Implications of gauge symmetry for precision Higgs physics

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1601.02006 with Axel Maas
1605.01512
(proposal (also 1607.05860
1701.02881)

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(2009-2011)
H+ searches
in tt decays
with
CMS-LIP
Deviations from the Standard Model

(CMS 2016) “Limits are set on scalar resonances produced through gluon-gluon fusion, and on Randall-Sundrum gravitons. A modest excess of events compatible with a narrow resonance with a mass of about 750 GeV is observed.”

⇒ >500 Models of New Physics: parametrizations of deviations from SM.

Always a region of parameters where models compatible with SM.

Alternatives? Yes, within the SM
Summary

• No spontaneous breakdown of gauge symmetry

• Higgs field in the structure of leptons and hadrons

• Shortcomings of Mean-field approximation to the Higgs PDFs

• Higgs PDFs from lattice simulations

• Higgs PDFs from experimental data

After discussion:

• New Physics searches + Higgs PDFs studies
Spontaneous breakdown of global symmetry

Expectation value $\langle \phi \rangle_{J,N}$

$\phi$: observable
$J$: parameter affected by transformation $g$

Finite size $N$:

$$\lim_{J \to 0} \langle (\phi - g(\phi)) \rangle_{J,N} = 0$$

Spontaneous symmetry breaking when:

$$\lim_{J \to 0} \left\{ \lim_{N \to \infty} \langle (\phi - g(\phi)) \rangle_{J,N} \right\} \neq 0$$

Limits do not commute

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Spontaneous breakdown of gauge symmetry?

Local gauge transformation affects small region near each space-time point: \( \lim_{N \to \infty} \)

Mainstream view:

Anderson (1958) “longitudinal and transverse excitations are different in the superconductor... this difference allows a gauge-invariant explanation of the Meissner effect.”

Nambu (1960) “The Meissner effect calculation is thus rendered strictly gauge invariant, but essentially keeping the BCS result unaltered for transverse fields.”

Higgs (1966) “our result suggests that it must be possible to rewrite the theory in a form in which only gauge-invariant variables appear.”
Elitzur (1975) “a spontaneous breaking of local symmetry for a symmetrical gauge theory without gauge fixing is impossible.”

‘t Hooft (1980) “the words spontaneous breakdown are formally not correct for local gauge theories. The vacuum never breaks local gauge invariance. The neutral intermediate vector boson is the “meson”

\[
\phi^\dagger D_\mu \phi = i g v^2 4 W^3_\mu + \text{total derivative} + \text{higher orders}
\]

The \( W^{\pm}_\mu \) are obtained from the “baryons” \( \epsilon_{ij} \phi^i D_\mu \phi^j \), and the Higgs particle can also be obtained from \( \phi^\dagger \phi \).”

Englert (2014) “strictly speaking there is no spontaneous symmetry breaking of a local symmetry. One uses perturbation theory to select at zero coupling a scalar field configuration from global SSB; but this preferred choice is only a convenient one.”
Mean-field approximation ⇒ Broken gauge symmetry.
Mean-field approximation

Kadanoff (2009) “the concept of mean field forms the basis of much of modern condensed matter physics and also of particle physics.

sometimes an infinite statistical system has a phase transition, and that transition involves a discontinuous jump in a quantity we call the order parameter.

But we have given no indication of how big the jump might be, nor of how the system might produce it. Mean field theory provides a partial, and partially imprecise, answer to that question.”
Mean-field approximation

Every pair of molecules interact at a close distance. To take into account all these interactions is computationally expensive.

Mean-field approach: every molecule interacts only with the average distribution of other molecules.
Strocchi (2013) “If the potential has a non-trivial minimum $\phi = \overline{\phi}$, one can consider a semiclassical approximation based on the expansion $\phi = \overline{\phi} + \varphi$, treating $\overline{\phi}$ as a classical constant field and $\varphi$ as small.”

Strocchi (2013) “Thus, the expansion can be seen as an expansion around a (symmetry breaking) mean field ansatz, and it is very important that a renormalized perturbation theory based on it exists and yields a non vanishing symmetry breaking order parameter $<\phi> \neq 0$ at all orders. This is the standard (perturbative) analysis of the Higgs mechanism.”
Structure of leptons

$SU(2)_L$ gauge doublets:

$\phi$: Higgs field

$\Psi_{e\nu}$: electron and neutrino

$O^{\nu e} = \phi^\dagger \Psi_{e\nu}$ gauge-invariant

Propagator:

\[
< O^{\nu e}(x) \bar{O}^{\nu e}(y) > = v^2 < e\bar{e} > + \\
+ v < (\varphi^*(x) + \varphi(y))e\bar{e} > + < \varphi^* \varphi e\bar{e} >
\]

$\phi_0 = v + \varphi$ (neutral Higgs component in the unitary gauge)

(’t Hooft (1980); Frohlich et al. (1981)) Mean-field $\sim$ 1-particle state

(Maas and Mufti (2015)) 2-particle states seen on lattice simulations
Higgs PDFs from Mean-Field

PDFs (parton density functions): composition of gauge-invariant bound states in terms of elementary fields (in fixed gauge)

Mean field approximation: physical electron $\Rightarrow ve$
(at the minimum of the Higgs potential $\phi_0 = v$)

$\Rightarrow$ Trivial Higgs PDFs are a good approximation

Support from: 
- Experimental data;
- Theory Elitzur (1975); De Angelis et al. (1978); Osterwalder and Seiler (1978); Frohlich et al. (1981);
- Lattice simulations Fradkin and Shenker (1979); Caudy and Greensite (2008); Bonati et al. (2010); Wurtz and Lewis (2013); Maas (2013); Maas and Mufti (2014, 2015).
Electron structure???

