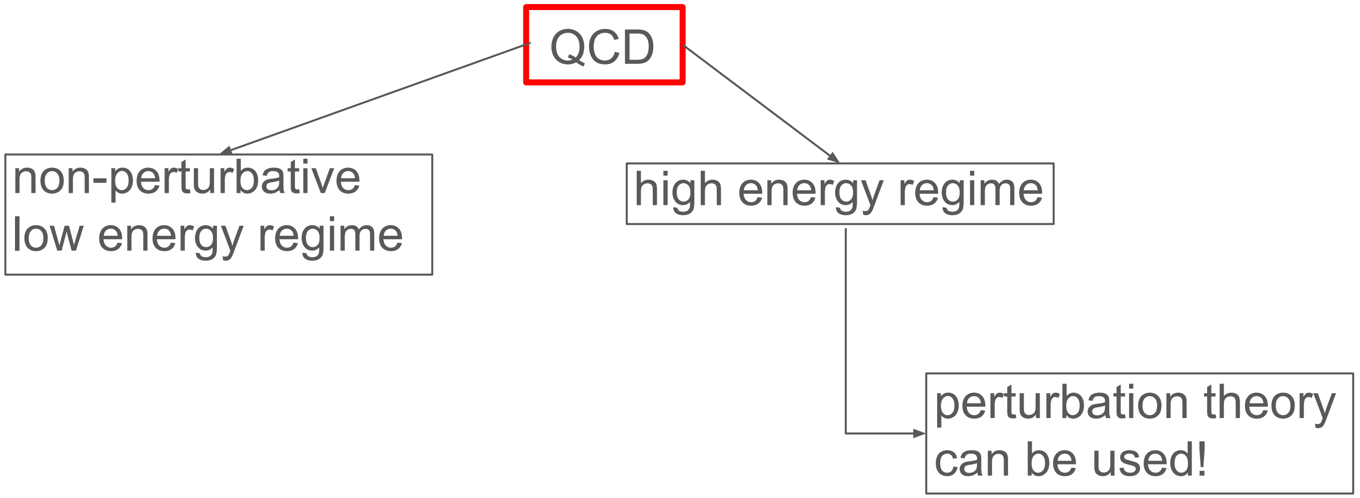
Running of α_s (the concept of the Renormalisation)

High Energy Physics
Student: Riccardo Campara
Professor: Ricardo Gonçalo

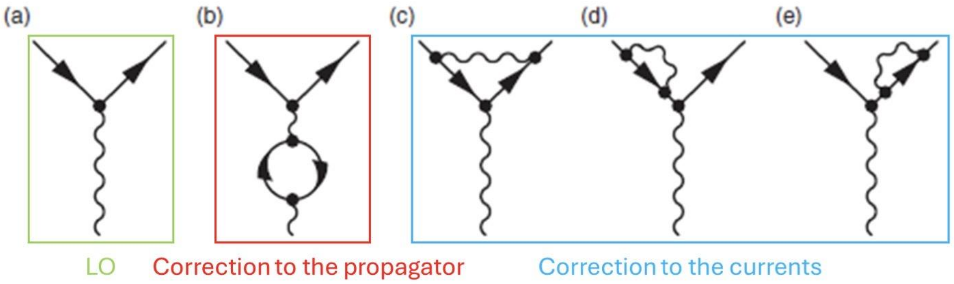
Introduction

At low-energy scales, the coupling constant of QCD is large, $\alpha_s \sim O(1)$

Thanks to some computational techniques of lattice QCD, it turns out that $\alpha_s \neq k$



Renormalisation in QED



For a QED vertex we have an infinite set of higher order diagrams. All the second order diagram modify the strength of the interaction relative to the LO diagram

Ward-Takashi identity → the effects of the corrections on the currents cancel at all orders in perturbation theory

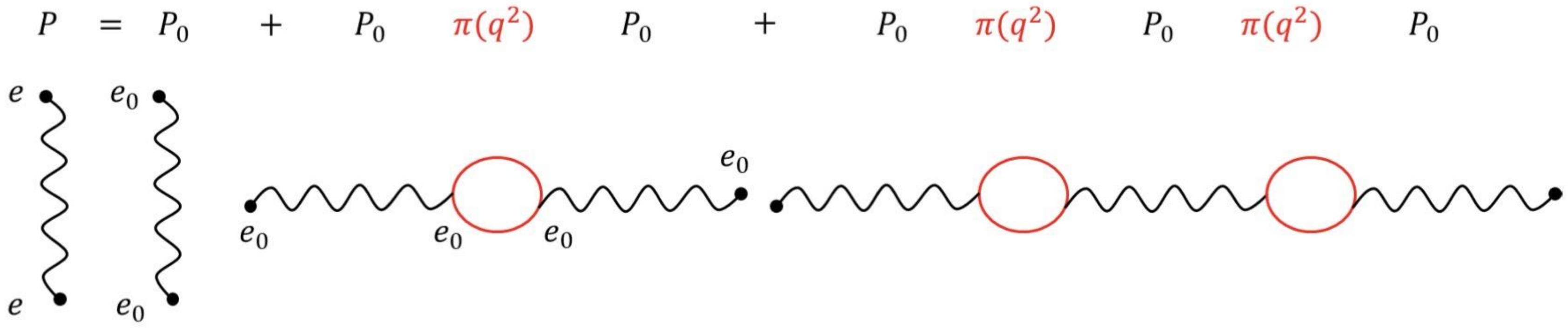
Need of RENORMALISATION → the loop term gives rise to a divergence

