# Relativistic phenomenology of meson spectra with a covariant quark model in Minkowski space

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# Outline

- From QCD to Hadron Physics
- Motivation
- Covariant Spectator Theory Bethe-Salpeter (CST-BS) Formalism
- Heavy and heavy-light mesons with CST-BS
- Numerical solution of CST-BS
- Results and the predictive power of covariant interaction kernels
- Summary and Outlook

For further details see: arXiv:1608.08065

# From QCD to Hadron Physics

- The dynamical content of QCD as a *local* quantum field theory of quarks and gluons is described by its Lagrangian or, equivalently, its action.

- *But*, to unfold
  the *physical content* of QCD Lagrangian *towards* a *quantum-field theoretical* description of hadrons, is a *very* difficult task!

- @ large energies & small distances (ultraviolet region)
  interaction between quarks and gluons is weak and perturbative methods *can* be used;

  @ low energies & large distances (infrared region)
  coupling becomes strong and perturbation theory "fails"

  - dynamical chiral symmetry breaking
  - generation of large constituent quark masses from almost massless quarks
  - formation of hadrons (mesons, baryons)
  - confinement of quarks and gluons inside hadrons

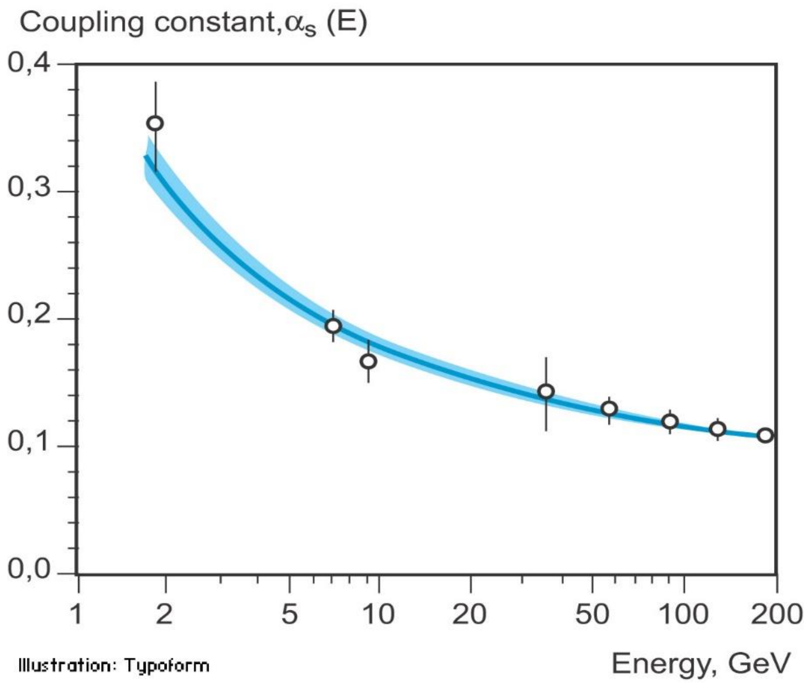


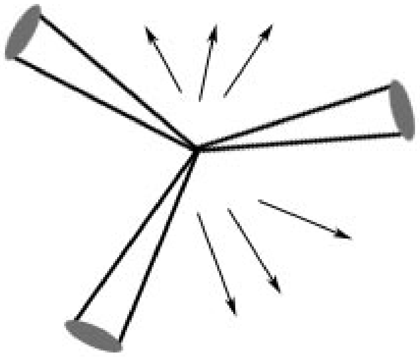
Illustration: Typoform

# Experiments addressing QCD

- From the 1960s to 1990s, $e^+e^-$ colliders evolved from low center-of-mass energies $\sqrt{s}\sim 1 GeV$ with modest luminosity to the Large Electron Positron (LEP) collider with $\sqrt{s}$ up to $209\ GeV$ and a vastly greater luminosity.

$e^+e^-$ colliders

→ Along the way, the $e^+e^-$colliders PETRA (at **DESY**) and PEP (at **SLAC**) saw the first three-jet events.



Since jets are ordinarily produced when quarks hadronize, and quarks are produced only in pairs, an **additional particle** is required to explain events containing an odd number of jets.

Quantum chromodynamics indicates that this particle is a particularly energetic **gluon**, radiated by one of the quarks, which hadronizes much as a quark does.

- End of 1990s – two $B$-factories at **KEK** and **SLAC** and the operation of low energy, high-intensity colliders in Beijing, Cornell, Frascati, and Novosibirsk:

$B$-factories

→ good for studies of *quarkonium* physics and decays of open *charm* and *bottom* mesons;

# Experiments addressing QCD

→ copious production of τ leptons at $e^+e^-$ colliders led to a way to measure $\alpha s$ via their hadronic decays. Measurements of the hadronic cross section at various energy ranges play a useful role in understanding the interplay of QCD and QED.

$e^+e^-$ colliders

- Experiments with $e^-$, μ, $\nu$, $\gamma$, or hadron beams impinging on a fixed target have been a cornerstone of QCD:

Fixed target experiments

→ Early studies of deep inelastic scattering at **SLAC** led to the parton model. This technique and the complementary production of charged lepton pairs (the so-called Drell–Yan production) - understanding proton structure.

→ Later, HERA continued this theme with $e^-\,p$ and $e^+\,p$ colliding beams: besides nucleon structure and it made important contributions to strangeness and charm physics, as well as to the spectroscopy of light mesons and non-SM particles such as leptoquarks. This line of research continues to this day at **Jefferson Lab**, **J-PARC**, Mainz, **Fermilab**, and **CERN**; future, post-HERA *ep* colliders are under discussion.

# Experiments addressing QCD

Hadron colliders

- The history of hadron colliders started in 1971 with *pp* collisions at **CERN**'s Intersecting Storage Rings (ISR) [center-of-mass energy of 30 $GeV$]

  → The ISR ran for more than 10 years with *pp* and $p\bar{p}$ collisions, as well as with ion beams: *pd*, *dd*, *pα*, and *αα*. During this time, its luminosity increased by three orders of magnitude. This machine paved the way for the successful operation of proton–antiproton colliders:
    - ✓ the $Sp\,\bar{p}S$ at **CERN** [$\sqrt{s} = 630\ GeV$] in the 1980s,
    - ✓ $p\bar{p}$ Tevatron at **Fermilab** [$\sqrt{s} = 1.96\ TeV$], which ran until 2011.

- Currently, the Large Hadron Collider (LHC) collides *pp* beams at the highest energies in history, with a design energy of *14 TeV* and luminosity four orders of magnitude higher than the ISR.

  → Physics at these machines started from studies of jets at the ISR and moved to diverse investigations including proton structure, precise measurements of the W mass, searches for heavy fundamental particles leading to discoveries of the *top quark* and *Higgs*, *production of quarkonia*, and *flavor physics*.

# Experiments addressing QCD

- At the same time, pioneering experiments with light ions ($A \sim 14$) at relativistic energies started in the 1970s at **LBNL** in the United States and at **JINR** in Russia. The program continued in the 1980s with fixed-target programs at the CERN SPS and BNL AGS. These first experiments employed light-ion beams (A ~ 30) on heavy targets (A ~ 200). In the 1990s, the search for the quark–gluon plasma continued with truly heavy-ion beams (A ~ 200)

Ion colliders

- In this era, the maximum $\sqrt{s_{NN}} \sim 20\ GeV$. With the new millennium the heavy-ion field entered the collider era, first with the Relativistic Heavy-Ion Collider (RHIC) at **BNL** at $\sqrt{s_{NN}} \sim 200\ GeV$ and, in 2010, the LHC at **CERN**, reaching the highest currently available energy, $\sqrt{s_{NN}} \sim 2.76\ TeV$.

Heavy-ion colliders

→ The goal of heavy-ion physics is to map out the nuclear matter phase diagram. Proton-proton collisions occur at zero temperature and baryon density, while heavy-ion collisions can quantify the state of matter of bulk macroscopic systems. The early fixed-target experiments probed moderate values of temperature and baryon density. The current collider experiments reach the zero baryon density, high-temperature regime, where the quark–gluon plasma can be studied under conditions that arose in the early universe.

- To reach the needed temperature and baryon density, two new facilities—FAIR at **GSI** and NICA at **JINR**—are being built.

# Motivation

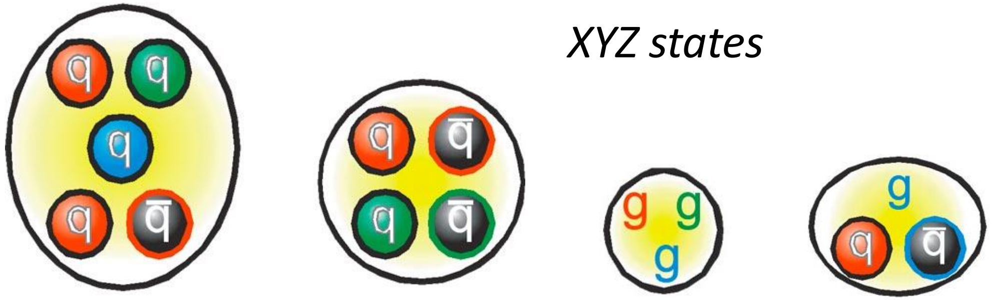
- The physics of mesons, in particular, is a very active area of research, especially due to the ample amount of new experimental data measured at facilities such as the LHC, BaBaR, Belle, CLEO, and more exciting results can also be expected from **Jefferson Lab** (GlueX) and FAIR (PANDA) in the near future.

  → *spectroscopy: classification of mesons (new states)*

Normal meson

??

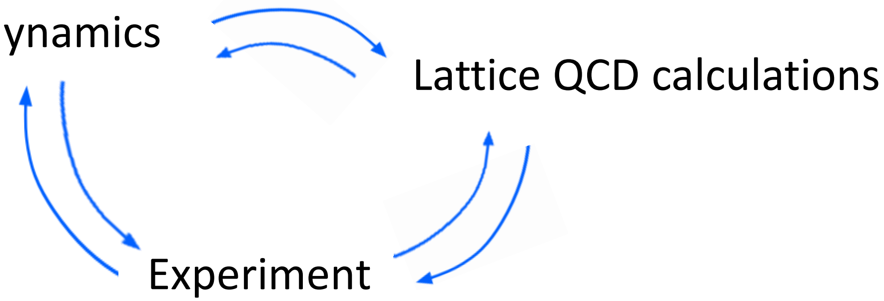
*XYZ states*



Pentaquark Tetraquark Glueball Hybrid meson

  → *structure: form factors (*Minkowski can be more convenient than Euclidean formulations because form factors can be computed directly in the timelike region with no need for analytical continuations).

