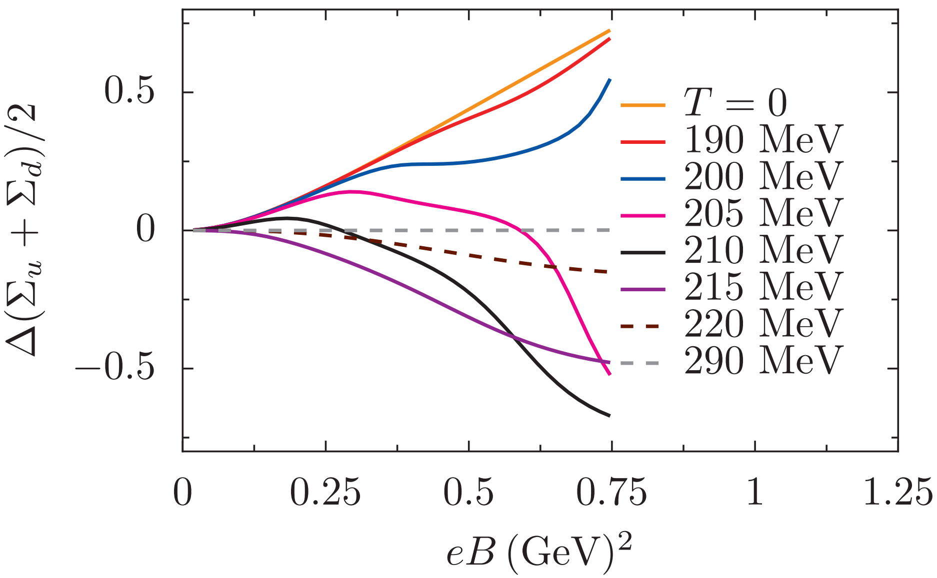

- For $200 < T < 215$ MeV: strong interplay between partial restoration of χ_S and magnetic catalysis
  - smaller $eB \longmapsto$ magnetic catalysis is dominant;
  - higher $eB \longmapsto$ restoration of χ_S is dominant;