Relevant for:


- Initial-state radiation at LEP
  (Schael et al. (2006); Yennie et al. (1961); Kuraev and Fadin (1985); Skrzypek and Jadach (1991);
  Abdallah et al. (2014); Slominski and Szwed (2001)).

The contribution from pure QED to the electron structure functions is calculated using perturbation theory.
Example: photon PDF in the proton

Pagani et al. (2016)

\[ \frac{d\sigma}{dm} \text{ [pb/GeV]} \]

\[ \text{data (ATLAS, arXiv:1511.04716)} \]

**tt\( \rightarrow \) (QCD+EW, NNDPF2.3QED), LHC8**

\[ \mu = m(tt) \]
\[ \mu = H_T/2 \]
\[ \mu = m_t \]

\[ m(tt) \text{ [GeV]} \]

0.95 1 1.05 400 600 800 1000 1200 1400 1600

\[ (QCD+EW)/data; \text{ scale+PDF unc.} \]

\[ (QCD+EW)/QCD \quad \text{data unc.} \]

\[ \text{NNPDF30} \quad \text{MRST2004 (0,1)} \quad \text{CT14qed_inc (0,5)} \quad \text{LUXqed} \]

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top quark PDF in the proton

Han et al. (2015)
Maltoni et al. (2012)
“simpler and allow the resummation of possibly large initial state logarithms”
How does an experimentalist estimate the systematic uncertainty of a mean field approximation?

**Using an alternative approximation**

(Analogy in QCD: Lund string model vs. Cluster model)
Gattringer and Lang (2010) “Replacing space-time by a Euclidean lattice has proven to be an efficient approach which allows for both theoretical understanding and computational analysis. Lattice QCD has become a standard tool in elementary particle physics.”

Monte-Carlo simulations, distribution of events is function of the Lagrangian
Strocchi (2013) “mean field expansions may yield misleading results about the occurrence of symmetry breaking and the energy spectrum.

The Euclidean functional integral approach [Lattice simulations] gives symmetric correlation functions and in particular $\langle \phi \rangle = 0$.

This means that the mean field ansatz is incompatible with the non-perturbative quantum effects and the approximation leading to the quadratic Lagrangian is not correct.”
when \( m_H/m_W < 1 \):

- lattice \( \neq \) perturbation theory
- Not a weak \( \rightarrow \) strong coupling transition

\( (m_H/m_W \propto \sqrt{\lambda}) \)

---

Evertz et al. (1986); Langguth and Montvay (1985); Maas and Mufti (2014)
shortcomings of Mean-field approximation

- mean-field approximations in nuclear and solid-state physics can be improved in many ways, fake breaking of symmetries common (Grasso et al. (2016); Egido (2016));
- in a grand-unified theory at weak coupling the spectrum in the lattice \( \neq \) mean-field Maas and Torek (2016);
- in a theory with abelian Higgs mechanism mean field fails Tada and Koma (2016);
- trivial PDFs require + assumptions (e.g. on the gravity sector) to sidestep a non-perturbative gauge dependence (Maas and Mufti (2014); Ilderton et al. (2010); Faddeev (2008));
- phase diagram is different for different gauge-fixings Caudy and Greensite (2008).
PDFs from Lattice simulations

PDFs calculable (in principle) $\neq$ model of new physics

Ji (2013) “studying a large momentum hadron on lattice is computationally still challenging, but at least this could be achieved when computational power continues improving.”

Bacchetta et al. (2017) “In the future, the method can be used to produce PDFs entirely based on lattice QCD results.”
Higgs PDFs from Lattice simulations

Today, *indications* from the lattice.
Sufficient, if *combined* with experimental data.
Experimental constraints also allow *faster* lattice simulations

Maas and Mufti (2014)

Alternatives: symmetry conserving mean-field (Grasso et al. (2016)), etc.
Egger et al. (2017) Proof of concept (CEPC/ILC)

- $e^-\text{-H bound state}$
- $e^+\text{-H bound state}$
- $\mu^-\text{-H bound state}$
- $\mu^+\text{-H bound state}$
- $Z\text{-H-H bound state}$

Implications of gauge symmetry
Egger et al. (2017) Proof of concept (CEPC/ILC)

Higgs PDFs can be constrained at colliders. Below $2M_H$ threshold: off-shell suppression
Higgs PDFs from LHC

$h^*$: valence higgs boson (on-shell or off-shell)

- constraining the photon and gluon PDF using $t\bar{t}$ events Pagani et al. (2016); Czakon et al. (2016)
- constraining the underlying event using anomalous $Z$ production Aad et al. (2014); Mucibello (2012)
- anomalous on-shell Higgs production due to an anomalous trilinear coupling Degrassi et al. (2016); Englert and Spannowsky (2014); Logan (2015)
Higgs PDFs from LEP

- Anomalous Bhabha scattering at LEP Bourilkov (1999); Alcaraz et al. (2006); Schael et al. (2013)

- Fragmentation: Searches for the Higgs boson at LEP in the $H Z \rightarrow \mu^+\mu^- b\bar{b}$ final state

Dittmaier and Schumacher (2013)
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New Physics + Higgs PDFs

“Model independent” analysis ⇒ Mean-field approximation independent?

Maybe not.

Extension of the software for PDFs from QCD, to Higgs PDFs (goal)

Adapt Hera PDF studies to the LHC Higgs PDF studies
New Physics+ Higgs PDFs

Many theories are motivated by problems of the perturbative approach to the Standard Model.

Problem may be in the mean-field approximation, not in the Standard Model.

Example (Supersymmetry): The (gauge-invariant) bound state masses are independent of the renormalization $\Rightarrow$ not affected as the gauge-dependent elementary fields by hierarchy problem Frohlich et al. (1981); Maas (2013)

Example (Flavour puzzle): In principle, the families $(e, \mu, \tau)$ can be excited bound states of $e - H$. Impossible to know soon Egger et al. (2017).
Decouple systematic uncertainties from the experimental results is possible Cranmer et al. (2015); Brehmer et al. (2016); Boudjema et al. (2013); Kraml et al. (2012).