- Models with *testable* dynamics



Lattice QCD calculations

Experiment

# New states

- (Mini-review) Developments in heavy quarkonium spectroscopy:

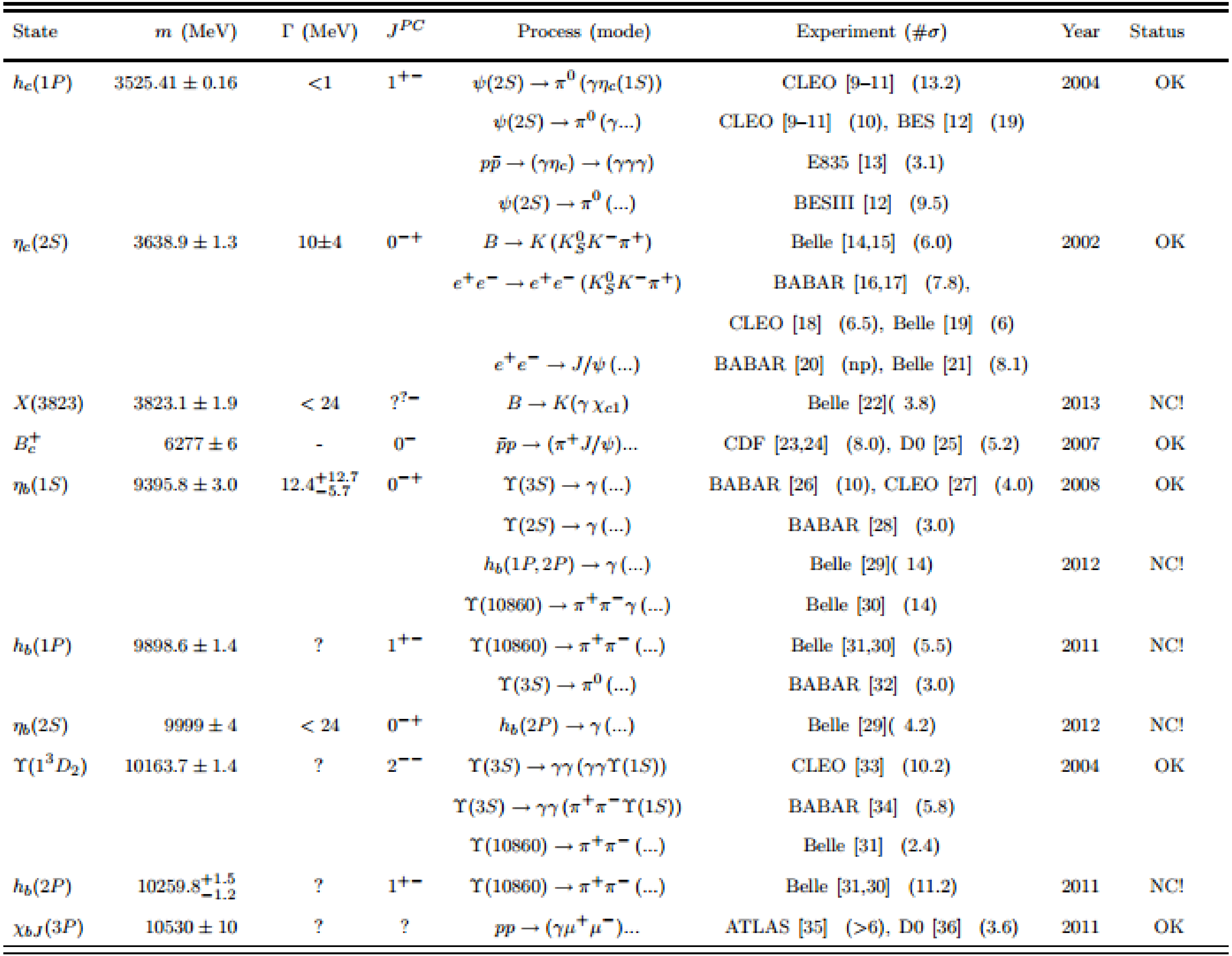
> "A golden age for heavy quarkonium physics dawned at the turn of this century, initiated by the confluence of exciting advances in quantum chromodynamics (QCD) and an explosion of related experimental activity."

K.A. Olive *et al.* (Particle Data Group), Chin. Phys. C, **38**, 090001 (2014) and 2015

# New states

- New states **below the open flavor thresholds** in the $c\bar{c}$, $b\bar{c}$, and $b\bar{b}$ regions, ordered by mass.

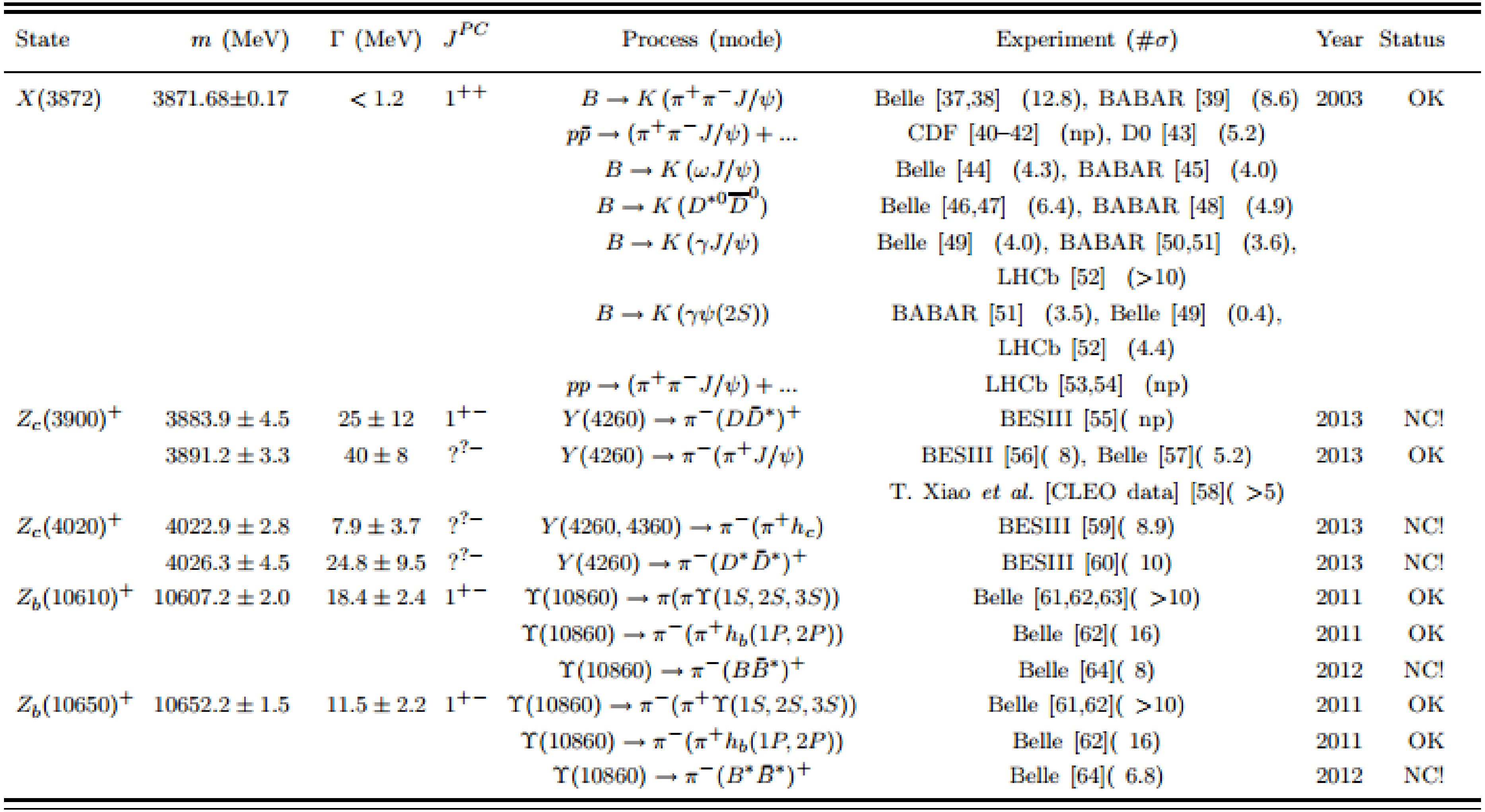
Normal meson



| State | $m$ (MeV) | $\Gamma$ (MeV) | $J^{PC}$ | Process (mode) | Experiment (#$\sigma$) | Year | Status |
|---|---|---|---|---|---|---|---|
| $h_c(1P)$ | $3525.41 \pm 0.16$ | $<1$ | $1^{+-}$ | $\psi(2S) \to \pi^0 (\gamma\eta_c(1S))$ | CLEO [9–11] (13.2) | 2004 | OK |
| | | | | $\psi(2S) \to \pi^0 (\gamma ...)$ | CLEO [9–11] (10), BES [12] (19) | | |
| | | | | $p\bar{p} \to (\gamma\eta_c) \to (\gamma\gamma\gamma)$ | E835 [13] (3.1) | | |
| | | | | $\psi(2S) \to \pi^0 (...)$ | BESIII [12] (9.5) | | |
| $\eta_c(2S)$ | $3638.9 \pm 1.3$ | $10\pm4$ | $0^{-+}$ | $B \to K (K_S^0 K^- \pi^+)$ | Belle [14,15] (6.0) | 2002 | OK |
| | | | | $e^+e^- \to e^+e^- (K_S^0 K^- \pi^+)$ | BABAR [16,17] (7.8), | | |
| | | | | | CLEO [18] (6.5), Belle [19] (6) | | |
| | | | | $e^+e^- \to J/\psi (...)$ | BABAR [20] (np), Belle [21] (8.1) | | |
| $X(3823)$ | $3823.1 \pm 1.9$ | $< 24$ | $?^{?-}$ | $B \to K(\gamma\,\chi_{c1})$ | Belle [22]( 3.8) | 2013 | NC! |
| $B_c^+$ | $6277 \pm 6$ | - | $0^-$ | $\bar{p}p \to (\pi^+ J/\psi)...$ | CDF [23,24] (8.0), D0 [25] (5.2) | 2007 | OK |
| $\eta_b(1S)$ | $9395.8 \pm 3.0$ | $12.4^{+12.7}_{-5.7}$ | $0^{-+}$ | $\Upsilon(3S) \to \gamma (...)$ | BABAR [26] (10), CLEO [27] (4.0) | 2008 | OK |
| | | | | $\Upsilon(2S) \to \gamma (...)$ | BABAR [28] (3.0) | | |
| | | | | $h_b(1P, 2P) \to \gamma (...)$ | Belle [29]( 14) | 2012 | NC! |
| | | | | $\Upsilon(10860) \to \pi^+\pi^-\gamma (...)$ | Belle [30] (14) | | |
| $h_b(1P)$ | $9898.6 \pm 1.4$ | ? | $1^{+-}$ | $\Upsilon(10860) \to \pi^+\pi^- (...)$ | Belle [31,30] (5.5) | 2011 | NC! |
| | | | | $\Upsilon(3S) \to \pi^0 (...)$ | BABAR [32] (3.0) | | |
| $\eta_b(2S)$ | $9999 \pm 4$ | $< 24$ | $0^{-+}$ | $h_b(2P) \to \gamma (...)$ | Belle [29]( 4.2) | 2012 | NC! |
| $\Upsilon(1^3D_2)$ | $10163.7 \pm 1.4$ | ? | $2^{--}$ | $\Upsilon(3S) \to \gamma\gamma (\gamma\gamma\Upsilon(1S))$ | CLEO [33] (10.2) | 2004 | OK |
| | | | | $\Upsilon(3S) \to \gamma\gamma (\pi^+\pi^-\Upsilon(1S))$ | BABAR [34] (5.8) | | |
| | | | | $\Upsilon(10860) \to \pi^+\pi^- (...)$ | Belle [31] (2.4) | | |
| $h_b(2P)$ | $10259.8^{+1.5}_{-1.2}$ | ? | $1^{+-}$ | $\Upsilon(10860) \to \pi^+\pi^- (...)$ | Belle [31,30] (11.2) | 2011 | NC! |
| $\chi_{bJ}(3P)$ | $10530 \pm 10$ | ? | ? | $pp \to (\gamma\mu^+\mu^-)...$ | ATLAS [35] (>6), D0 [36] (3.6) | 2011 | OK |

