High Energy Physics

Quantum Electrodynamics (QED)

Mariana Mendes Nº2021216335

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FACULDADE DE CIÊNCIAS E TECNOLOGIA
UNIVERSIDADE D COIMBRA

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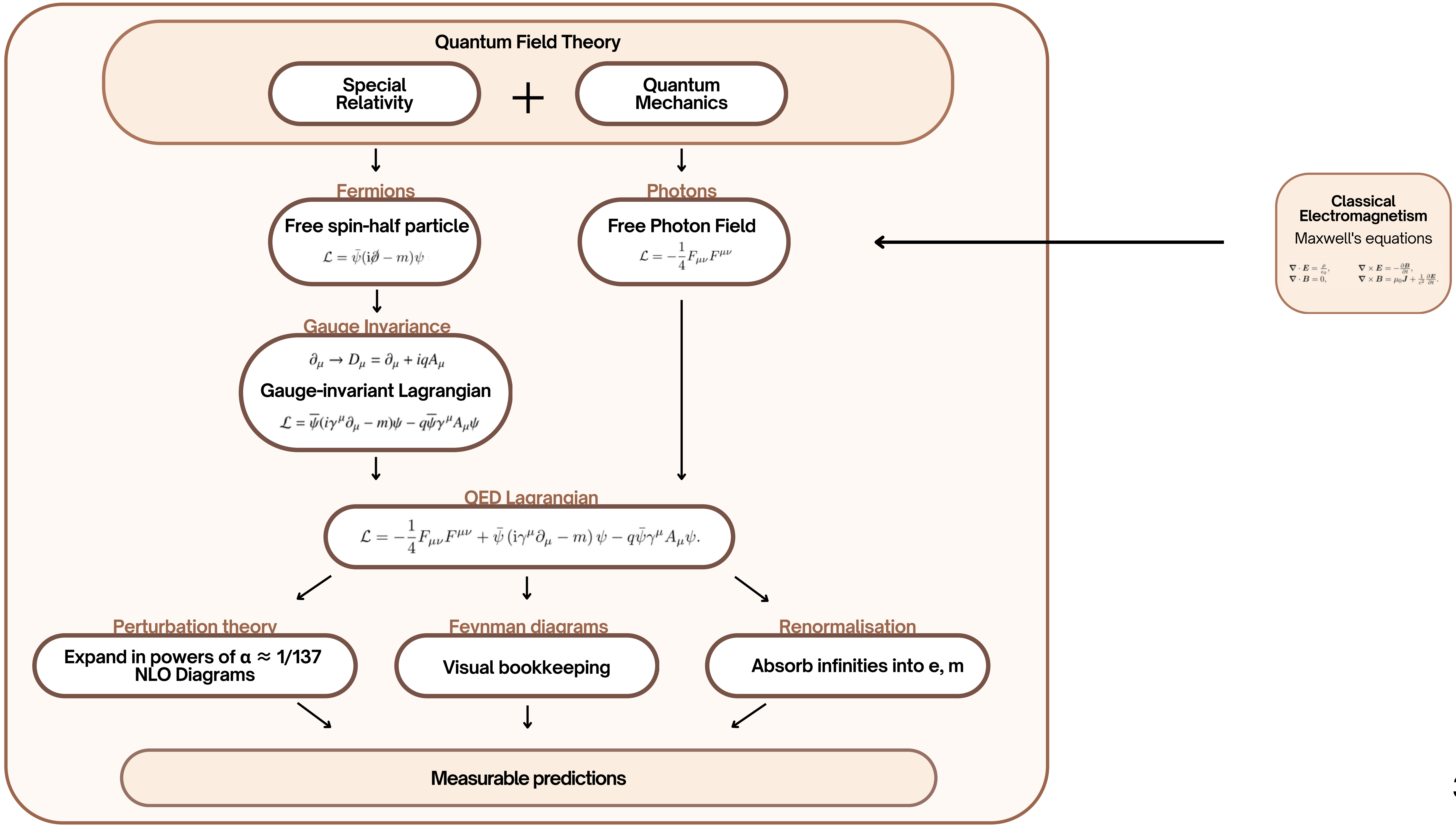
An Overview of the theory

Quantum Electrodynamics: The theory of light interacting with charged matter.