After decoupling theoretical uncertainties, an experimental result is a numerical function (whose arguments are the parameters affected by the theoretical uncertainties).

Functional Package Management GNU’s Guix and Guile Courtès (2013); Courtès and Wurmus (2015), would allow to publish, access and combine (many) experimental results and manage their dependencies.
Obrigado

References


A. Maas and P. Torek. Predicting the singlet vector channel in a partially Higgsed gauge theory. 2016, 1607.05860.


Giuseppe Degrassi, Pier Paolo Giardino, Fabio Maltoni, and Davide Pagani. Probing the Higgs self coupling via single Higgs production at the LHC. JHEP, 12:080, 2016, 1607.04251.


Erhard Seiler. On the Higgs-Confinement Complementarity. 2015, 1506.00862.


“The federal government recommended flossing in 1979-2015. In 2016 the government acknowledged the effectiveness of flossing had never been researched, as required.” (Associated Press)

(Feynman 1973)

All the people are doing the same ritual brush, brush, brush—for no good reason? Think about it.
Prelude

Spontaneous symmetry breaking (SSB) $\rightarrow$ disjoint phases in a system (local interactions, e.g. the Ising model or gauge theories)

Expectation value $\omega_{J,N} : \mathcal{A} \rightarrow \mathbb{R}$ (positive linear functional)

$J$: intensity of external source breaking a group of symmetries $G$.  
$\mathcal{A}$: set of observables.

Finite size $N$: continuous expectation values

\[
\left\{ \begin{array}{ll} 
\omega_{J,N}(A - g(A)) = 0 & \text{if } J = 0 \\
\lim_{J \to 0} \omega_{J,N}(A - g(A)) = 0 & 
\end{array} \right.
\]

any observable $A \in \mathcal{A}$ and any transformation $g \in G$. 
Definition 1. Spontaneous symmetry breaking when:

\[ \lim_{J \to 0} \left\{ \lim_{N \to \infty} \omega_{J,N}(A - g(A)) \right\} \neq 0 \]

for some \( A \in \mathcal{A} \) and some \( g \in G \).

Limit of a convergent sequence of continuous functions is not necessarily continuous.

Other definitions in statistical mechanics are not based on explicit symmetry breaking.
SSB possible for a global symmetry in a system with infinite size.

Elitzur (1975) “a spontaneous breaking of local symmetry for a symmetrical gauge theory without gauge fixing is impossible.”

local gauge transformation affects only a small sized system near each space-time point.
Outline

Higgs potential in multi-Higgs-doublet models

Confinement

Gauge-invariant operators in 2HDM (no $U(1)_Y$)

Majorana construction

Observable states of 2HDM

Spontaneous symmetry breaking in 2HDMs

The FMS mechanism

$Spin(4)$ symmetric 2HDM for the lattice
1 Higgs potential

Holomorphic functions $\frac{\partial f(z,z^*)}{\partial z^*} = 0$

central objects of study in complex analysis

The Higgs potential is not an holomorphic function

$\frac{\partial V(\phi_j,\phi_j^*)}{\partial \phi_j^*} \neq 0$.

No advantage in the Higgs field being a complex vector space

$V(\phi_j,\phi_j^*) = V(Re(\phi_j), Im(\phi_j))$

Complex irreducible representations of $G \times H$ are a direct product of complex irreducible representations of $G$ and of $H$.

Not the case for real irreducible representations.

(wiki/Representation_theory_of_finite_groups,arXiv:1309.5280)
Standard Higgs field (4 real components) $V(\phi, \phi^*) = V(\phi^\dagger \phi)$

$SO(4) \simeq (SU(2)_R \times SU(2)_L)/\mathbb{Z}_2$ (generators $\tau^j$ and $\sigma^j$)

$SU(2)_L$ gauge symmetry
Global symmetry $SO(4)/SU(2)_L \simeq SO(3)$ (E.g. $\phi^\dagger D_\mu \tau^j \phi$)

$N$-Higgs-doublets ($4N$ real components):

- different global symmetry $G/SU(2)_L$
- charged scalars;
- mixing between neutral scalar particles;
- Spontaneous/explicit global symmetry violation in the Higgs potential;
- Rich flavour phenomenology (e.g. meson decays, oscillations)
Definition 2. (Electroweak symmetry breaking)

After perturbative gauge-fixing, the Higgs vev minimizes the Higgs potential.

The symmetries broken by the Higgs vev are the spontaneously broken symmetries.

Perturbation theory can only deal with small perturbations of the Higgs field → non-null Higgs vev.

**Challenge:** spontaneous breaking of global symmetries in the presence of the Higgs mechanism
• Find an absolute minimum of the potential $\phi = \frac{v}{\sqrt{2}} \phi_0$;

• Projector $P_0$ on the $SU(2)_L$-orbit of $\phi_0$ such that $P_0 \phi_0 = \phi_0$;

• Modify the Higgs potential $W = V + \epsilon U$,
  $\epsilon > 0$ is arbitrarily small and $U = -v^2 \phi^\dagger P_0 \phi + (\phi^\dagger \phi)^2$;

• The absolute minima of $W = V + \epsilon U$ is the $SU(2)_L$-orbit of $\phi_0$;

The perturbation theory then implies that in the limit $\epsilon \to 0$, there are finite vevs breaking the global symmetries $\Rightarrow$ SSB by Def. 1

Evaluating vevs of $SU(2)_L$-invariant observables, we make no assumptions about SSB of gauge symmetry.
Classical minimization: no limit on the order of the potential ⇒
effective field theory, no assumptions on the ultra-violet completion
(appropriate for experimental data Eichhorn et al. (2015))

Consistency: Let $p(\phi)$ be a $G_f$-invariant polynomial in the Higgs field $\phi$.

