Gluões escaldados numa folha de alface

Como se propagam os gluões num meio quente e muito pouco denso?

Paulo Silva, Orlando Oliveira
Center for Physics, University of Coimbra

David Dudal
KU Leuven, campus Kortrijk

Pedro Bicudo, Nuno Cardoso
CFTP, IST, University of Lisbon

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Outline

1. Introduction and Motivation

2. Gluon propagator @ finite T
   - Positivity violation and spectral densities
   - Gluon mass scales
   - Z_3 dependence

3. Conclusions and Outlook

Cromodinâmica Quântica

- Teoria das interacções fortes
- Interacção entre quarks mediada por gluões
- Gluões possuem cor $\Rightarrow$ teoria não-linear
- Dificuldades no limite de baixas energias
  - regime não perturbativo
  - confinamento, simetria quiral
- Lagrangeano

$$\mathcal{L}_{QCD} = -\frac{1}{4} F^a_{\mu\nu} F^{a\mu\nu} + \bar{q}^{af} (i\gamma^\mu D^{ab}_\mu - m_f \delta^{ab}) q^{bf}$$

Funções de Green

- valores expectáveis no vácuo de produtos de operadores de campo ordenados no tempo

$$\mathcal{G}^{(n)}_{j_1\cdots j_n}(x_1,\cdots,x_n) = \langle 0|\mathcal{T}(\hat{\phi}_{j_1}(x_1)\cdots\hat{\phi}_{j_n}(x_n))|0\rangle$$

- Formulação dos integrais de caminho
  - Funcional gerador

$$Z[J] = \int\prod_k \mathcal{D}\phi_k \exp\left(i\int d^4x(\mathcal{L}[\phi] + J_k(x)\phi_k(x))\right)$$

  - Funções de Green: derivação funcional

$$\mathcal{G}^{(n)}_{j_1\cdots j_n}(x_1,\cdots,x_n) = \left.\frac{\delta^n Z[J]}{i\delta J_1(x_1)\cdots i\delta J_n(x_n)}\right|_{J_1,\cdots,J_n=0}$$

$$= \int\prod_k \mathcal{D}\phi_k(\phi_{j_1}(x_1)\quad\cdots\quad\phi_{j_n}(x_n))\exp(iS[\phi])$$

Cromodinâmica Quântica na rede

- Formulação que permite:
  - estudar regime não perturbativo
  - simular a QCD num computador
- Ingredientes fundamentais:
  - rede finita de pontos (espaçamento *a*)
    $\Rightarrow$ integrais funcionais com dimensão finita
  - formalismo do tempo imaginário $t = -ix_4$
    Teorias de Campo $\equiv$ Mecânica Estatística 4D

$$\int \mathcal{D}\phi \, e^{iS[\phi]} \to \int \mathcal{D}\phi \, e^{-S^{(E)}[\phi]}$$

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Campos de gauge na rede

- Links
  - matrizes SU(3) situadas nas linhas orientadas que unem os pontos da rede
  - campos de gauge fundamentais em teorias de gauge na rede.

$$U_\mu(x) = e^{iag_0 A_\mu(x+a\hat{e}_\mu/2)}$$

- Transformação de gauge nas links

$$U_\mu(x) \to g(x) U_\mu(x) g^\dagger(x+\hat{\mu}).$$

- Acção de gauge na rede

$$S_W = \beta \sum_{x,\mu>\nu} \left(1 - \frac{1}{N} \mathrm{ReTr}[\Box_{\mu\nu}(x)]\right) \xrightarrow[a\to 0]{} \int d^4x \tfrac{1}{4} \sum_{a,\mu,\nu} (F^a_{\mu\nu})^2 + \mathcal{O}(a^2)$$

- Plaquette: $\Box_{\mu\nu}(x) = U_\mu(x) U_\nu(x+\hat{\mu}) U^\dagger_\mu(x+\hat{\nu}) U^\dagger_\nu(x)$

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Cálculo de valores expectáveis na rede

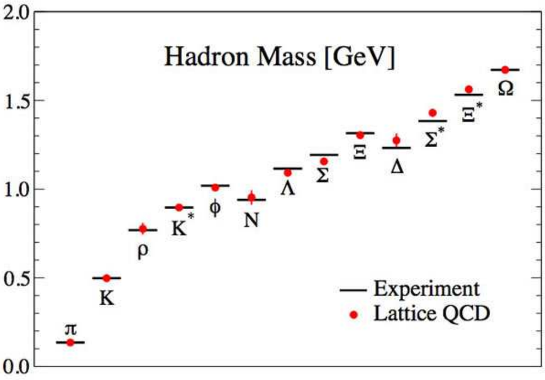
$$\langle A \rangle = \frac{\int \mathcal{D}U A[U] \exp(-S_W[U])}{\int \mathcal{D}U \exp(-S_W[U])}$$

