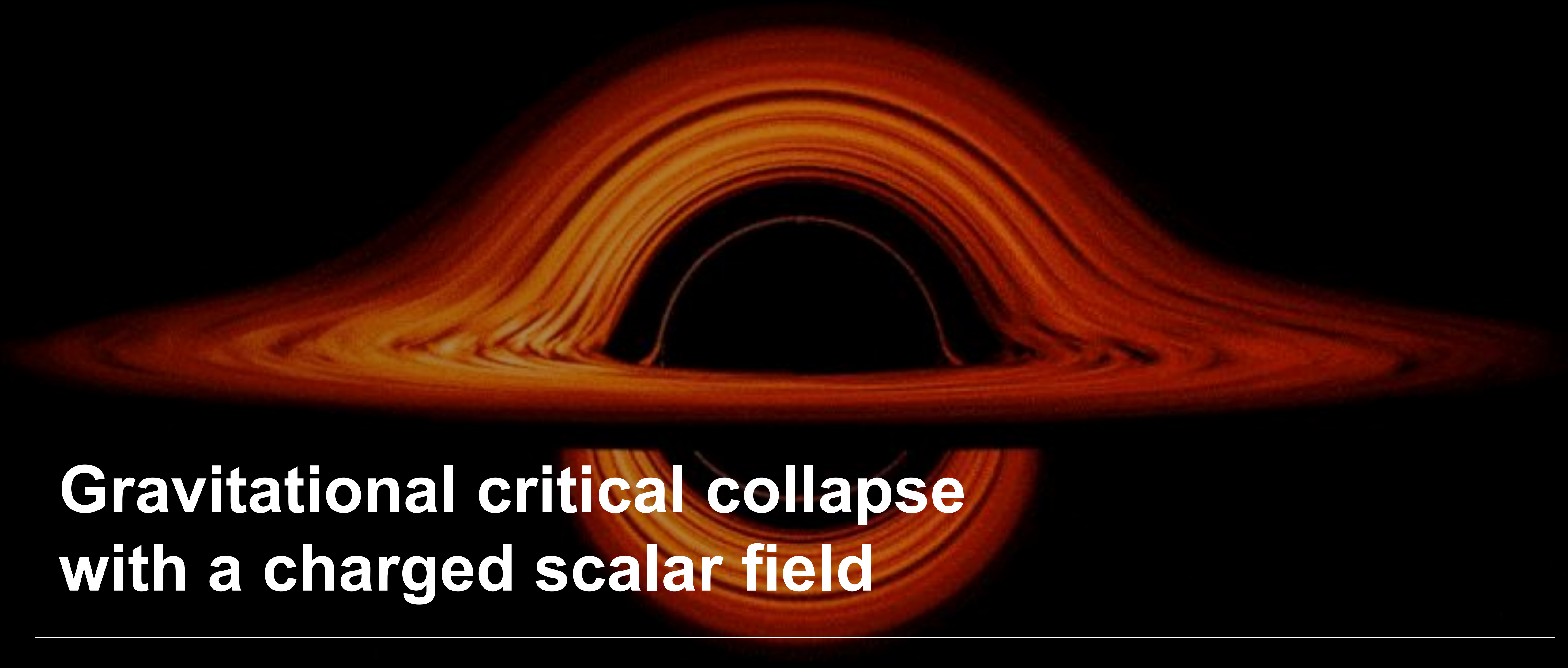

Gravitational critical collapse with a charged scalar field

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Introduction

Extremal black hole formation as a critical phenomenon

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Proof of new regime of critical collapse:

Critical solution is an **extremal RN black hole**

1.5 Extremal critical collapse of a charged scalar field and in vacuum

It is natural to conjecture the analog of Theorem 1 for a massless charged scalar field in spherical symmetry:

Conjecture 4. *Extremal critical collapse occurs in the spherically symmetric Einstein–Maxwell-charged scalar field model and there exist critical solutions which are isometric to extremal Reissner–Nordström in the domain of outer communication after sufficiently large advanced time.*

In [KU22], the present authors showed that a black hole with an extremal Reissner–Nordström domain of outer communication and containing no trapped surfaces can arise from regular one-ended Cauchy data in the spherically symmetric charged scalar field model (see Corollary 3 of [KU22]). The proof is based on a *characteristic gluing* argument, in which we glue a late ingoing cone in the interior of extremal Reissner–Nordström to an ingoing cone in Minkowski space. The desired properties of the spacetime are obtained softly by Cauchy stability arguments. In particular, the method is inadequate to address whether the solution constructed in [KU22, Corollary 3] is critical.

It is also natural to conjecture the analog of Theorem 1 for the Einstein vacuum equations,

Conjecture:

Extremal critical collapse happens for **charged scalar field**

Field Equations

Einstein-Maxwell-Klein-Gordon System:

Charged SF

$$\mathcal{D}_a \mathcal{D}^a \phi = 0$$

with $\mathcal{D}_a := \nabla_a + iqA_a$

Maxwell Eqs

$$\nabla_a \mathcal{F}^{ab} = -4\pi j^b$$
$$\nabla_a \mathcal{F}^{*ab} = 0$$

Einstein Eqs

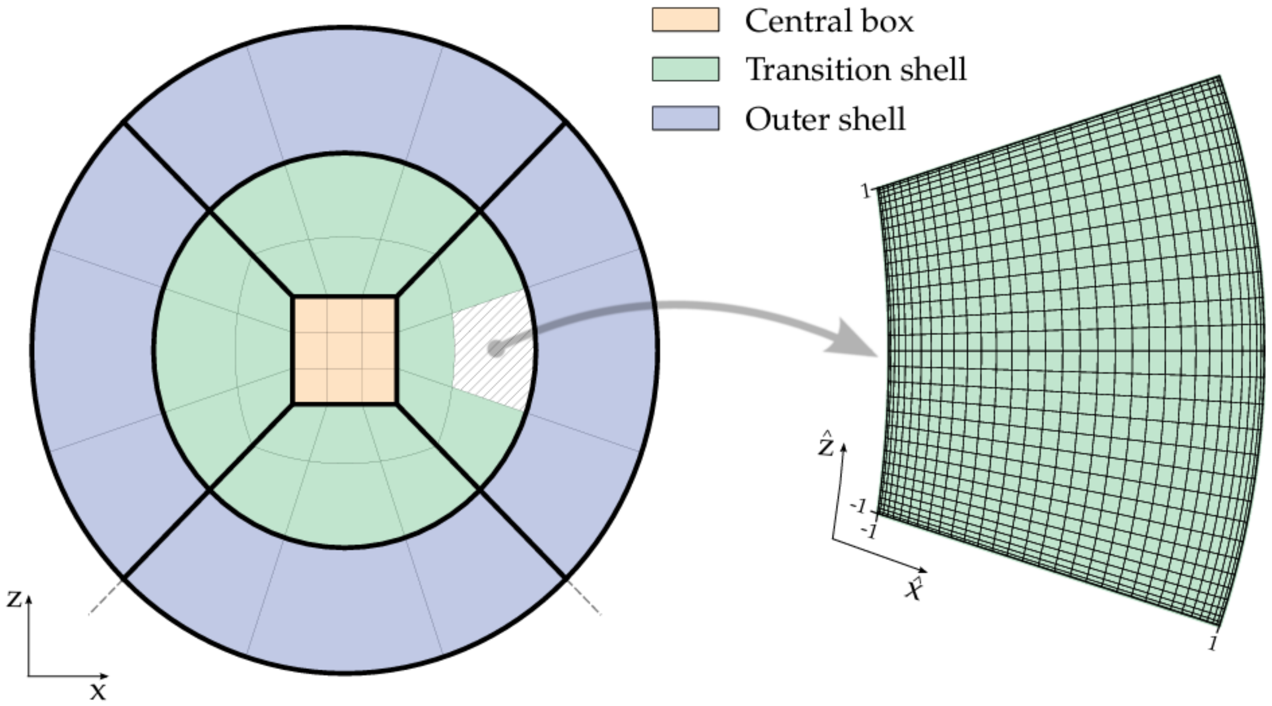
$$R_{ab} - \frac{1}{2} R g_{ab} = 8\pi T_{ab}$$

3+1 Decomposition

$$\partial_t \phi = \alpha \Pi + \beta^i \Phi_i + iq\varphi\phi$$
$$\partial_t \Phi_i = \Pi \partial_i \alpha + \alpha \partial_i \Pi + \Phi_j \partial_i \beta^j + \beta^j \partial_j \Phi_i + iq\phi\partial_i \varphi + iq\varphi\Phi_i$$
$$\partial_t \Pi = \beta^i \partial_i \Pi + \alpha K \Pi + \gamma^{ij} (\Phi_j \partial_i \alpha + \alpha \partial_i \Phi_j - \alpha\, {}^{(3)}\Gamma^k{}_{ij} \Phi_k) + iq(\alpha \gamma^{ij}\, {}^{(3)}A_i \Phi_j - \varphi \Pi) \,.$$

$$\partial_t A_i = -\alpha E_i - D_i \varphi + \beta^j \partial_j A_i + A_j \partial_i \beta^j \,,$$
$$\partial_t E^i = \beta^j \partial_j E^i - E^j \partial_j \beta^i + \alpha K E^i + (\partial \times \alpha B)^i - 4\pi \alpha j^i \,,$$
$$\partial_t B^i = \beta^j \partial_j B^i - B^j \partial_j \beta^i + \alpha K B^i - (\partial \times \alpha E)^i \,,$$

Numerical Implementation: BAMPS



[Hilditch et al. arXiv:1504.04732v2]

Pseudospectral

Derivatives are calculated with all points

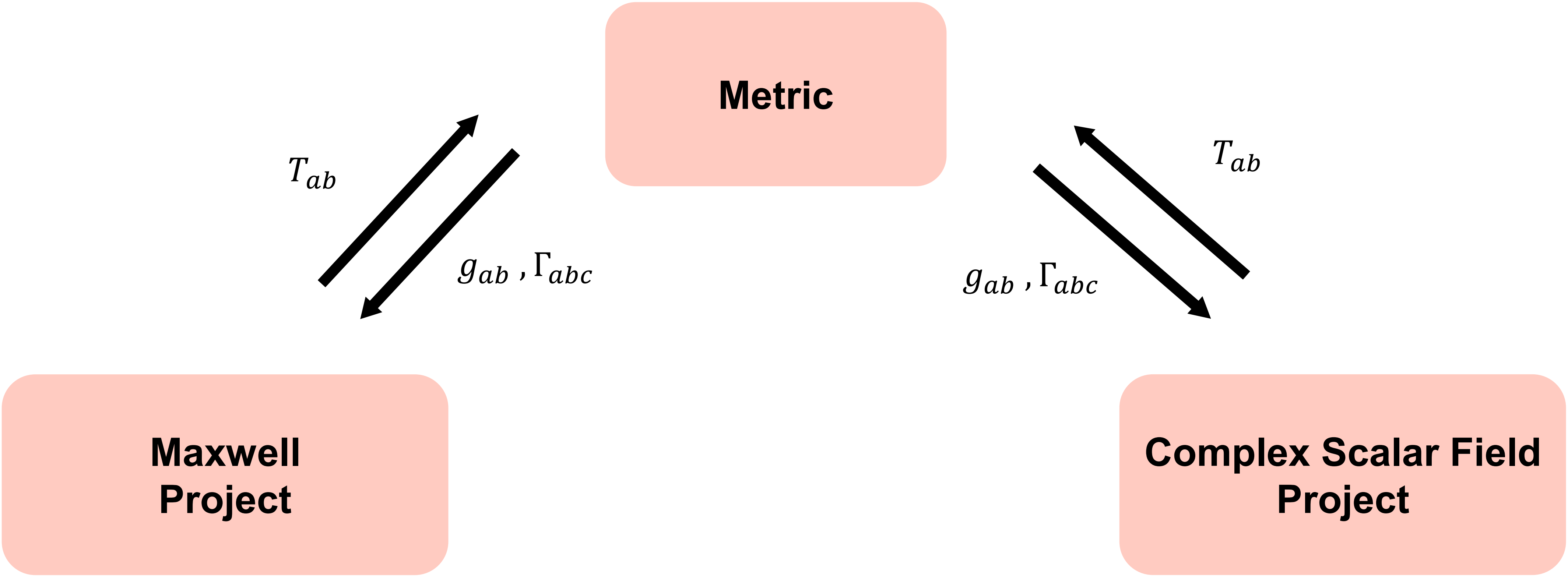
Multidomain

Domain is divided in patches, where each patch solves its own initial boundary value problem