# New states

- New states **near the first open flavor thresholds** in the $c\bar{c}$ and $b\bar{b}$ regions, ordered by mass



| State | $m$ (MeV) | $\Gamma$ (MeV) | $J^{PC}$ | Process (mode) | Experiment (#$\sigma$) | Year | Status |
|---|---|---|---|---|---|---|---|
| $X(3872)$ | 3871.68±0.17 | $< 1.2$ | $1^{++}$ | $B \to K(\pi^+\pi^- J/\psi)$ | Belle [37,38] (12.8), BABAR [39] (8.6) | 2003 | OK |
| | | | | $p\bar{p} \to (\pi^+\pi^- J/\psi) + ...$ | CDF [40–42] (np), D0 [43] (5.2) | | |
| | | | | $B \to K(\omega J/\psi)$ | Belle [44] (4.3), BABAR [45] (4.0) | | |
| | | | | $B \to K(D^{*0}\overline{D}^0)$ | Belle [46,47] (6.4), BABAR [48] (4.9) | | |
| | | | | $B \to K(\gamma J/\psi)$ | Belle [49] (4.0), BABAR [50,51] (3.6),<br>LHCb [52] (>10) | | |
| | | | | $B \to K(\gamma\psi(2S))$ | BABAR [51] (3.5), Belle [49] (0.4),<br>LHCb [52] (4.4) | | |
| | | | | $pp \to (\pi^+\pi^- J/\psi) + ...$ | LHCb [53,54] (np) | | |
| $Z_c(3900)^+$ | $3883.9 \pm 4.5$ | $25 \pm 12$ | $1^{+-}$ | $Y(4260) \to \pi^-(D\bar{D}^*)^+$ | BESIII [55]( np) | 2013 | NC! |
| | $3891.2 \pm 3.3$ | $40 \pm 8$ | $?^{?-}$ | $Y(4260) \to \pi^-(\pi^+ J/\psi)$ | BESIII [56]( 8), Belle [57]( 5.2)<br>T. Xiao *et al.* [CLEO data] [58]( >5) | 2013 | OK |
| $Z_c(4020)^+$ | $4022.9 \pm 2.8$ | $7.9 \pm 3.7$ | $?^{?-}$ | $Y(4260, 4360) \to \pi^-(\pi^+ h_c)$ | BESIII [59]( 8.9) | 2013 | NC! |
| | $4026.3 \pm 4.5$ | $24.8 \pm 9.5$ | $?^{?-}$ | $Y(4260) \to \pi^-(D^*\bar{D}^*)^+$ | BESIII [60]( 10) | 2013 | NC! |
| $Z_b(10610)^+$ | $10607.2 \pm 2.0$ | $18.4 \pm 2.4$ | $1^{+-}$ | $\Upsilon(10860) \to \pi(\pi\Upsilon(1S, 2S, 3S))$ | Belle [61,62,63]( >10) | 2011 | OK |
| | | | | $\Upsilon(10860) \to \pi^-(\pi^+ h_b(1P, 2P))$ | Belle [62]( 16) | 2011 | OK |
| | | | | $\Upsilon(10860) \to \pi^-(B\bar{B}^*)^+$ | Belle [64]( 8) | 2012 | NC! |
| $Z_b(10650)^+$ | $10652.2 \pm 1.5$ | $11.5 \pm 2.2$ | $1^{+-}$ | $\Upsilon(10860) \to \pi^-(\pi^+\Upsilon(1S, 2S, 3S))$ | Belle [61,62]( >10) | 2011 | OK |
| | | | | $\Upsilon(10860) \to \pi^-(\pi^+ h_b(1P, 2P))$ | Belle [62]( 16) | 2011 | OK |
| | | | | $\Upsilon(10860) \to \pi^-(B^*\bar{B}^*)^+$ | Belle [64]( 6.8) | 2012 | NC! |

Normal meson

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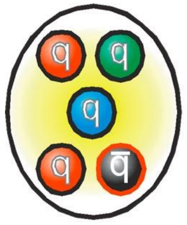


Pentaquark

Tetraquark

Hybrid meson

Glueball

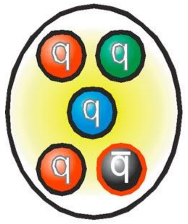
For a review on gluonium and other non-$q\bar{q}$ candidates see PDG 2006, Journal of Physics G **33** 1 (2006). See also the "Note on scalar mesons" in the $f0(500)$ Particle Listings, our note "New charmonium-like states" in PDG 2008, Physics Letters B**667** 1 (2008), and the extensive chapter on Spectroscopy in N. Brambilla *et al*. (Quarkonium Working Group), The European Physical Journal C **71** 1534 (2011).

# New states

- New states **above the first open flavor thresholds** in the $c\bar{c}$ and $b\bar{b}$ regions, ordered by mass

Normal meson

??

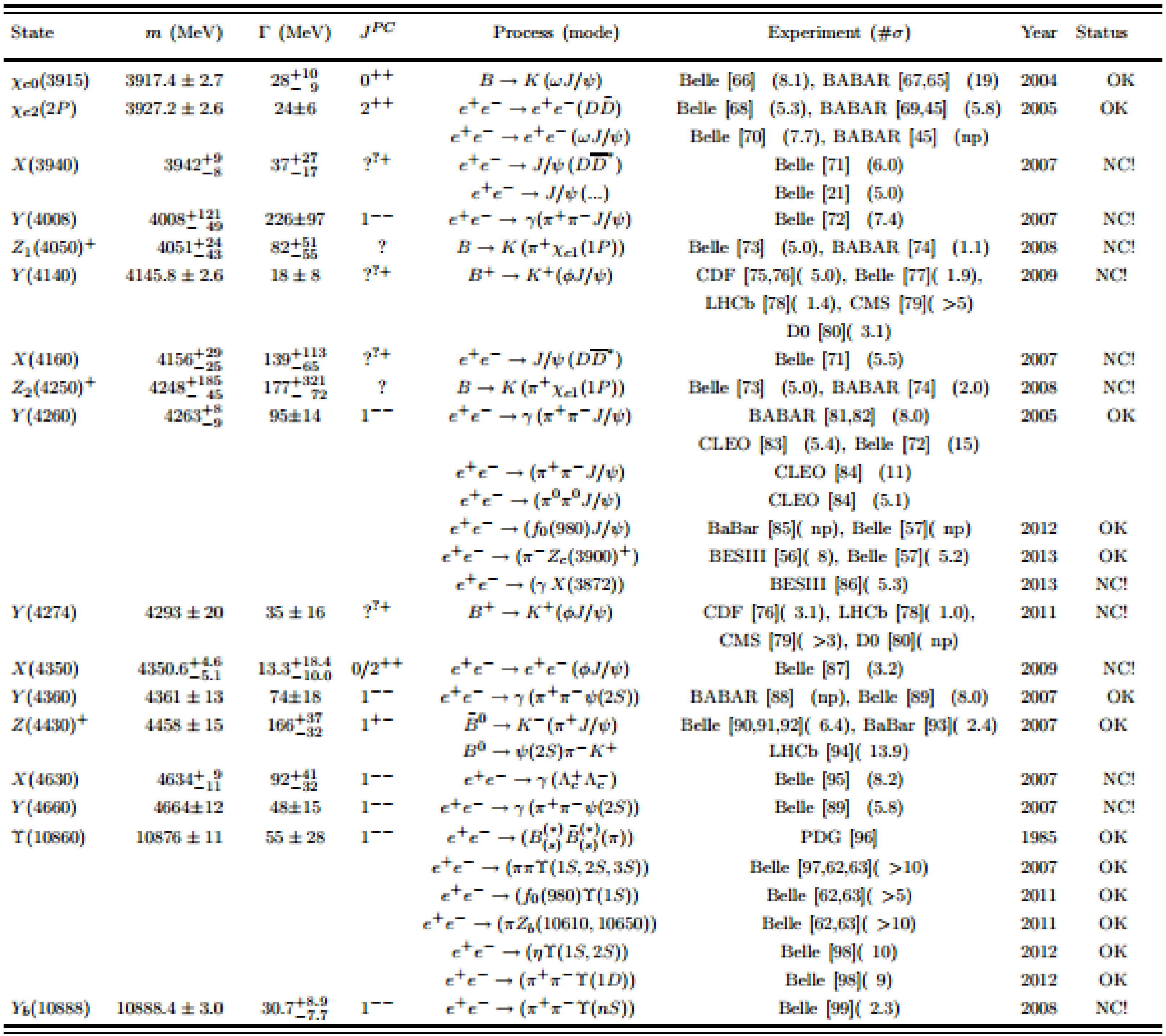


Pentaquark

Tetraquark

Hybrid meson

Glueball



| State | $m$ (MeV) | $\Gamma$ (MeV) | $J^{PC}$ | Process (mode) | Experiment (#$\sigma$) | Year | Status |
|---|---|---|---|---|---|---|---|
| $\chi_{c0}(3915)$ | $3917.4 \pm 2.7$ | $28^{+10}_{-9}$ | $0^{++}$ | $B \to K(\omega J/\psi)$ | Belle [66] (8.1), BABAR [67,65] (19) | 2004 | OK |
| $\chi_{c2}(2P)$ | $3927.2 \pm 2.6$ | $24 \pm 6$ | $2^{++}$ | $e^+e^- \to e^+e^-(D\bar{D})$ | Belle [68] (5.3), BABAR [69,45] (5.8) | 2005 | OK |
| | | | | $e^+e^- \to e^+e^-(\omega J/\psi)$ | Belle [70] (7.7), BABAR [45] (np) | | |
| $X(3940)$ | $3942^{+9}_{-8}$ | $37^{+27}_{-17}$ | $?^{?+}$ | $e^+e^- \to J/\psi(D\overline{D}^*)$ | Belle [71] (6.0) | 2007 | NC! |
| | | | | $e^+e^- \to J/\psi(...)$ | Belle [21] (5.0) | | |
| $Y(4008)$ | $4008^{+121}_{-49}$ | $226 \pm 97$ | $1^{--}$ | $e^+e^- \to \gamma(\pi^+\pi^- J/\psi)$ | Belle [72] (7.4) | 2007 | NC! |
| $Z_1(4050)^+$ | $4051^{+24}_{-43}$ | $82^{+51}_{-55}$ | ? | $B \to K(\pi^+\chi_{c1}(1P))$ | Belle [73] (5.0), BABAR [74] (1.1) | 2008 | NC! |
| $Y(4140)$ | $4145.8 \pm 2.6$ | $18 \pm 8$ | $?^{?+}$ | $B^+ \to K^+(\phi J/\psi)$ | CDF [75,76]( 5.0), Belle [77]( 1.9), | 2009 | NC! |
| | | | | | LHCb [78]( 1.4), CMS [79]( >5) | | |
| | | | | | D0 [80]( 3.1) | | |
| $X(4160)$ | $4156^{+29}_{-25}$ | $139^{+113}_{-65}$ | $?^{?+}$ | $e^+e^- \to J/\psi(D\overline{D}^*)$ | Belle [71] (5.5) | 2007 | NC! |
| $Z_2(4250)^+$ | $4248^{+185}_{-45}$ | $177^{+321}_{-72}$ | ? | $B \to K(\pi^+\chi_{c1}(1P))$ | Belle [73] (5.0), BABAR [74] (2.0) | 2008 | NC! |
| $Y(4260)$ | $4263^{+8}_{-9}$ | $95 \pm 14$ | $1^{--}$ | $e^+e^- \to \gamma(\pi^+\pi^- J/\psi)$ | BABAR [81,82] (8.0) | 2005 | OK |
| | | | | | CLEO [83] (5.4), Belle [72] (15) | | |
| | | | | $e^+e^- \to (\pi^+\pi^- J/\psi)$ | CLEO [84] (11) | | |
| | | | | $e^+e^- \to (\pi^0\pi^0 J/\psi)$ | CLEO [84] (5.1) | | |
| | | | | $e^+e^- \to (f_0(980) J/\psi)$ | BaBar [85]( np), Belle [57]( np) | 2012 | OK |
| | | | | $e^+e^- \to (\pi^- Z_c(3900)^+)$ | BESIII [56]( 8), Belle [57]( 5.2) | 2013 | OK |
| | | | | $e^+e^- \to (\gamma X(3872))$ | BESIII [86]( 5.3) | 2013 | NC! |
| $Y(4274)$ | $4293 \pm 20$ | $35 \pm 16$ | $?^{?+}$ | $B^+ \to K^+(\phi J/\psi)$ | CDF [76]( 3.1), LHCb [78]( 1.0), | 2011 | NC! |
| | | | | | CMS [79]( >3), D0 [80]( np) | | |
| $X(4350)$ | $4350.6^{+4.6}_{-5.1}$ | $13.3^{+18.4}_{-10.0}$ | $0/2^{++}$ | $e^+e^- \to e^+e^-(\phi J/\psi)$ | Belle [87] (3.2) | 2009 | NC! |
| $Y(4360)$ | $4361 \pm 13$ | $74 \pm 18$ | $1^{--}$ | $e^+e^- \to \gamma(\pi^+\pi^- \psi(2S))$ | BABAR [88] (np), Belle [89] (8.0) | 2007 | OK |
| $Z(4430)^+$ | $4458 \pm 15$ | $166^{+37}_{-32}$ | $1^{+-}$ | $\bar{B}^0 \to K^-(\pi^+ J/\psi)$ | Belle [90,91,92]( 6.4), BaBar [93]( 2.4) | 2007 | OK |
| | | | | $B^0 \to \psi(2S)\pi^- K^+$ | LHCb [94]( 13.9) | | |
| $X(4630)$ | $4634^{+9}_{-11}$ | $92^{+41}_{-32}$ | $1^{--}$ | $e^+e^- \to \gamma(\Lambda_c^+\Lambda_c^-)$ | Belle [95] (8.2) | 2007 | NC! |
| $Y(4660)$ | $4664 \pm 12$ | $48 \pm 15$ | $1^{--}$ | $e^+e^- \to \gamma(\pi^+\pi^- \psi(2S))$ | Belle [89] (5.8) | 2007 | NC! |
| $\Upsilon(10860)$ | $10876 \pm 11$ | $55 \pm 28$ | $1^{--}$ | $e^+e^- \to (B_{(s)}^{(*)}\bar{B}_{(s)}^{(*)}(\pi))$ | PDG [96] | 1985 | OK |
| | | | | $e^+e^- \to (\pi\pi\Upsilon(1S,2S,3S))$ | Belle [97,62,63]( >10) | 2007 | OK |
| | | | | $e^+e^- \to (f_0(980)\Upsilon(1S))$ | Belle [62,63]( >5) | 2011 | OK |
| | | | | $e^+e^- \to (\pi Z_b(10610,10650))$ | Belle [62,63]( >10) | 2011 | OK |
| | | | | $e^+e^- \to (\eta\Upsilon(1S,2S))$ | Belle [98]( 10) | 2012 | OK |
| | | | | $e^+e^- \to (\pi^+\pi^-\Upsilon(1D))$ | Belle [98]( 9) | 2012 | OK |
| $Y_b(10888)$ | $10888.4 \pm 3.0$ | $30.7^{+8.9}_{-7.7}$ | $1^{--}$ | $e^+e^- \to (\pi^+\pi^-\Upsilon(nS))$ | Belle [99]( 2.3) | 2008 | NC! |