- Integral de dimensão elevada $\rightarrow$ método Monte Carlo
- Estimativa na rede:
  - conjunto de configurações $\{U^{(i)}\}_{i=1}^{N}$
  - densidade de probabilidade $P(U) = \exp(-S_W[U])$

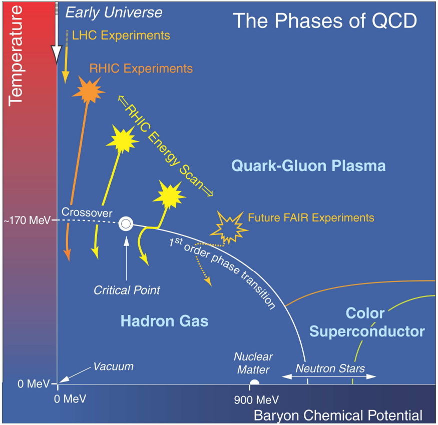
$$\langle A \rangle = \frac{1}{N} \sum_{i=1}^{N} A(U^{(i)}) + \mathcal{O}(\frac{1}{\sqrt{N}}).$$

  - erro estatístico $\sim \frac{1}{\sqrt{N}}$ garantido pelo teorema do limite central

Hadronic Spectrum from Lattice QCD



QCD Phase Diagram



Lattice QCD at finite temperature

- expectation values in a heat bath

$$\langle A \rangle = \frac{1}{Z} \mathrm{Tr}\left[e^{-\beta H} A \right]$$

- thermal Green's function for bosons:

$$\begin{aligned} G(x,y;\tau,0) &= Z^{-1}\mathrm{Tr}\left[e^{-\beta H} \hat{\phi}(x,\tau)\hat{\phi}(y,0) \right] \\ &= Z^{-1}\mathrm{Tr}\left[\hat{\phi}(y,0) e^{-\beta H} \hat{\phi}(x,\tau) \right] \\ &= Z^{-1}\mathrm{Tr}\left[e^{-\beta H} e^{\beta H} \hat{\phi}(y,0) e^{-\beta H} \hat{\phi}(x,\tau) \right] \\ &= Z^{-1}\mathrm{Tr}\left[e^{-\beta H} \mathcal{T}_\tau \left(\hat{\phi}(y,\beta)\hat{\phi}(x,\tau) \right) \right] \\ &= G(x,y;\tau,\beta) \end{aligned}$$

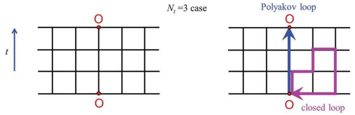
- temperature plays the role of imaginary time $T = \frac{1}{aL_t}$
- $\phi(y,0) = \phi(y,\beta) \Rightarrow$ Matsubara frequencies $\omega_n = 2\pi n T$

QCD Phase Diagram

- study of the phase diagram of QCD relevant e.g. for heavy ion experiments
- QCD has phase transition where quarks and gluons become deconfined for sufficiently high T

- Polyakov loop
  - order parameter
  - $L = \langle L(\vec{x}) \rangle \propto e^{-F_q/T}$
  - On the lattice:

$$L(\vec{x}) = \mathrm{Tr} \prod_{t=0}^{N_t - 1} \mathcal{U}_4(\vec{x}, t)$$



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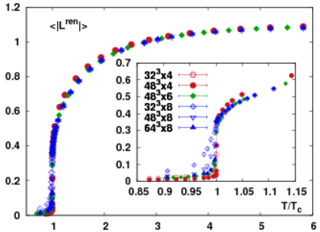

- $T < T_c : L = 0$ (center symmetry)
- $T > T_c : L \neq 0$ (spontaneous breaking)

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Center symmetry

- Wilson gauge action is invariant under a center transformation
- temporal links on a hyperplane $x_4 = const$ multiplied by

$$z \in Z_3 = \{e^{-i2\pi/3}, 1, e^{i2\pi/3}\}$$

- Polyakov loop $L(\vec{x}) \to zL(\vec{x})$
- $T < T_c$
  - local P_L phase equally distributed among the three sectors

$$L = \langle L(\vec{x}) \rangle \approx 0$$

- $T > T_c$
  - Z_3 sectors not equally populated: $L \neq 0$

G. Endrödi, C. Gattringer, H.-P. Schadler, arXiv:1401.7228
C. Gattringer, A. Schmidt, JHEP **01**, 051 (2011)
C. Gattringer, Phys. Lett. **B 690**, 179 (2010)