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Dirac equation

$$(i\gamma^\mu \partial_\mu - m)\psi = 0$$

The Lagrangian that leads to the Dirac equation is given by

$$\mathcal{L} = \bar{\psi}(\mathrm{i}\partial\!\!\!/ - m)\psi = \bar{\psi}(\mathrm{i}\gamma^\mu \partial_\mu - m)\psi \qquad \bar{\psi} = \psi^\dagger \gamma^0$$

It describes fermion matter fields whose equation of motion is the Dirac equation.
By looking for **free-particle/antiparticle plane wave solutions** of the form

Particle $\psi(\mathbf{x}, t) = u(E, \mathbf{p})e^{i(\mathbf{p}\cdot\mathbf{x} - Et)}$

Antiparticle $\psi(\mathbf{x}, t) = v(E, \mathbf{p})e^{-i(\mathbf{p}\cdot\mathbf{x} - Et)}$

$$\psi(x) = \begin{pmatrix} \psi_1 \\ \psi_2 \\ \psi_3 \\ \psi_4 \end{pmatrix} = \begin{pmatrix} \Psi_1 + i\Phi_1 \\ \Psi_2 + i\Phi_2 \\ \Psi_3 + i\Phi_3 \\ \Psi_4 + i\Phi_4 \end{pmatrix}$$

The field ψ(x) is a four-component complex spinor

The **first two solutions** of the **free-particle and antiparticle** to the Dirac equation are:

$$u_1(p) = \sqrt{E+m}\begin{pmatrix} 1 \\ 0 \\ \frac{p_z}{E+m} \\ \frac{p_x + ip_y}{E+m} \end{pmatrix} \text{ and } u_2(p) = \sqrt{E+m}\begin{pmatrix} 0 \\ 1 \\ \frac{p_x - ip_y}{E+m} \\ \frac{-p_z}{E+m} \end{pmatrix} \qquad v_1(p) = \sqrt{E+m}\begin{pmatrix} \frac{p_x - ip_y}{E+m} \\ \frac{-p_z}{E+m} \\ 0 \\ 1 \end{pmatrix} \text{ and } v_2(p) = \sqrt{E+m}\begin{pmatrix} \frac{p_z}{E+m} \\ \frac{p_x + ip_y}{E+m} \\ 1 \\ 0 \end{pmatrix}$$

source: Lancaster, T., & Blundell, S. J. (2014). Quantum field theory for the gifted amateur.
Thomson, M. (2013). Modern Particle Physics.

Gauge Invariance

The required local gauge symmetry is expressed as the invariance of the Lagrangian under a U(1) **local phase transformation** of the fields.

$$\psi(x) \to \psi'(x) = e^{iq\chi(x)}\psi(x)$$

With this transformation the **Lagrangian for a free spin-half particle,** becomes:

$$\mathcal{L} \to \mathcal{L}' = \mathcal{L} - q\bar{\psi}\gamma^{\mu}\left(\partial_{\mu}\chi\right)\psi$$

So this means that the Lagrangian is not invariant under U(1) local phase trasformations. The required **gauge invariance can be restored** by replacing the derivative $\partial\mu$, with:

$$\partial_{\mu} \to D_{\mu} = \partial_{\mu} + iqA_{\mu}$$

The **covariant derivative** and where $A\mu$ is a new field.