$\alpha = (e^2/4\pi) \approx 1/137$ charge of the electron → effective electron charge
e0 → bare charge → in Feynman diagrams

$P = e^2/q^2 \quad P_0 = e_0^2/q^2$

Renormalisation in QED

$$P = P_0 + P_0\pi(q^2)P_0 + P_0\pi(q^2)P_0\pi(q^2)P_0 + ... = P_0\sum_{k=0}^{\infty}\Big(\pi(q^2)P_0\Big)^k = P_0\frac{1}{1-\pi(q^2)P_0}$$



1. we don't know P_0 because we don't know the bare charge
2. we are hiding the divergence, but it still present inside $\pi(q^)$.

$$\Pi(q^2) = \frac{\pi(q^2)}{q^2}$$

Renormalisation in QED

$$P = \frac{e^2(q^2)}{q^2} = \frac{e_0^2}{q^2}\frac{1}{1-e_0^2\Pi(q^2)} \qquad \Rightarrow e^2(q^2) = \frac{e_0^2}{1-e_0^2\Pi(q^2)}$$

if the physical electron charge is known at some scale $q^2 = \mu^2$ we can rearrange the expression and obtain the bare charge

$$e_0^2 = \frac{e^2(\mu^2)}{1+e^2(\mu^2)\Pi(\mu^2)}$$

It can be sobstituted back

$$e^2(q^2) = \frac{e^2(\mu^2)}{1-e^2(\mu^2)\cdot[\Pi(q^2)-\Pi(\mu^2)]}$$

Consequently, the coupling strength is no longer constant, it runs with the q^2 scale of the virtual photon. For q^2 and μ^2 larger than m_e^2

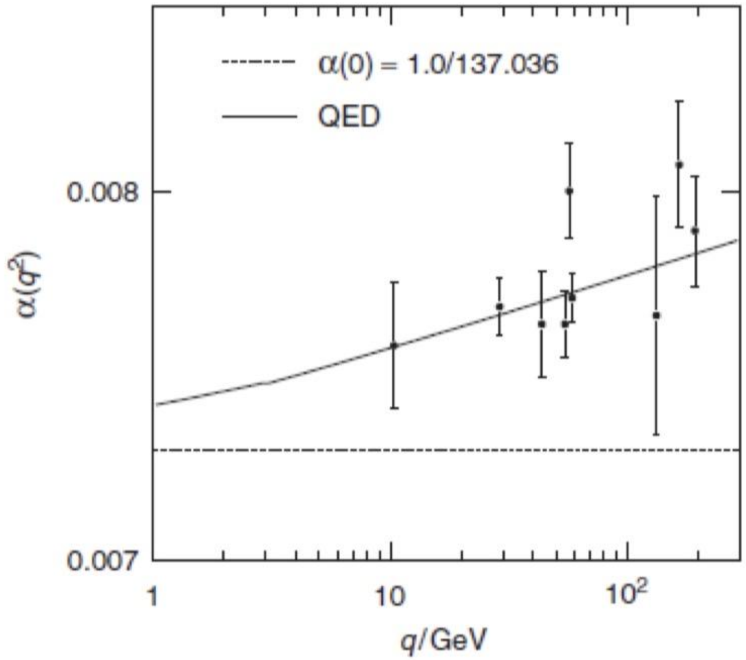
$$\Pi(q^2) - \Pi(\mu^2) \simeq \frac{1}{12\pi^2}\ln\left(\frac{q^2}{\mu^2}\right)$$

$$\Rightarrow \alpha(q^2) = \frac{\alpha(\mu^2)}{1-\alpha(\mu^2)\frac{1}{3\pi}\ln\left(\frac{q^2}{\mu^2}\right)}$$

Renormalisation in QED

$$\alpha(\sim 0) \approx \frac{1}{137} \qquad \alpha(q^2 = m_Z^2) = \frac{1}{127.4 \pm 2.1}$$

