# Measurement-Induced Phase Transition for Free Fermions

## Integrated Project of the 2nd Cycle

*Author*: João Chaveiro

*Supervisors*: Prof. Giuseppe De Tomasi
Prof. Pedro Ribeiro

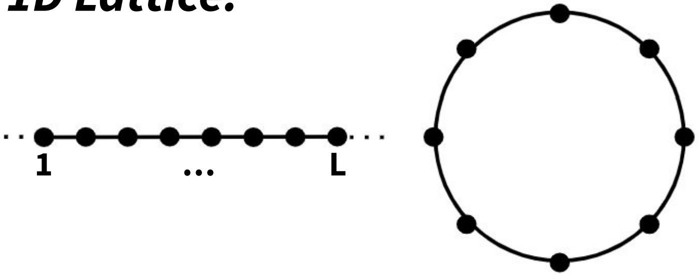
# Setup:

**Hamiltonian:**

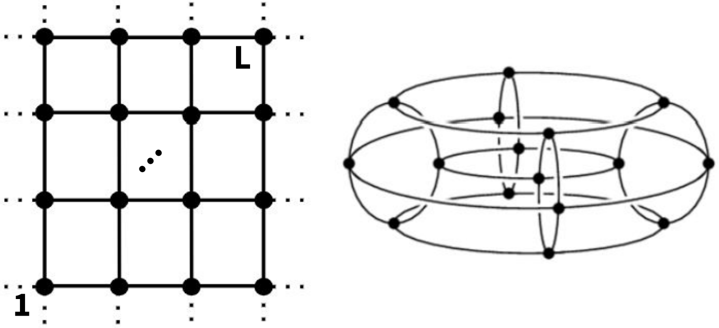
$$H = -J \sum_{\langle i,j \rangle} (c_i^\dagger c_j + c_j^\dagger c_i)$$

**Topologies Considered:**

***1D Lattice:***

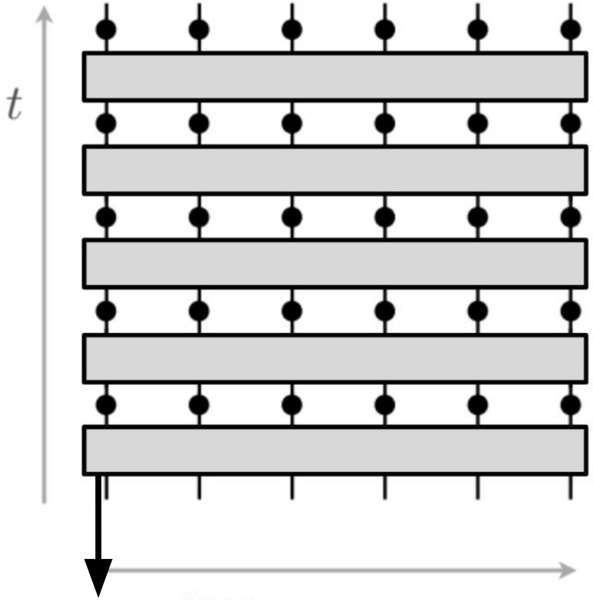



***2D Lattice:***



# Simulation:

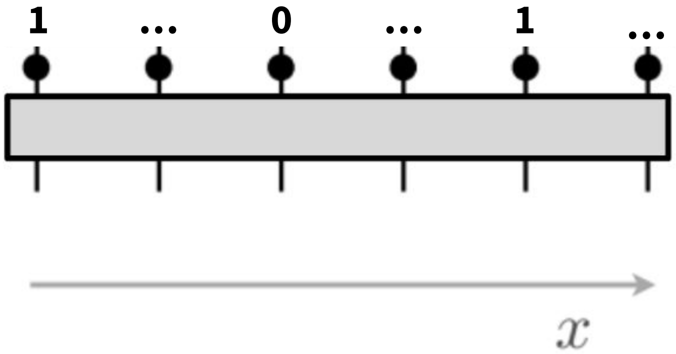
## 1 - Unitary Evolution Operator:



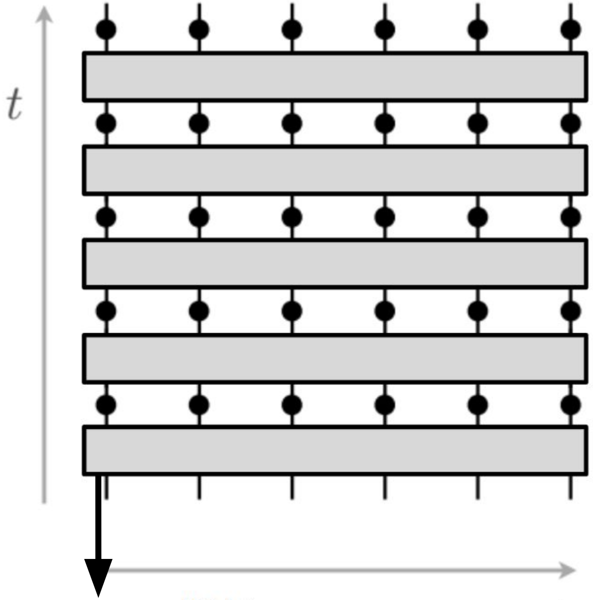

$$U(\Delta t) \approx e^{-iH\Delta t} \rightarrow C_{t+\Delta t} = U(0,\Delta t)^{\dagger} C_t U(0,\Delta t)$$

# Simulation:

## 2 - Site Selection/Measurement:



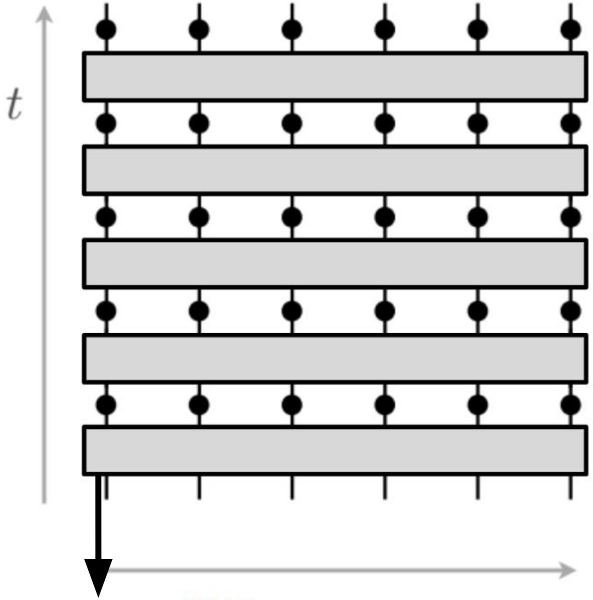

## 1 - Unitary Evolution Operator:




$$U(\Delta t) \approx e^{-iH\Delta t} \longrightarrow C_{t+\Delta t} = U(0,\Delta t)^{\dagger} C_t U(0,\Delta t)$$

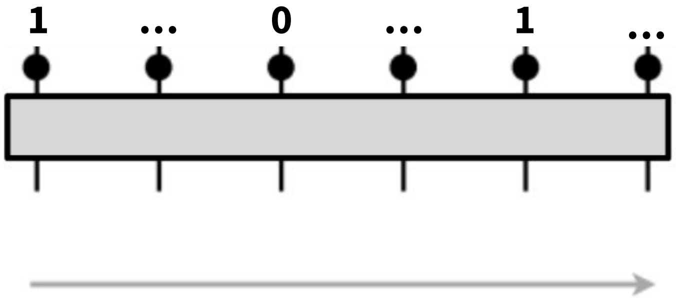
# Simulation:

## 1 - Unitary Evolution Operator:




$$U(\Delta t) \approx e^{-iH\Delta t} \longrightarrow C_{t+\Delta t} = U(0,\Delta t)^{\dagger} C_t U(0,\Delta t)$$