The cancellation of the unwanted term is achieved provided **the new field transforms as**: $A_{\mu} \to A'_{\mu} = A_{\mu} - \partial_{\mu}\chi$

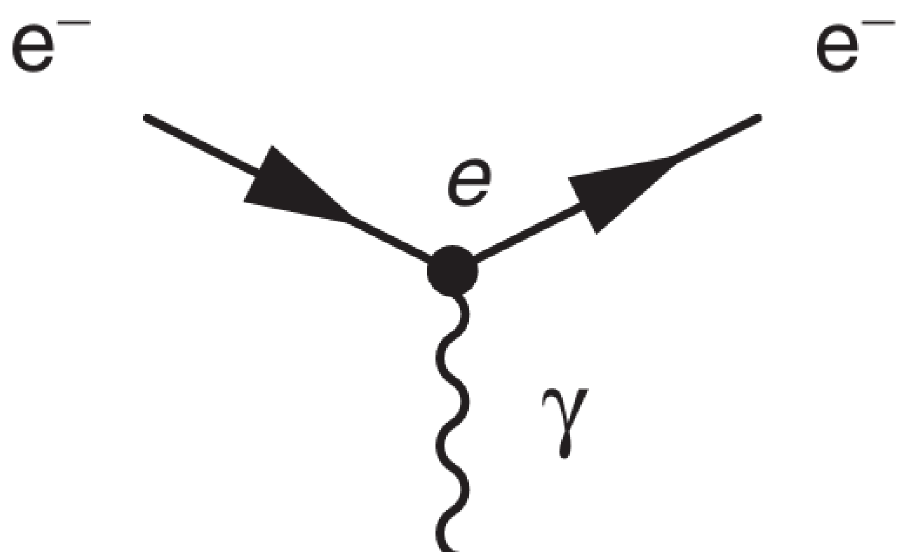
source: Lancaster, T., & Blundell, S. J. (2014). Quantum field theory for the gifted amateur.
Thomson, M. (2013). Modern Particle Physics.

For Fermions

The gauge-invariant Lagrangian for a spin-half fermion, can be achieved only by the introduction of the field A_μ:

$$\mathcal{L} = \bar{\psi}(i\gamma^\mu \partial_\mu - m)\psi - q\bar{\psi}\gamma^\mu A_\mu \psi$$

Which can be identified as the photon.



source: Lancaster, T., & Blundell, S. J. (2014). Quantum field theory for the gifted amateur.
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For Photons

Maxwell's equations for the electromagnetic field Aμ=(φ,A) can be expressed as:

$$\partial_\mu F^{\mu\nu} = j^\nu$$

j=(ρ,J) is the four-vector current associated with the charge and current densities ρ and J. The corresponding Lagrangian is

$$\mathcal{L}_{EM} = -\tfrac{1}{4}F^{\mu\nu}F_{\mu\nu} - j^\mu A_\mu$$

In the absence of sources jμ=0, and the Lagrangian for the free photon field is

$$\mathcal{L}_{EM} = -\tfrac{1}{4}F^{\mu\nu}F_{\mu\nu}$$

The free photon field Aμ can be written in terms of a plane wave and a four-vector ε(λ)

$$A_\mu = \varepsilon_\mu^{(\lambda)} e^{i(\mathbf{p}\cdot\mathbf{x}-Et)}$$

Recall

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$$

$$= \begin{pmatrix} 0 & E_x & E_y & E_z \\ -E_x & 0 & -B_z & B_y \\ -E_y & B_z & 0 & -B_x \\ -E_z & -B_y & B_x & 0 \end{pmatrix}$$

