

Constraints on multi-scalar models

Perturbative unitarity for models with singlet and doublet scalars

Carolina T. Lopes

Supervisors:

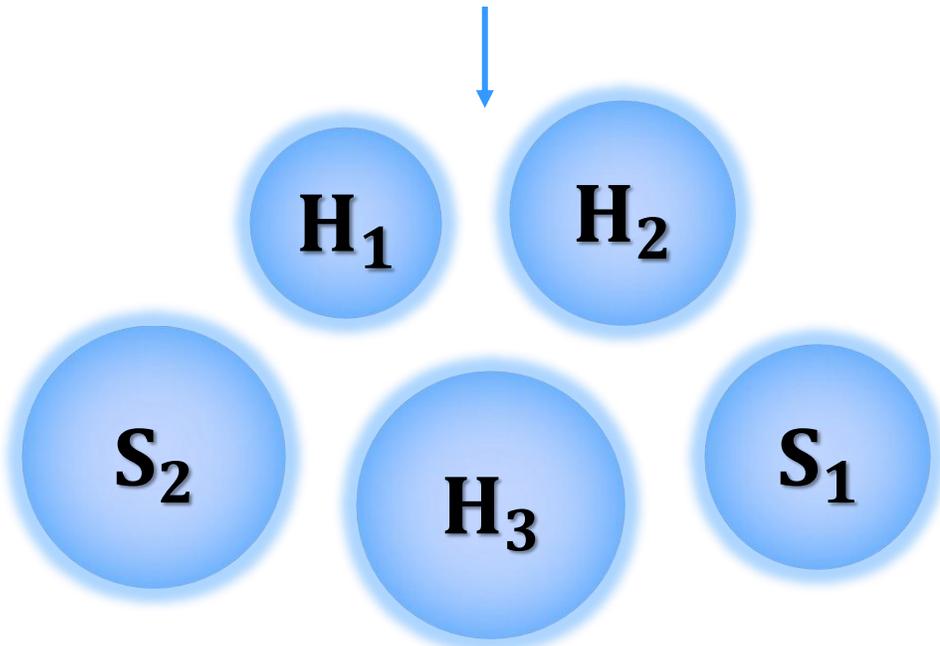
Prof. João Paulo Silva

André Milagre

[arXiv:2510.02434v2 \[hep-ph\]](https://arxiv.org/abs/2510.02434v2), *C. T. Lopes*, *A. Milagre* and *J. P. Silva*