## 2 - Site Selection/Measurement:



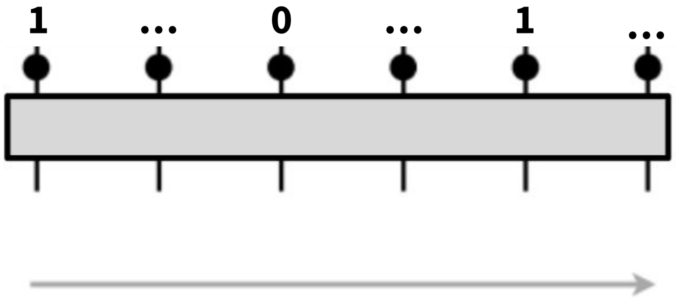

$x$

## 3 - State Update:

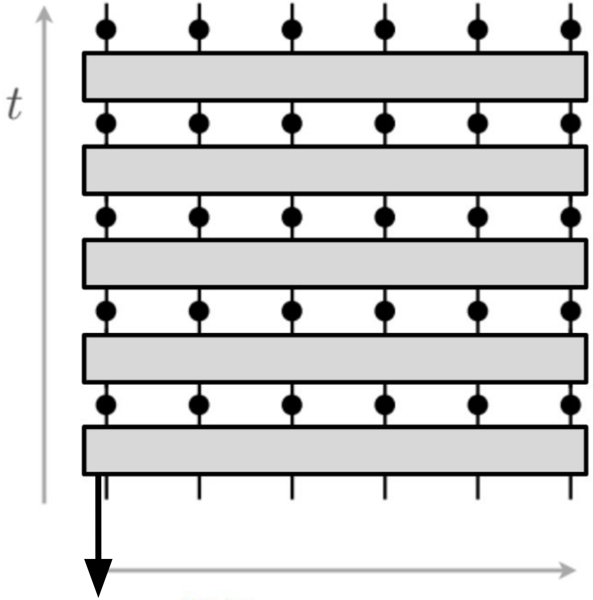
$$[C']_{ij} = \begin{cases} \mathbf{C_{ij}} + \frac{(\delta_{ik} - \mathbf{C_{ik}})(\delta_{jk} - \mathbf{C_{kj}})}{\mathbf{1} - \mathbf{C_{kk}}} & \text{if } \hat{n}_k = 1 \text{ (Occupied)} \\ \mathbf{C_{ij}} - \frac{\mathbf{C_{ik}}\mathbf{C_{kj}}}{\mathbf{C_{kk}}} & \text{if } \hat{n}_k = 0 \text{ (Empty)} \end{cases}$$

# Simulation:

## 2 - Site Selection/Measurement:




## 1 - Unitary Evolution Operator:




$$U(\Delta t) \approx e^{-iH\Delta t} \longrightarrow C_{t+\Delta t} = U(0,\Delta t)^{\dagger} C_t U(0,\Delta t)$$

## 3 - State Update:

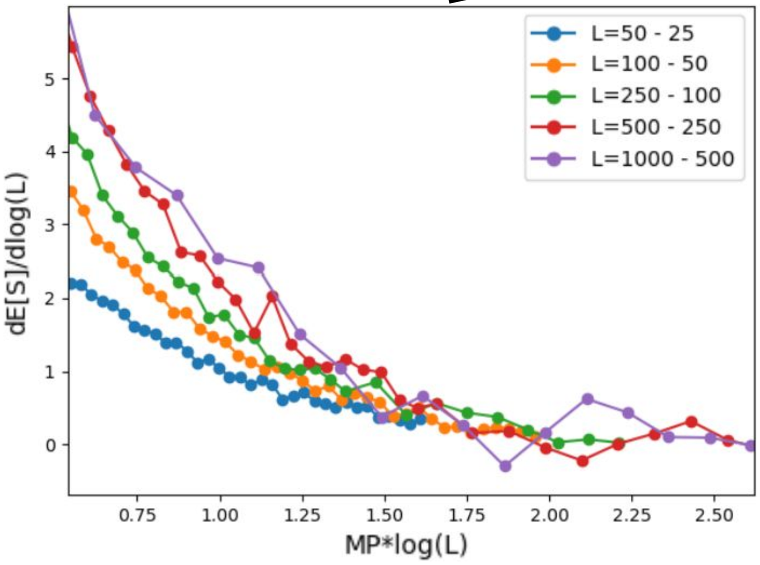
$$[C']_{ij} = \begin{cases} \mathbf{C_{ij}} + \frac{(\delta_{ik} - \mathbf{C_{ik}})(\delta_{jk} - \mathbf{C_{kj}})}{1 - \mathbf{C_{kk}}} & \text{if } \hat{n}_k = 1 \text{ (Occupied)} \\ \mathbf{C_{ij}} - \frac{\mathbf{C_{ik}} \mathbf{C_{kj}}}{\mathbf{C_{kk}}} & \text{if } \hat{n}_k = 0 \text{ (Empty)} \end{cases}$$