# Theoretical tools for QCD

- Effective field theories (EFTs)
  grew out of the operator-product expansion (OPE) and the formalism of phenomenological Lagrangians and, thus, provide a standard way to analyze physical systems with many different energy scales.
- Lattice gauge theory
  speedily progressing in what concerns systematic finite volume effects as well as increasingly small quark masses
- Other non-perturbative approaches
  among the most used techniques are: the limit of the large number of colors, generalizations of the original Shifman–Vainshtein–Zakharov sum rules, QCD vacuum models and effective string models, the AdS/CFT conjecture, and **Schwinger–Dyson equations**, …

More recently… →

*Heavy Quarkonium in a Light-Front Holographic Basis,*

*Heavy-quarkonium states within the renormalization-group procedure for effective particles*

*Nakanishi integral representation of the BS amplitude,*

*etc. …*

} Bound-state problem

# Theoretical tools for QCD

**Schwinger–Dyson equations**

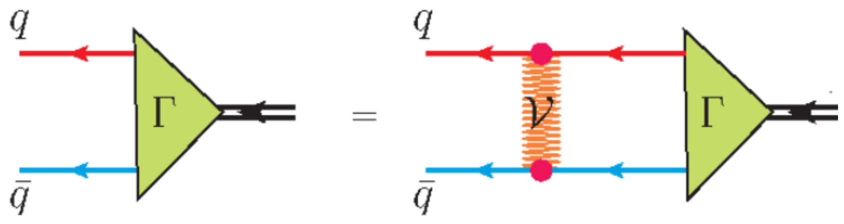
close in spirit, we aim a *self-consistent* quantum field theoretical approach, *but* in Minkowski space, designed for *all* $q\bar{q}$-type mesons and satisfying:

**Covariant Spectator Theory**
**Bethe-Salpeter (CST-BS)**

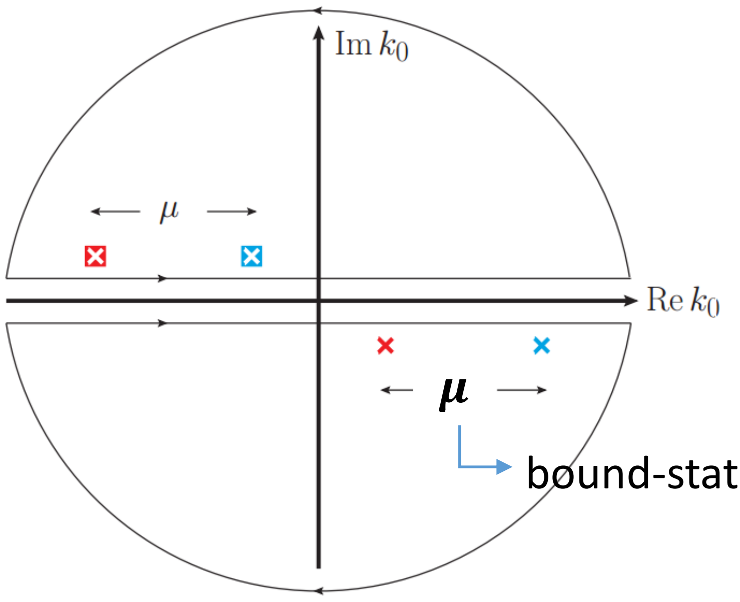
1. Poincaré covariance
   *in general* quarks require relativistic treatment
2. Confinement
   linear: suggested from nonrelativistic potential models and lattice QCD studies
3. Spontaneous chiral symmetry breaking
   existence of massless Goldstone pion and dynamical generation of constituent (dressed) quark mass from self-interactions
   Nambu-Jona-Lasinio-type mechanism

# CST-BS Formalism overview

- Bethe-Salpeter (BS) Equation

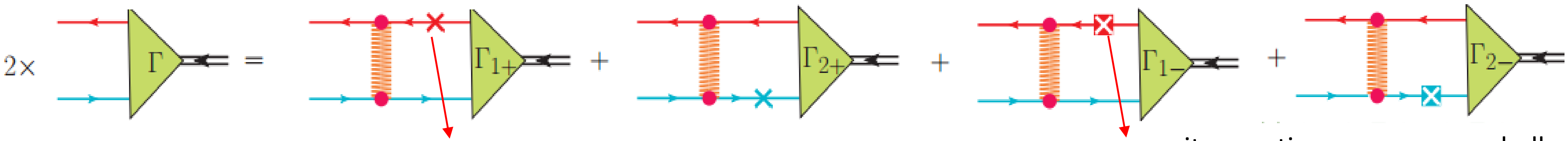


- Covariant Spectator Theory (CST): propagator pole contributions *approximate* sum of ladder and crossed ladders (to be seen later)



bound-state mass

- light equal-mass quarks and deeply bound states ($\boldsymbol{\mu}$ small) like pion, require a charge-conjugation symmetric equation, the so-called four-channel (4CST-BS) equation:

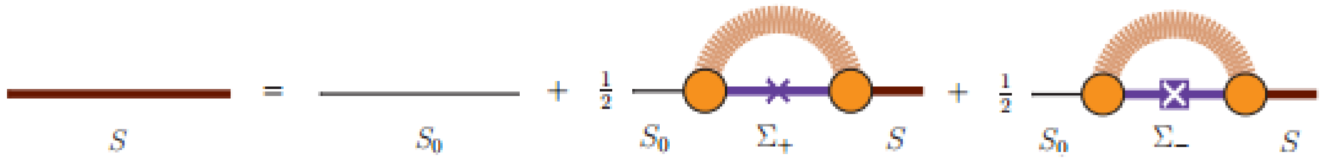


$q$ on its positive energy mass-shell

$q$ on its negative energy mass-shell

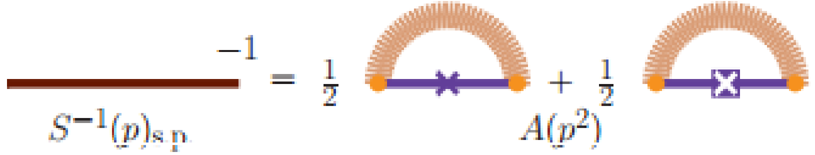
- calculate dynamical CST quark mass function $M(k^2)$ with one-body CST-Dyson (mass gap equation)

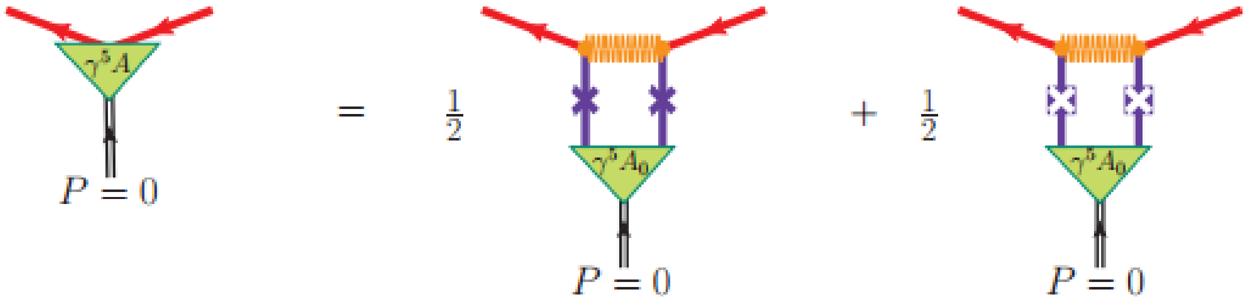
self-energy $\Sigma(p) = A(p^2) + \not{p}\, B(p^2)$

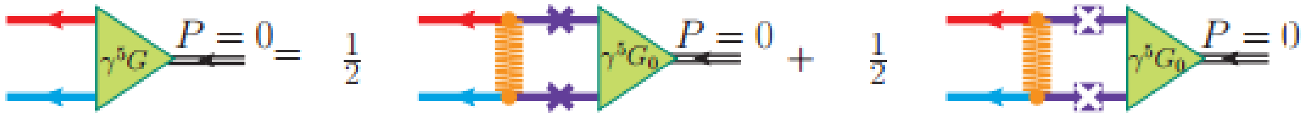


# CST-BS Formalism overview

- chiral limit ($m_0 = 0$): scalar part (s. p.) of one-body equation for $A$ and bound-state equation for a massless pion are identical







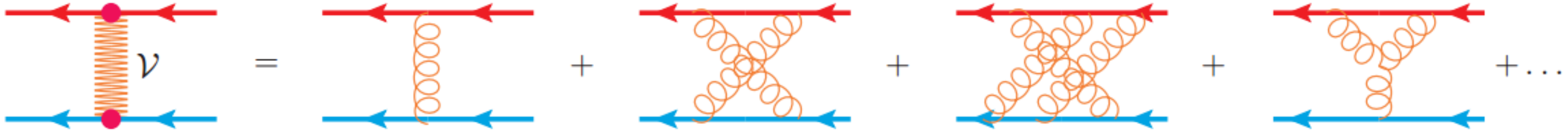
→ a massless pion state exists! Goldstone pion in chiral limit associated with spontaneous chiral symmetry breaking.

- CST-BS formalism has also been applied recently to compute $\pi$ e.m. form factor and study of $\pi - \pi$ scattering

E. P. Biernat, M. T. Peña, A. Stadler, F. Gross: PRD **89**, 016005, 016006 (2014); PRD **92**, 076011 (2015);
& also with J. E. Ribeiro: PRD **90**, 096008 (2014).