If any $G_f$-invariant Higgs potential is necessarily $G$-invariant,
the observable $p(\phi)$ must also be invariant under $G$,
since $p(\phi)$ can appear in a $G_f$-invariant Higgs potential.
If $G_f$ is a classical $\times$ finite group ⇒ No spontaneous symmetry breaking of
$G/G_f$ since all $G_f$-invariant observables are also $G$-invariant.

Example: Global symmetry $G/SU(2)_L$ for one-Higgs-doublet cannot be
explicitly broken ⇒ no spontaneous symmetry breaking of $G/SU(2)_L$

More examples with CP Branco and Ivanov (2016)
2 Confinement

Options for Electroweak Theory:

1) Define the theory with gauge fixing (standard in perturbation theory), Gribov (1978); Singer (1978) non-perturbative ambiguity, the local non-abelian gauge-fixing condition is insufficient

2) gauge-invariant gauge charge, e.g. dressed elementary operators (photons are neutral), non-abelian (global) gauge charges cannot be (locally) gauge-invariant Haag (1992).

3) Fröhlich, Morchio, and Strocchi (1981): FMS mechanism inspired in the confinement mechanism, effectively matches gauge fixing + perturbation theory under some assumptions

4) Technicolor 5) ? (next) ...
Maas and Mufti (2015) SU(2) Yang-Mills-Higgs on the lattice phase diagram

\[
\frac{1}{g} \propto \text{gauge coupling} \\
\alpha \propto m_h^2 v^2 \text{ dashed lines: break global subgroup remaining after incomplete gauge-fixing.}
\]

Osterwalder and Seiler (1978)
Fradkin and Shenker (1979)
Caudy and Greensite (2008)
Seiler (2015)

Bonati et al. (2010) “hints that the above transitions are not related to confinement”

Englert (2005) “Electric-magnetic dualities suggest that, at some fundamental level, confinement is a condensation of magnetic monopoles and constitutes the magnetic dual of the BEH mechanism”

However Englert does not cite FMS mechanism
3 Gauge-invariant operators in 2HDM (no $U(1)_Y$)

$SU(2)_L$ Higgs doublets $\phi_1, \phi_2$

gauge field $W^j_\mu$ with $j, k, l = 1, 2, 3$

Higgs Potential $V(\phi_1, \phi_2)$,

coupling constant $g$,

\[
\mathcal{L} \equiv ((D^\mu \phi_1)^\dagger (D_\mu \phi_1) + ((D^\mu \phi_2)^\dagger (D_\mu \phi_2) - V(\phi_1, \phi_2) - \frac{1}{4} W^j_\mu W^{j\mu\nu}
\]

\[
D_\mu \equiv \partial_\mu + igW^j_\mu \sigma^j
\]

\[
W^j_{\mu\nu} \equiv -\frac{i}{g} \text{tr}([D_\mu, D_\nu] \sigma^j) = \partial_\mu W^j_\nu - \partial_\nu W^j_\mu - g\epsilon^{jkl}W^k_\mu W^l_\nu
\]

Levi-Civita $\epsilon^{jkl}$, Pauli matrices in gauge space $\sigma^j$
Karassiov (1992) (+ general Wineman and Pipkin (1964))

Any polynomial of \(\phi_1, \phi_2\) which is gauge invariant is a polynomial on

\[\phi_1^* \phi_1^a, \phi_2^* \phi_2^a, \phi_2^* \phi_1^a; \epsilon_{ab} \phi_1^a \phi_2^b; \epsilon_{ab} \phi_1^a \phi_2^b\]

\[\phi_{jb}^* \equiv (\phi_j^b)^* \quad a, b = 1, 2\] are gauge indices.

Also parallel transport \(U(x, y, C)\) from \(y\) to \(x\) along line \(C\).

for infinitesimal line elements

\[U(x, y, C) \approx (1 + D_\mu(x)dl_1^\mu)(1 + D_\nu(x)dl_2^\nu)...(1 + D_\alpha(x)dl_n^\alpha)\]

\(dl_1, dl_2, ..., dl_n\) (\(n\) finite) are infinitesimal Lorentz vectors forming \(C\) by concatenation.
Set of primitive (algebraically ind.) gauge-invariant operators for 2HDM:

- $tr(U(x,x,C'))$  Giles (1981)

- $\phi^\dagger_j(x)U(x,y,C)\phi_k(y)$

- $\phi^\dagger_j(x)U(x,y,C)\bar{\phi}_k(y)$

- $\bar{\phi}^\dagger_j(x)U(x,y,C)\phi_k(y)$

- $\bar{\phi}^\dagger_j(x)U(x,y,C)\bar{\phi}_k(y)$

$\bar{\phi}^a_j(x) \equiv \epsilon^{ab}\phi^*_j(x)$,

indices $j,k = 1,2$ are Higgs flavor indices,
4 Majorana construction

Shirokov (2015): $A^a, B^a$ are $2^n \times 2^n$ complex unitary matrices

\[ A^a A^b + A^b A^a = 2g^{ab}1 \]
\[ B^a B^b + B^b B^a = 2g^{ab}1 \]

$a \in \{1, \ldots, 2n\}$, $n < 4$, $g \equiv \text{diag}(-1, \ldots, +1, \ldots)$ ($n$ entries $-1$ and $n+1$)

Generalized Pauli’s theorem:

1. $B^a = S A^a S^{-1}$. $S$ is unitary and unique up to a phase;
2. there is a basis where all $A^a$ are real;
3. Clifford algebra generated by $A^a$ is isomorphic to the algebra of $2^n \times 2^n$ matrices.