EPNJL *Inverse* magnetic catalysis

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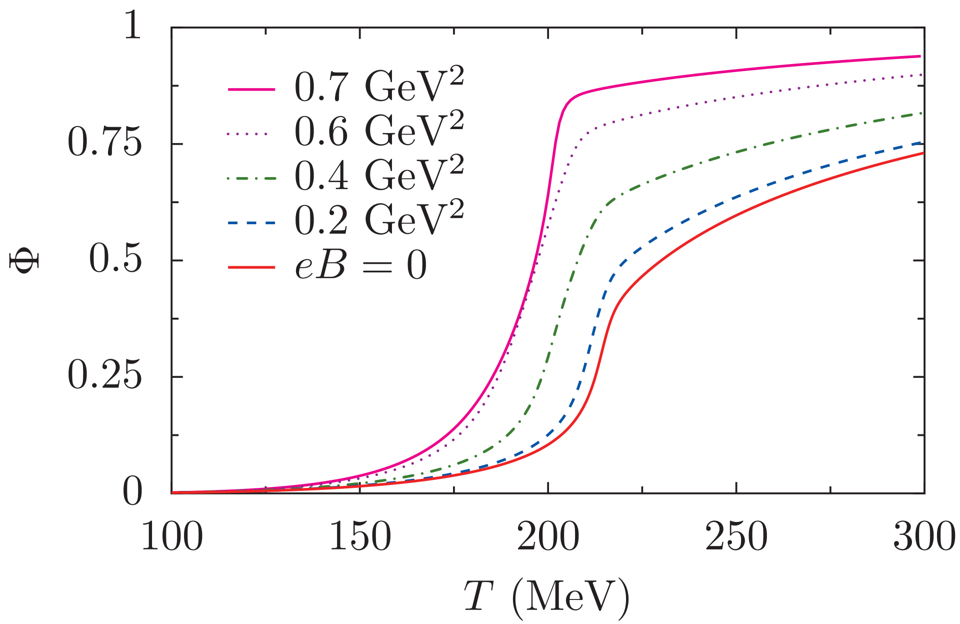
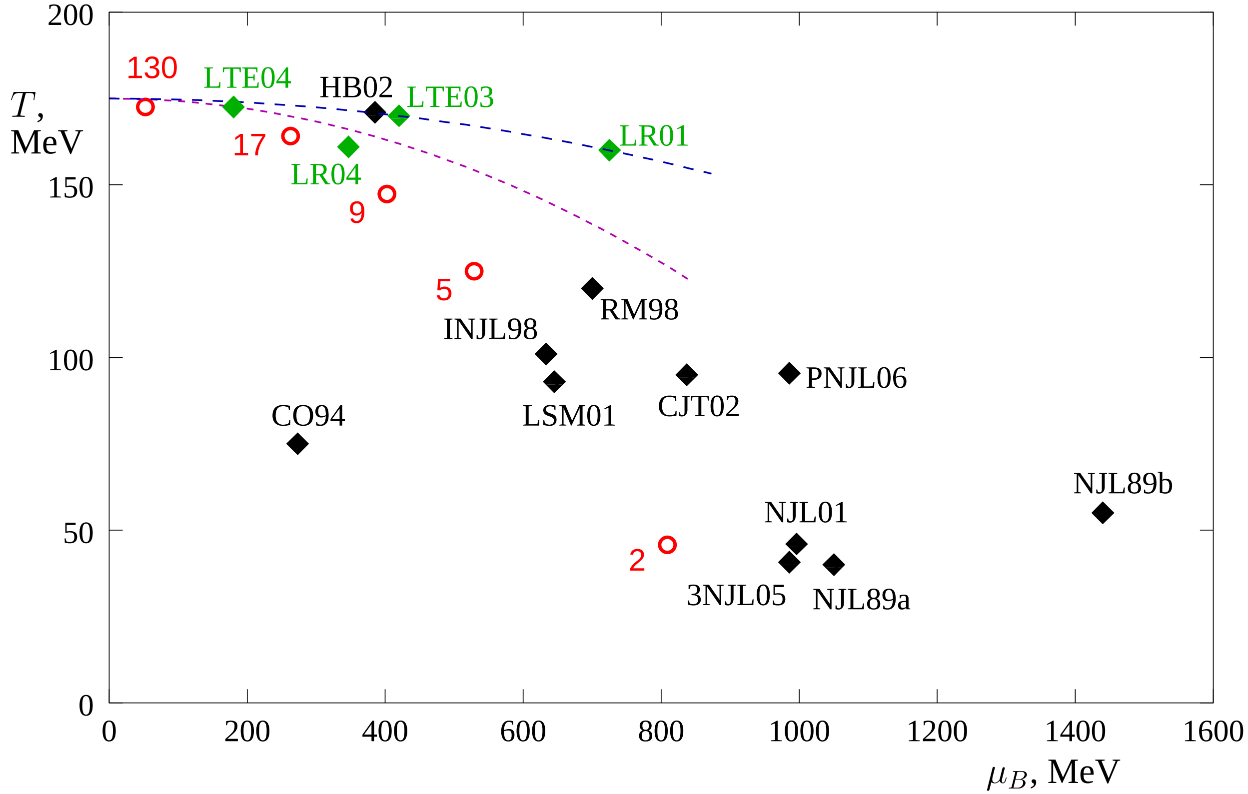


- The Polyakov Loop follows the same tendency predicted by lattice calculations[1]: T_c^{Φ} decreases as eB increases;
- For eB above 0.75 GeV^2 $T_0(eB)$ becomes too small and the deconfinement transition becomes of $1^{st}-$ order.