F. M. Stokes, W. Kamleh, D. B. Leinweber, arXiv:1312.0991

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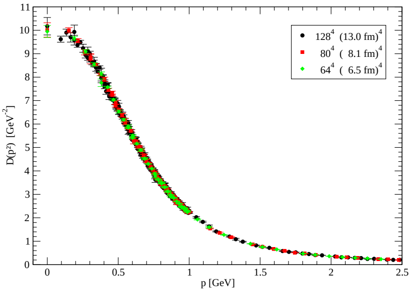
QCD Green's functions

- In a Quantum Field Theory, knowledge of all Green's functions allows a complete description of the theory
- In QCD, propagators of fundamental fields (e.g. quark, gluon and ghost propagators) encode information about non-perturbative phenomena
  - In particular, gluon propagator encodes information about confinement/deconfinement
- Since the gluon propagator is a gauge dependent quantity, we need to choose a gauge
  - in our works: Landau gauge $\partial_\mu A_\mu = 0$

Gluon propagator at zero temperature



- Definition

$$D^{ab}_{\mu\nu}(\hat{q}) = \frac{1}{V}\langle A^a_\mu(\hat{q}) A^b_\nu(-\hat{q})\rangle$$

- Tensor structure

$$D^{ab}_{\mu\nu}(q) = \delta^{ab}\left(\delta_{\mu\nu} - \frac{q_\mu q_\nu}{q^2}\right) D(q^2)$$

A. G. Duarte, O. Oliveira, PJS, Phys.Rev. D94 (2016) 014502

Gluon propagator at finite temperature

- Two components:
  - transverse D_T
  - longitudinal D_L

$$D^{ab}_{\mu\nu}(\hat{q}) = \delta^{ab}\left(P^T_{\mu\nu}D_T(q_4^2,\vec{q}) + P^L_{\mu\nu}D_L(q_4^2,\vec{q})\right)$$

$$D_T(q^2) = \frac{1}{2V(N_c^2-1)}\left(\langle A_i^a(q)A_i^a(-q)\rangle - \frac{q_4^2}{\vec{q}^2}\langle A_4^a(q)A_4^a(-q)\rangle\right)$$

$$D_L(q^2) = \frac{1}{V(N_c^2-1)}\left(1+\frac{q_4^2}{\vec{q}^2}\right)\langle A_4^a(q)A_4^a(-q)\rangle$$

- Finite temperature on the lattice: $L_t << L_s$

$$T = \frac{1}{aL_t}$$

Lattice setup finite T

Temp. (MeV)	β	L_s	L_t	a [fm]	1/a (GeV)
121	6.0000	64	16	0.1016	1.943
162	6.0000	64	12	0.1016	1.943
194	6.0000	64	10	0.1016	1.943
243	6.0000	64	8	0.1016	1.943
260	6.0347	68	8	0.09502	2.0767
265	5.8876	52	6	0.1243	1.5881
275	6.0684	72	8	0.08974	2.1989
285	5.9266	56	6	0.1154	1.7103
290	6.1009	76	8	0.08502	2.3211
305	5.9640	60	6	0.1077	1.8324
305	6.1326	80	8	0.08077	2.4432
324	6.0000	64	6	0.1016	1.943
366	6.0684	72	6	0.08974	2.1989
397	5.8876	52	4	0.1243	1.5881
428	5.9266	56	4	0.1154	1.7103
458	5.9640	60	4	0.1077	1.8324
486	6.0000	64	4	0.1016	1.943

- Simulations: use of Chroma and PFFT libraries
- keep a constant (spatial) physical volume $\sim (6.5fm)^3$
- all data renormalized at $\mu = 4GeV$