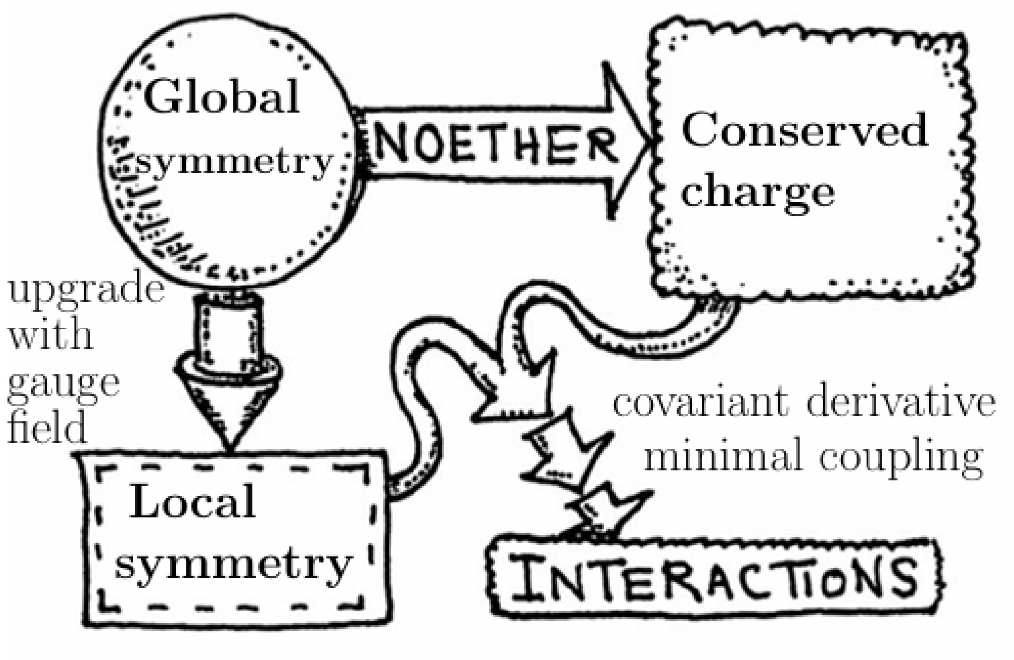
source: Lancaster, T., & Blundell, S. J. (2014). Quantum field theory for the gifted amateur.
Thomson, M. (2013). Modern Particle Physics.

QED Lagrangian

Looking at both of the results gotten in the previus slides, we get to write down perhaps the most successful theory in modern physics: **Quantum electrodynamics** or QED.

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \bar{\psi}\left(\mathrm{i}\gamma^{\mu}\partial_{\mu} - m\right)\psi - q\bar{\psi}\gamma^{\mu}A_{\mu}\psi.$$

The QED Lagrangian includes the **contributions from** the **gauge field of electromagnetism** and from the **locally gauge invariant Dirac Lagrangian** describing fermions and their interactions.



In order to have local gauge invariance, minimal coupling tells us that we must add to L an interaction term Li = $- q\bar{\psi}\gamma^{\mu}A_{\mu}\psi$

source: Lancaster, T., & Blundell, S. J. (2014). Quantum field theory for the gifted amateur.

For Photons

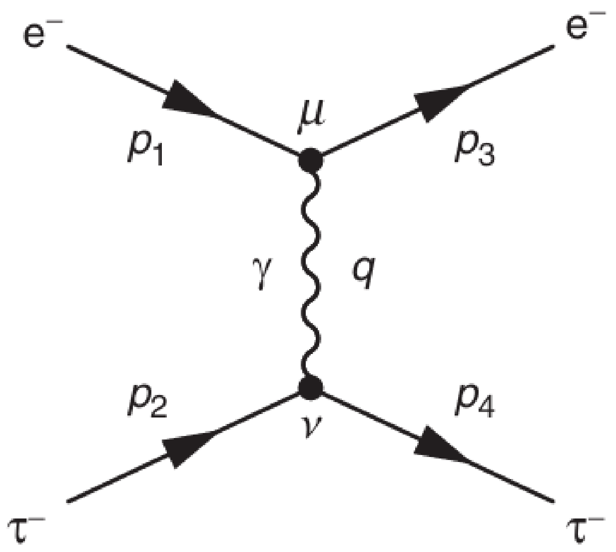
For the exchange of the photon, which is a spin-1 particle, it is necessary to sum over the quantum-mechanical amplitudes for the possible polarisation states.

