CP-odd (basis) invariants

Ivo de Medeiros Varzielas [ivo.de@udo.edu]

CFTP, IST, UL

Sesimbra, 2017/02/08

Why study CP Violation (CPV) in Portugal?

CPV is extremely important:
necessary ingredient for matter-antimatter asymetry
- we wouldn't exist without it!

And... There must be additional sources of CPV:
One of few guarantees of Physics Beyond the Standard Model

Top experts on CPV (in the world) work in Portugal!

CP in Portugal...

But not this...

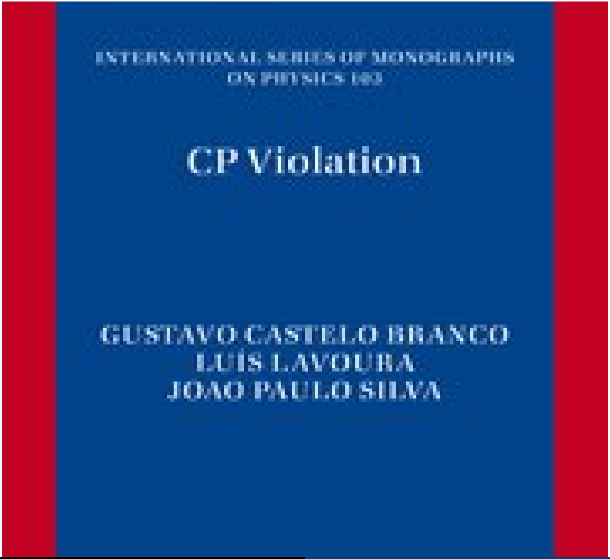
COMBOIOS DE PORTUGAL

Fermion CP...

Scalar CP...

There is even a book about it

By Portuguese authors B. L. S.



How to study CPV

You want to study CPV. But... How?

Previously on "Symmetries in particle physics"

Vectors transform in specific ways under frame transformations (e.g. rotation by an angle θ)
$(x, y) \rightarrow (x', y') = (x\cos\theta + y\sin\theta, -x\sin\theta + y\cos\theta)$

But one can build, from vectors, quantities that are frame-independent:
Squared length of a vector (x, y), is $x^2 + y^2 = x'^2 + y'^2$

More generally, scalar product of two vectors
$(x_1, y_1).(x_2, y_2) = x_1x_2 + y_1y_2 = (x'_1, y'_1).(x'_2, y'_2) = x'_1x'_2 + y'_1y'_2$

Invariants are very important as Physics should not depend on "arbitrary" choice of frame!

Electron mass Lagrangian

$$-\mathcal{L}_m = m_e \overline{e}_L e_R + h.c. \,, \tag{1}$$

where the LH electron e_L is inside the “doublet” $L = (e_L, \nu_L)$ with the LH neutrino in a (weak interaction) basis;
e_R, the RH counterpart

In QED, m_e is just a number (1×1 matrix)
In SM, with 3 generations, it gets generalized to a 3×3 matrix

General CP transformations

In SM, L and e_R are 3-d "vectors" in "generation space" (triplets)
e.g. $e_R = (e_{R1}, e_{R2}, e_{R3})$

General CP transformation

$$L \to X_L L^* \tag{2}$$
$$e_R \to X_R e_R^* \tag{3}$$

X_L and X_R are unitary matrices
(e.g. $X_L^\dagger X_L = X_L X_L^\dagger = 1$)
X_L, X_R need not be the unit matrix

CP invariance requirements

In order for $\mathcal{L}_m$ to be CP invariant, the term shown in:

$$-\mathcal{L}_m = m_e \overline{e}_L e_R + h.c. \,, \quad (4)$$

goes into the respective h.c. under CP, and vice-versa, i.e.:

$$X_L^\dagger m_e X_R = m_e^* \quad (5)$$

(the requirement is $m_e = m_e^*$ for the trivial case)

Building CP-odd invariants (CPIs)

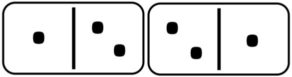
CPIs need to be combinations where X_L and X_R do not appear

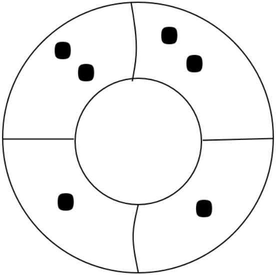
E.g. do $m_e m_e^\dagger$:
$m_e m_e^\dagger \rightarrow X_L^\dagger m_e X_R X_R^\dagger m_e^\dagger X_L = X_L^\dagger m_e m_e^\dagger X_L$

Taking the trace allows to cycle X_L around:
$\mathrm{Tr}\,(X_L^\dagger m_e m_e^\dagger X_L) = \mathrm{Tr}\,(X_L X_L^\dagger m_e m_e^\dagger) = \mathrm{Tr}\,(m_e m_e^\dagger)$

So for any CP transformation X_L, X_R
a requirement for CP conservation is
$\mathrm{Tr}\,(m_e m_e^\dagger) = (\mathrm{Tr}\,(m_e m_e^\dagger))^*$

It is kind of like playing with (mouldable) dominos





CPI for SM

Here is one that was prepared earlier

This is a necessary condition for CP conservation:

$$\left(\mathrm{Tr}\left[m_u m_u^\dagger, m_d m_d^\dagger\right]^3\right) = 0\,, \tag{6}$$

It works for any number of quark generations;
for 3 generations, it is sufficient.
This CPI gives the area of the quark unitarity triangle
(i.e. it is proportional to the Jarlskog invariant)

Thank you for your attention

E-mail:
ivo.de@udo.edu

Any questions?