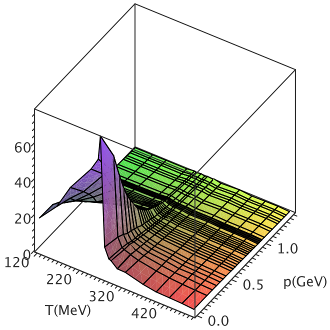
O. Oliveira, PJS, PoS(LATTICE2012)216

Acta Phys.Polon.Supp. 5 (2012) 1039

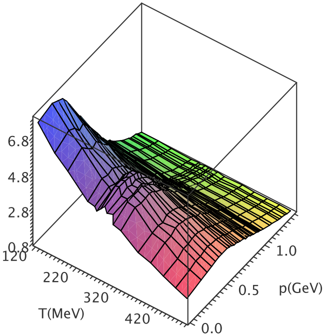
PoS(Confinement X)045

Surface plots

Longitudinal component



Transverse component



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Spectral density

- Euclidean momentum-space propagator of a (scalar) physical degree of freedom

$$\mathcal{G}(p^2) \equiv \langle \mathcal{O}(p)\mathcal{O}(-p) \rangle$$

- Källén-Lehmann spectral representation

$$\mathcal{G}(p^2) = \int_0^\infty \mathrm{d}\mu \frac{\rho(\mu)}{p^2 + \mu} , \qquad \text{with } \rho(\mu) \geq 0 \text{ for } \mu \geq 0 .$$

- spectral density contains information on the masses of physical states described by the operator $\mathcal{O}$

$$\rho(\mu) = \sum_\ell \delta(\mu - m_\ell^2) \, |\langle 0|\mathcal{O}|\ell_0 \rangle|^2 ,$$

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Spectral density: motivation

- Main goal: compute the spectral density of gluons and other (un)physical degrees of freedom
  - important for e.g. DSE/BSE spectrum studies (Minkowski space)
  - spectral density is not strictly positive
  - traditional Maximum Entropy Method does not allow negative spectral densities
- Way out: Tikhonov regularization plus Morozov discrepancy principle

D. Dudal, O. Oliveira, PJS, PRD 89 (2014) 014010

Positivity violation

Spectral representation

$$D(p^2) = \int_0^{+\infty} d\mu \frac{\rho(\mu)}{p^2 + \mu^2}$$

On the lattice: study the temporal correlator

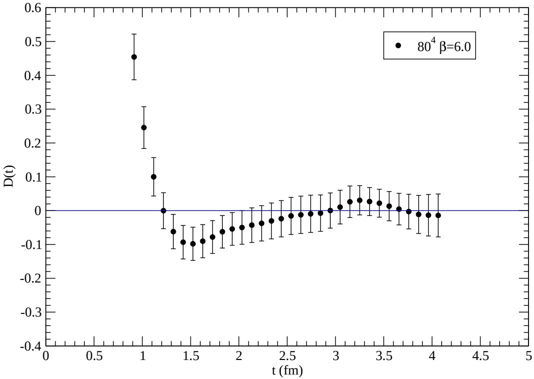
$$C(t) = \int_{-\infty}^{\infty} \frac{dp}{2\pi} D(p^2) \exp(-ipt) = \int_0^{\infty} d\omega \rho(\omega^2) e^{-\omega t}$$

$C(t) < 0$

- negative spectral density
- positivity violation
- gluon confinement

$C(t) > 0$ says nothing about $\rho(\mu)$

Positivity violation for the gluon propagator



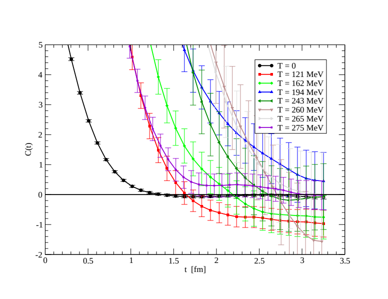
Already observed in lattice simulations

C. Aubin, M. C. Ogilvie, Phys. Rev D70, 074514 (2004)

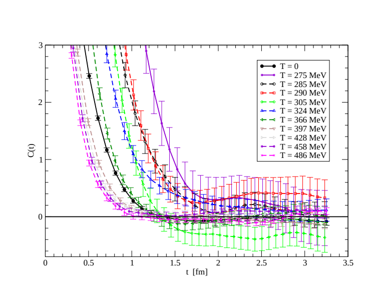
A. Cucchieri, T. Mendes, A. R. Taurines, Phys. Rev. D71, 051902 (2005)