For a real (as opposed to virtual) photon, the polarisation vector is always transverse to the direction of motion. a photon propagating in the z-direction can be described by

$$\varepsilon^{(1)} = (0, 1, 0, 0) \quad \text{and} \quad \varepsilon^{(2)} = (0, 0, 1, 0)$$

QED matrix element - An example

For the QED scattering process e−τ- → e−τ-



The Lorentz-invariant matrix element can be obtained by using the potential of for the interaction at the e−γ vertex

$$u_e^{\dagger}(p_3)\, Q_e e \gamma^0 \gamma^{\mu} \varepsilon_{\mu}^{(\lambda)}\, u_e(p_1)$$

Similarly, the interaction at the τ−γ vertex

$$u_\tau^{\dagger}(p_4)\, Q_\tau e \gamma^0 \gamma^{\nu} \varepsilon_{\nu}^{(\lambda)*}\, u_\tau(p_2)$$

source: Thomson, M. (2013). Modern Particle Physics.

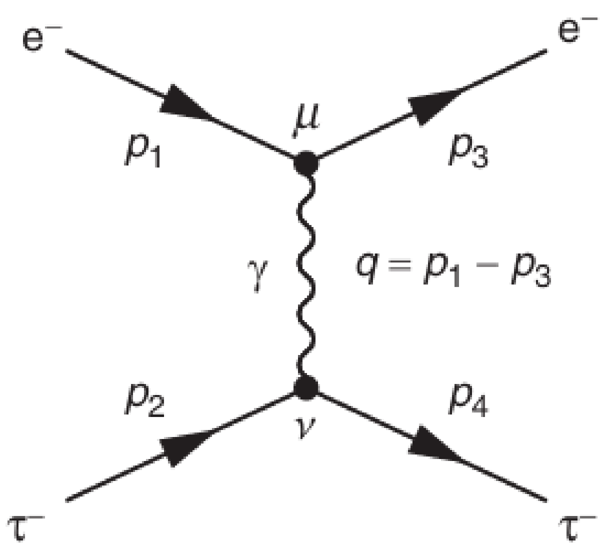
QED matrix element - An example

The QED matrix element is obtained by summing over both the two possible time orderings and the possible polarisation states of the virtual photon.

$$\mathcal{M} = \sum_{\lambda} \left[u_e^{\dagger}(p_3) Q_e e \gamma^0 \gamma^{\mu} u_e(p_1)\right] \varepsilon_{\mu}^{(\lambda)} \frac{1}{q^2} \varepsilon_{\nu}^{(\lambda)*} \left[u_{\tau}^{\dagger}(p_4) Q_{\tau} e \gamma^0 \gamma^{\nu} u_{\tau}(p_2)\right]$$

$$\sum_{\lambda} \varepsilon_{\mu}^{(\lambda)} \varepsilon_{\nu}^{(\lambda)*} = -g_{\mu\nu}$$

$$\mathcal{M} = -\left[Q_e e\, \overline{u}_e(p_3) \gamma^{\mu} u_e(p_1)\right] \frac{g_{\mu\nu}}{q^2} \left[Q_{\tau} e\, \overline{u}_{\tau}(p_4) \gamma^{\nu} u_{\tau}(p_2)\right]$$



$$\overline{u}(p_3)[ie\gamma^{\mu}]u(p_1)$$

$$\frac{-ig_{\mu\nu}}{q^2}$$

$$\overline{u}(p_4)[ie\gamma^{\nu}]u(p_2)$$

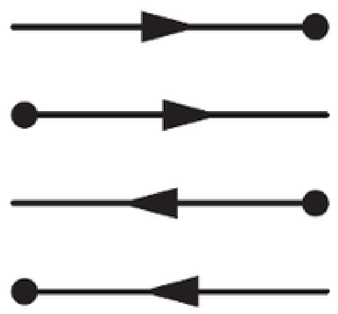
source: Lancaster, T., & Blundell, S. J. (2014). Quantum field theory for the gifted amateur.
Thomson, M. (2013). Modern Particle Physics.

Feynman Rules for QED

The product of all of these terms is equivalent to −iM.