[1] F. Bruckmann, et al., JHEP 1304 (2013) 112

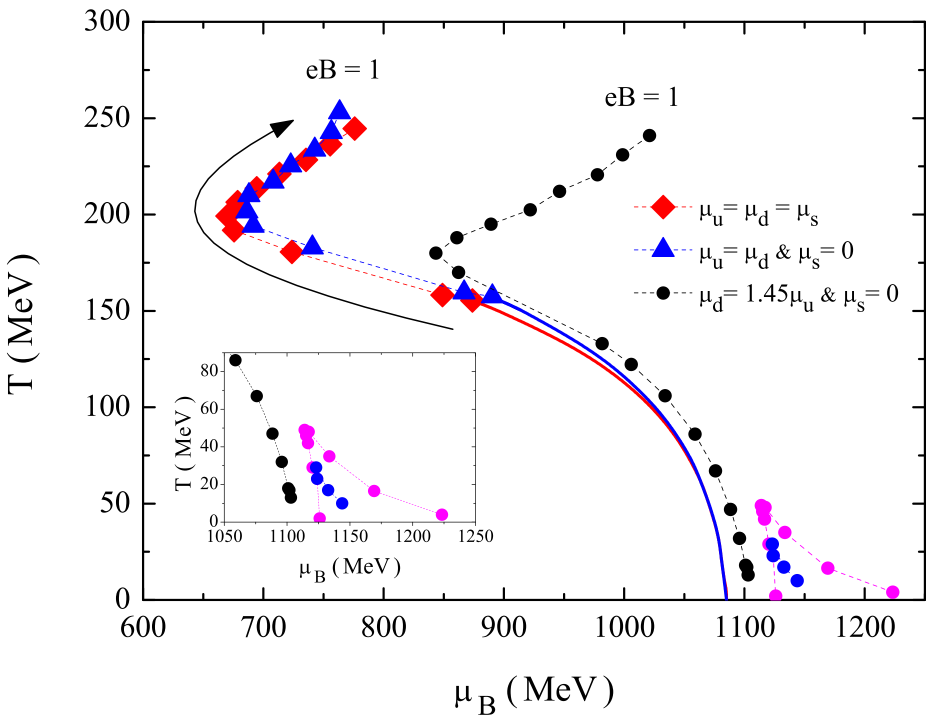
Where is the CEP?



Predictions for the location of the QCD critical point on the phase diagram

– M. Stephanov, PoS(LAT2006)024

PNJL phase diagram



- The trend is very similar for different scenarios:
  - as the intensity of the magnetic field increases, T^{CEP} increases and μ_B^{CEP} decreases until $eB \sim 0.3$ GeV^2;
  - for stronger magnetic fields both T^{CEP} and μ_B^{CEP} increase;
- Phase transition driven by the magnetic field will occur (for $\mu_d > 1.45\mu_u$):
  - possible appearance of multiple CEPs for for sufficiently small values of eB and T.

Summary

- In the presence of an external magnetic field at $T = 0$, the quantitative behavior of SU(3) PNJL and EPNJL is closer to the lattice results;

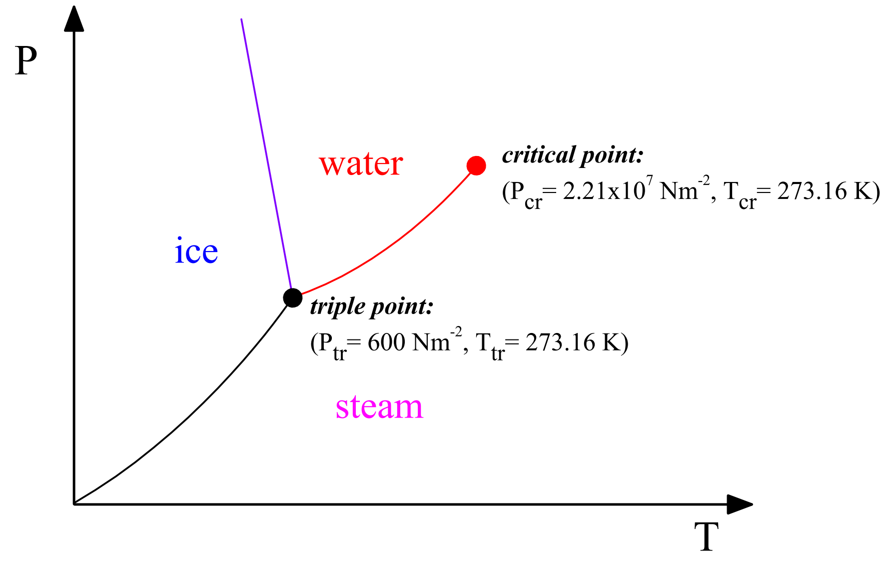
- Chiral and deconfinement transition temperatures increase in the presence of an external magnetic field, although the deconfinement transition temperature suffers a much weaker effect;
- In the EPNJL model the magnetic field destroys the coincidence of the deconfinement and chiral transition temperatures at $eB = 0$;

- In the EPNJL model, a magnetic field dependent parametrization of the Polyakov loop can invert the magnetic catalysis and the pseudo-critical temperatures decreases with the increase of the magnetic field strength, reproducing lattice calculations.