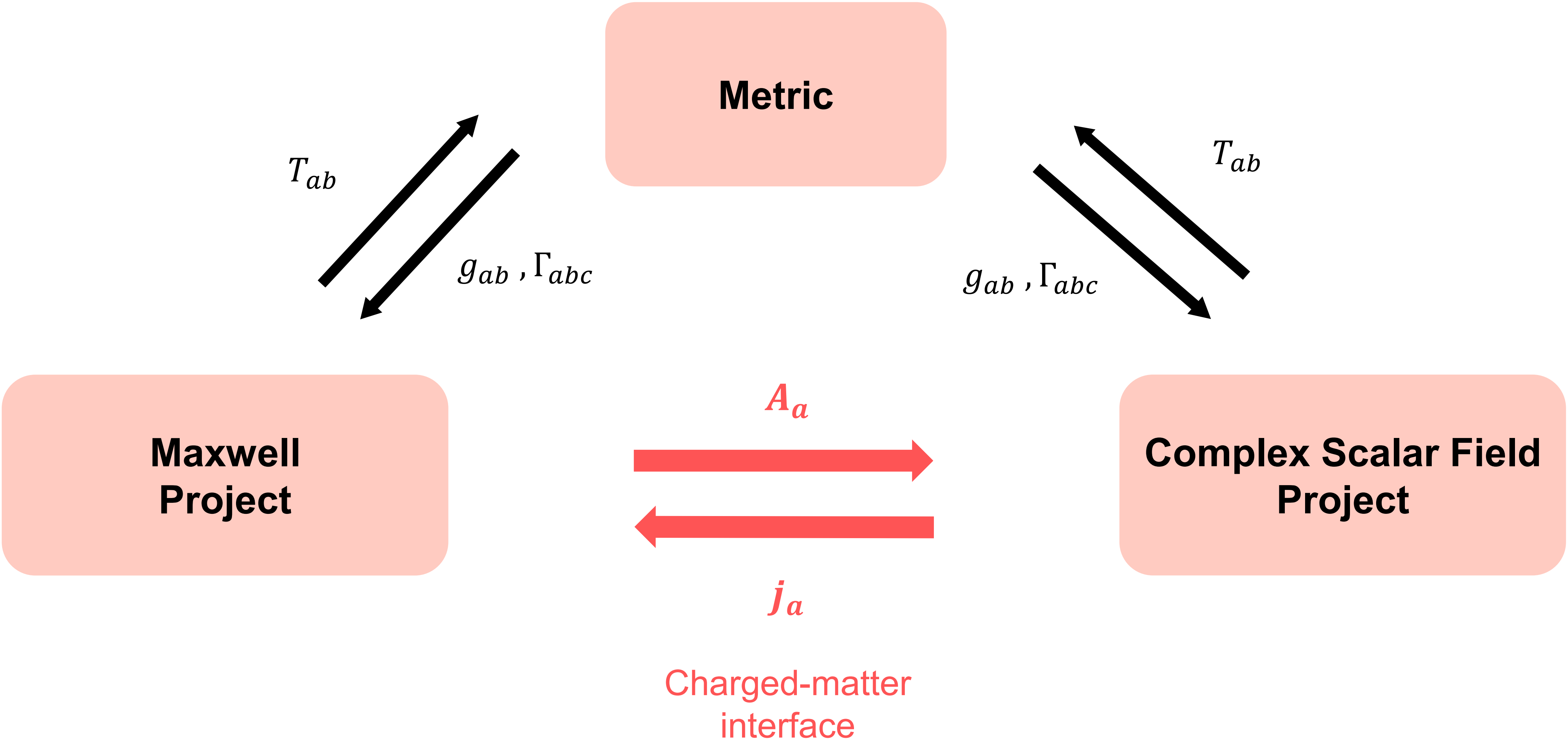
Full 3D code

Cartoon method is used for reductions in symmetry cases

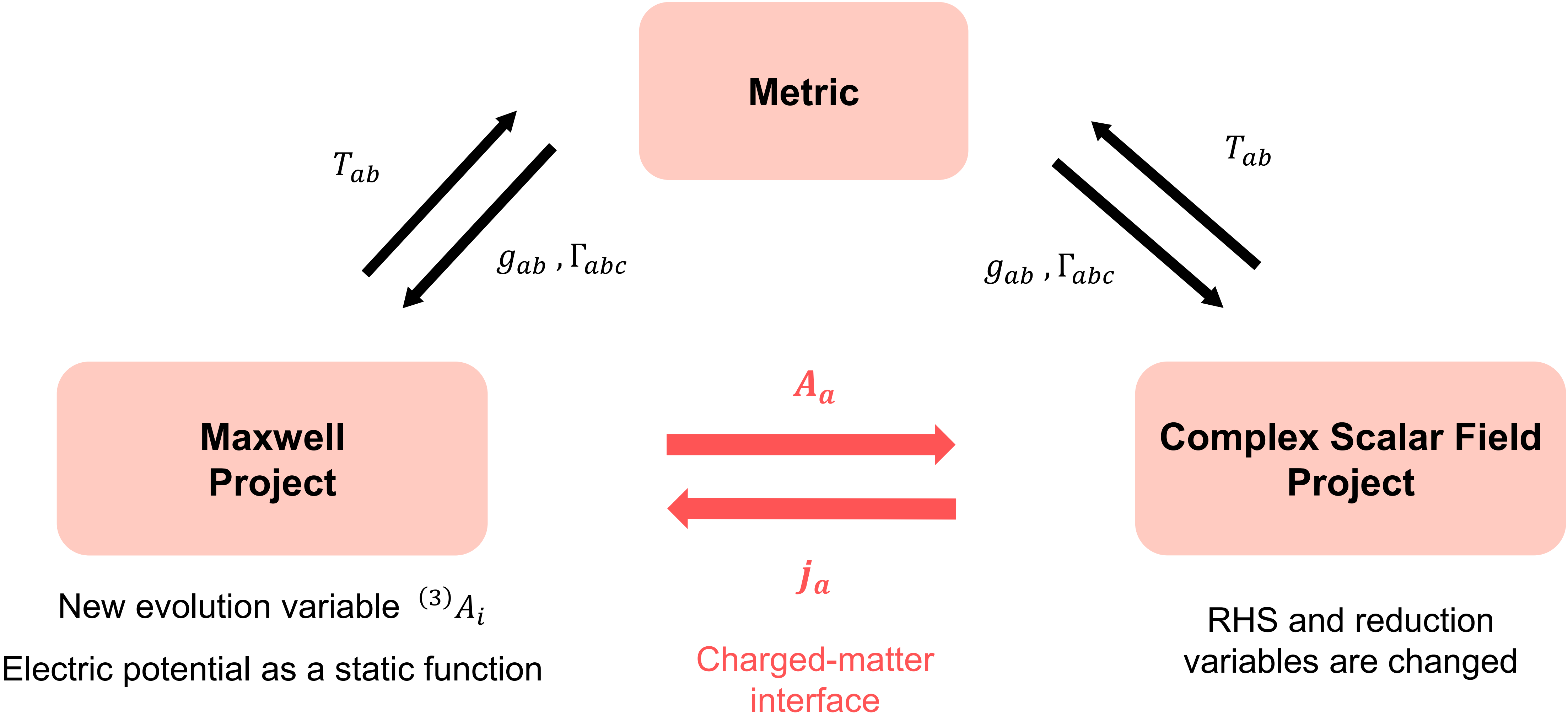
Numerical Implementation: BAMPS



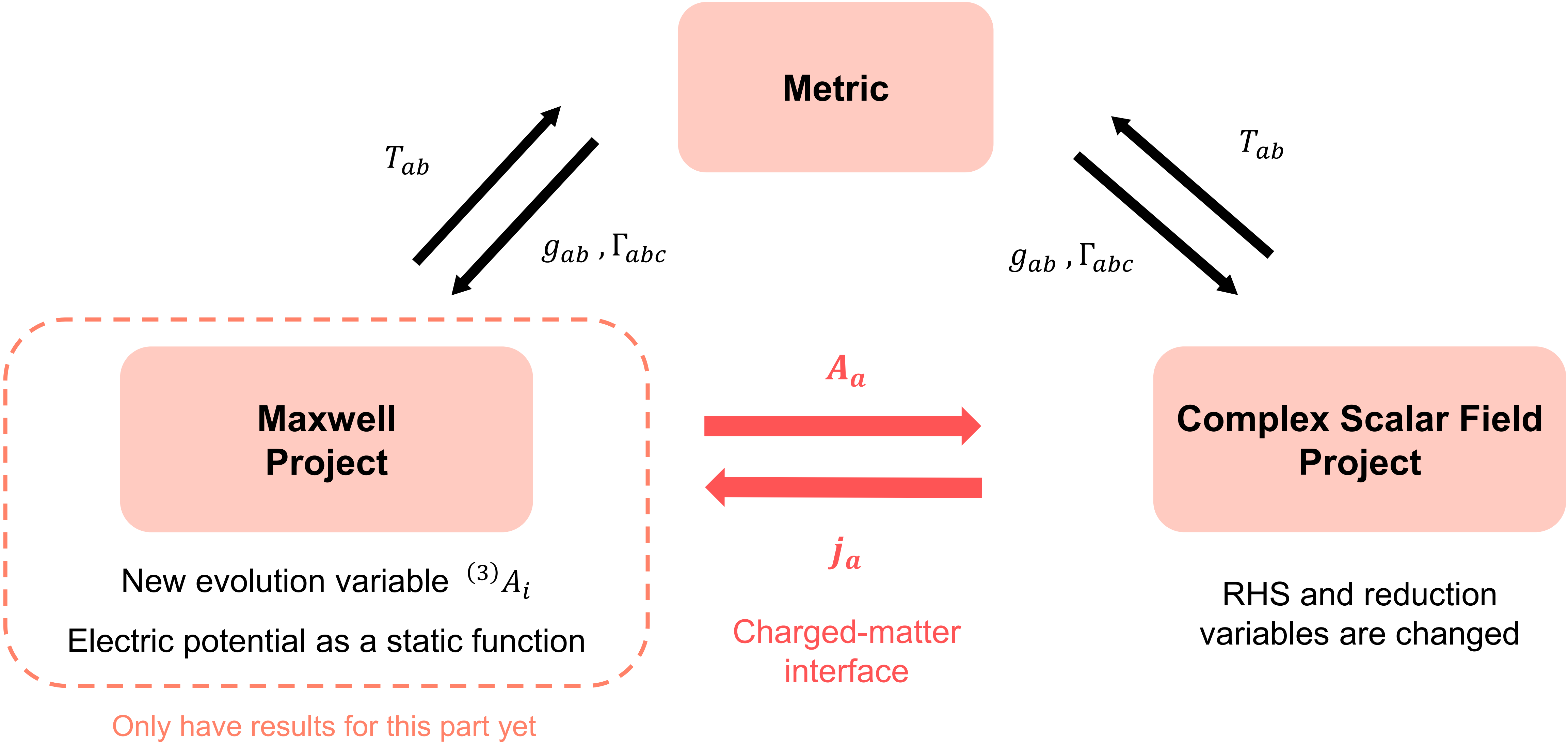
Numerical Implementation: BAMPS



Numerical Implementation: BAMPS



Numerical Implementation: BAMPS



Results and Future work

The implementation of the vector potential A introduces a new constraint:

Curl constraint $\mathcal{G}_A^i := (D \times A)^i - B^i = 0$

Simulations

EM dipole in flat background

- Error with exact solution and $\mathcal{G}_A$ converges exponentially

