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Exciting Black Holes with Stars

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MEFT Project

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1) The physics behind Black Hole observations

- BHs are effectively ‘’invisible’’, depending on **external agents** for observation;

- Light remains orbiting the BH forever when shot with an **impact parameter** of $3\sqrt{3}MG/c^2$;

- The **light ring** controls which light particles **leave** the BH and which **fall**.

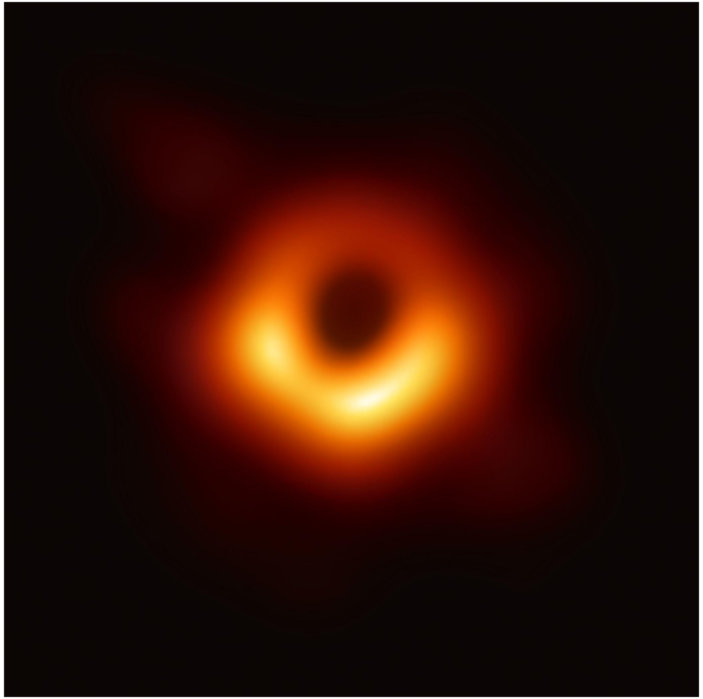

Figure 1: Image of the M87* BH taken by the Event Horizon Telescope [1]

[1] 1906.11238

2) Objectives

- **Reproduce** the results obtained by V. Cardoso, F. Duque and A. Foschi in [2] for Schwarzschild BHs:
  - **Luminosity** detected by a far away observer in Schwarzschild;

- **Expand** their approach to Kerr BHs and find new physical phenomena.

[2] 2102.07784v2

3) Setup: Zero initial data with an emitter

- We set the emitter's velocity to v=0.4 in different directions;
- New phenomena arise, such as the **relativistic Doppler effect**.