Appendix: H_2O phase diagram

- **Thermodynamical information can be presented in the form of a phase diagram: the different manifestations or phases of a substance occupy different regions**

- **Familiar example: H_2O P.D.**



(no to scale)

Appendix: H_2O phase diagram

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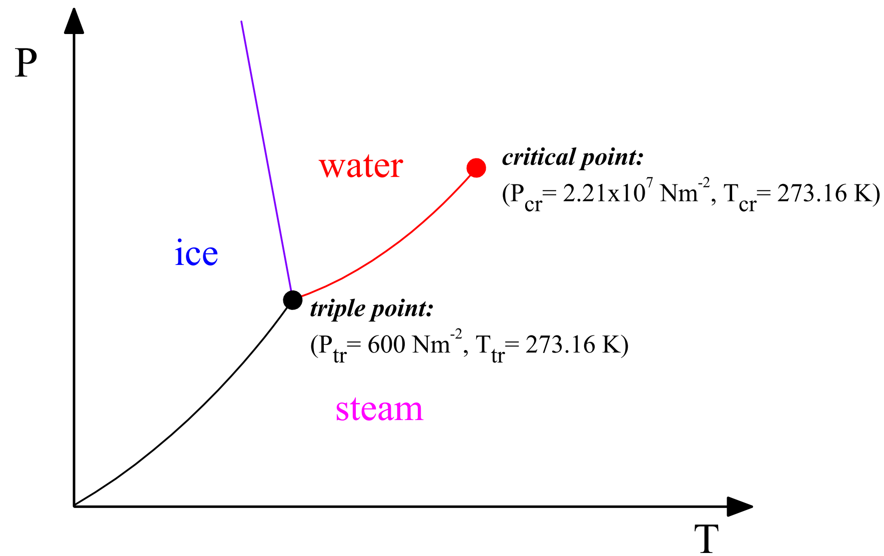


(no to scale)

– The lines mark the various coexistence curves $P(T)$

⇓

two phases are in equilibrium

– moving along a path in the (T, P) plane which intersects the curves

⇓

phase transition (e.g.: melting or boiling)

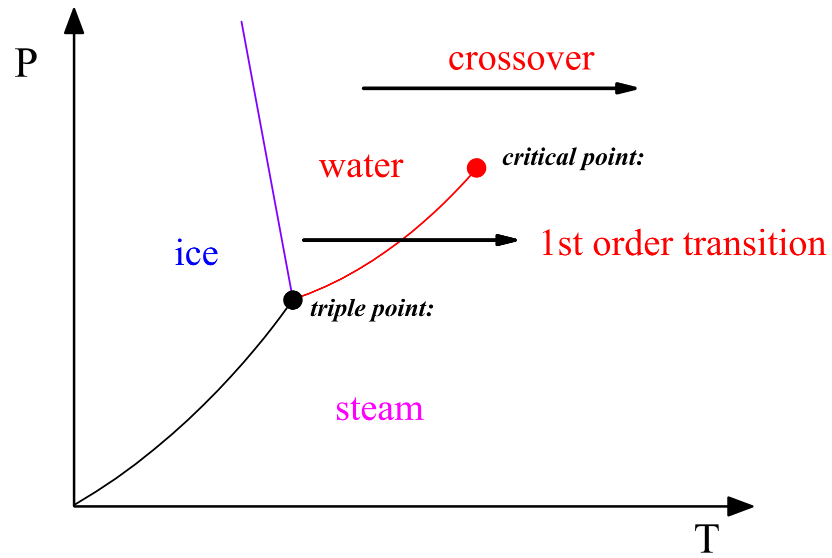
– Two special points in the P.D.

- **triple point** (where all three phases coexist)
- **critical point** (where the liquid–vapour separation disappears)

Appendix: H_2O phase diagram

- **Thermodynamical information can be presented in the form of a phase diagram: the different manifestations or phases of a substance occupy different regions**

- **Familiar example: H_2O P.D.**



(no to scale)

— **transition between liquid and vapour:**

- $T < T_c$: **first-order**

$\Downarrow$

discontinuities in the entropy ΔS and in the volume ΔV

- *critical point:* **second order**

$\Downarrow$

singularities in the specific heat C_P

- *beyond the critical point:* **crossover region**

$\Downarrow$

thermodynamic observables still vary very rapidly

Quantum Chromodynamics – QCD

- **Theory of strong interactions**
- **Describes the structure of hadrons and (ultimately) nuclei**
- **Elementary degrees of freedom are quarks and gluons**
  - Carry new charge: Color charge
  - Quarks come in red, green, blue, and anti-colors
  - Interactions mediated by gluons
  - Gluons are also charged