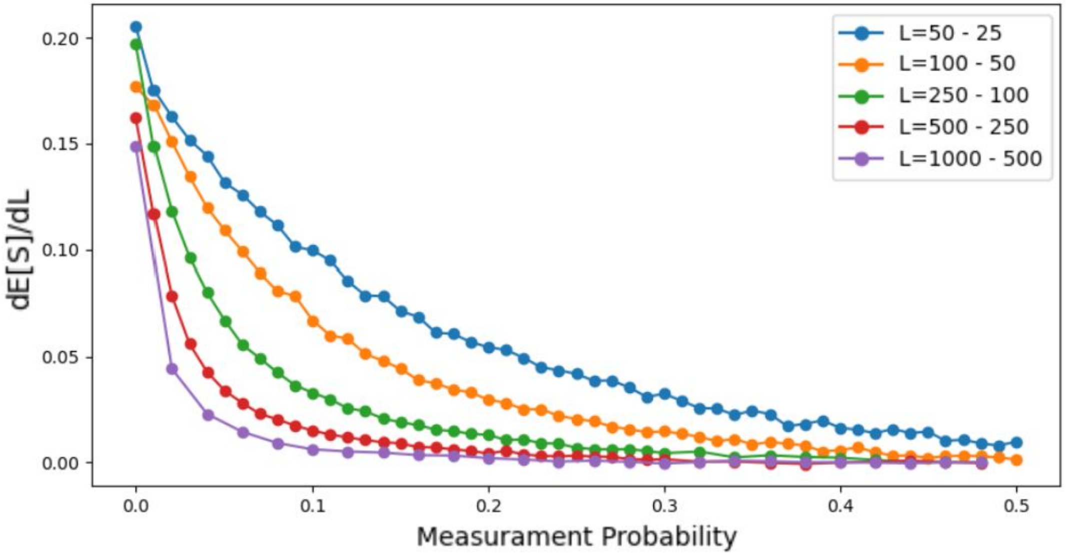
**4 - REPEAT !**

# 1D Lattice:

$p_c = 0!$ **No transition!**

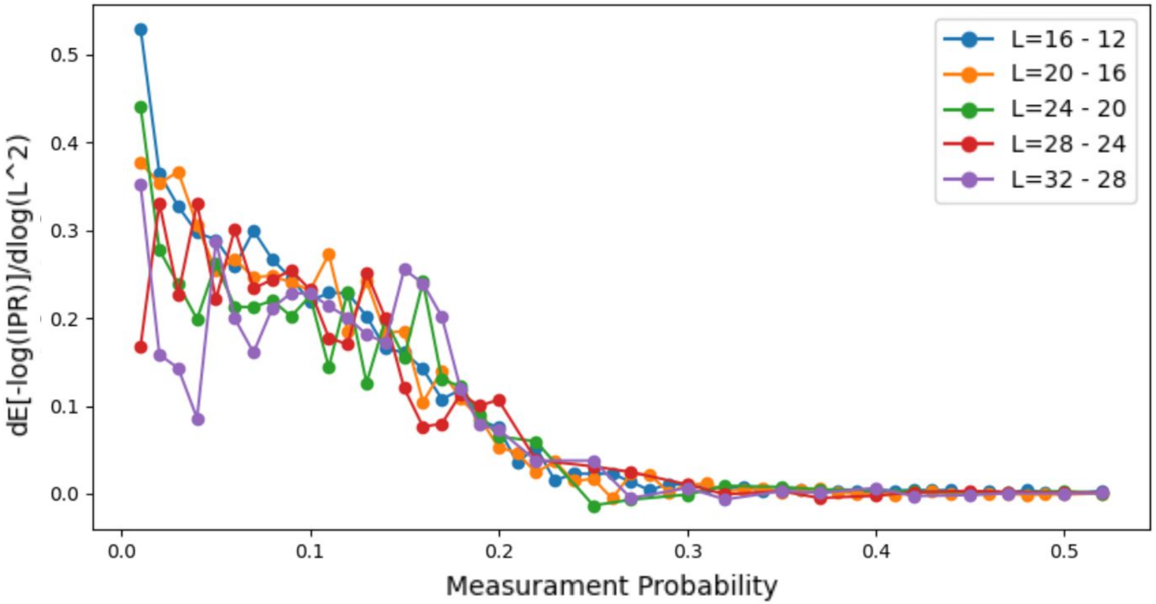
**Derivatives of S:**



# 2D Lattice:

**-Log(IPR):**



## Future Work

- ➔ Optimize the algorithm and construct better ways to evaluate critical phenomena;
- ➔ Increase resolution and simulate bigger systems;
- ➔ Explore different topologies and unitary evolution operators.