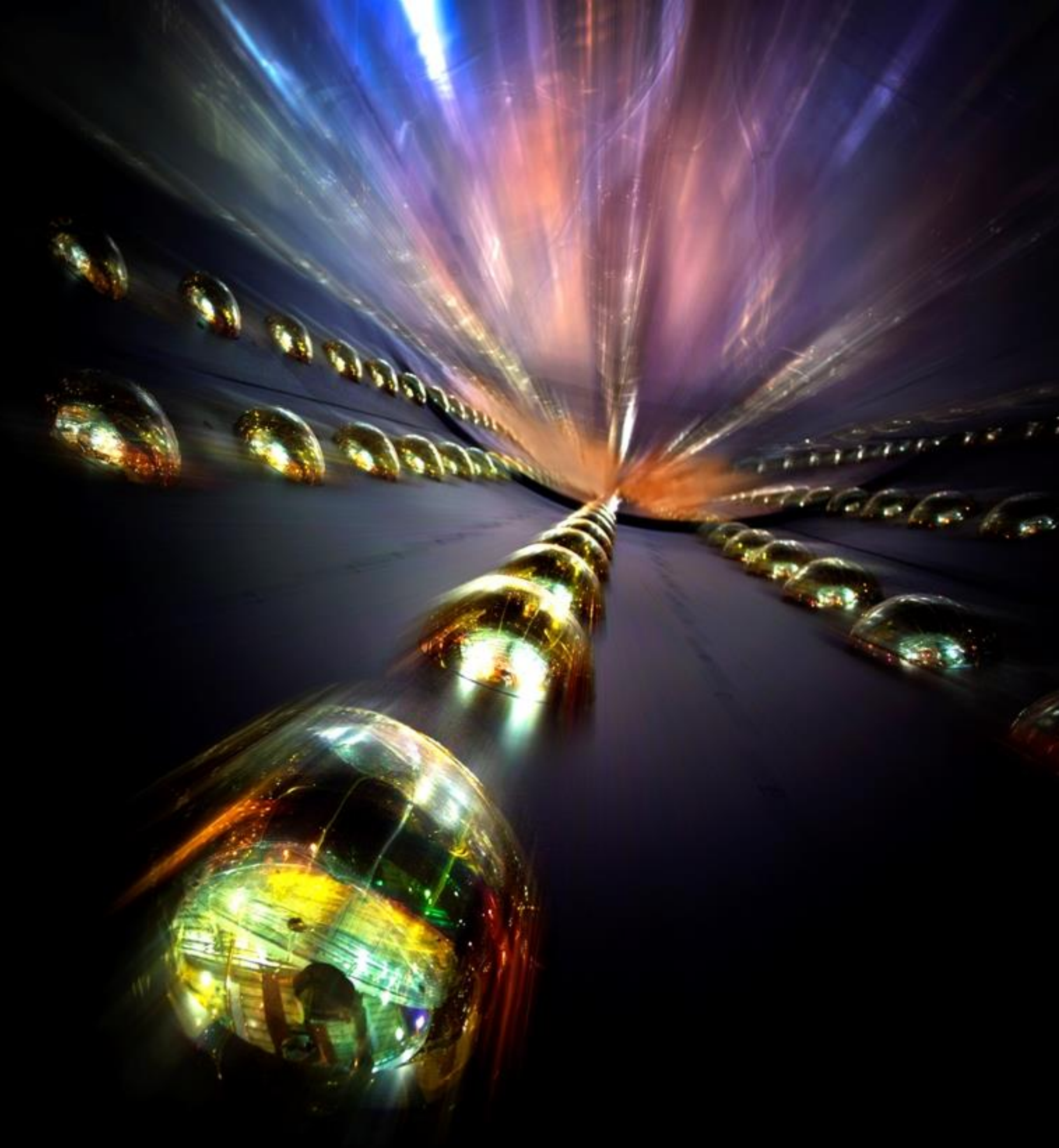

1st Lisbon Mini-School on Particle and astroparticle Physics

2-4 February 2016

Hotel Ever – Costa da Caparica

Hands on Neutrinos

I. BRIEF HISTORY OF NEUTRINOs

The neutrino was first postulated by Wolfgang Pauli in 1930 to explain how beta particles emitted in beta decay could have a continuous energy spectrum, without violating the principle of energy, linear and angular momentum. Pauli hypothesized a neutral (and, therefore, undetected) particle that he called “neutron”. This new particle would be emitted together with the electron and share its energy, thus explaining the continuous spectrum of the electron energy (see Fig.1).