RN black hole with fixed background

- Error with exact solution converges exponentially
- But $\mathcal{G}_A$ grows exponentially with resolution and in time

Future work:

- Implement constraint damping for $\mathcal{G}_A$
- Follow similar procedure for charged scalar field
- Do critical collapse for charged scalar field

Backup Slides

3 + 1 Decomposition

$$n_a = -\alpha \nabla_a t$$

$$n^a = \frac{1}{\alpha}(\partial_t - \beta^i \partial_i)^a$$

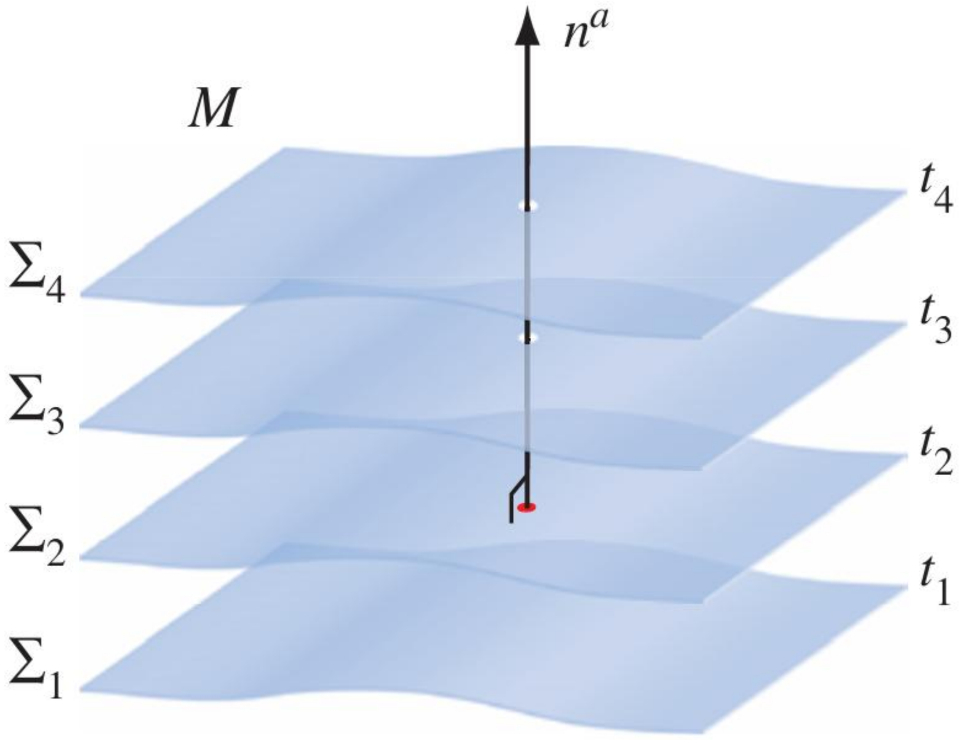
n – unit normal vector to Σ_t
α – lapse
β – shift