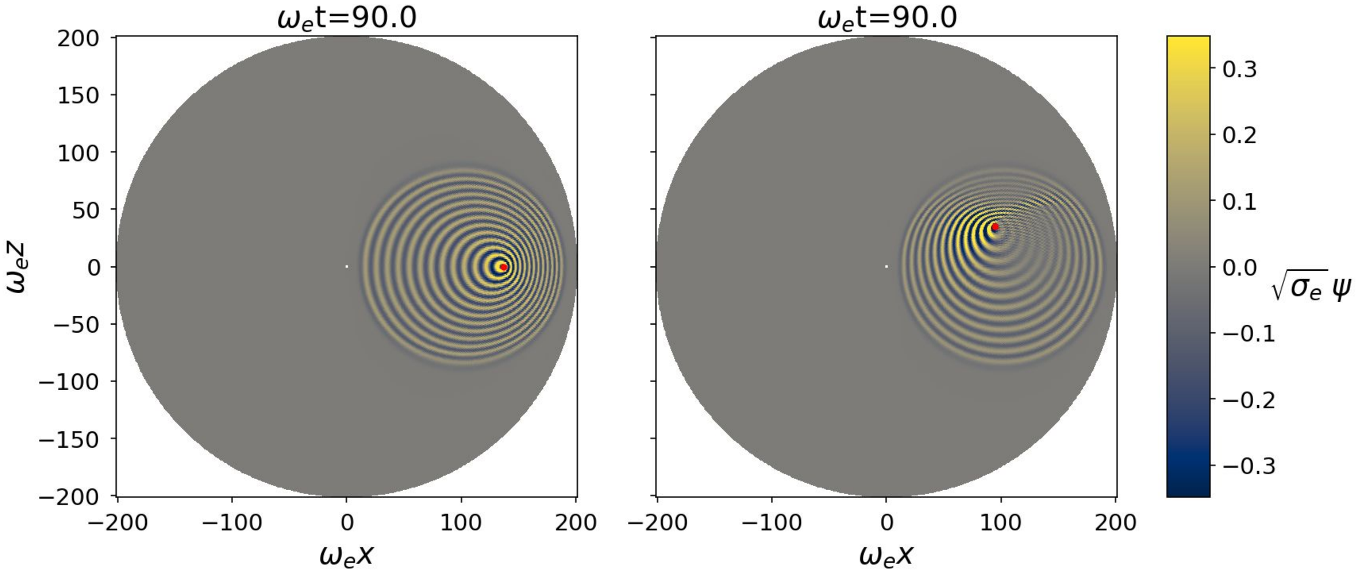

Figure 2: Wave emitted by a moving source of scalar radiation.

4) Future work

- Simulate **geodesic** motion in **Kerr**;
- Test a **free initial data** setup in **Kerr**;
- Implement the **emitter** in the **Teukolsky** equation;
- Find the **luminosity** measured by a far away observer.

Thanks for your attention

Extra Slides

A) Equations of motion

- In this thesis, we are interested in the **equations of motion**:

For particles

$$\ddot{x}^{\mu} + \Gamma^{\mu}_{\alpha\beta}\dot{x}^{\alpha}\dot{x}^{\beta} = 0 \quad (1)$$

For waves

$$\Box\Psi = 4\pi \sin\left(\omega_e \tau\right) T \quad (2)$$

A) The Kerr solution

- We are interested in studying **astrophysical** BHs.
- When isolated, they are described by the **Kerr spacetime.**
- The Christoffel symbols can be calculated from the Kerr metric and the wave equation becomes the **Teukolsky equation** for scalar fields:

$$\left[\frac{(r^2+a^2)^2}{\Delta} - a^2\sin^2\theta\right]\frac{\partial^2\Psi}{\partial t^2} + \frac{4Mar}{\Delta}\frac{\partial^2\Psi}{\partial t\,\partial\phi} + \left[\frac{a^2}{\Delta} - \frac{1}{\sin^2\theta}\right]\frac{\partial^2\Psi}{\partial\phi^2}$$

$$-\frac{\partial}{\partial r}\left(\Delta\frac{\partial\Psi}{\partial r}\right) - \frac{1}{\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial\Psi}{\partial\theta}\right) = 4\pi\Sigma\sin(\omega_e\tau)T\,. \qquad (3)$$

A) Emitter term explained

- The **emitter term** is defined as

$$T^{\mu\nu} = m_e \int_{-\infty}^{+\infty} \delta^{(4)}(x - z(\tau)) \frac{dz^\mu}{d\tau} \frac{dz^\nu}{d\tau} d\tau = m_e \frac{dt}{d\tau_e} \frac{dz^\mu}{dt} \frac{dz^\nu}{dt} \delta^{(3)}(\vec{r} - \vec{r}_e) \tag{11}$$

- When **simplifying** to flat spacetime, it becomes

$$T = A\, \delta^{(3)}(\vec{r} - \vec{r}_e(t))\,, \qquad A = m_e \sqrt{1 - v^2} = \frac{m_e}{\gamma} \tag{12}$$

- To compute the **Dirac delta function**, we approximate

$$\delta^{(3)}(\vec{r} - \vec{r}_e(t)) \approx \frac{1}{\pi^{\frac{3}{2}} \epsilon^3} \exp\left(-\frac{d^2}{\epsilon^2}\right) \tag{13}$$

B) The Minkowski spacetime toy model

- We want to start the study with **simple** toy models and progressively **increase** the complexity of the system.