Majorana spinors: $2^n$ complex vectors $u$ satisfying $\Theta u = u$.
$\Theta$: anti-linear involution commuting with $A^a$, unique up to a phase.
$n = 3$: 8-dimensional Majorana spinor $\phi$ (Pilaftsis (2012)).

Generators of $SU(2)_L$: $i\sigma^j \equiv \epsilon^{jkl} A_k A_l$ ($j, k, l = 1, 2, 3$)

\[ \Sigma_j \equiv A^j + 3 \quad (j = 1, 2, 3), \quad \Sigma_4 \equiv A^1 A^2 A^3 \quad \text{and} \quad \Sigma_5 \equiv \Sigma_1 \Sigma_2 \Sigma_3 \Sigma_4 = -A^7. \]

1, $\Sigma_a$ ($a, b = 1, \ldots, 5$): basis of hermitian matrices conserved by $SU(2)_L$. $\Sigma_a$ anti-commute with each other.

$[\Sigma_a, \Sigma_b]$: basis of skew-hermitian matrices conserved by $SU(2)_L$, generators of $Spin(5)$ (double cover of $SO(5)$).

Rewrite set of primitive gauge-invariant operators:

- $\phi^\dagger(x) U(x, y, C) \phi(y)$ (singlet under $SO(5)$);
- $\phi^\dagger(x) U(x, y, C) \Sigma_a \phi(y)$ ($5$ representation of $SO(5)$);
- $\phi^\dagger(x) U(x, y, C) [\Sigma_a, \Sigma_b] \phi(y)$ ($10$ representation of $SO(5)$);
5 Observable states of 2HDM

Higgs potential (basis-invariant formalism O’Neil (2009)):

\[ V(\phi) = \mu_a \phi^\dagger \Sigma a \phi + \frac{1}{2} \lambda_{ab}(\phi^\dagger \Sigma a \phi)(\phi^\dagger \Sigma b \phi) \]

parameters of the potential ⇒ background fields (spurions)

Ivanov (2006); Botella et al. (2013)

\( \mu_0, \lambda_{00} \) singlets,
\( \mu_a, \lambda_{0a} \) are 5-dim representations of \( SO(5) \)
\( \lambda_{ab} \) is a tensor of \( SO(5) \)

Lagrangian invariant under gauge \( SU(2)_L \) and background \( Spin(5) \).
Let \( V(\phi = \frac{v}{\sqrt{2}} \phi_0) \) be absolute minimum, \( (v \equiv \text{vev}, \phi_0^\dagger \phi_0 = 1) \).

by reparametrization \( \Sigma_5 \phi_0 = \phi_0 \) \((Spin(5) \rightarrow Spin(4))\)

\( H_1 \equiv \frac{1+\Sigma_5}{2} \phi \) \( H_2 \equiv \Sigma_4 \frac{1-\Sigma_5}{2} \phi \), at the minimum \( H_2 = 0 \).

isomorphism \( Spin(4) \simeq (SU(2)_{R1} \times SU(2)_{R2}) \)

\( SU(2)_{R1} \) generators \( \Sigma_j \Sigma_4 (1 + \Sigma_5) / 2 \)

\( SU(2)_{R2} \) generators \( \Sigma_j \Sigma_4 (1 - \Sigma_5) / 2 \)
after (suitable) gauge fixing, constant \( \frac{v}{\sqrt{2}} \phi_0 \) minimizing the potential

\[ i\sigma_j \phi_0 = \Sigma_4 \Sigma_j \phi_0 \quad (j = 1, 2, 3), \]

\( \phi^0 \) conserves \( SO(3) \times Spin(3) \cong (SU(2)_{R1} \times SU(2)_{R2})/Z_2, \)

generators \( (\Sigma_4 \Sigma_j (1 + \Sigma_5)/2 - i\sigma_j) \) and \( \Sigma_4 \Sigma_j (1 - \Sigma_5), \) respectively.
φ₀ fixes a system of gauge coordinates.

4 projections φ₀φ₀† and −Σ₄Σⱼφ₀φ₀†Σ₄Σⱼ (fixed j = 1, 2, 3) sum to 1 and decompose the 4 dim real spinor space of $SU(2)_L$

subspace $\propto φ₀$: $φ₀ H₁, φ₀ H₂$.

subspace $\propto Σ₄Σⱼφ₀$: would-be Goldstone bosons $φ₀ Σ₄Σⱼ H₁$ and $φ₀ Σ₄Σⱼ H₂$.

3 projections for the triplet of $SU(2)_L$,

subspace $\propto (φ₀ ⊗ Σ₄Σⱼφ₀ − Σ₄Σⱼφ₀ ⊗ φ₀)$: $φ₀ † D_µ Σ₄Σⱼφ₀ = \frac{g}{2} W_µ^j$. 
expand $\sqrt{2}\phi = v\phi_0 + \varphi$

Assuming the fluctuations $\varphi$ small in average compared to $v$

<table>
<thead>
<tr>
<th>$J(SU(2)_{R1})$</th>
<th>$J(SU(2)_{R2})$</th>
<th>Operator</th>
<th>Expansion</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>$H_1^\dagger H_1$</td>
<td>$\frac{v^2}{2} + v\phi_0^\dagger \varphi$</td>
</tr>
<tr>
<td>1/2</td>
<td>1/2</td>
<td>$H_1^\dagger \Sigma_a \Sigma_4 H_2$</td>
<td>$\frac{v}{2} \phi_0^\dagger \Sigma_a \varphi$</td>
</tr>
<tr>
<td>1</td>
<td>0</td>
<td>$H_1^\dagger D_\mu \Sigma_j \Sigma_4 H_1$</td>
<td>$\frac{gv^2}{4} W^j_\mu$</td>
</tr>
</tbody>
</table>

($j = 1, 2, 3$ and $a = 1, 2, 3, 4$, first terms in expansion only)

other primitive invariants involving $\leq 1$ covariant derivative expand to $\geq 2$ elementary fields at leading order, since the vev contribution to $H_2$ is null.