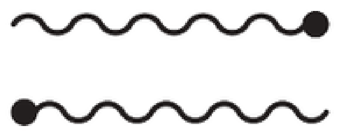
Dirac spinor

initial-state particle:	$u(p)$	
final-state particle:	$\overline{u}(p)$	
initial-state antiparticle:	$\overline{v}(p)$	
final-state antiparticle:	$v(p)$	



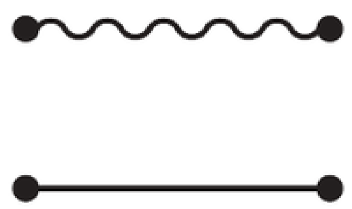
Polarization vector

initial-state photon:	$\varepsilon_\mu(p)$	
final-state photon:	$\varepsilon^*_\mu(p)$	

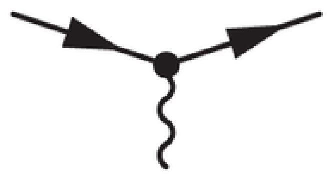


Propagator factor for each internal line

photon propagator:	$-\frac{ig_{\mu\nu}}{q^2}$	
fermion propagator:	$-\frac{i(\gamma^\mu q_\mu + m)}{q^2 - m^2}$	



QED vertex:	$-iQe\gamma^\mu$	



source: Thomson, M. (2013). Modern Particle Physics.

Calculations in perturbation theory

In QED, the dominant contribution to a cross section or decay rate is usually the Feynman diagram with the fewest number of interaction vertices, known as the lowest-order (LO) diagram.

In addition to the lowest-order diagram there are an infinite number of higher-order-diagrams resulting in the same final state. Next-to-leading-order (NLO) diagrams.

The total amplitude Mfi for a particular process is the sum of all individual amplitudes giving the same final state.

$$\mathcal{M}_{fi} = \mathcal{M}_{\mathrm{LO}} + \sum_{j} \mathcal{M}_{1,j} + \cdots$$

In general, the individual amplitudes are complex.
The contributions from different diagrams can interfere:

- **positively**
- **negatively**

source: Lancaster, T., & Blundell, S. J. (2014). Quantum field theory for the gifted amateur.
Thomson, M. (2013). Modern Particle Physics.