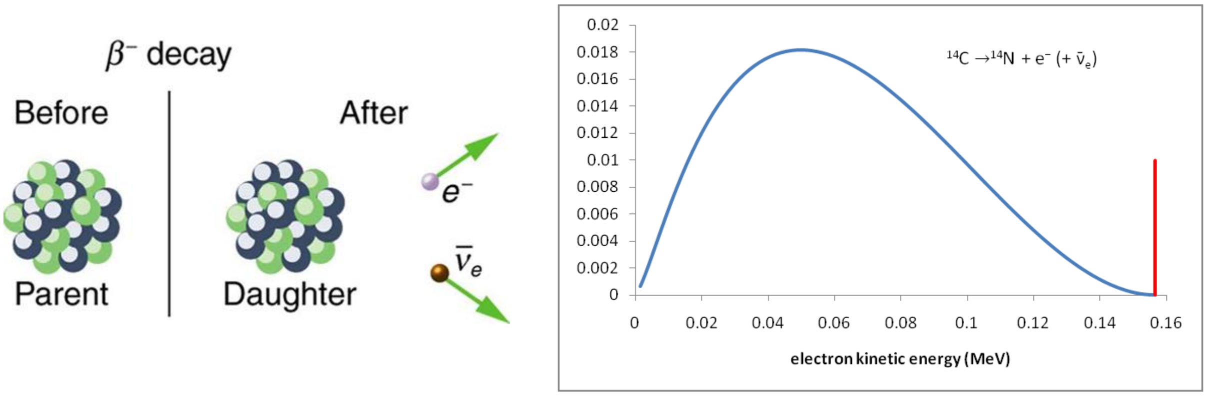

Fig. 1 – (Left) Representation of β decay. A parent Nucleus decays into a daughter nucleus with emission of a very light neutral particle, the (anti)neutrino $\bar{\nu}_e$. (Right) β-decay spectrum (blue solid line) and the would-be spectrum if only the electron is emitted (vertical red line).

In 1932, James Chadwick discovered a “heavy” nuclear neutral particle and also named it a neutron. The name "neutrino" (which in Italian would mean “little neutral one”) for Pauli’s particle was proposed by Enrico Fermi, who started using it during a conference in Paris in July 1932 and at the Solvay Conference in October 1933.

The publication of the experimental neutrino detection happened in the 20 July 1956 issue of the journal *Science*, where Clyde Cowan, Frederick Reines and collaborators confirmed that they had detected Pauli’s neutrino. For this discovery Reines was awarded the 1995 Nobel Prize (shared with Martin Perl). In this experiment, known today as the Cowan–Reines neutrino experiment, antineutrinos produced in a nuclear reactor by β decay interact with protons to produce neutrons (n) and positrons (e^+): $\bar{\nu}_e + p \rightarrow n + e^+$. Posteriorly, the positron finds an electron, producing two gamma rays (γ) which are detectable. The neutron is subsequently captured by a nucleus, releasing another photon. The coincidence of both events – positron annihilation and neutron capture –provides a unique signature of an (anti)neutrino interaction.

In 1962, Leon M. Lederman, Melvin Schwartz and Jack Steinberger found that, besides the electron one, there is another kind of neutrino. This new neutrino was first detected by looking muon interactions, and it was therefore called muon neutrino (the 1988 Nobel Prize in Physics was awarded for the discovery of this particle). Later, in 1975, a third type of lepton, the tau, was discovered at the Stanford Linear Accelerator Center (SLAC). Similarly to what happened for the muon and the electron, it was expected that the tau neutrino would also exist. The first evidence for this particle arose from the observation of missing energy and momentum in tau decays (which are analogous to beta decay), and the actual detection of tau-neutrino interactions was announced in 2000 by the DONUT collaboration at Fermilab in the United States. Before that, the existence of this particle had already been inferred by both theoretical consistency and experimental data from the Large Electron–Positron Collider (LEP) at CERN. At this point, it was established that neutrinos come in three flavours: the electron (ν_e), the muon (ν_μ), and the tau (ν_τ) neutrino.

Starting in the late 1960s, several experiments concluded that the number of electron neutrinos arriving from the Sun was between 1/3 and 1/2 of the number predicted by the model which describes the dynamics of the Sun: the Standard Solar Model. Such discrepancy, which rapidly became known as the solar neutrino problem, lacked from a definite solution for about thirty years. Only recently the problem was solved. The solution relies on the fact that neutrinos oscillate between flavours and, therefore (as you will show), must be massive (contrarily to what is predicted by the Standard Model of particle physics).