Possible to use further covariant derivatives, but cannot expand to 1 elementary field, as there are none with other Lorentz quantum numbers.
6 Spontaneous symmetry breaking in 2HDMs

If absolute minimum not unique (up to gauge transformations), fixing $\Sigma_5 \phi_0 = \phi_0$ is in conflict with a global symmetry.

If the symmetry is spontaneously broken, then such a would-be global symmetry of the model is explicitly broken by an infinitesimal parameter.

It may occur in 2HDMs (lattice Lewis and Woloshyn (2010)) depending on the Higgs potential.

Finite lattice, no spontaneous symmetry breaking: estimate the results for the infinite-volume limit and then extrapolate the estimates to $J \rightarrow 0$

Lewis and Woloshyn (2010): Done for continuous symmetry breaking in a 2HDM.
7 The FMS mechanism

Frohlich, Morchio, and Strocchi (1981) (group-theory) correspondence:

\[ H_1 \Leftrightarrow \frac{v}{\sqrt{2}} \phi_0 \text{(fixes gauge coordinate system)} \]

gauge – invariant states \( \Leftrightarrow \) elementary gauge – dependent fields

one-to-one (set of primitive states), except for would-be Goldstone bosons

\( \phi_0^\dagger \Sigma_4 \Sigma_j H_1 \) disappear from the spectrum,

\( \Sigma_4 \Sigma_j \) is skew-adjoint so \( H_1^\dagger \Sigma_4 \Sigma_j H_1 = 0 \).
Complete expansion:

\[ 2H_1^\dagger H_1 = v^2 + 2v\phi_0^\dagger \phi + \phi^\dagger \phi \]

\(v\phi_0^\dagger \phi\), \(\phi^\dagger \phi\) same quantum numbers, to distinguish:

- approximately by the energy spectrum
- or in perturbation theory.

recall KLN theorem sum all initial and final states (incl. soft photons) with same quantum number in a energy window \(\Rightarrow\) infrared finite.
Assuming $\phi^\dagger \phi \approx$ scattering state, energy spectrum $\gtrsim 2m_H \neq m_H$.

In perturbation theory, for asymptotic state the mass is on-shell $m_H$, so contribution from $\phi^\dagger \phi$ negligible.

For intermediate states, since the (gauge-invariant) Lagrangian is the same no deviations expected.

Calculating the spectrum and testing the FMS mechanism in the lattice is an extension of Maas (2013); Wurtz and Lewis (2013); Maas and Mufti (2014, 2015)

We must still account for precision electroweak observables

In any case, Frohlich et al. (1981) “standard perturbation expansion cannot be asymptotic to gauge-dependent correlation functions.”
8 \( \text{Spin}(4) \) symmetric 2HDM for the lattice

\[
V(\phi) = \mu_0 \phi^\dagger \phi + \mu_5 \phi^\dagger \Sigma_5 \phi \\
+ \frac{1}{2} \lambda_{00} (\phi^\dagger \phi)^2 + \lambda_{05} (\phi^\dagger \phi)(\phi^\dagger \Sigma_5 \phi) + \frac{1}{2} \lambda_{55} (\phi^\dagger \Sigma_5 \phi)^2
\]

To avoid breaking the \( \text{Spin}(4) \) group, \( \pm \Sigma_5 \phi_0 = \phi_0 \). For \( \lambda_{05} = 0 \):

1. (“control sample”) \( \mu_5 > 0, \lambda_{55} = 0 \), realistic Bhupal Dev and Pilaftsis (2014) Maximally-Symmetric 2HDM.

2. \( \mu_5 \to 0 \) with \( \mu_5 > 0 \) and \( \lambda_{55} \neq 0 \), spontaneous symmetry breaking of the discrete \( Z_4 \).

3. \( \mu_5 \to 0 \) with \( \mu_5 > 0 \) and \( \lambda_{55} = 0 \), spontaneous symmetry breaking of the continuous \( \text{Spin}(5) \to \text{Spin}(4) \). 4 massless Goldstone bosons.
\[ < H_1^\dagger(y) H_1(y) H_1^\dagger(x) H_1(x) > \text{ and } < H_2^\dagger(y) H_2(y) H_2^\dagger(x) H_2(x) > \]

After gauge fixing, we can expand them as:

\[ < H_1^\dagger(y) H_1(y) H_1^\dagger(x) H_1(x) > \approx \frac{v^4}{4} + \frac{v^2}{2} < \varphi^\dagger(y) \phi_0 \phi_0^\dagger \varphi(x) > + \ldots \]
\[ < H_2^\dagger(y) H_2(y) H_2^\dagger(x) H_2(x) > = < \varphi_2^\dagger(y) \varphi_2(y) \varphi_2^\dagger(x) \varphi_2(x) > \]

where \( \varphi_2 \equiv \phi_0^\dagger \Sigma_4 \varphi \).

Neglecting interactions, energy spectrum \( \gtrsim m_h \) and \( \gtrsim 2m_H \).

\( \mu_5 \to 0 \), check if \( Z_4 \) symmetry is recovered.

If \( \mu_5 = 0 \) by definition the correlations are \( Z_4 \) symmetric.
9 Summary

Assuming gauge symmetry breaking or using only complex representations of groups is not enough to study the phenomenology of multi-Higgs-doublet models.

For multi-Higgs-doublets, the FMS mechanism justifies that the spectrum is well described by the gauge-dependent elementary states.