- **Flat spacetime** is the simplest reduction of Kerr spacetime.

- Geodesics become **straight lines** with constant velocity and the Teukolsky equation for scalar waves becomes the massless **Klein-Gordon equation**.

$$\boxed{\left(\frac{\partial^2}{\partial t^2} - \nabla^2\right)\Psi = 4\pi \sin(\omega_e \tau_e) T} \quad (4)$$

B) Mode separation

- Solving a 3+1 equation is very expensive, so we **separate into modes**:

$$\Psi = \psi(t, r, \theta) e^{im\phi} \tag{14}$$

- In **spherical coordinates**, equation (4) becomes:

$$\left[\frac{\partial^2}{\partial t^2} - \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 \frac{\partial}{\partial r}\right) - \frac{1}{r^2 \sin\theta}\frac{\partial}{\partial \theta}\left(\sin\theta \frac{\partial}{\partial \theta}\right) + \frac{m^2}{r^2 \sin^2\theta}\right]\Psi = 4\pi \sin(\omega_e \tau_e) T \tag{15}$$

- The **emitter term** also changes, becoming

$$T = \frac{A}{\pi \epsilon^2 \sqrt{\sigma_e}} \exp\left(-\frac{d^2}{\epsilon^2}\right) \exp\left(im\phi_e\right) \qquad \sigma_e = r\, r_e \sin\theta \sin\theta_e \tag{16}$$

C) Setup: free initial data

- We consider an initial outgoing spherically symmetric gaussian package:

$$\psi = \frac{\sin r}{r} e^{-\left(\frac{r-r_0}{\epsilon}\right)^2} \qquad \partial_t \psi = \pm(\partial_r \psi + \psi/r) \tag{17}$$

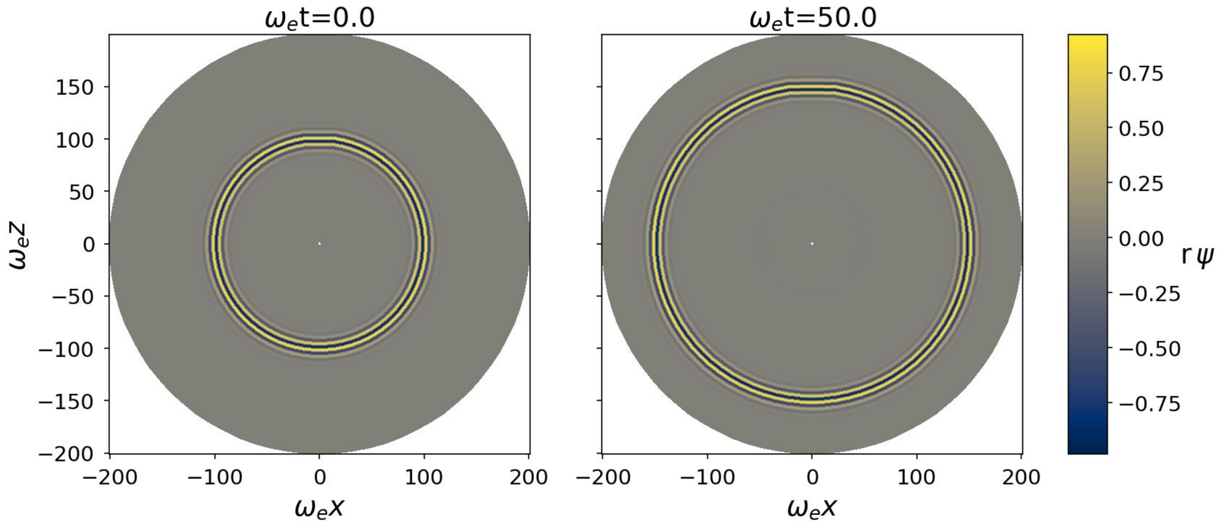

Figure 5: Initial outgoing spherically symmetric wave propagation

C) Convergence tests

- We apply an L2 norm to the error to verify convergence:

$$L^2(\Delta\psi) = \sqrt{\int_{\mathcal{D}} |\Delta\psi|^2} \quad (18)$$

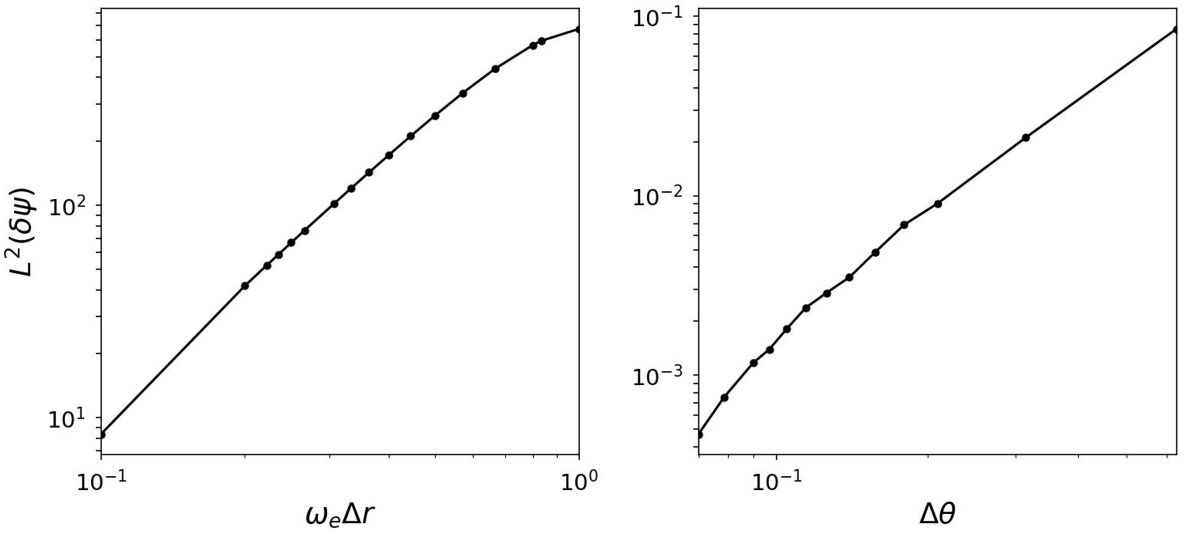

Figure 6: The convergence for different radial and angular resolutions

D) Setup: Zero initial data with an emitter

- We added a **static emitter** to the setup;
- Many theoretical predictions were **verified**.