In this activity we want you to study the basics of neutrino oscillations.

II. HANDS ON NEUTRINO OSCILLATIONS

As you have learned in Quantum Mechanics I, quantum systems are described by states. Therefore, at the level of elementary particles, where quantum mechanics is obviously at work, we can describe a neutrino of a given flavor X as being represented by a state $|\nu_X\rangle$. We call these states "flavor eigenstates". Let us take the case of only two flavors: ν_e and ν_μ, associated to quantum states $|\nu_e\rangle$ and $|\nu_\mu\rangle$. Moreover, we will consider that these states are not mass eigenstates in the sense that they do not coincide with the eigenstates of the Hamiltonian for a free particle with mass m_i and energy $E_i^2 = p_i^2 c^2 + m_i^2 c^4$. Therefore, we will consider that the two flavor eigenstates $|\nu_e\rangle$ and $|\nu_\mu\rangle$ are quantum superpositions of the two mass eigenstates $|\nu_1\rangle$ and $|\nu_2\rangle$.

1. The first thing we want you to do is to write $|\nu_e\rangle$ and $|\nu_\mu\rangle$ as a combination of $|\nu_1\rangle$ and $|\nu_2\rangle$. You should choose a parameterization with only one parameter (think about the best way to do it). Remember… These quantum superpositions should obey the probability conservation law of quantum mechanics and should be orthogonal.

Suppose now that at $t = 0$ an electron neutrino described by the state $|\nu_e\rangle$ is produced (for instance at the Sun) as a result of some some nuclear reaction. Taking into account that the propagation of mass eigenstates follows the time-dependent Schrödinger equation:

$$i\hbar \frac{\partial \,|\, \nu_i\,(t)\rangle}{\partial t} = H|\,\nu_i(t)\rangle\,,$$

2. Obtain $|\nu_e(t)\rangle$, which represents your flavor state at any instant of time t.
3. What is the probability that, at a time t your electron neutrino has oscillated into a muon neutrino?
4. What are the necessary conditions for neutrino oscillations to occur?

CONGRATULATIONS! YOU HAVE JUST DONE NOBEL PRIZE PHYSICS!

Consider now that you have at the Earth an experiment which is able to detect electron neutrinos coming from the Sun with a certain energy E.

5. Considering that neutrinos travel a distance L from the Sun to Earth in vacuum, and that the flux of electron neutrinos coming out from the Sun is $\Phi_e^{\odot}$, how do you express the flux of neutrinos detected by your (supposedly perfect) experiment $\Phi_e^{\oplus}$?

As you must have concluded by now, the neutrino oscillation frequency is given by

$$\frac{\Delta m^2 L}{2E} \ , \qquad \Delta m^2 = m_2^2 - m_1^2 \ ,$$

where $m_{1,2}$ are the neutrino masses (fixed by Nature), L (the distance travelled by neutrinos), and E (their energy). In principle, it is not possible to measure oscillation probabilities for precise values of the propagation distance L and the neutrino energy E. This is so because in a real experiment both the source and detection processes have uncertainties. Namely, the source is not monochromatic and the energy resolution of the detector is finite. Therefore, in practice one has to average the oscillation probabilities by an appropriate distribution $\phi(L/E)$.

6. To illustrate the effect of this averaging, consider that $\phi(L/E)$ is a Gaussian distribution with mean value $\langle L/E\rangle$ and standard deviation $\sigma_{L/E}$. Obtain the averaged oscillation probability $\langle P_{\nu_e\to\nu_\mu}\rangle$ as a function of $\langle L/E\rangle\, \Delta m^2$. Plot the oscillation probabilities and the averaged ones for $\sigma_{L/E} = 0.2\,\langle L/E\rangle$, as a function of $\langle L/E\rangle$[Km/GeV] Δm^2[eV^2] in the case where your initial flavor neutrino state is a maximal admixture of $|\nu_1\rangle$ and $|\nu_2\rangle$. Discuss the results with your colleagues.

If your experiment does not observe any oscillation this means that your data imply an upper bound on the averaged transition probability, i.e $\langle P_{\nu_e\to\nu_\mu}(L/E)\rangle \leq P_{\nu_e\to\nu_\mu}^{max}$.

7. Show that that this imposes constraints on how neutrinos mix among each other. If for a certain experiment $P^{max}_{\nu_e \to \nu_\mu} = 0.1$, and considering that $\sigma_{L/E} = 0.2\, \langle L/E \rangle$, show how your experiment constrains the neutrino mixing angle as a function of $\langle L/E \rangle$[Km/GeV] Δm^2[eV^2].