Positivity violation finite T - longitudinal component

Below T_C

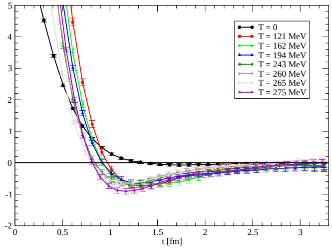


Above T_c

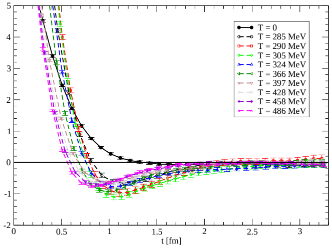


Positivity violation finite T - transverse component

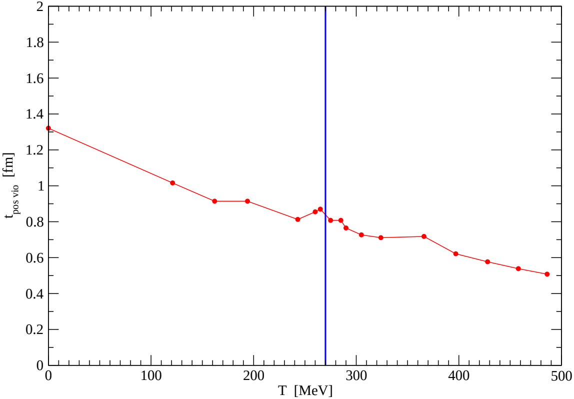
Below T_C



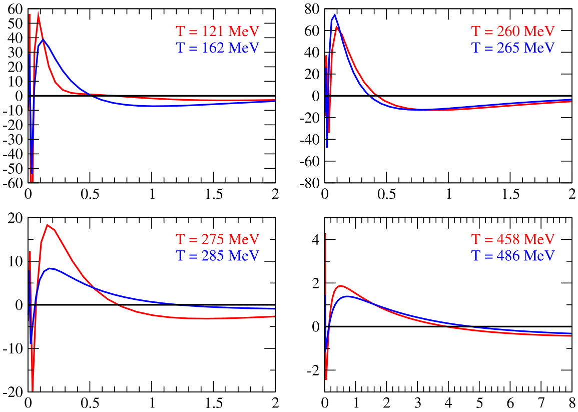
Above T_c



Positivity violation scale – transverse component



Longitudinal propagator spectral densities



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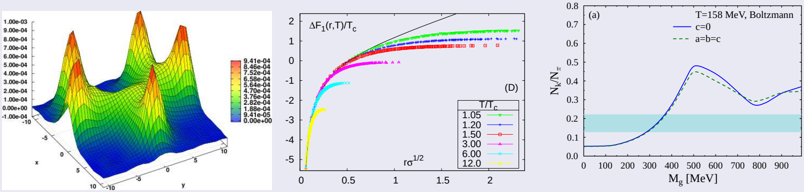
Why gluon mass?



- At $T = 0$ we have colour screening and flux tubes,

  J. M. Cornwall, Phys. Rev. D 26, 1453 (1982)
  N. Cardoso, P. Bicudo, Phys. Rev. D 87, 034504 (2013)
  N. Cardoso, M. Cardoso, P. Bicudo [arXiv:1302.3633 [hep-lat]]

- at large T Debye screening,

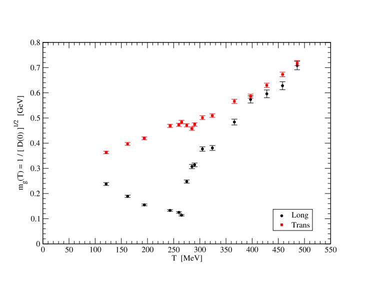
  M. Doring, K. Hubner, O. Kaczmarek, and F. Karsch, Phys. Rev. D 75, 054504 (2007)
  M. Bluhm, B. Kampfer and K. Redlich, Phys. Rev. C 84, 025201 (2011)

- at T_c a mass scale in the π and K multiplicities in heavy ions

  P. Bicudo, F. Giacosa, E. Seel Phys.Rev. C86, 034907 (2012)

Gluon mass at finite T

naive M_L and M_T function of T



Interpretation

- The simplest ansatz for a massive propagator is,

$$D(p) = \frac{1}{p^2 + M^2}$$

$$\Rightarrow M = 1/\sqrt{D(0)}$$

PJS, O. Oliveira, P. Bicudo, N. Cardoso, Phys.Rev. D89 (2014) 074503

Gluon mass at finite T

- for a better IR ansatz, we fit D_i using a Yukawa fit with mass M

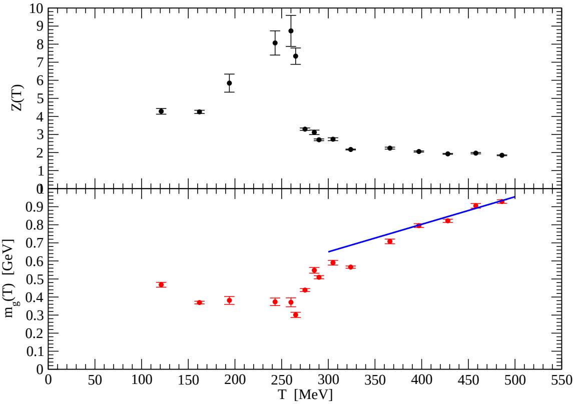
$$D_i(p^2) = \frac{Z}{p^2 + m^2}$$

  and look for the largest fitting range p_{max}
- this fits quite well D_L
- the Yukawa does not fit D_T