$$\gamma_a{}^b = \delta_a{}^b + n_a n^b$$

Projection operator

$$w^a = {}^{(3)}w^a + n^b w_b n^a$$

Decomposition of 4-vector



[Baumgarte and Shapiro. “Numerical Relativity”]

Penalty method

Well-posedness

A PDEs system is well-posed if there exists a unique solution that depends continuously on the initial data. Equivalent to being stronly hyperbolic.

$\partial_t u = A^p \partial_p u + Bu$ ← If this is strongly hyperbolic, then for every spatial vector s $\quad P^s \equiv A^s = A^p s_p$

has real eigenvalues & complete set of eigenvectors

Characteristic variables are travelling waves $\quad l_\lambda \cdot u$ → l_λ are left-eigenvectors of P^s with eigenvalue λ

Penalty method

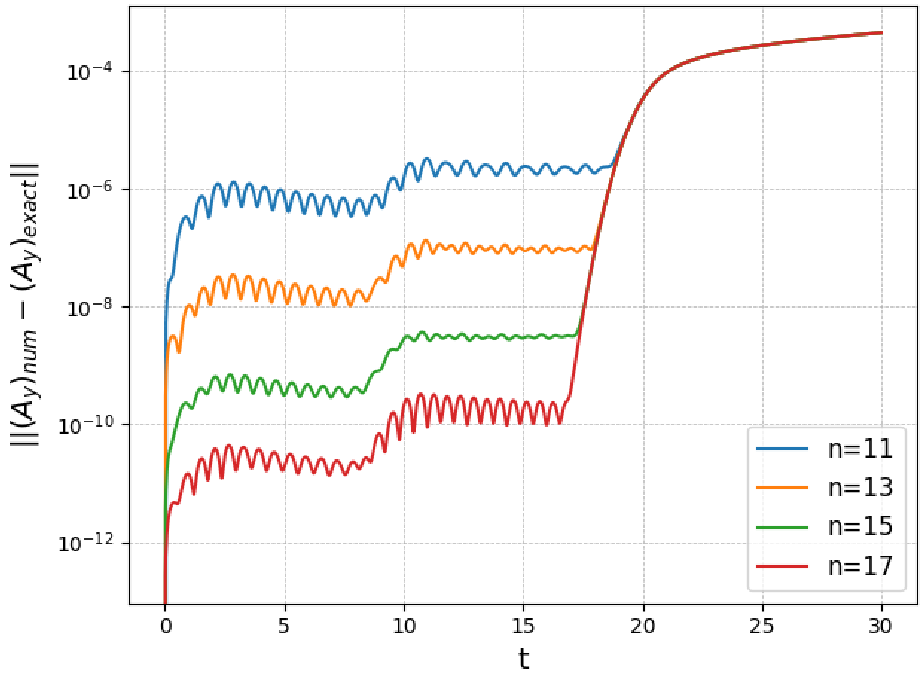
Change rhs at boundary

$$\partial_t u \hat{=} (\partial_t u)_{\hat{\Omega}} + p\lambda(u - u_{BC})\Theta(-\lambda)$$

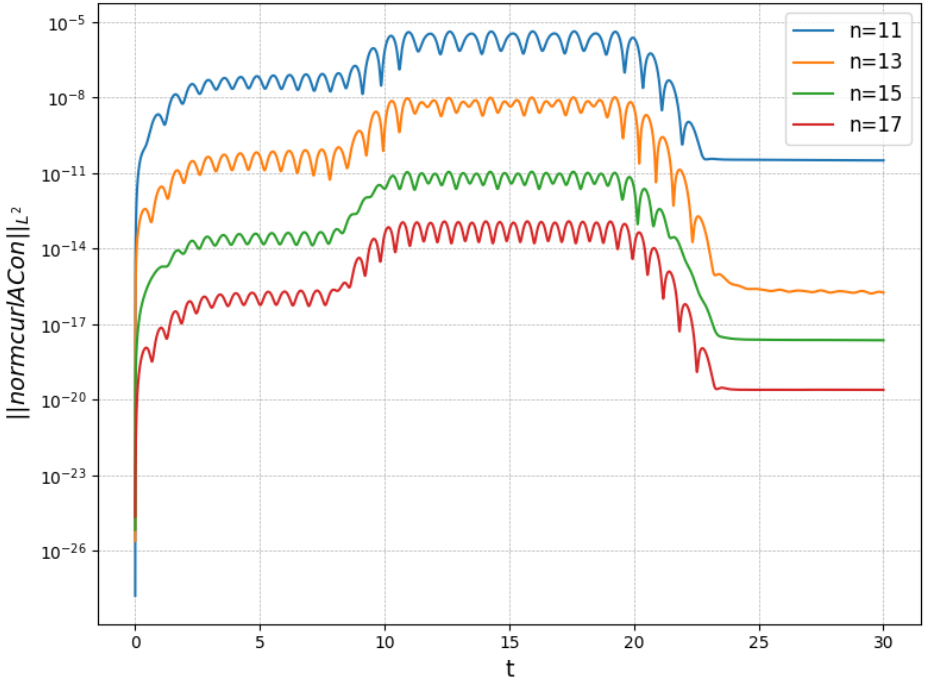
Force outgoing charcteristic var to be incoming characteristic var in next patch

Electromagnetic dipole in flat background

Error of Ay with exact solution



Norm of $\mathcal{G}_A$



Both converge exponentially with resolution.

$\mathcal{G}_E$ and $\mathcal{G}_B$ also converge.