If not, the physical states would, as in QCD, require non-perturbative methods, even at weak coupling.
The assumptions:
the field fluctuations around the vacuum are small in average and there is spontaneous symmetry breaking of the global symmetry when the gauge orbit minimizing the Higgs potential is not unique.

To confirm the FMS mechanism and assumptions requires non-perturbative calculations, next step.

(Addition of photons and fermions in 1601.02006)
Next?

Classical electrodynamics: gauge-invariant local states

Quantum U(1) gauge: either
gauge-invariant non-local states or gauge-dependent local states

Quantum SU(2) gauge: gauge-invariant local states (Higgs mechanism)
Quantum SU(3) gauge: gauge-invariant local states (confinement)

To me, we need to look for gauge-invariant local states in U(1).
Implies probabilities instead of amplitudes Weinberg (2014), in-in formalism
And to work with phaseless (real) operators Pedro (2013)

In the mean time, gauge-dependent local states in U(1)
option 1) ok for abelian Higgs mechanism (Ginzburg-Landau Superconductivity),

the Higgs mechanism is based in the fact:

breaking local gauge symmetries \neq global symmetries

the Goldsone theorem does not apply
the Nambu-Goldstone bosons may be absent.

Englert (2014) “The vacuum is no more degenerate and strictly speaking there is no spontaneous symmetry breaking of a local symmetry.[...]

The disappearance of the NG boson is thus an immediate consequence of local symmetry. The above argument (Englert, 2005) was formalized much later (Elitzur, 1975)”
10 Introducing Photons

$U(1)_Y$ gauge symmetry with generator $\Sigma_1 \Sigma_2$:

background symmetry: $(U(1)_Y \times Spin(3)) \rtimes Z_4$
custodial $Spin(3)$ generators $\Sigma_3 \Sigma_4$, $\Sigma_3 \Sigma_5$, $\Sigma_4 \Sigma_5$
$Z_4$ generated by the charge reversal transformation $\phi \rightarrow \Sigma_2 \Sigma_3 \phi$.

The $U(1)_Y \times Spin(3)$ is a normal subgroup.
Any transformation is the product of:
element of $U(1)_Y \times Spin(3)$ and element of $Z_4$.

Parity and charge reversal are conserved separately in the absence of fermions
under charge reversal $B_\mu \rightarrow -B_\mu$.

Neutral vacuum condition: $\phi_0$ aligned along linear combination of $\Sigma_{3,4,5}$. 

Implications of gauge symmetry | Leonardo Pedro
11 Introducing Fermions

Quark field $Q_L, \Sigma_1 \Sigma_2 Q_L = iQ_L, \Sigma_5 Q_L = Q_L$ and transforming under $SU(2)_L$ as $\phi$.

Most general Yukawa couplings with the quarks:

$$-\mathcal{L}_{Y_Q} = \overline{Q_L} \Gamma_d \phi \ d_R + \overline{Q_L} \ \Sigma_3 \Sigma_1 \Gamma_u \phi \ u_R + \text{h.c.}$$

$$\Gamma_w \equiv \Gamma_w^0 + \Gamma_w^1 \Sigma_3 \Sigma_4 + \Gamma_w^2 \Sigma_4 \Sigma_5 + \Gamma_w^3 \Sigma_5 \Sigma_3)$$

$\Gamma_{wa}$ self-conjugate and acting as real scalars on $\phi$

$w = u, d$ and $a = 0, 1, 2, 3$.

The custodial $Spin(3)$ group acts on $\phi$ and $\Gamma_w^\dagger$ in the same way, the product $\Gamma_w \phi$ is $Spin(3)$ invariant.
By reparametrization of $\Gamma_w$, $\Sigma_5 \phi_0 = \phi_0$. In this basis

$$H_1 \equiv \frac{1-i\Sigma_1 \Sigma_2}{2} \Sigma_5 \phi,$$

$$H_2 \equiv \Sigma_4 \Sigma_5 \frac{1-i\Sigma_1 \Sigma_2}{2} \Sigma_2 \phi,$$

$$\tilde{H}_j \equiv \Sigma_3 \Sigma_1 H_j^*.$$

$$-\frac{\nu}{\sqrt{2}} \mathcal{L}_{YQ} = \overline{Q}_L H_1 M_d d_R + \overline{Q}_L H_2 N^0_d d_R$$

$$+ \overline{Q}_L \tilde{H}_1 M_u u_R + \overline{Q}_L \tilde{H}_2 N^0_u u_R + \text{h.c.},$$

where $M_w \equiv \Gamma_{w0} + i\Gamma_{w1}$, $N^0_w \equiv \Gamma_{w3} + i\Gamma_{w4}$.

Majorana masses in seesaw I ($\nu$MSM) gauge singlets $\Rightarrow$ nonperturbative $\checkmark$
\[ H_1^\dagger iD_\mu \Sigma_1 \Sigma_3 H_1 \ (W^-_\mu) \]
\[ \cos \theta_W H_1^\dagger iD_\mu H_1 - \sin \theta_W \frac{g v^2}{4} B_\mu \ (Z_\mu) \]
\[ A_\mu \equiv \sin \theta_W H_1^\dagger iD_\mu H_1 + \cos \theta_W \frac{g v^2}{4} B_\mu \ (A_\mu) \]
\[ H_1^\dagger H_1 \ (h) \]
\[ H_1^\dagger \Sigma_4 H_2 \ (R) \]
\[ H_1^\dagger \Sigma_3 H_2 \ (I) \]
\[ H_1^\dagger \Sigma_1 H_2 \ (H^+) \]
\[ \tilde{H}_1^\dagger Q \ (d_L) \]
\[ \tilde{H}_1^\dagger Q \ (u_L) \]
\[ H_1^\dagger L \ (e_L) \]
\[ \tilde{H}_1^\dagger L \ (\nu_L) \]
\[ \Sigma_5 \phi_0 = \phi_0 \]
\[ H_1 \equiv \frac{1-i\Sigma_1 \Sigma_2}{2} \frac{1+\Sigma_5}{2} \phi \]
\[ H_2 \equiv \Sigma_4 \Sigma_5 \frac{1-i\Sigma_1 \Sigma_2}{2} \frac{1-\Sigma_5}{2} \phi \]
Yang (1952) “It is the purpose of the present paper to calculate the spontaneous magnetization (i.e., the intensity of magnetization at zero external field) of a two-dimensional Ising model of a ferromagnet.”