Motivation

Beyond the Standard Model

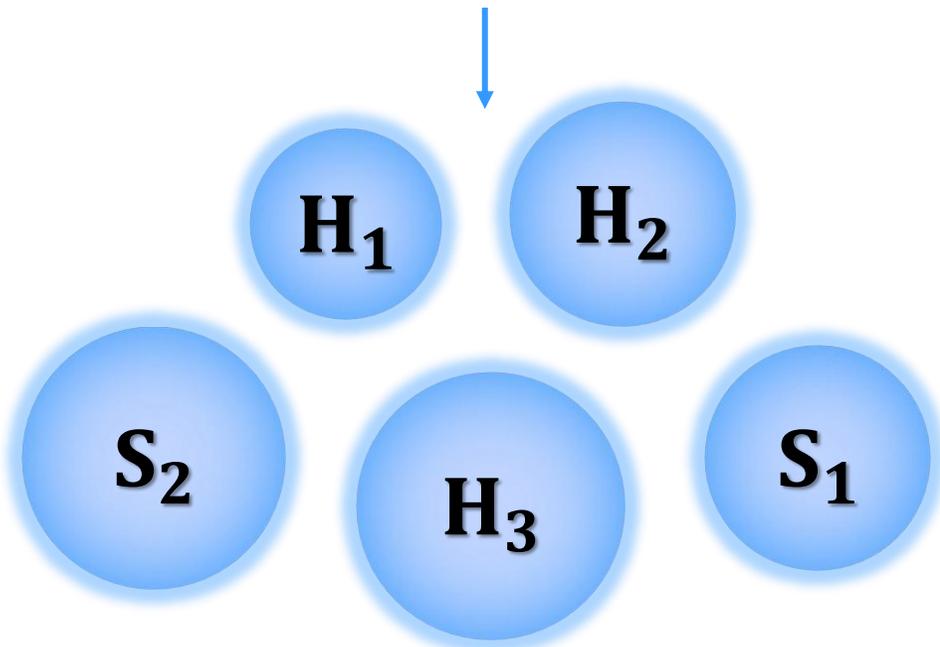


Scalar Extensions
of the SM

Many **Beyond-Standard-Model (BSM)** theories introduce **extra scalar fields** to address the **SM** limitations

Motivation

Beyond the Standard Model



Scalar Extensions
of the SM

Many **Beyond-Standard-Model (BSM)** theories introduce **extra scalar fields** to address these limitations

But, once we extend the field content, we must ensure that the model remains **theoretically consistent!**

Motivation

Theoretical consistency

Motivation

Theoretical consistency

Unitarity



Probability Conservation

Motivation

Theoretical consistency

Unitarity



Probability Conservation

Unitarity Bounds have been worked out for specific models:

SM

2HDM

SM + S

...

Motivation

Theoretical consistency

Unitarity



Probability Conservation

Unitarity Bounds have been worked out for specific models:

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But there was still no framework that works for **any number of singlets and doublets...**

Motivation

Theoretical consistency

Unitarity

→ Probability Conservation

Unitarity Bounds have been worked out for specific models:

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But there was still no framework that works for **any number of singlets and doublets...**

→ **done** by **Carolina T. Lopes, André Milagre and João P. Silva**, [arXiv:2510.02434v2 \[hep-ph\]](https://arxiv.org/abs/2510.02434v2)

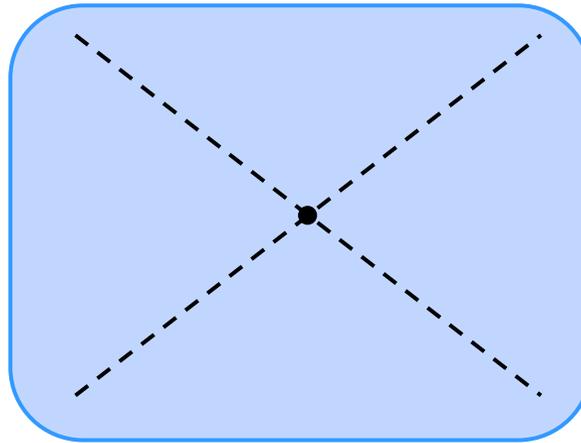
Partial-wave unitarity bounds

$2 \rightarrow 2$ scattering

- Let's consider a $2 \rightarrow 2$ scattering process between complex scalar fields, with flavour indices a, b, c, d ,

$$A_a B_b \rightarrow C_c D_d$$

High-energy limit



contact

Partial-wave unitarity bounds

Amplitude – tree-level & high-energy limit

- Consequently, in the **high-energy limit**, only the **quartic interactions** involving the external scalars contribute to the tree-level amplitude,

$$\mathcal{M}_{A_a B_b \rightarrow C_c D_d} = - \frac{\partial^4 V_4}{\partial A_a \partial B_b \partial C_c^* \partial D_d^*}$$

Partial-wave unitarity bounds

Unitarity

$$\mathcal{M}_{A_a B_b \rightarrow C_c D_d} = - \frac{\partial^4 V_4}{\partial A_a \partial B_b \partial C_c^* \partial D_d^*}$$

Perturbative Unitarity \Rightarrow

Imposes a **bound** on the amplitude!