Electromagnetic dipole in flat background

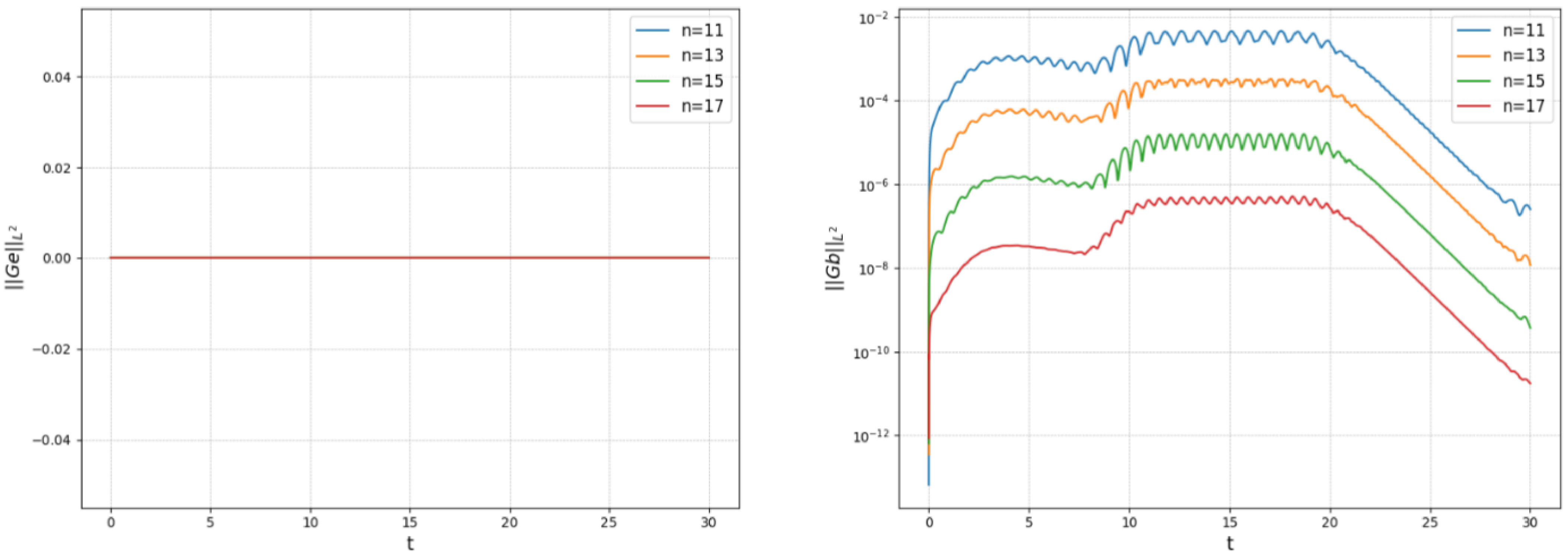


FIGURE 5: Convergence of two error norms as function of time t with the number of collocation points n for an electromagnetic dipole with flat background. Left: Norm of $\mathcal{G}_{\mathcal{E}}$. Right: Norm of the curl constraint $\mathcal{G}_B$.

Electromagnetic dipole in flat background

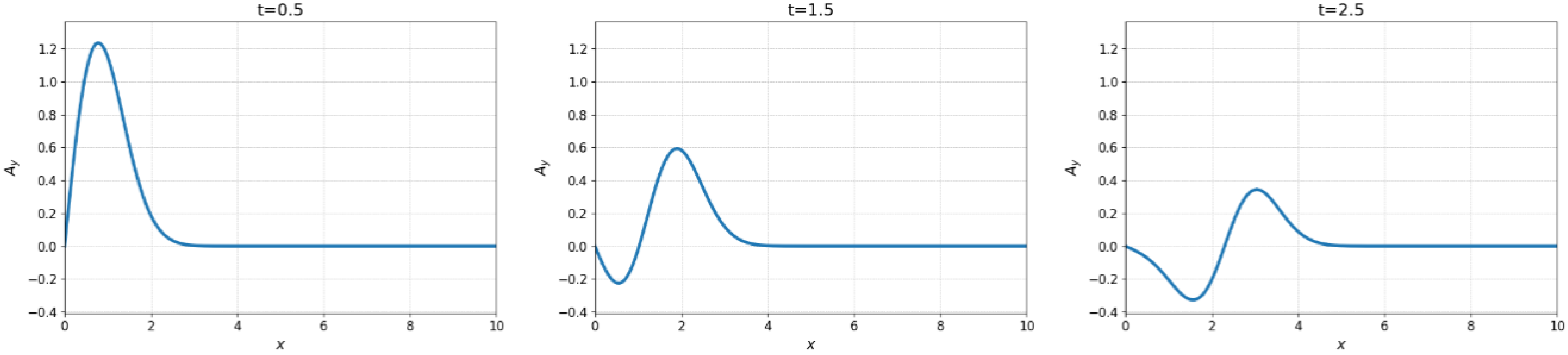
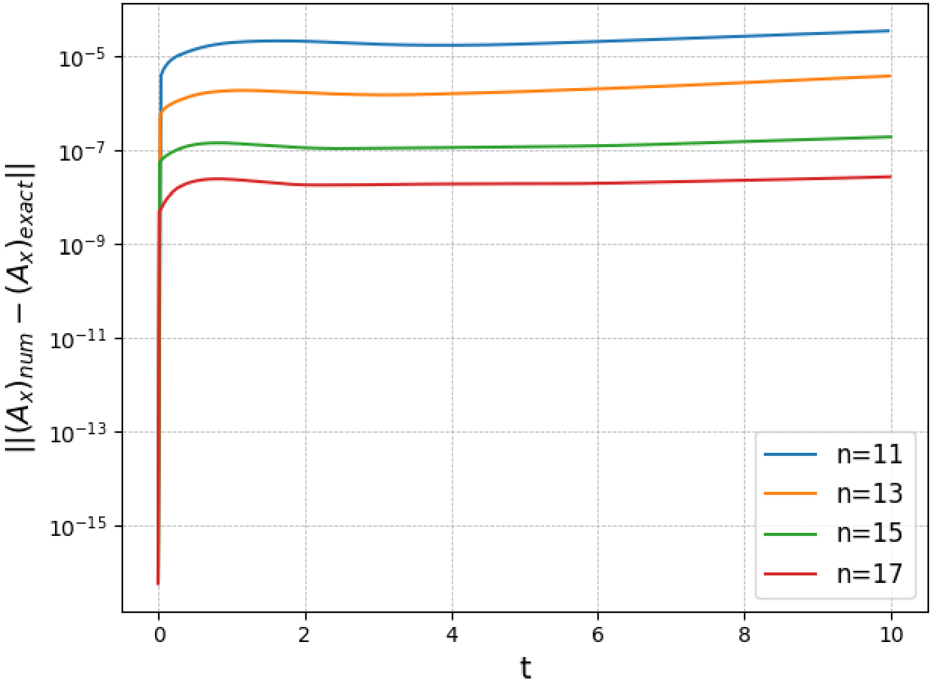


FIGURE 2: Snapshots of ${}^{(3)}A_y$ at different evolution times in the x axis. Initial data is not shown as ${}^{(3)}A_y(t=0)=0$. The whole evolution domain is not plotted, which actually goes to $x=20$.