Fits of the longitudinal propagator

T	p_{max}	Z_L	M_L	$\chi^2/d.o.f.$
121	0.467	4.28(16)	0.468(13)	1.91
162	0.570	4.252(89)	0.3695(73)	1.66
194	0.330	5.84(50)	0.381(22)	0.72
243	0.330	8.07(67)	0.374(21)	0.27
260	0.271	8.73(86)	0.371(25)	0.03
265	0.332	7.34(45)	0.301(14)	1.03
275	0.635	3.294(65)	0.4386(83)	1.64
285	0.542	3.12(12)	0.548(16)	0.76
290	0.690	2.705(50)	0.5095(85)	1.40
305	0.606	2.737(80)	0.5900(32)	1.30
324	0.870	2.168(24)	0.5656(63)	1.36
366	0.716	2.242(55)	0.708(13)	1.80
397	0.896	2.058(34)	0.795(11)	1.03
428	1.112	1.927(24)	0.8220(89)	1.30
458	0.935	1.967(37)	0.905(13)	1.45
486	1.214	1.847(24)	0.9285(97)	1.55

Gluon mass at finite T



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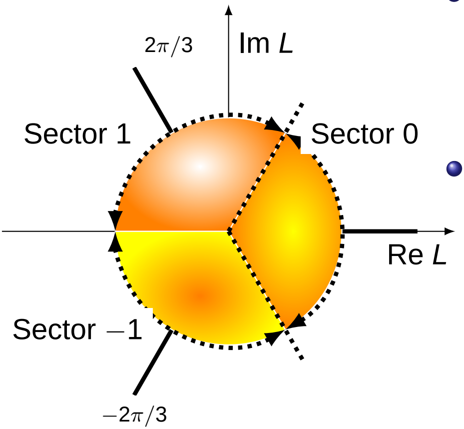
Z_3 dependence

- D_L and D_T show quite different behaviours with T
- Usually, the propagator is computed such that $\arg(P_L) < \pi/3$ (Z_3 sector 0)
- what happens in the other sectors?

PJS, O. Oliveira, PRD **93** (2016) 114509

Z_3 dependence



- for each configuration, 3 gauge fixings after a Z_3 transformation

$$\mathcal{U}'_4(\vec{x}, t=0) = z\,\mathcal{U}_4(\vec{x}, t=0)$$

- configurations classified according to $\langle L \rangle = |L| e^{i\theta}$

$$\theta = \begin{cases} -\pi < \theta \le -\frac{\pi}{3}, & \text{Sector -1,} \\ -\frac{\pi}{3} < \theta \le \frac{\pi}{3}, & \text{Sector 0,} \\ \frac{\pi}{3} < \theta \le \pi, & \text{Sector 1} \end{cases}$$

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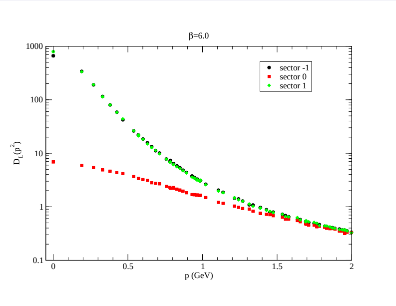
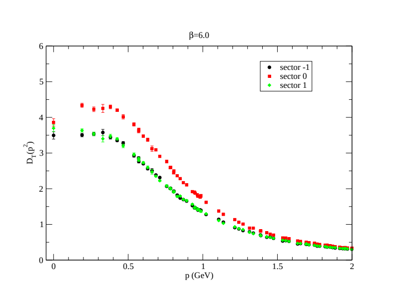
Typical result at high T (324 MeV)

Longitudinal component



Transverse component



What happens near T_c ?