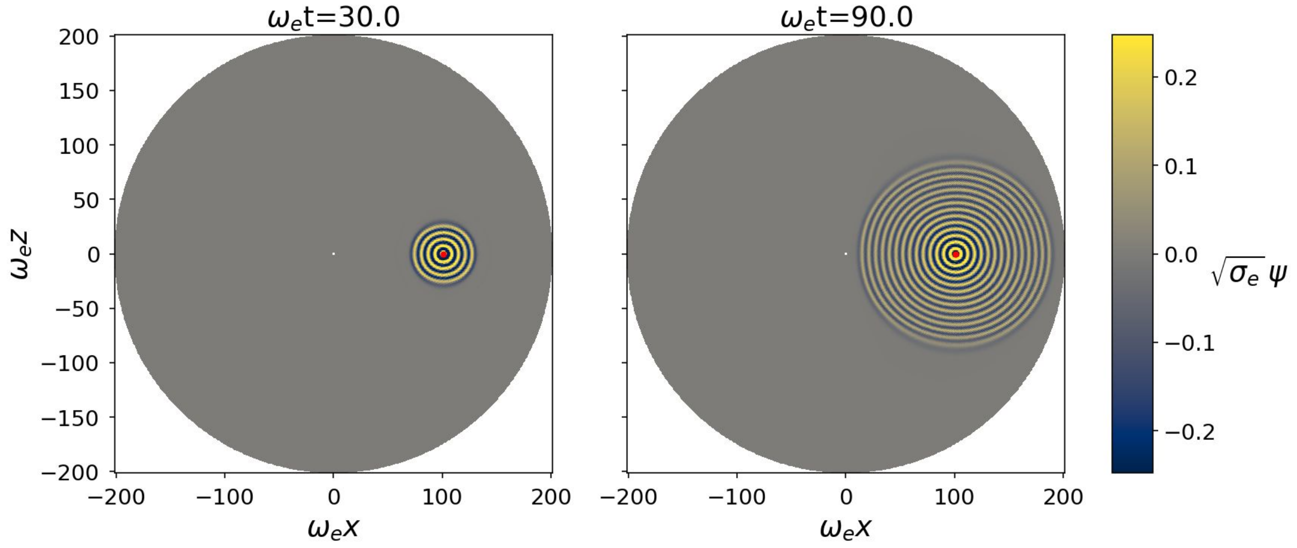

Figure 2: Wave emitted by a static source of scalar radiation.

D) Relativistic Doppler effect verified!

- Radial motion with observer, both at the equator, can be reduced to **1D** problem

- The longitudinal formulation of the **relativistic Doppler effect** is:

$$z = \sqrt{\frac{1+v}{1-v}} = \frac{\omega_e}{\omega_o} = \frac{T_o}{2\pi} \quad (5)$$

$$\begin{cases} z_{\text{theo}} \approx 1.53 \\ z_{\text{obs}} \approx 1.59 \end{cases} \quad (6)$$

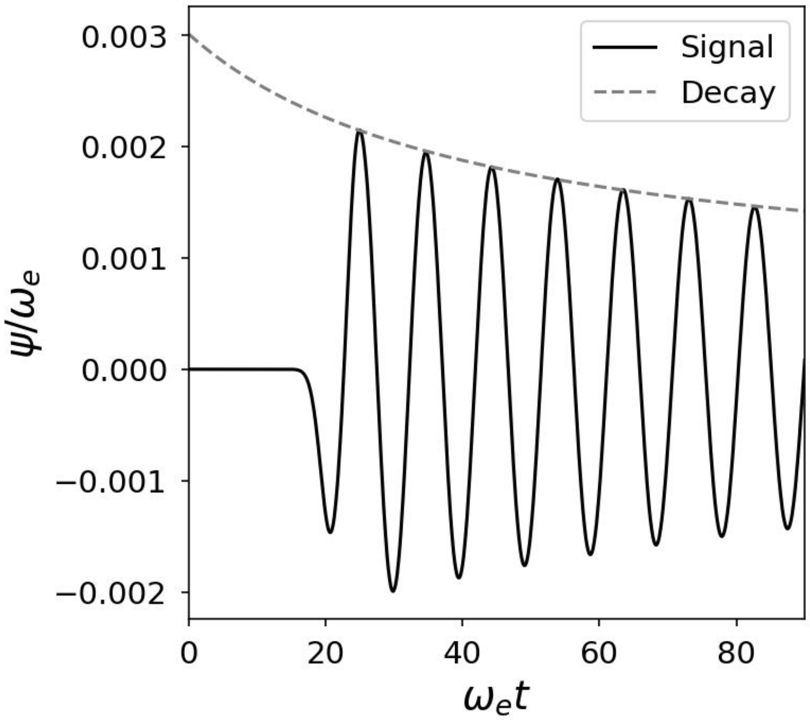

Figure 4: Time profile of the wave seen by an observer in the radial motion

D) Decay model

- The theoretical prediction of the wave decay is:

$$\psi(t,r) \propto \frac{1}{(r_o - r_{e0})/v - t} \quad (7)$$

- So we fit the peaks with a simple model:

$$\psi(t,r) = \frac{a_1}{a_2 - t} + a_3 \quad (8)$$

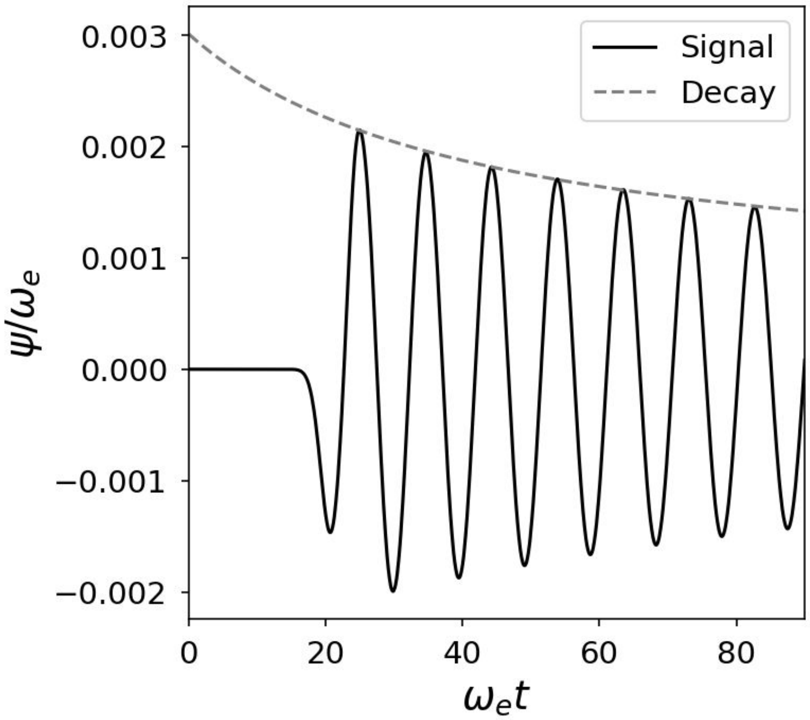

Figure 4: Time profile of the wave seen by an observer in the radial motion

D) Decay model

- We obtain the following results:

$$\begin{cases} (r_o - r_{e0})/v = \text{-}50 \\ \omega_e a_2 \approx \text{-}42.5 \end{cases} \quad (9)$$

- There is a clear difference of 7.5.

- Hypothesis:

$$\omega_e \epsilon / v = 7.5 \quad (10)$$

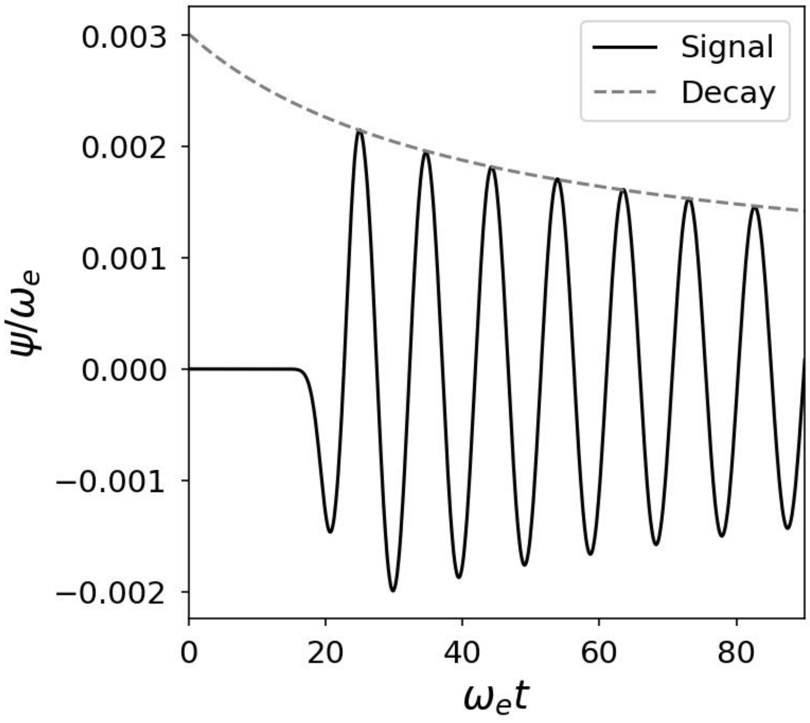

Figure 4: Time profile of the wave seen by an observer in the radial motion