# Interaction kernel truncation — *CST* key feature

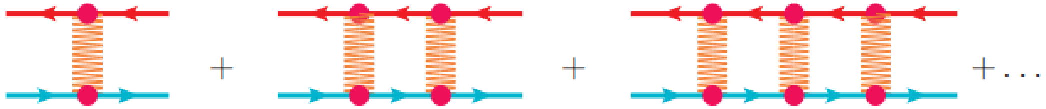
- The kernel contains all two-body irreducible diagrams



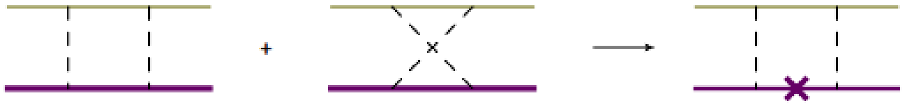
- In the BS equation the kernel is effectively iterated to all orders
  *But* the complete kernel is a sum of an infinite number of irreducible diagrams has to be truncated
  (most often: ladder approximation)

  However,
  - No one-body limit (missing crossed ladders)
  - Not best suited to describe bound states (crossed-ladder contributions are significant)

    see Nieuwenhuis and Tjon, PRL77, 814 (1996)



- In a $\phi^3$-theory the sum of box and crossed box diagrams is approximated by heavy particle pole contribution of box diagram.



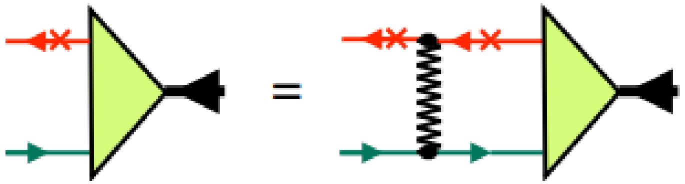
proof: Gross, Relativistic Quantum Mechanics and Field Theory, (2004)

cancellation in all orders and exact in heavy mass limit ⇒ one-boson-exchange kernel with heavy particle on-mass shell produces exact sum of all ladder and crossed ladder diagrams!

- CST prescription of placing particles on their mass-shell, *effectively*, goes beyond rainbow-ladder approximation!

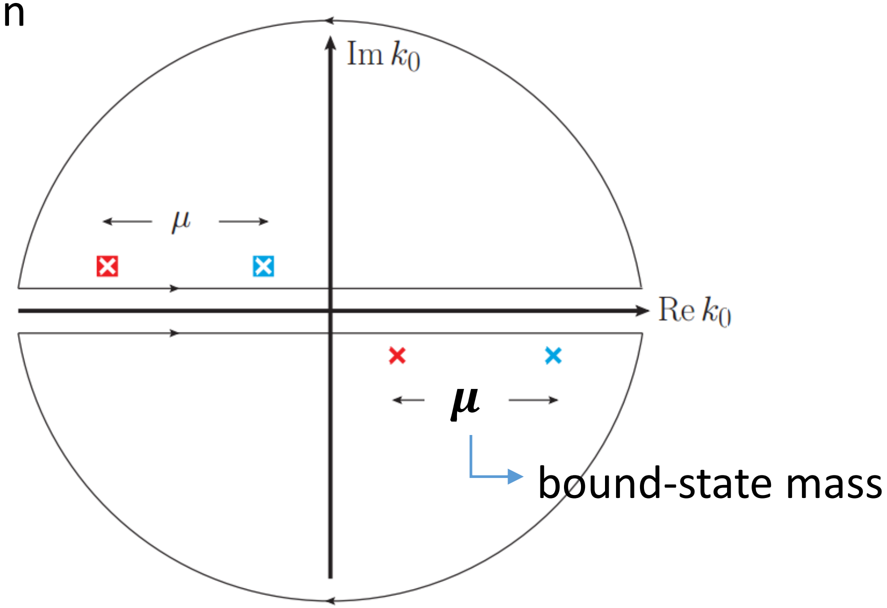
# Heavy and heavy-light mesons with CST-BS

- If $\boldsymbol{\mu}$ is large, the one-channel (1CST-BS) equation is a good approximation



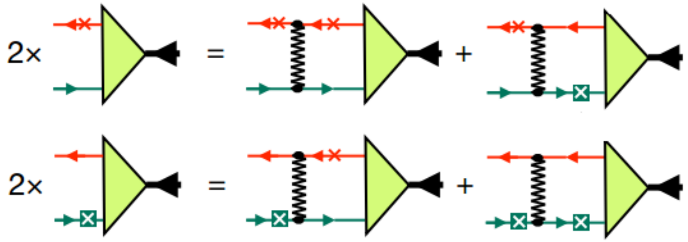
and possesses *important* features:

√ smooth nonrelativistic limit (to the Schrödinger equation),
√ correct one-body limit,
√ it is manifestly covariant (despite its loop integrations being 3-dimensional)!



- *However*, heavy quarkonium states calculated with the 1CST-BS equation have no definite C-parity.

*This would be the correct system of equations:*



- *Not a problem!* only the axial-vector mesons have both parities (separated *only* by $5-6\ MeV$ in $b\bar{b}$, $14\ MeV$ in $c\bar{c}$)

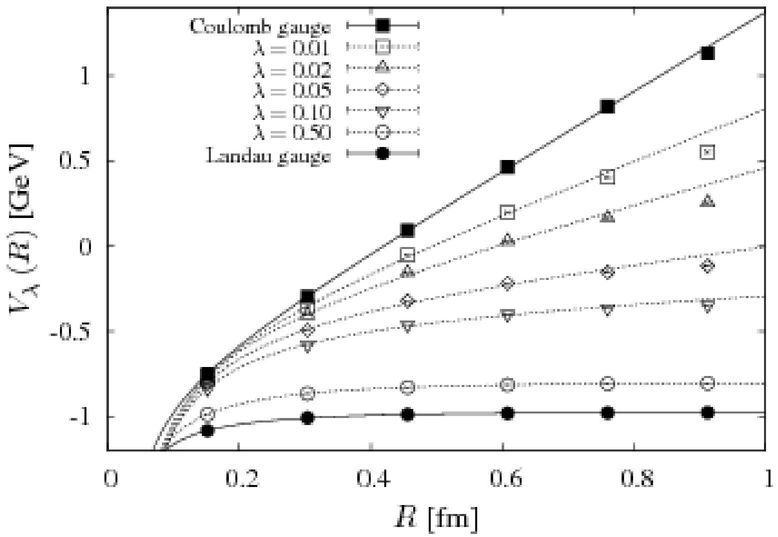
# Confining potential in momentum space

- Phenomenological $q\bar{q}$ kernel is inspired by Cornell potential:

$$\boxed{V(r) = \sigma r - {}^{\alpha_s}/_{r} - C}$$

A. Laschka *et al.* PRD **83**, 094002 (2011)

Static QCD potential from lattice



- NR linear potential in momentum space:
  Fourier transform of screened potential

Usually: $$\sigma r = \lim_{\epsilon\to 0} \sigma \frac{\partial^2}{\partial\epsilon^2}\frac{e^{-\epsilon r}}{r}$$

But simpler: $$\sigma r = \lim_{\epsilon\to 0} -\frac{\sigma}{\epsilon}\left(e^{-\epsilon r} - 1\right) \equiv \tilde{V}_A(r) - \tilde{V}_A(0)$$

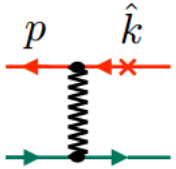
FT: $$V_L(\boldsymbol{q}) = V_A(\boldsymbol{q}) - (2\pi)^3\delta(\boldsymbol{q})\int\frac{d^3\boldsymbol{q}'}{(2\pi)^3}V_A(\boldsymbol{q}') \quad \text{with} \quad V_A(q) = -\frac{8\pi\sigma}{\boldsymbol{q}^4}$$

$$\langle V_L\phi\rangle(\boldsymbol{q}) = \int\frac{d^3\boldsymbol{k}}{(2\pi)^3}V_L(\boldsymbol{p}-\boldsymbol{k})\phi(\boldsymbol{k}) = -8\pi\sigma\int\frac{d^3\boldsymbol{k}}{(2\pi)^3}\frac{\phi(\boldsymbol{k})-\phi(\boldsymbol{p})}{(\boldsymbol{p}-\boldsymbol{k})^4}$$

only a Cauchy principal value singularity remains
(and this one can also be explicitly removed)

- Covariant generalization: $\boldsymbol{q} \to -q^2$

S.L *et al.* PRD **90**, 096003 (2014)



Initial state: either quark or antiquark on-shell

$$\langle V_L\phi\rangle(p) = \int\frac{d^3k}{(2\pi)^3}\frac{m}{E_k}V_L(p,\hat{k})\phi(\hat{k}) = -8\pi\sigma\int\frac{d^3k}{(2\pi)^3}\frac{m}{E_k}\frac{\phi(\hat{k})-\phi(\hat{p}_R)}{\left(p-\hat{k}\right)^4}$$

$$\hat{k} = (E_k, \boldsymbol{k}) \qquad \hat{p}_R = (E_{p_R}, \boldsymbol{p_R})$$

value $\boldsymbol{k}$ at which kernel becomes singular

# Lorentz structure of the kernel

- We use a kernel of the general form

$$\mathcal{V} = \left[(1-y)\left(1_1 \otimes 1_2 + \gamma_1^5 \otimes \gamma_2^5\right) - y\gamma_1^\mu \otimes \gamma_{\mu 2}\right]V_L - \gamma_1^\mu \otimes \gamma_{\mu 2}\left[V_{Coul.} + V_C\right]$$

where $V_L, V_{Coul.}, V_C$ are relativistic generalizations of a linear conning potential, a short-range Coulomb term and a global constant potential.

- The parameter $y$ dials continuously between the two extreme cases $y = 1$ being pure vector coupling, and $y = 0$ pure scalar+pseudoscalar coupling.

- The reason for the presence of a pseudoscalar component is chiral symmetry. Although in general scalar interactions break chiral symmetry, it was shown that the CS equation with our relativistic linear confining kernel satisfies the axial-vector Ward-Takahashi identity when it is accompanied by an equal-weight pseudoscalar interaction.

PRD **90**, 096008 (2014).

- Finally, for any interaction kernel $K$, the 1CST-BS equation for the vertex function $\Gamma$, reads

$$\Gamma(p) = -\sum_K \int \frac{d^3k}{(2\pi)^3} \frac{m_1}{E_{1k}} V_K(p,k)\Theta_1^{K(\mu)} \frac{m_1 + \hat{\not k}_1}{2m_1} \Gamma(k) \frac{m_2 + \not k_2}{m_2^2 - k_2^2 - i\epsilon} \Theta_{2(\mu)}^K$$

# Numerical solution of CST-BS

- Instead of solving the 1CST-BS directly for these structure functions, we prefer to first introduce relativistic "wave functions", defined as Dirac spinor matrix elements of the vertex function multiplied by the off-shell quark propagator and with definite orbital angular momentum (*why*? important when comparing to experimentally determined states).

- The 1CST-BS for the relativistic wave functions can be written as a generalized linear eigenvalue problem for the total bound-state mass $\mu$.

- We solve this system by expanding the wave functions in a basis of $B$-splines.