Spontaneous as particular case of Explicit symmetry breaking. Other equivalent (for the Ising model) definitions:

“The spontaneous magnetization $I$ per atom is exactly the usual long-range order parameter $s$ which may be defined as the average of the absolute value of the total spin of the lattice divided by the number of atoms.

That $I$ is equal to $s$ is easily seen-from the fact that the introduction of a vanishingly weak positive magnetic field merely cuts out all states of the lattice for which the total spin is negative.”
’t Hooft (1980) “the words “spontaneous breakdown” are formally not correct for local gauge theories. The vacuum never breaks local gauge invariance because it itself is gauge invariant.

The neutral intermediate vector boson is the “meson”

\[ \phi^\dagger D_\mu \phi = i \frac{g v^2}{4} W_\mu^3 + \text{total derivative} + \text{higher orders} \]

The \( W^\pm_\mu \) are obtained from the “baryons” \( \epsilon_{ij} \phi^i D_\mu \phi^j \), and the Higgs particle can also be obtained from \( \phi^\dagger \phi \).

Is there no fundamental difference then between a theory with spontaneous breakdown and a theory with confinement? Sometimes there is. In the above example the Higgs was a faithful representation of \( SU(2) \). This is why the above procedure worked.”

Fröhlich et al. (1981) “the relevant feature is the structure of the residual group.” (defined by the minimizing orbit of the Higgs potential)
Implications for one Higgs doublet from lattice sim.:

Maas and Mufti (2015) $m_h < m_W \Rightarrow$ non-perturbative effects rule

Gies and Sondenheimer (2015) (top-bottom-Higgs system)

non-perturbative effects affect (in)stability of the Higgs potential

...and a big unknown
mostly unexplored, due to technology and mathematical limitations

Careful, non-perturbative effects for small couplings are common
e.g. Hydrogen energy levels
despite that for larger couplings, larger non-perturbative effects expected
Background symmetries

background field or spurion:

- fixed when minimizing the action
- non-trivial representation of background symmetries

when calculating observables, spurions $\Rightarrow$ numerical values.

observables invariant under group of background symmetries

Ivanov (2006) reparametrization
Haber and Surujon (2012) basis transformation that do not change the Lagrangian’s functional form or spurion analysis
Botella et al. (2013) weak-basis transformations
Georgi (2009) spurions as source fields
11.1 A contribution for a systematic search for FCNCs

BGL analysis code: cftp.ist.utl.pt/~leonardo

Test on 2HDM type II:

<table>
<thead>
<tr>
<th>$M_{H^+}$ (GeV)</th>
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<tr>
<td>900</td>
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<td>200</td>
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<td>100</td>
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</tbody>
</table>

$\log_{10}(\tan\beta)$

Giac and Ginac (C++ algebra systems) available (also Flavour Kit, etc.);

Using LLVM the code generation of GiNaC can be improved;

Library containing many known formulas for decays important for FCNCs; (contribution)

Library making global fits, from models, from formulas, from experimental data (contribution)

Library containing the experimental data distributions.

(CERN-based ROOSTATS-like package for FCNCs?)
Beyond the Standard Model

Branco and Emmanuel-Costa (2014) the simplest scheme to break spontaneously:

\[ SU(3)_C \times SU(2)_L \times U(1)_Y \rightarrow SU(3)_C \times U(1)_{em} \]

⇒ One Higgs Doublet

Ellis (2013) QCD, ElectroWeak, Flavour ⇒ Experiments ✓

\[ \nu \text{ masses and mixing, baryon asymmetry, dark matter, CMB fluctuations} \]

⇒ New Physics

Altarelli (2014) νMSM(3 ν_R, Seesaw I) + inflaton field ⇒ Experiments ✓

gravity; cosmological const. (dark energy); hierarchy; strong CP; arbitrariness; meta-stability; non-perturbative definition; accidental suppression of FCNCs, EDMs, p^+ decay
Contributions from Social Sciences

PDG (2014)

Wanke (2013) “significant jumps, pointing either to a common systematic shift or to the effect of biased analyses.”

Borrelli (2013) presented at CERN “As far as theorists are concerned, the role of personal skills seems to be a major factor in the choice of models to work on.”

Kahnemann (2002) Nobel in Economics Lecture “people rely on a limited number of heuristic principles which reduce the complex tasks of assessing probabilities and predicting values to simpler judgmental operations. In general, these heuristics are quite useful, but sometimes they lead to severe and systematic errors.”
Correlations are important in data analysis to attribute to new physics a deviation in data.
Top-down and Bottom-up approaches

Buras and Girrbach (2014)

models (e.g. BGL) vs. effective field theory (e.g. MFV)

- correlations between observables
  - low/high energy, all flavours, hadronic/leptonic
- less sensitive to free parameters
- patterns of flavour violation
- may differ from the SM and MFV

For BGL:

\[ \text{Br}(B_s \rightarrow \mu\mu)/10^{-9} \]

\[ \text{Br}(B \rightarrow \mu\mu)/10^{-10} \]

Extend the scalar sector to study Higgs mechanism and SM problems.
But constrain FCNCs, Flavour and CP violation pattern accounted by SM.