Calculations in perturbation theory

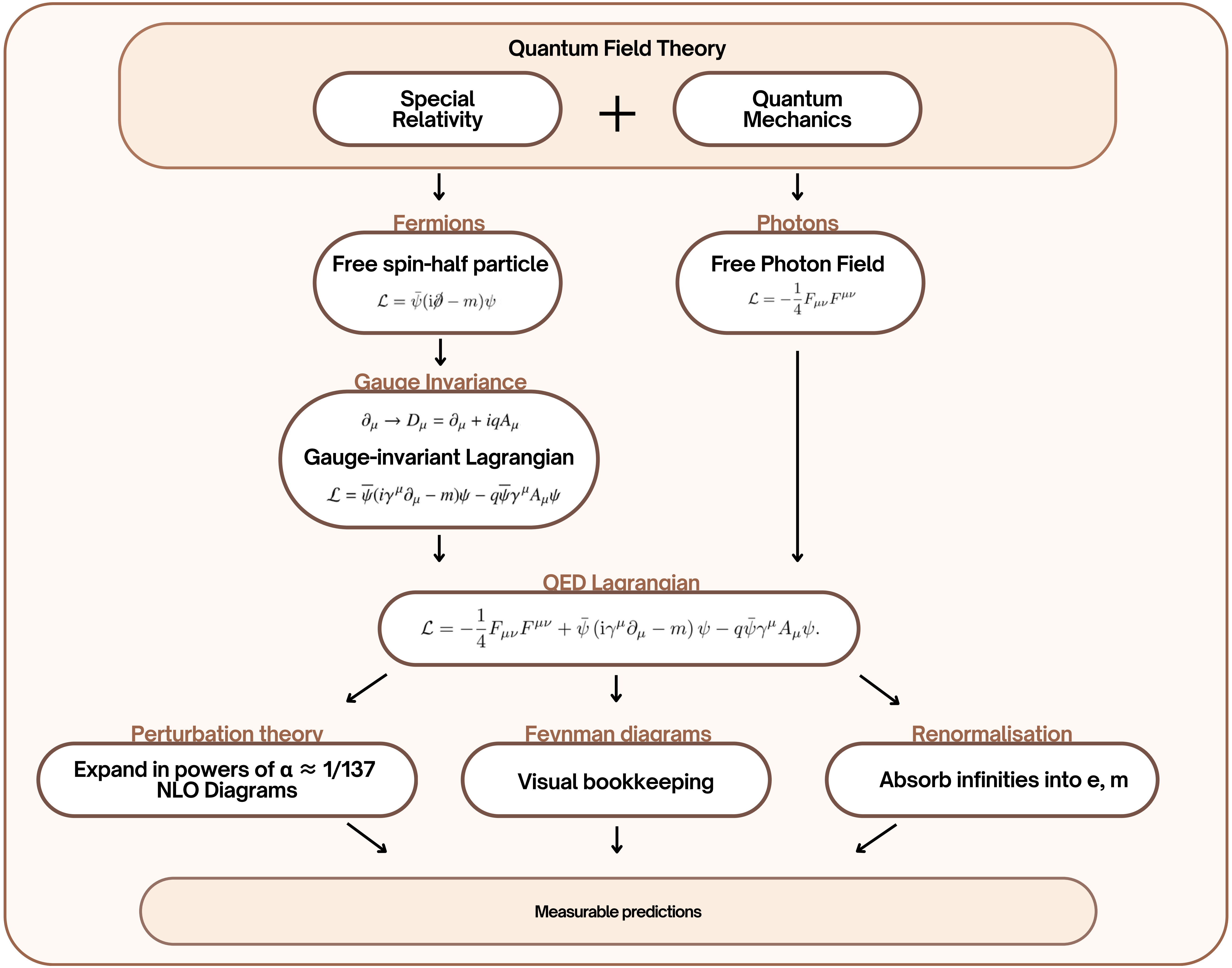
Physical observables, such as decay rates and cross sections, depend on the matrix element squared given by

$$|\mathcal{M}_{fi}|^2 = \left(\alpha M_{\mathrm{LO}} + \alpha^2 \sum_j M_{1,\mathrm{j}} + \cdots\right)\left(\alpha M^*_{\mathrm{LO}} + \alpha^2 \sum_k M^*_{1,\mathrm{k}} + \cdots\right)$$
$$= \alpha^2 |M_{\mathrm{LO}}|^2 + \alpha^3 \sum_j \left(M_{\mathrm{LO}} M^*_{1,j} + M^*_{\mathrm{LO}} M_{1,j}\right) + \alpha^4 \sum_{jk} M_{1,j} M^*_{1,\mathrm{k}} + \cdots$$

For QED, the dimensionless coupling constant $\alpha \approx 1/137$ is sufficiently small that this series converges rapidly and is dominated by the LO term.