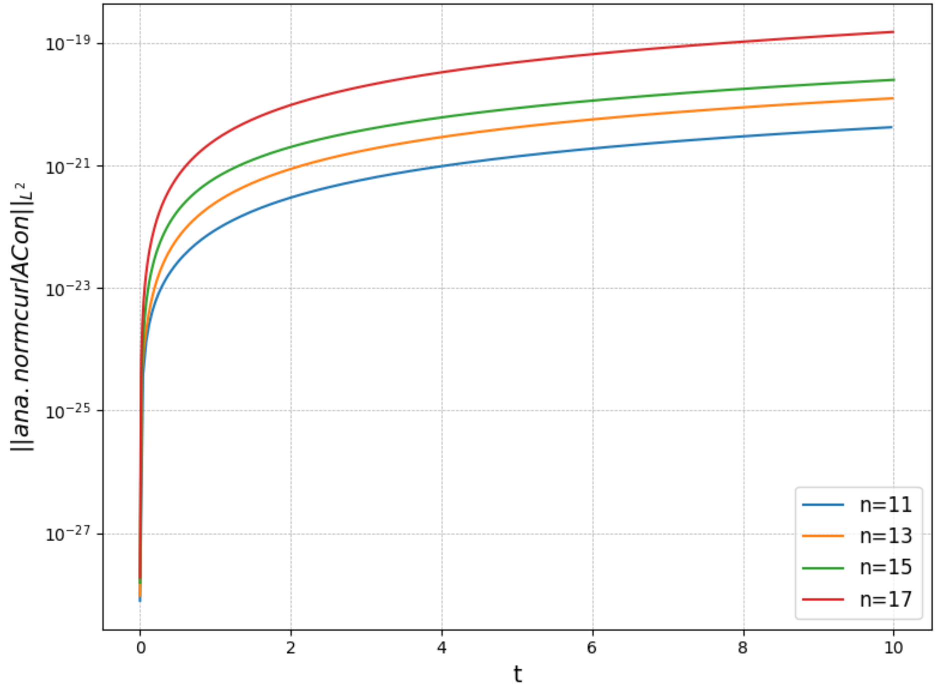
Reissner-Nordström BH in fixed background

Error of Ax with exact solution



Converges exponentially with resolution.