$$|\mathcal{M}_{A_a B_b \rightarrow C_c D_d}| \leq 8\pi$$

The Model

Particle Content

$$SU(2)_L \times U(1)_Y$$

$$\Phi_i = (\phi_i^+, \phi_i^0)^T, \quad i = 1, \dots, n_D$$

$SU(2)$ doublets with $Y = \frac{1}{2}$

$$\varphi_i^+, \quad i = 1, \dots, n_c$$

$SU(2)$ singlets with $Y = 1$

$$\chi_i, \quad i = 1, \dots, n_n$$

$SU(2)$ singlets with $Y = 0$

The Model

Conserved Quantum Numbers

$$SU(2)_L \times U(1)_Y \rightarrow U(1)_Q$$

- Quantum numbers corresponding to the symmetries of the underlying theory remain **conserved**
- Therefore, we can constrain the possible initial and final states
 - **Coupled-channel analysis** \Rightarrow **coupled-channel matrix**
 - **Unitarity bound:** largest eigenvalue $\leq 8\pi$

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 - **Coupled-channel analysis** \Rightarrow **coupled-channel matrix**
 - **Unitarity bound:** largest eigenvalue $\leq 8\pi$

Our approach aims to take advantage of three conserved quantities: **Q, Y, T**

Mathematica Notebook: BounDS

- To automate the generation of scattering matrices, compute their eigenvalues, and ensure these values remain below 8π , we have developed a Mathematica notebook, **BounDS**

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The **user** simply inputs:

Step 1

Specify the values n_D, n_C , and n_n

e.g.

```
nD = 2; nC = 0; nn = 0;
```

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e.g.

```
nD = 2; nC = 0; nn = 0;
```

Step 2

Additional flavor and/or generalized CP symmetries (abelian and non-abelian)

e.g.

```
nSym = 1; Sym[1] = {Phi[1] -> Phi[1], Phi[2] -> Exp[Ia] Phi[2]};
```

Mathematica Notebook: BounDS

- To automate the generation of scattering matrices, compute their eigenvalues, and ensure these values remain below 8π , we have developed a Mathematica notebook, **BounDS**

BounDS outputs:

The **quartic part** of the scalar potential and the **scattering matrices**

e.g.

$$\begin{aligned} &(\Phi_1^\dagger \cdot \Phi_1) \lambda_{11,11} + (\Phi_2^\dagger \cdot \Phi_2) \lambda_{22,22} + 2(\Phi_1^\dagger \cdot \Phi_1) \\ &(\Phi_2^\dagger \cdot \Phi_2) \lambda_{11,22} + 2(\Phi_1^\dagger \cdot \Phi_2) (\Phi_2^\dagger \cdot \Phi_1) \lambda_{12,21} \end{aligned}$$

e.g.

$$(\phi_1^+ \cdot \phi_1^+ \quad \phi_1^+ \cdot \phi_2^+ \quad \phi_2^+ \cdot \phi_2^+)$$

$$\begin{pmatrix} 2\lambda_{11,11} & 0 & 0 \\ 0 & 2(\lambda_{11,22} + \lambda_{12,21}) & 0 \\ 0 & 0 & 2\lambda_{22,22} \end{pmatrix}$$

Closed-form expressions for the eigenvalues of the scattering matrices whenever possible

e.g.

$$\begin{aligned} &2\lambda_{11,11} \\ &2(\lambda_{11,22} + \lambda_{12,21}) \\ &2\lambda_{22,22} \end{aligned}$$

<https://github.com/andremilagre/BounDS.git>

Thank You!

EXTRA

Motivation

Requirements

For any number of SU(2) singlet and doublet scalars:

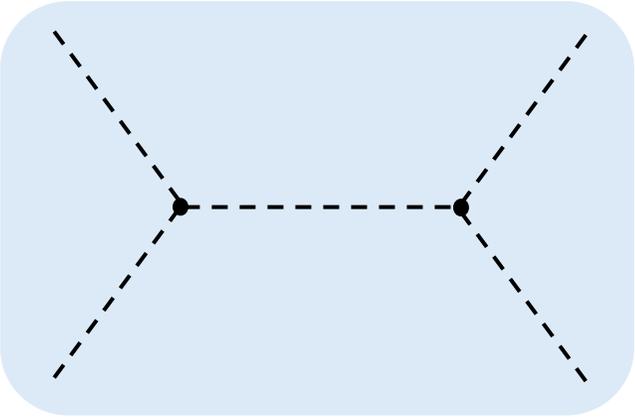
Vacuum stability → **case-by-case**

Boundedness from below → **case-by-case**

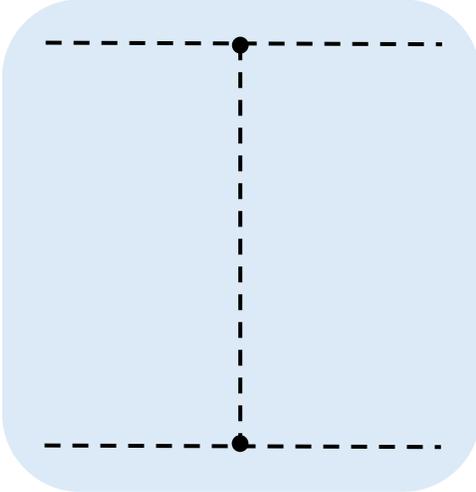
S, T, U oblique parameters → **done** by W. Grimus, L. Lavoura, O.M. OGREID and P. OSLAND, *Nucl.Phys.B* 801 (2008) 81-96

Partial-wave unitarity bounds

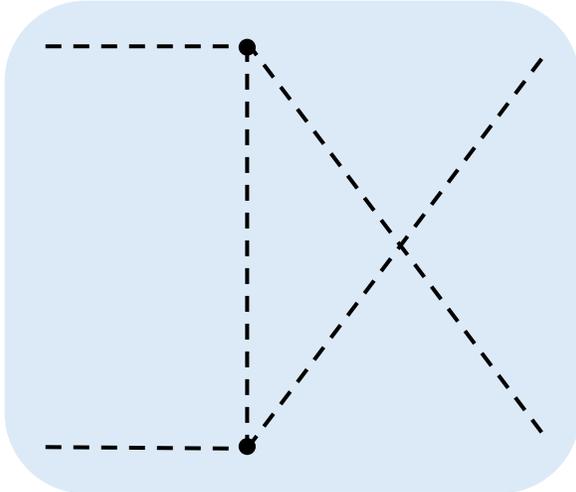
Amplitude



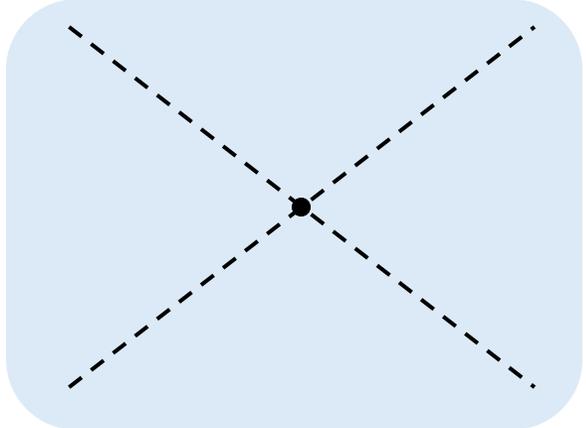
s - channel



t - channel



u - channel



contact

$$i\mathcal{M} = i\mathcal{M}_s + i\mathcal{M}_t + i\mathcal{M}_u + i\mathcal{M}_c$$

SM and @ tree-level

High-energy limit
($s \gg M_h^2$)

$$\mathcal{M}_s \propto \frac{M_h^2}{M_Z^2} \frac{1}{s - M_h^2}$$

$$\mathcal{M}_t \propto \frac{M_h^2}{M_Z^2} \frac{1}{t - M_h^2}$$

$$\mathcal{M}_u \propto \frac{M_h^2}{M_Z^2} \frac{1}{u - M_h^2}$$

$$\mathcal{M}_c \propto \frac{M_h^2}{M_Z^2}$$

0

Partial-wave unitarity bounds

Partial wave decomposition

The amplitude can be written as a sum over angular momentum components:

$$\mathcal{M}(\cos \theta) = 16\pi \sum_{J=0}^{\infty} a_J (2J + 1) P_J(\cos \theta)$$