Quark flavor	Mass[1]	Bound state [MeV]	Spin	Charge [Q/e]
Up	$2.3^{+0.7}_{-0.5}$ MeV	~ 300	$\frac{1}{2}$	2/3
Down	$4.8^{+0.7}_{-0.3}$ MeV	~ 300	$\frac{1}{2}$	-1/3
Strange	95 ± 5 MeV	~ 450	$\frac{1}{2}$	-1/3
Charm	1275 ± 0.025 MeV	~ 1800	$\frac{1}{2}$	2/3
Bottom	4.18 ± 0.03 GeV	~ 4800	$\frac{1}{2}$	-1/3
Top	173 ± 0.6 GeV	No known bound states	$\frac{1}{2}$	2/3
Gluon	0	Not applicable?	1	0

[1] Particle Data Group, PRD 86 (2012) 010001

Appendix: QCD

QCD Lagrangian

$$\mathcal{L}_{QCD} = \bar{\psi}\,(\,i\gamma^\mu D_\mu - \hat{m}\,)\,\psi - \frac{1}{4}F^a_{\mu\nu}\,F_a^{\mu\nu}$$

- ψ; $\bar{\psi}$: quarks with six flavors (u, d, s, c, b, t) and three colors $(N_c = 3)$;
- $\hat{m}$: current quark mass matrix $(\hat{m} = \mathrm{diag}_f(m_u, m_d, \ldots, m_t))$;
- $D_\mu = \partial_\mu - ig\frac{\lambda^a}{2}A^a_\mu$: covariant derivative;
  - A^a_μ $(a = 1, 2, ..., 8)$: gluon field carrying color
- $F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + g_S\, f^{abc}\, A^b_\mu\, A^c_\nu$: Gluonic field strength tensor;
  - λ^a: 8 generators of the $\mathrm{SU}_c(3)$ group (Gell-Mann matrices)
  - g_S: strong coupling constant

Appendix: Symmetries of QCD

Symmetry structure of QCD:

- (approximate) global symmetry $\mathrm{SU}_L(\mathrm{N}_f)\times\mathrm{SU}_R(\mathrm{N}_f)$, which is spontaneously broken to $\mathrm{SU}_V(\mathrm{N}_f)$

$\mathrm{N}_f = 3$:

Symmetry	Transformation	Current	Name	Manifestation in nature
$\mathrm{SU}_V(3)$	$\psi \to \exp(i\frac{\lambda_a\alpha_a}{2})\psi$	$V_\mu^a = \bar{\psi}\gamma_\mu\frac{\lambda_a}{2}\psi$	isospin	approximately conserved
$\mathrm{U}_V(1)$	$\psi \to \exp(i\alpha_V)\psi$	$V_\mu = \bar{\psi}\gamma_\mu\psi$	baryonic	always conserved
$\mathrm{SU}_A(3)$	$\psi \to \exp(i\frac{\gamma_5\lambda_a\theta_a}{2})\psi$	$A_\mu^a = \bar{\psi}\gamma_\mu\gamma_5\frac{\lambda_a}{2}\psi$	**chiral**	**spontaneously broken**
$\mathrm{U}_A(1)$	$\psi \to \exp(i\gamma_5\alpha_A)\psi$	$A_\mu = \bar{\psi}\gamma_\mu\gamma_5\psi$	Axial	“$\mathrm{U}_A(1)$ problem”