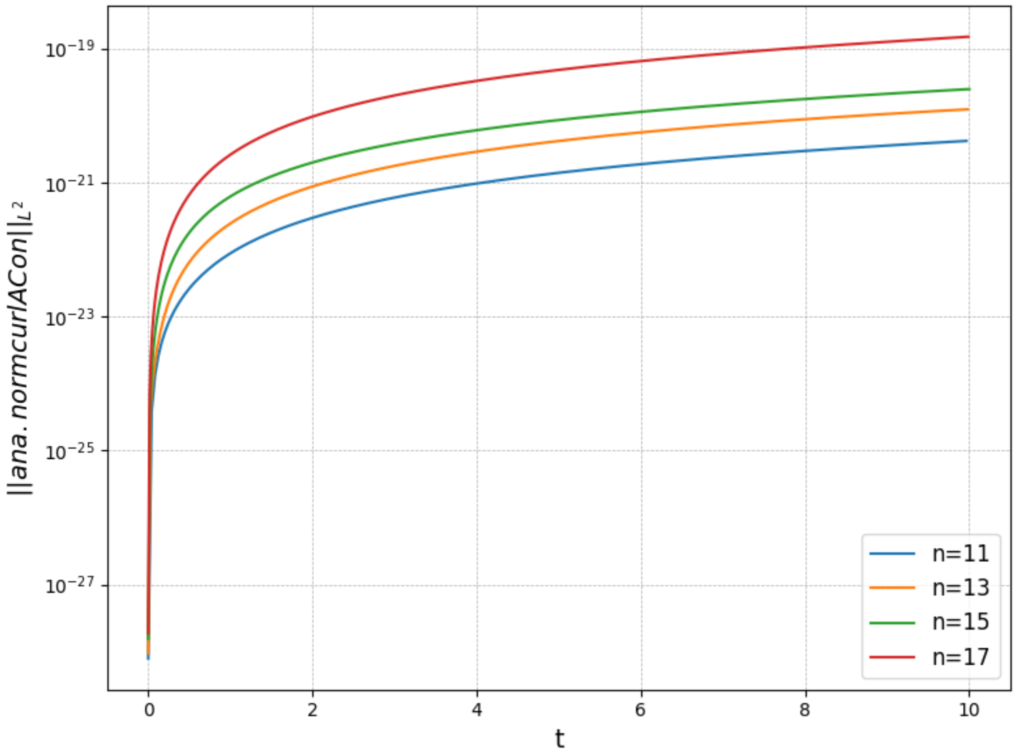
Norm of $\mathcal{G}_A$



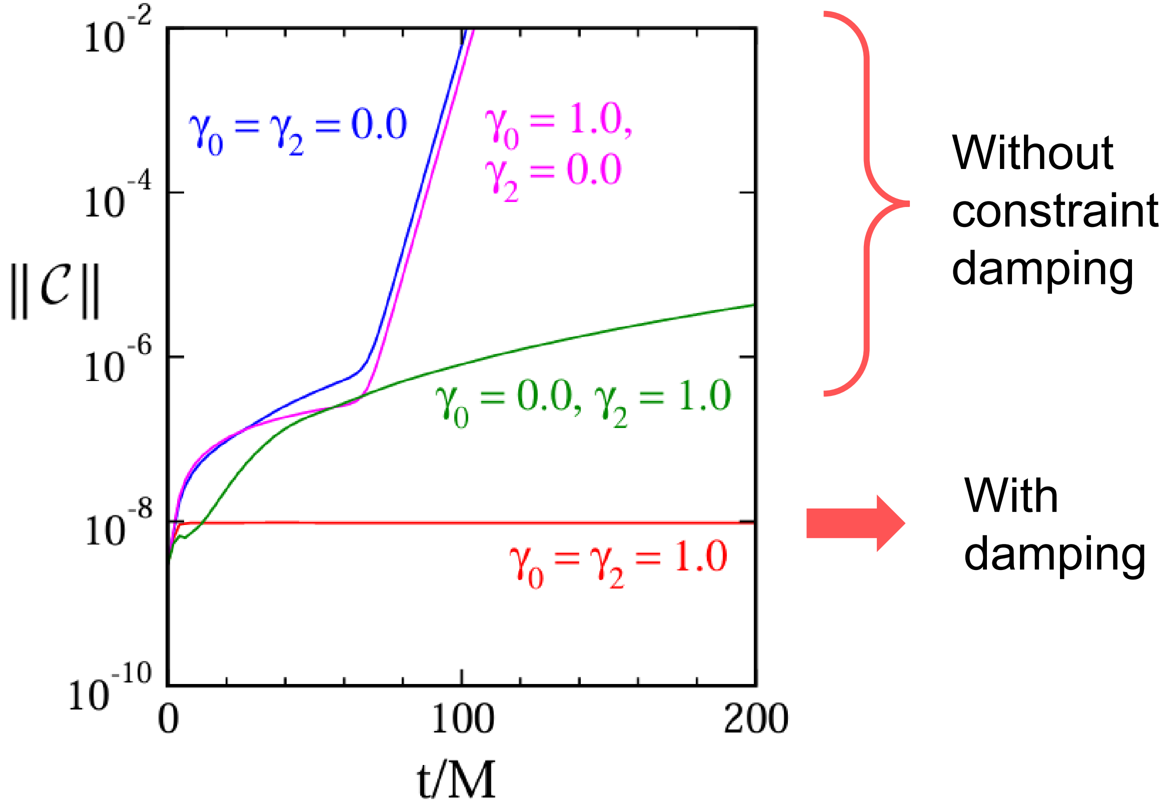
Grows exponentially with resolution and in time!

Reissner-Nordström BH in fixed background

Norm of $\mathcal{G}_A$



Probably need to add **constraint damping** to equations



[Lee Lindblom et al. arXiv:gr-qc/0512093]

Reissner-Nordström BH in fixed background

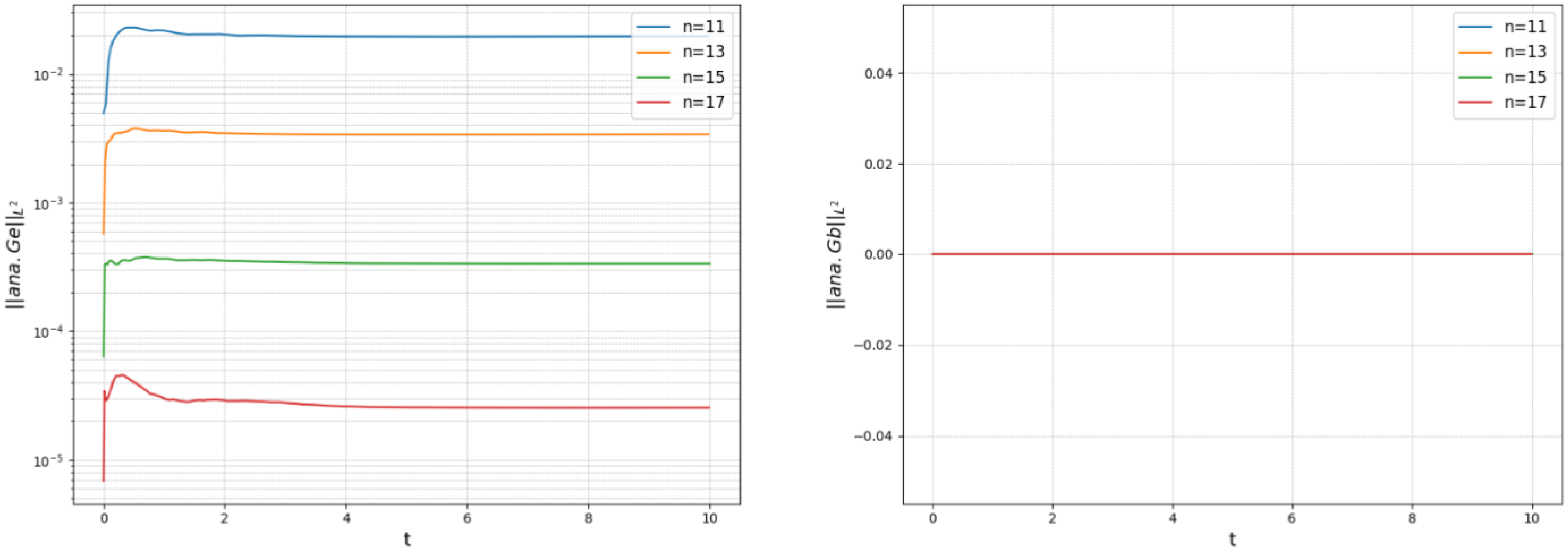


FIGURE 6: Convergence of two error norms as function of time t with the number of collocation points n for a Reissner-Nordström with fixed background. Left: Norm of $\mathcal{G}_{\mathcal{E}}$. Right: Norm of the curl constraint $\mathcal{G}_B$.

Reissner-Nordström BH Gauge change

Decomposition of A in Kerr-Schild coords

- $$A^a = \gamma^a{}_b A^b + n^a n^b A_b \equiv {}^{(3)}A^a + \frac{n^a}{\alpha}\varphi$$
- $$A = -\frac{Q}{r}\left(d\tau + \frac{H}{1-H}dr\right)$$

$$\varphi = \frac{Q}{r}\left(1 + \frac{H^2}{1-H^2}\right)$$

$$^{(3)}A = \frac{Q}{r}\frac{H}{1-H}\left(\frac{H}{1+H}d\tau + dr\right)$$

Singular at r_- and r_+

Gauge change $A \to A + d\chi$

- $$\varphi_{\text{new}} = \left(1 + \frac{H^2}{1-H^2}\right) + \beta^r \partial_r \chi = \varphi_{\text{old}} + \beta^r \partial_r \chi$$
- $$\varphi_{\text{old}} = \frac{Q}{r}\left(1 + \frac{r^2 H^2}{1+H}\frac{1}{r-r_+}\frac{1}{r-r_-}\right)$$
- $$\chi = C_+ \ln(r - r+) + C_- \ln(r - r_-)$$
- $$C_\pm = -\frac{1}{2}\frac{Q^3}{Q^2 - M(M \mp \sqrt{M^2 - Q^2})}$$

$$\varphi_{\text{new}} = \frac{Q}{r}\frac{1}{1+H}$$

$$^{(3)}A^{\text{new}}_r = \frac{Q}{r} \implies {}^{(3)}A^{\text{new}}_i = \frac{Qx^i}{r^2}$$