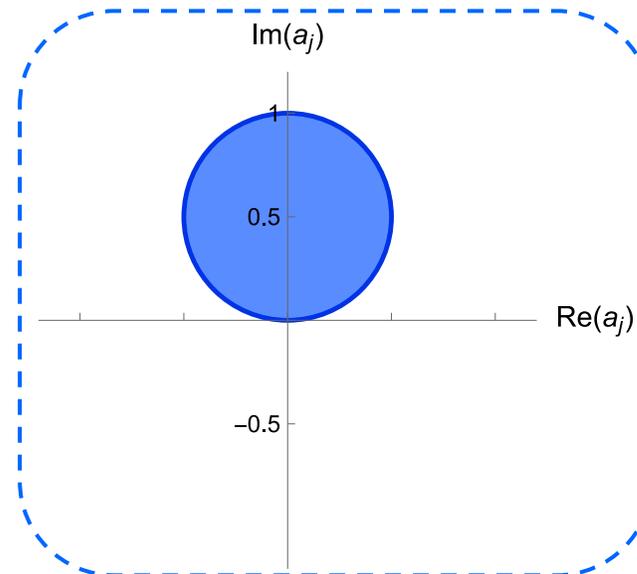
Partial Wave Expansion

$$a_J = \frac{2J + 1}{32\pi} \int_{-1}^1 \mathcal{M}(\cos \theta) P_J(\cos \theta) d\cos\theta$$

Partial Waves

Unitarity \Rightarrow

$$\text{Im}\{a_J\} \leq |a_J|^2$$



$$|a_J| \leq 1$$

$$|\text{Re}\{a_J\}| \leq \frac{1}{2}$$

$$0 \leq \text{Im}\{a_J\} \leq 1$$

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Partial Waves

$$|a_J| \leq 1$$

Tree-level \Rightarrow

$$a_J \in \mathbb{R}$$

$$|\operatorname{Re}\{a_J\}| \leq \frac{1}{2}$$

$$0 \leq \operatorname{Im}\{a_J\} \leq 1$$

Partial-wave unitarity bounds

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Partial Waves

Tree-level unitarity \Rightarrow

$$|a_J| \leq \frac{1}{2}$$

• In the **high-energy** limit, $\mathcal{M}(\cos \theta)$ is independent of $\theta \Rightarrow a_J = \frac{2J+1}{32\pi} \mathcal{M} \int_{-1}^1 P_J(\cos \theta) d\cos\theta$

• Given that $P_0(\cos \theta) = 1$ and $\int_{-1}^1 P_m(\cos \theta) P_n(\cos \theta) d\cos\theta = \frac{2}{2n+1} \delta_{mn} \Rightarrow \mathbf{a_0}$ gives the strongest bound

The Model

Hermiticity

$$\begin{aligned}
 V \supset V_4 = & \lambda_{ab,cd} (\Phi_a^\dagger \Phi_b) (\Phi_c^\dagger \Phi_d) + \alpha_{ab,cd} (\varphi_a^- \varphi_b^+) (\varphi_c^- \varphi_d^+) + \beta_{ab,cd} (\chi_a \chi_b) (\chi_c \chi_d) \\
 & + \delta_{ab,cd} (\Phi_a^\dagger \Phi_b) (\varphi_c^- \varphi_d^+) + \gamma_{ab,cd} (\Phi_a^\dagger \Phi_b) (\chi_c \chi_d) + \zeta_{ab,cd} (\varphi_a^- \varphi_b^+) (\chi_c \chi_d) \\
 & + \kappa_{ab,cd} (\Phi_a^T \sigma_2 \Phi_b) (\varphi_c^- \chi_d) + \kappa_{ab,cd}^* (\Phi_b^\dagger \sigma_2 \Phi_a^*) (\varphi_c^+ \chi_d)
 \end{aligned}$$