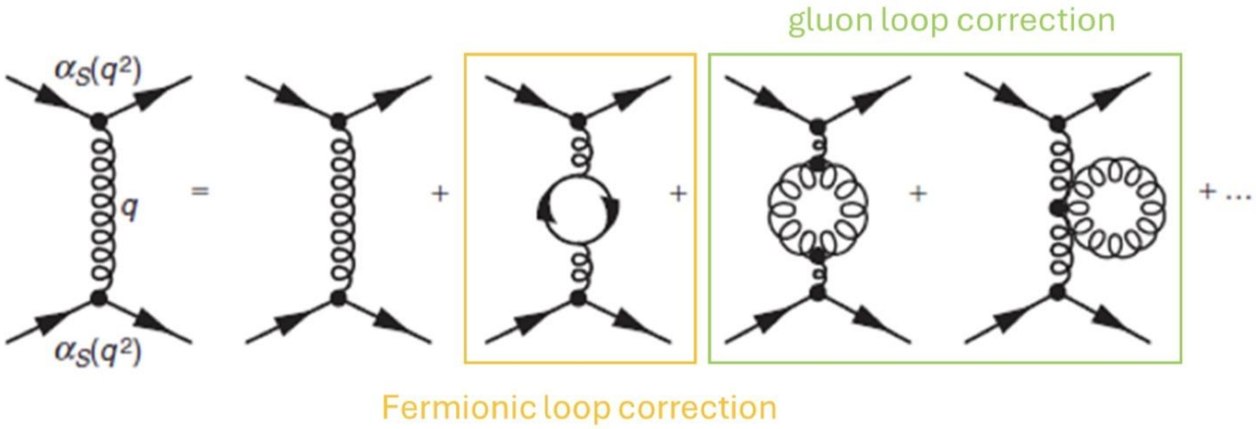
$$\alpha(q^2) = \frac{\alpha(\mu^2)}{1 - \alpha(\mu^2)\frac{1}{3\pi}\ln\left(\frac{q^2}{\mu^2}\right)}$$



The '-' sign implies that the c.c of QED increases with increasing $|q^2|$

Measurements of the coupling constant at different q^2 scales (OPAL experiment at LEP)

Running of α_s



$$\Pi(q^2) - \Pi(\mu^2) \approx -\frac{B}{4\pi} \ln \frac{q^2}{\mu^2}$$

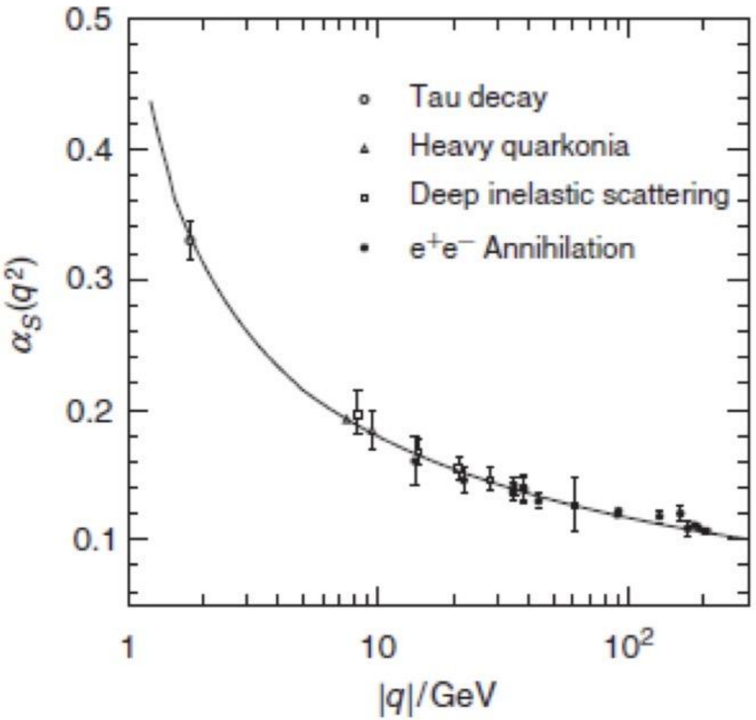
$$B = \frac{11N_c - N_f}{12\pi}$$

where N_f is the number of quarks flavours and N_c the number of colours

Running of α_s

$$\alpha_S(m_Z^2) = 0.1184 \pm 0.0007$$

$$\alpha_S(q^2) = \frac{\alpha_S(\mu^2)}{1 + B\alpha_S(\mu^2) \ln \frac{q^2}{\mu^2}}$$



The strength of the QCD coupling varies considerably over the range of energies relevant to particle physics

- |q| = 1 GeV c.c. of O(1)
- |q| > 100 GeV, c.c. about 0.1

This property of QCD is known as asymptotic freedom