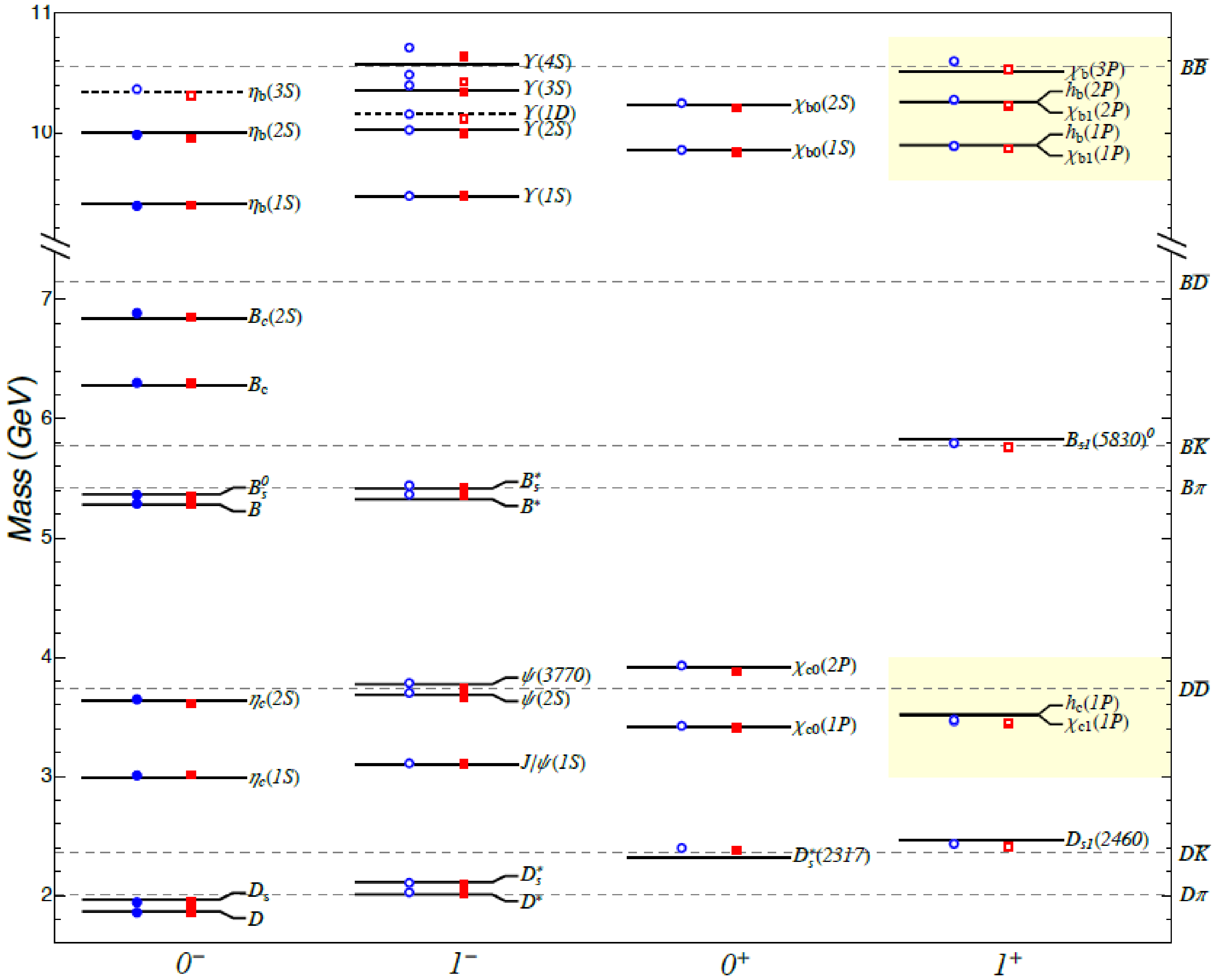
- Special attention is needed to treat the singularities in the kernel at $q^2 = \left(\hat{p}_1 - \hat{k}_1\right)^2 = 0$.

- Due to retardation effects, the loop integrals over the kernels do not converge. We use a standard Pauli-Villars regularization to cure this problem, at the expense of a momentum cut-off parameter $\Lambda$. It turns out that our results are very insensitive to this parameter ($\Lambda = 2\mathrm{m}_1$).

- We *set* the following masses

$$m_b = 4.892\ GeV, m_c = 1.600\ GeV, m_s = 0.448\ GeV \text{ and } m_u = m_d = 0.346\ GeV.$$

# Results



- Early results clearly favored pure scalar+pseudoscalar confinement, so throughout this work we set $y = 0$.

Model P1 (fitted to pseudoscalar states *only*)

$$\sigma = 0.2493\ GeV^2, \alpha_s = 0.3643,$$
$$C = 0.3491\ GeV$$

● states fitted

○ predicted states

$rms = 0.036\ GeV$

Model PSV1 (fitted to pseudoscalar, scalar and vector)

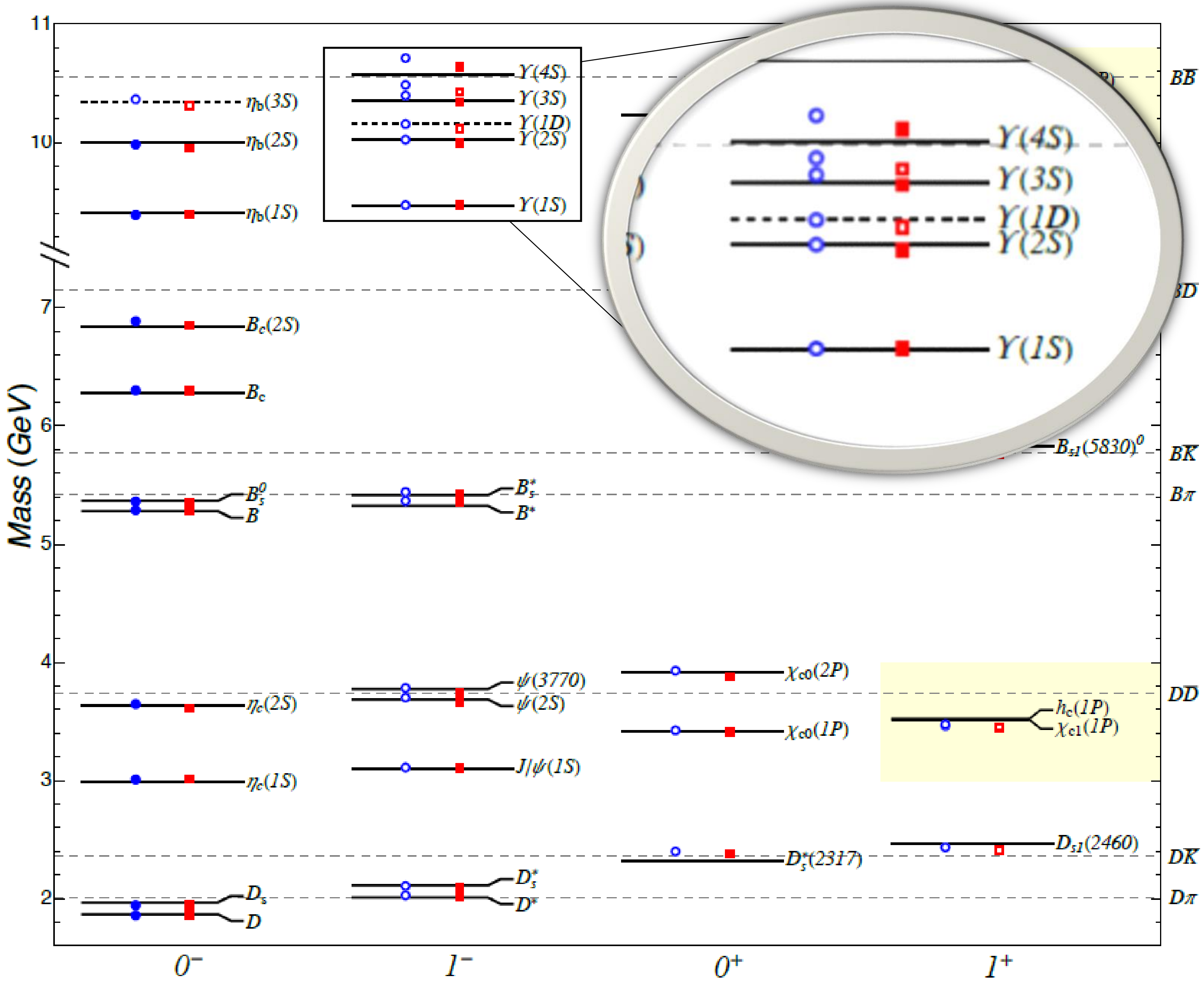
$$\sigma = 0.2247\ GeV^2, \alpha_s = 0.3614,$$
$$C = 0.3377\ GeV$$