- Not all couplings are independent,

Hermiticity + Field Swap \Rightarrow

$$\begin{aligned}
 \lambda_{ab,cd} &= \lambda_{ba,dc}^* = \lambda_{cd,ab} \\
 \alpha_{ab,cd} &= \alpha_{ba,dc}^* = \alpha_{cd,ab} = \alpha_{ad,cb} \\
 \beta_{ab,cd} &= \beta_{(ab,cd)}^* = \beta_{(ab,cd)} \\
 \delta_{ab,cd} &= \delta_{ba,dc}^* \\
 \gamma_{ab,cd} &= \gamma_{ba,cd}^* = \gamma_{ab,dc} \\
 \zeta_{ab,cd} &= \zeta_{ba,cd}^* = \zeta_{ab,dc} \\
 \kappa_{ab,cd} &= -\kappa_{ba,cd}
 \end{aligned}$$

\Rightarrow

Any permutation

$$\begin{aligned}
 \lambda_{aa,bb}, \lambda_{ab,ba} &\in \mathbb{R} \\
 \alpha_{aa,bb} = \alpha_{ab,ba} &\in \mathbb{R} \\
 \beta_{ab,cd} &\in \mathbb{R} \\
 \delta_{aa,bb} &\in \mathbb{R} \\
 \gamma_{aa,cd} &\in \mathbb{R} \\
 \zeta_{aa,cd} &\in \mathbb{R} \\
 \kappa_{aa,cd} &= 0
 \end{aligned}$$

The Model

Potential

$$SU(2)_L \times U(1)_Y$$

For $2 \rightarrow 2$ scattering, in the high-energy limit, we want to build the most general renormalizable quartic part of the scalar potential :

$SU(2)_L$ invariant

$U(1)_Y$ invariant

$$\Phi_i \sim \left(2, \frac{1}{2} \right)$$

$$\Phi_i^\dagger \sim \left(\bar{2}, -\frac{1}{2} \right)$$

$$\varphi_i^+ \sim (1, 1)$$

$$\varphi_i^- \sim (1, -1)$$

$$\chi_i \sim (1, 0)$$

$$2 \otimes 2 = 3 \oplus 1$$

$$\rightarrow \Phi_i^T \sigma_2 \Phi_j + h.c.$$

$$Y = \pm 1$$

$SU(2)_L$ invariant but **not** $U(1)_Y$ invariant

$$\bar{2} \otimes 2 = 3 \oplus 1$$

$$\rightarrow \Phi_i^\dagger \Phi_j$$

$$Y = 0$$

$SU(2)_L \times U(1)_Y$ invariant

The Model

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$$\varphi_i^+ \varphi_j^-$$

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$$\varphi_i^+ \chi_j + h.c.$$

$$Y = \pm 1$$

$SU(2)_L$ invariant but **not** $U(1)_Y$ invariant

$$\chi_i \chi_j$$

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$$\Phi_i^T \sigma_2 \Phi_j + h.c.$$

$$Y = \pm 1$$

$SU(2)_L$ invariant but **not** $U(1)_Y$ invariant

$$\varphi_i^- \chi_j + h.c.$$

$$Y = \mp 1$$

$SU(2)_L$ invariant but **not** $U(1)_Y$ invariant



$$(\Phi_i^T \sigma_2 \Phi_j)(\varphi_i^- \chi_j) + h.c.$$

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$SU(2)_L \times U(1)_Y$ invariant