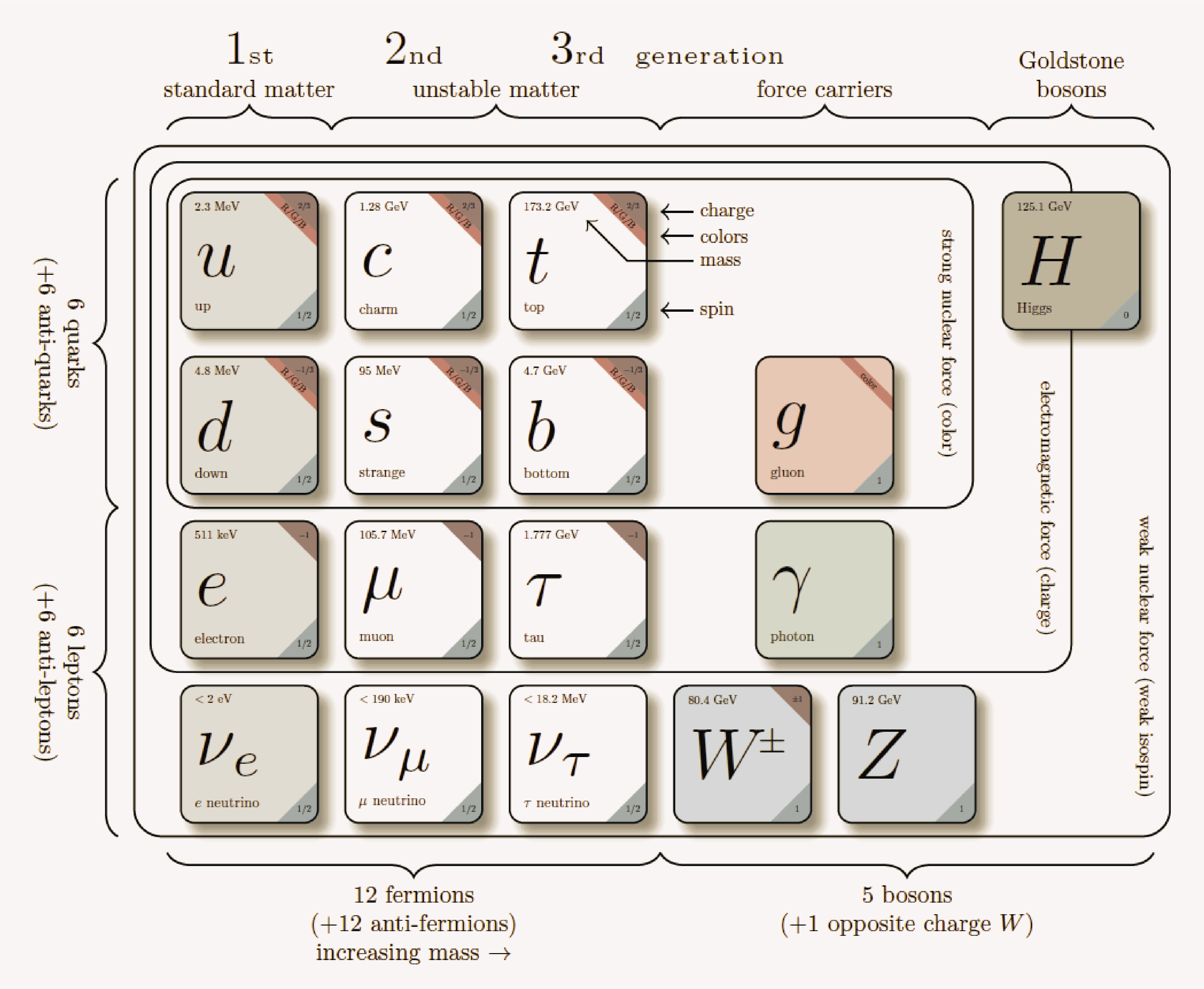
For this reason, usually only the lowest-order diagram(s) are considered for the calculations.

source: Lancaster, T., & Blundell, S. J. (2014). Quantum field theory for the gifted amateur.
Thomson, M. (2013). Modern Particle Physics.


Quantum Field Theory
Special Relativity
+
Quantum Mechanics
Fermions
Free spin-half particle
$\mathcal{L} = \bar{\psi}(i\partial\!\!\!/ - m)\psi$
Photons
Free Photon Field
$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu}$
Gauge Invariance
$\partial_\mu \to D_\mu = \partial_\mu + iqA_\mu$
Gauge-invariant Lagrangian
$\mathcal{L} = \overline{\psi}(i\gamma^\mu\partial_\mu - m)\psi - q\overline{\psi}\gamma^\mu A_\mu\psi$
QED Lagrangian
$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \bar{\psi}\left(i\gamma^\mu\partial_\mu - m\right)\psi - q\bar{\psi}\gamma^\mu A_\mu\psi.$
Perturbation theory
Expand in powers of α ≈ 1/137
NLO Diagrams
Feynman diagrams
Visual bookkeeping
Renormalisation
Absorb infinities into e, m
Measurable predictions

Questions

Standard Model



source: https://texample.net/model-physics/