- spatial physical volume $\sim (6.5\text{fm})^3$
- 100 configs per ensemble

Coarse lattices $a \sim 0.12fm$

Temp. (MeV)	$L_s^3 \times L_t$	β	a (fm)	$L_s a$ (fm)
265.9	$54^3 \times 6$	5.890	0.1237	6.68
266.4	$54^3 \times 6$	5.891	0.1235	6.67
266.9	$54^3 \times 6$	5.892	0.1232	6.65
267.4	$54^3 \times 6$	5.893	0.1230	6.64
268.0	$54^3 \times 6$	5.8941	0.1227	6.63
268.5	$54^3 \times 6$	5.895	0.1225	6.62
269.0	$54^3 \times 6$	5.896	0.1223	6.60
269.5	$54^3 \times 6$	5.897	0.1220	6.59
270.0	$54^3 \times 6$	5.898	0.1218	6.58
271.0	$54^3 \times 6$	5.900	0.1213	6.55
272.1	$54^3 \times 6$	5.902	0.1209	6.53
273.1	$54^3 \times 6$	5.904	0.1204	6.50

Fine lattices $a \sim 0.09fm$

Temp. (MeV)	$L_s^3 \times L_t$	β	a (fm)	$L_s a$ (fm)
269.2	$72^3 \times 8$	6.056	0.09163	6.60
270.1	$72^3 \times 8$	6.058	0.09132	6.58
271.0	$72^3 \times 8$	6.060	0.09101	6.55
271.5	$72^3 \times 8$	6.061	0.09086	6.54
271.9	$72^3 \times 8$	6.062	0.09071	6.53
272.4	$72^3 \times 8$	6.063	0.09055	6.52
272.9	$72^3 \times 8$	6.064	0.09040	6.51
273.3	$72^3 \times 8$	6.065	0.09025	6.50
273.8	$72^3 \times 8$	6.066	0.09010	6.49

How-to

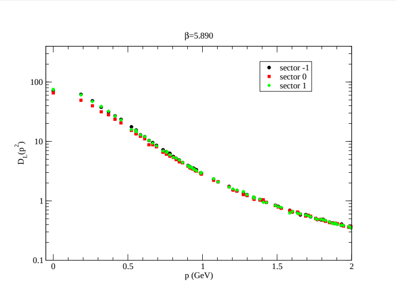
- Conical cut for momenta above 1GeV; all data below 1GeV
- Renormalization:

$$D_{L,T}(\mu^2) = Z_R D^{Lat}_{L,T}(\mu^2) = 1/\mu^2$$

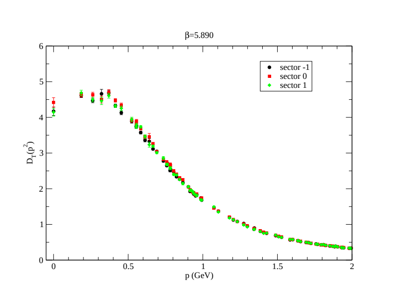
- Renormalization scale: $\mu = 4$ GeV
- D_L and D_T renormalized independently
  - within each $Z(3)$ sector, $Z_R^{(L)}$ and $Z_R^{(T)}$ agree within errors
- each Z_3 sector is renormalized independently
  - Z_R do not differ between the different $Z(3)$ sectors