■ states fitted

□ predicted states

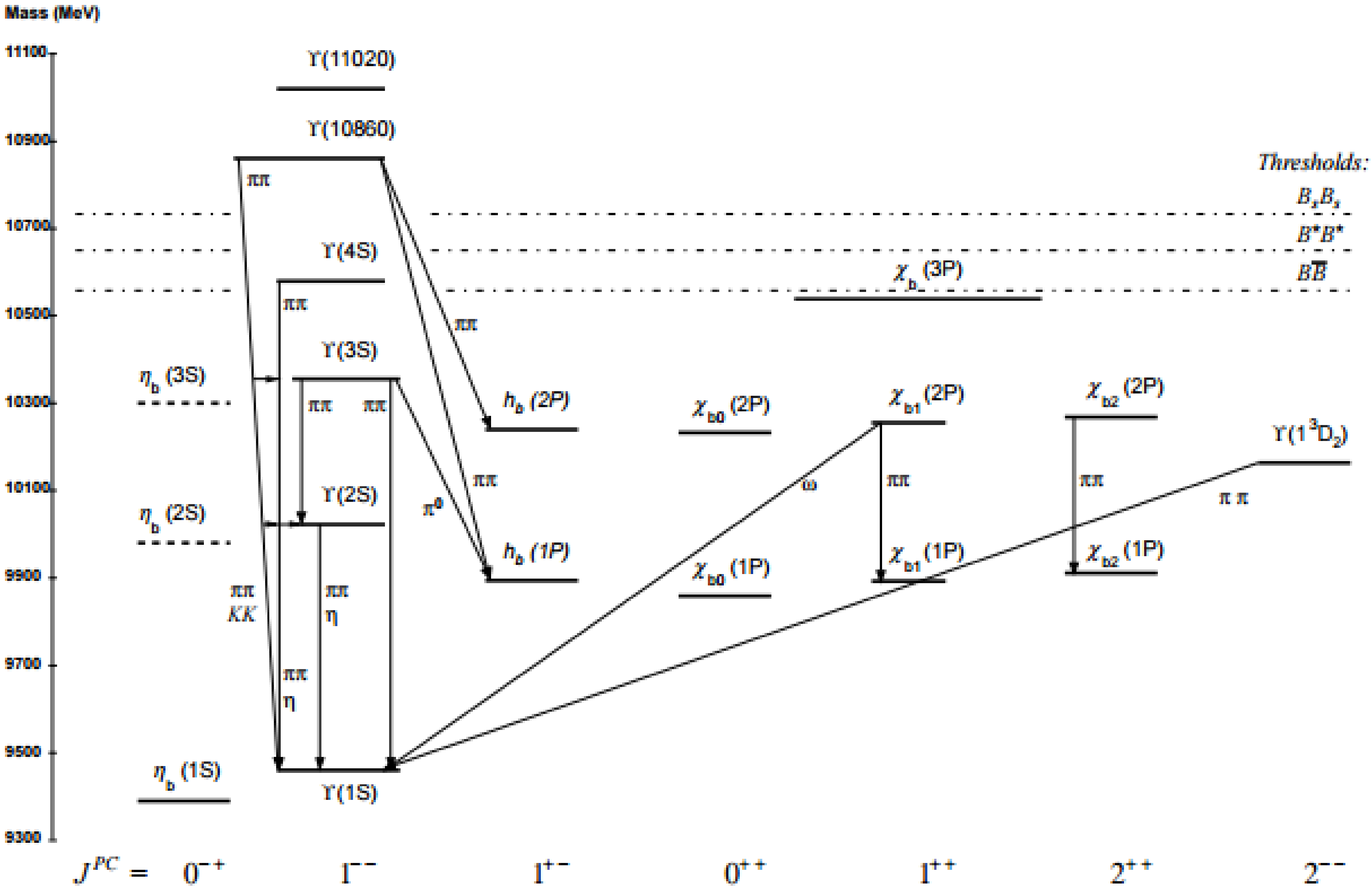
$rms = 0.030\ GeV$

# Predictive power of interaction kernels

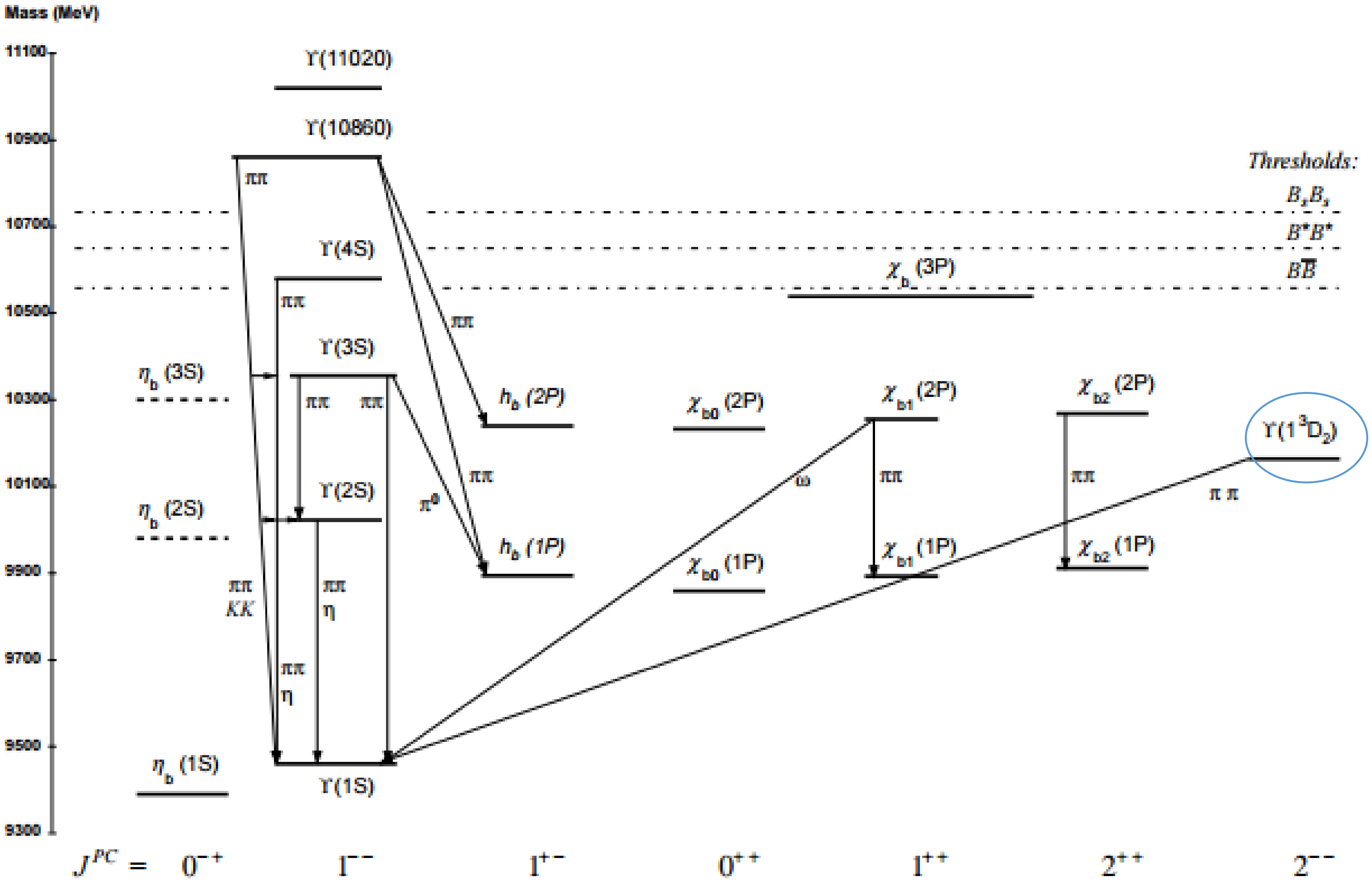


- Vector bottomonium: $S$ or $D$ states?