Appendix: Model and formalism

PNJL model at finite T, μ in an external magnetic field B

The thermodynamic potential is:

$$
\begin{aligned}
\Omega(T,\mu_f) &= \mathcal{U}(\Phi,\bar{\Phi},T) - N_c \sum_{f=u,d,s} \frac{|q_f|eB}{2\pi} \sum_{n=0}^{\infty} \alpha_n \int_{-\infty}^{\infty} \frac{dp_z}{2\pi} \Bigg(E_f \\
&+ \frac{T}{3} \ln\left\{1 + 3\bar{\Phi} e^{-(E_f-\mu_f)/T} + 3\Phi e^{-2(E_f-\mu_f)/T} + e^{-3(E_f-\mu_f)/T}\right\} \\
&+ \frac{T}{3} \ln\left\{1 + 3\Phi e^{-(E_f+\mu_f)/T} + 3\bar{\Phi} e^{-2(E_f+\mu_f)/T} + e^{-3(E_f+\mu_f)/T}\right\} \Bigg) \\
&+ g_S \sum_{\{f=u,d,s\}} \left\langle \bar{q}_f q_f \right\rangle^2 - 4 g_D \left\langle \bar{q}_u q_u \right\rangle \left\langle \bar{q}_d q_d \right\rangle \left\langle \bar{q}_s q_s \right\rangle
\end{aligned}
$$

with $E_f = \sqrt{2n|q_f|eB + p_z^2 + M_f^2}$

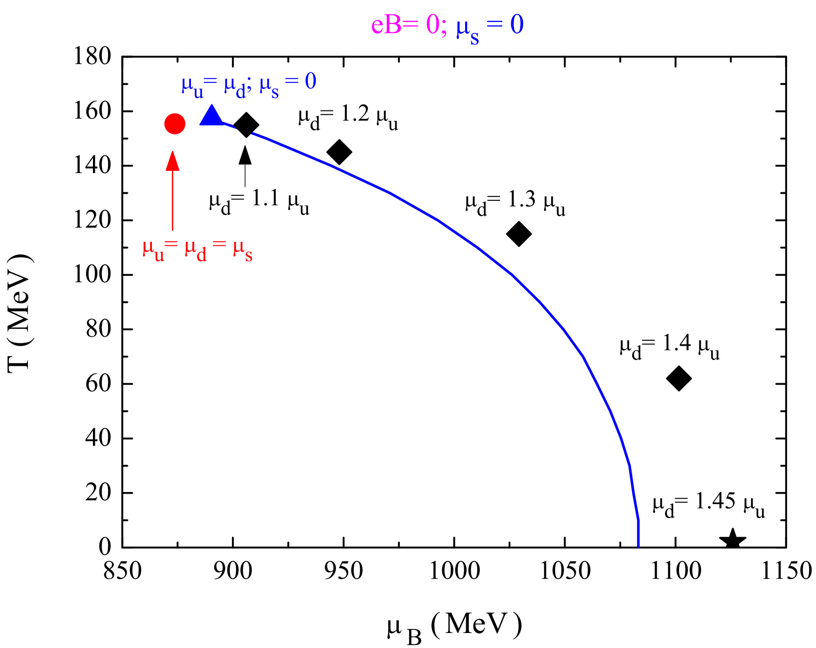
Model and formalism

Parameters and results:

Physical quantities	Parameter set and constituent quark masses
$f_\pi = 92.4$ MeV	$m_u = m_d = 5.5$ MeV
$M_\pi = 135.0$ MeV	$m_s = 140.7$ MeV
$M_K = 497.7$ MeV	$\Lambda = 602.3$ MeV
$M_{\eta'} = 960.8$ MeV	$g_S \Lambda^2 = 3.67$
$M_\eta = 514.8$ MeV*	$g_D \Lambda^5 = 12.36$
$f_K = 97.7$ MeV*	$M_u = M_d = 367.7$ MeV*
$M_\sigma = 728.8$ MeV*	$M_s = 549.5$ MeV*
$M_{a_0} = 873.3$ MeV*	
$M_\kappa = 1045.4$ MeV*	
$M_{f_0} = 1194.3$ MeV*	
$\theta_P = -5.8^{o\,*}$; $\theta_S = 16^{o\,*}$	

– S.P. Klevansky et al., PRC 53 (1996) 410
– P. Costa, et al., PRD 71 (2005) 116002

PNJL phase diagram



- symmetric matter: $\mu_u = \mu_d = \mu_s$;
- isopsin symmetric matter: $\mu_u = \mu_d$ and $\mu_s = 0$;
- presently, the isospin asymmetry of matter in HIC corresponds to $\mu_d < 1.1\mu_u$;
- neutron matter corresponds to $\mu_d \sim 1.2\mu_u$;
- increasing the isopsin asymmetry moves the CEP to smaller T^{CEP} and larger μ_B^{CEP}. For an asymmetry large enough ($\mu_d > 1.45\mu_u$) the CEP disappears.