Coarse lattices, below T_c

Longitudinal component

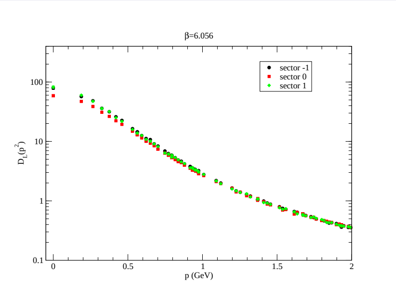


Transverse component

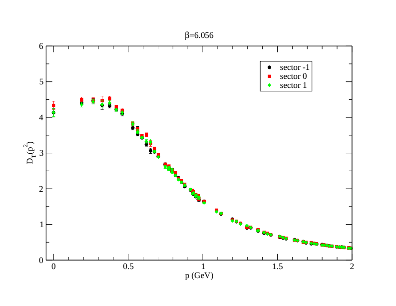


Fine lattices, below T_c

Longitudinal component

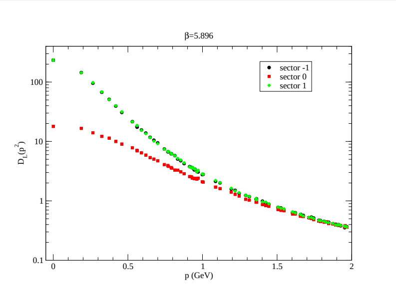


Transverse component

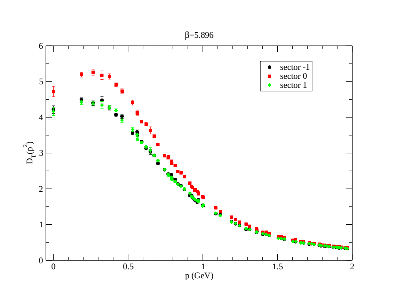


Coarse lattices, above T_c

Longitudinal component

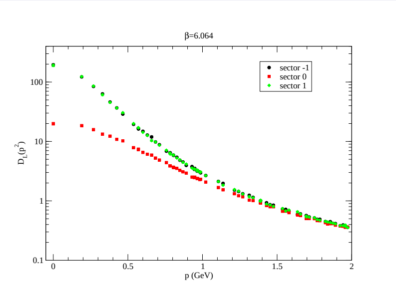


Transverse component

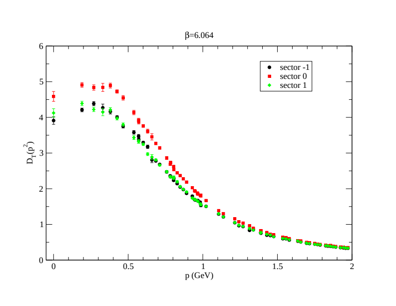


Fine lattices, above T_c

Longitudinal component

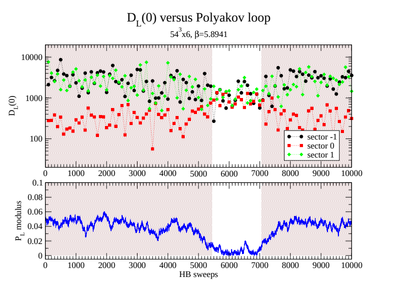


Transverse component

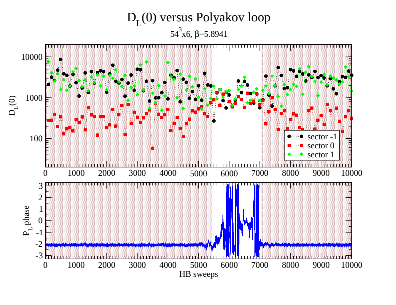


Polyakov loop history

Modulus

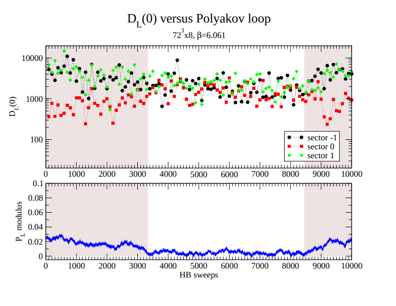


Phase

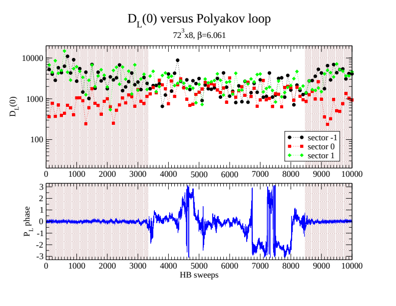


Polyakov loop history

Modulus

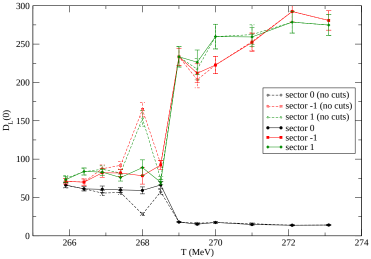


Phase

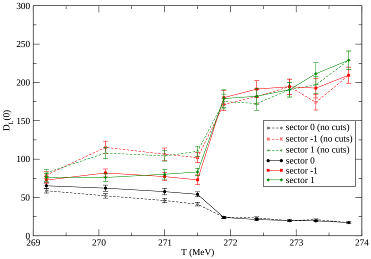


Removing configurations in wrong phase

Coarse lattices



Fine lattices



Conclusions and Outlook

- Extensive study of the gluon propagator at finite temperature
- Positive violation and spectral densities
- Mass scales
- Z_3 dependence
  - Correlation between L and the separation of D between the different sectors can be used to identify the phase transition
  - Possible existence of different phases near and above T_c
    - The dynamics differs in each sector
  - Outlook:
    - understand physics of different sectors (e.g. mass scales)
    - how quarks change the above picture?
      look at the distribution of eigenvalues of the Dirac operator

Gattringer, Rakow, Schafer, Soldner, PRD66(2002)054502