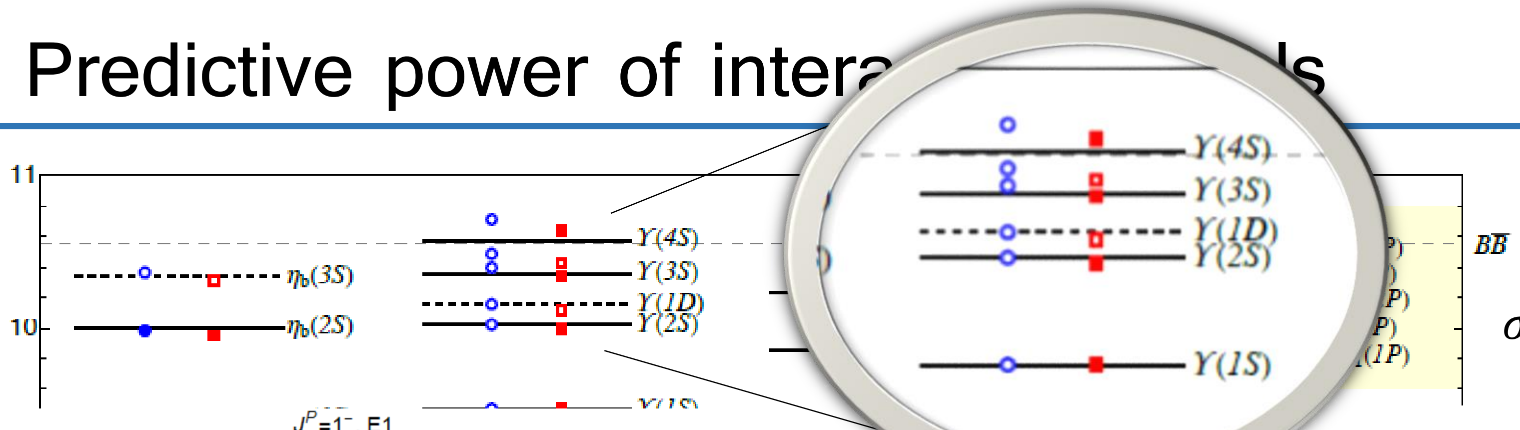
# The Bottomonium System



# The Bottomonium System

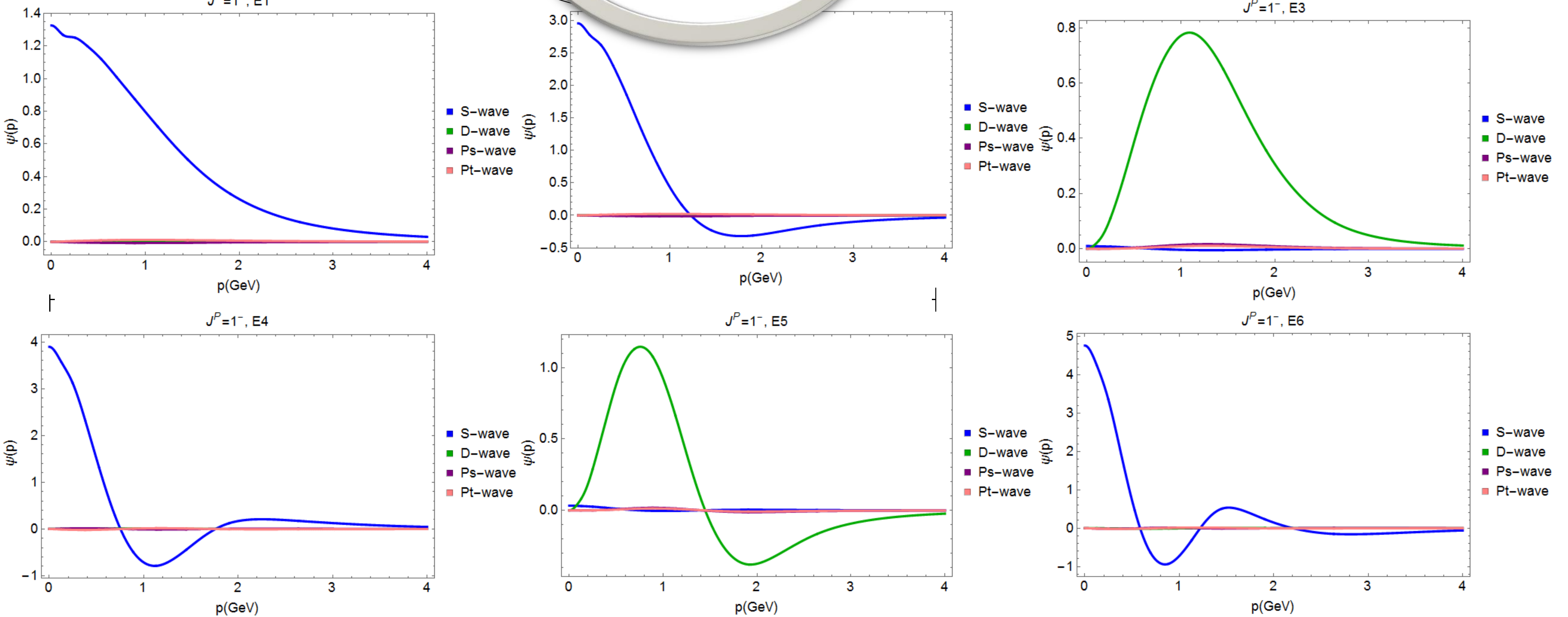


# Predictive power of intera ls

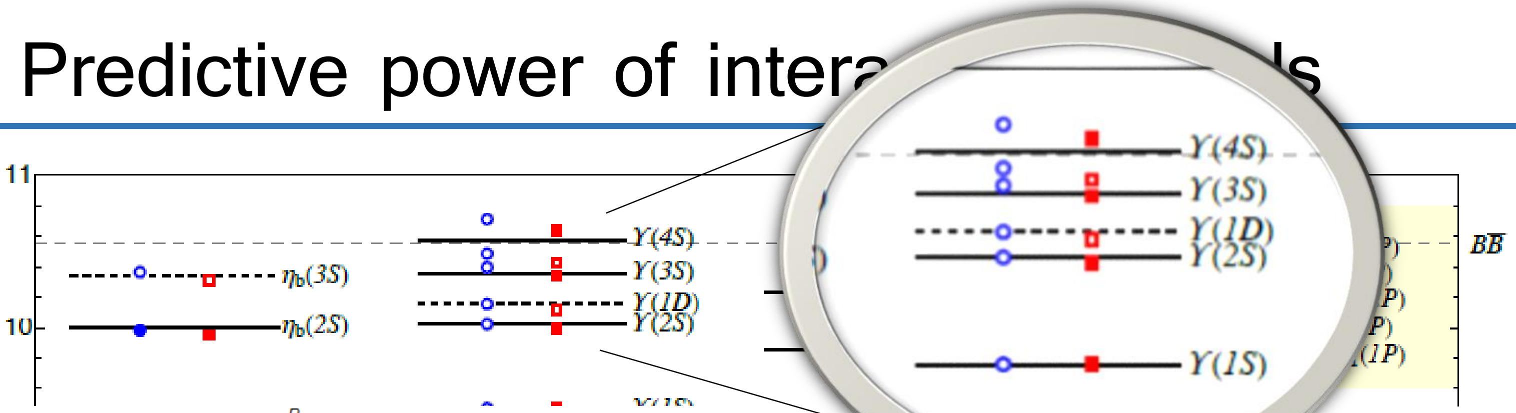


Model PSV1

$$\sigma = 0.2247\ GeV^2, \alpha_s = 0.3614, C = 0.3377\ GeV$$



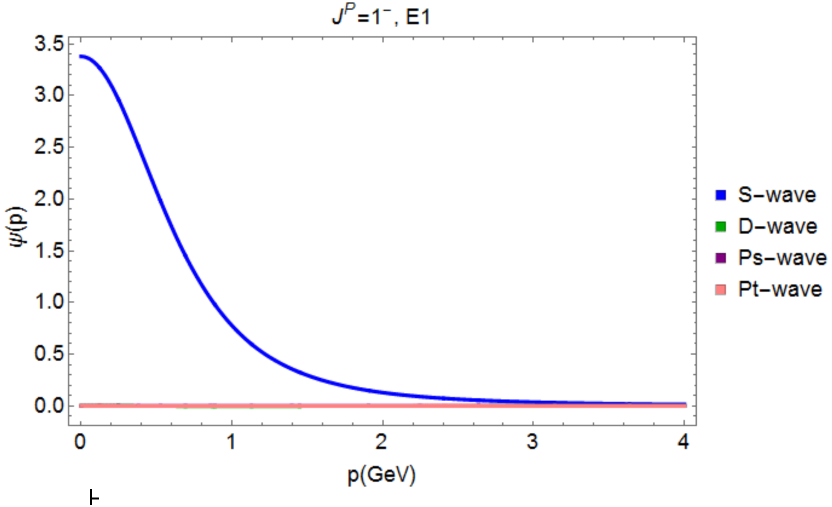
# Predictive power of intera ls

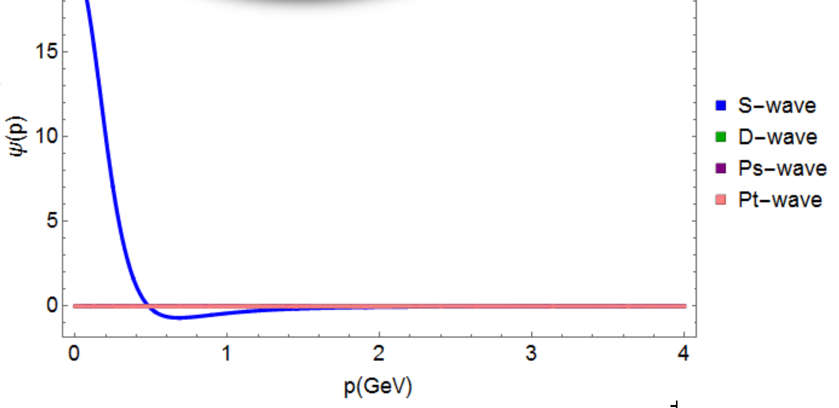


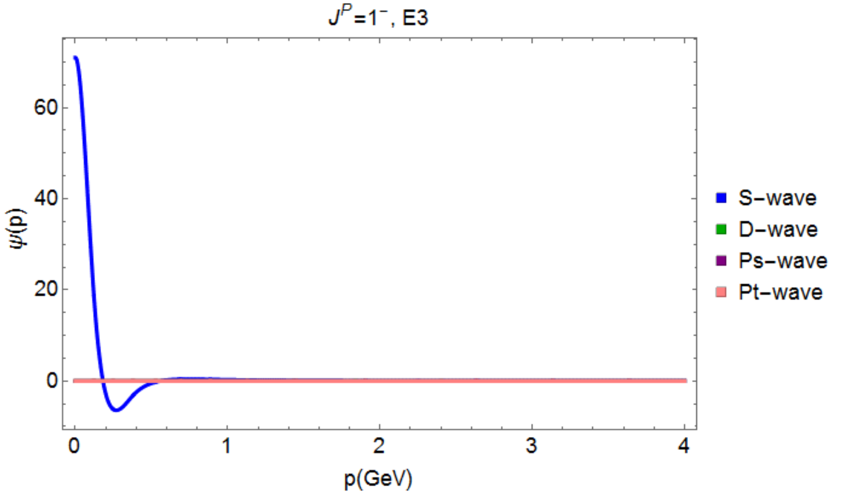
Just Coulomb

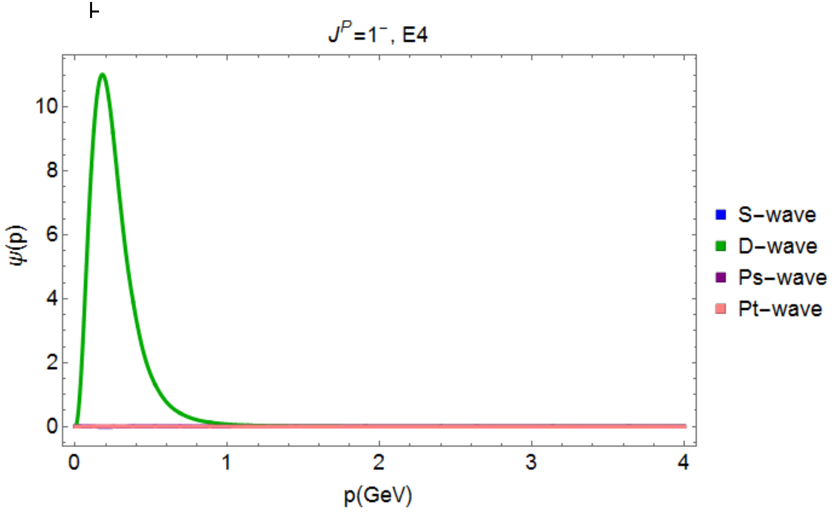
Switch off linear part

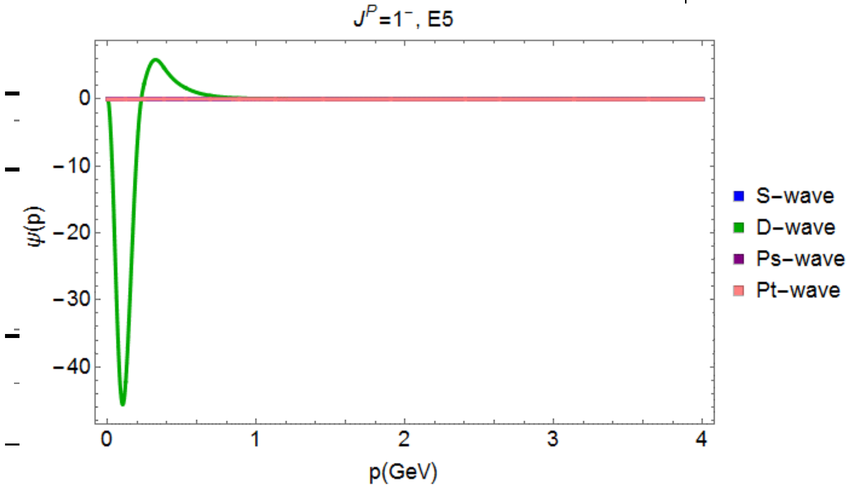
$\sigma = 0.0\ GeV^2, \alpha_s = 0.3614, C = 0.3377\ GeV$

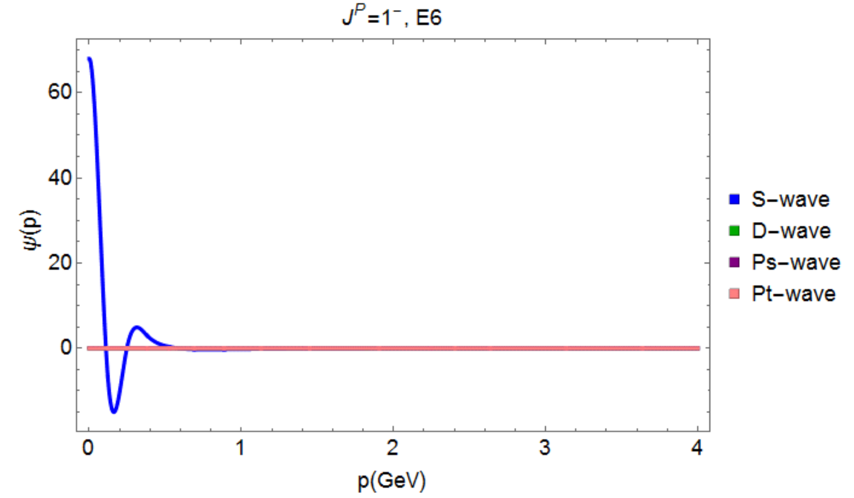




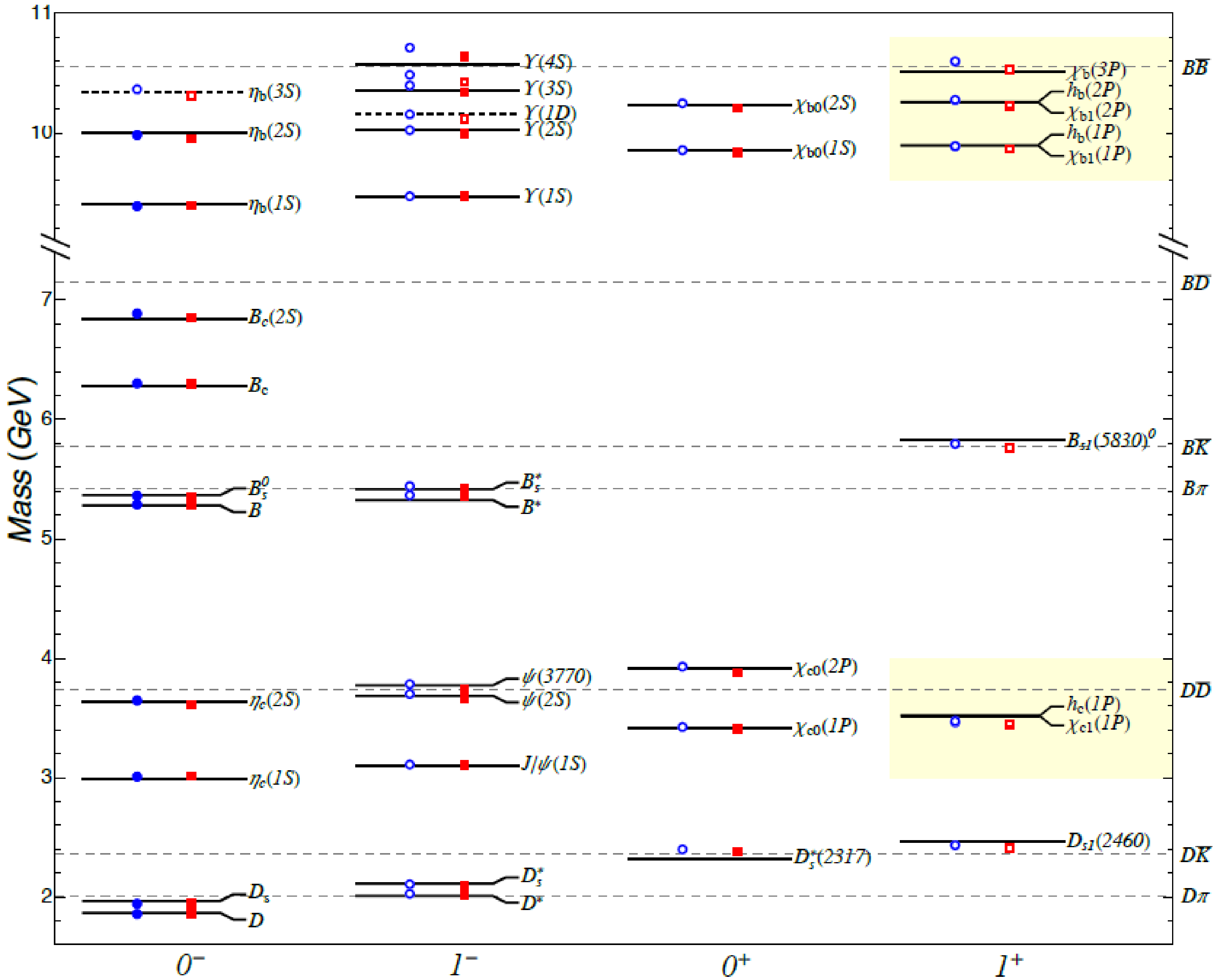








# Predictive power of *covariant* interaction kernels



- The observed meson spectrum is very well reproduced after setting a small number of model parameters (global fit).

- Remarkably, a fit to a few pseudoscalar meson states *only*, which are insensitive to spin-orbit and tensor forces and do not allow to separate the spin-spin from the central interaction, leads to essentially the same model parameters as a more general fit!

- Our covariant kernel correctly predicts the spin-dependent interactions solely based on their relation to the spin-independent interactions as dictated by covariance!

# Summary and Outlook

- We reported on the recent developments of CST-BS formalism applied to heavy and heavy-light mesons.
- Very good mass-spectrum was obtained with just a few parameters—we are inclined to believe that a *global* description is possible.
- We have *tested* that *covariance indeed* leads to an accurate prediction of the spin-dependent quark-antiquark interactions.

In a near future we aim to

- include also tensor mesons (*in progress*),
- extend the formalism to the light sector consistently, i.e., by solving the CST-Dyson (mass gap) & CST-BS equations together,
- compute other observables besides mass spectra (decay rates, form factors, etc. …).

Thank you!