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$$\chi_i \sim (1, 0)$$

$$\Phi_i^\dagger \Phi_j \Phi_k^\dagger \Phi_l$$

$$\Phi_i^\dagger \Phi_j \varphi_k^+ \varphi_l^-$$

$$\varphi_i^+ \varphi_j^- \varphi_k^+ \varphi_l^-$$

$$\Phi_i^\dagger \Phi_j \chi_k \chi_l$$

$$\varphi_i^+ \varphi_j^- \chi_k \chi_l$$

$$\chi_i \chi_j \chi_k \chi_l$$

$$(\Phi_i^T \sigma_2 \Phi_j)(\varphi_i^- \chi_j) + h. c.$$

$$SU(2)_L \times U(1)_Y \text{ invariant}$$

Quantum Numbers

- In any scattering process, the quantum numbers corresponding to the symmetries of the underlying theory remain **conserved**
- Therefore, we can constrain the possible initial and final states

Ex. 2HDM

$$SU(2)_L \times U(1)_Y \rightarrow U(1)_Q$$

In [arXiv:1708.09408v2 \[hep-ph\]](https://arxiv.org/abs/1708.09408v2), states are labelled by their **charge, Q** , and by their **hypercharge, Y**

This approach allows to organize all **22** basis 2-body states into **two 3x3, two 4x4** and one **8x8** scattering matrices

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In [arXiv:hep-ph/0508020v1](https://arxiv.org/abs/hep-ph/0508020v1), states are labelled by their **total isospin, T** , and by their **hypercharge, Y**

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Our approach aims to take advantage of all three conserved quantities: **Q, Y, T**

The Model

Basis of States, first using only Q and Y

$ Q, Y\rangle$	State	Conditions	Dimensionality
$ 2, 2\rangle$	$\phi_i^+ \phi_j^+$	$i \leq j$	$n_c(n_c + 1)/2$
$\left 2, \frac{3}{2}\right\rangle$	$\phi_i^+ \phi_j^+$	-	$n_D n_c$
$ 2, 1\rangle$	$\phi_i^+ \phi_j^+$	$i \leq j$	$n_D(n_D + 1)/2$
$\left 1, \frac{3}{2}\right\rangle$	$\phi_i^0 \phi_j^+$	-	$n_D n_c$
$ 1, 1\rangle$	$\{\phi_i^+ \phi_j^0, \phi_i^+ \chi_j\}$	-	$n_D^2 + n_n n_c$
$\left 1, \frac{1}{2}\right\rangle$	$\{\phi_i^+ \chi_j, \phi_i^{0*} \phi_j^+\}$	-	$n_D(n_n + n_c)$
$ 1, 0\rangle$	$\phi_i^+ \phi_j^{0*}$	-	n_D^2
$ 0, 1\rangle$	$\phi_i^0 \phi_j^0$	$i \leq j$	$n_D(n_D + 1)/2$
$\left 0, \frac{1}{2}\right\rangle$	$\{\phi_i^0 \chi_j, \phi_i^- \phi_j^+\}$	-	$n_D(n_n + n_c)$
$ 0, 0\rangle$	$\{\phi_i^+ \phi_j^-, \phi_i^0 \phi_j^{0*}, \phi_i^+ \phi_j^-, \chi_i \chi_j\}$	$\{-, -, -, i \leq j\}$	$2n_D^2 + n_c^2 + n_n(n_n + 1)/2$

The Model

Basis of States, first using only Q and Y

$$\mathcal{M}_{A_a B_b \rightarrow C_c D_d} = - \frac{\partial^4 V_4}{\partial A_a \partial B_b \partial C_c^* \partial D_d^*}$$

→

$$\begin{aligned} \mathcal{M}[\phi_a^+ \phi_b^+ \rightarrow \phi_c^+ \phi_d^+] &= \mathcal{M}[\phi_a^0 \phi_b^+ \rightarrow \phi_c^0 \phi_d^+] = \delta_{ca,db} \\ \mathcal{M}[\phi_a^+ \phi_b^+ \rightarrow \phi_c^+ \phi_d^+] &= \mathcal{M}[\phi_a^0 \phi_b^0 \rightarrow \phi_c^0 \phi_d^0] = 2\lambda_{ca,db} + 2\lambda_{da,cb} \\ \mathcal{M}[\phi_a^+ \chi_b \rightarrow \phi_c^+ \chi_d] &= \mathcal{M}[\phi_a^0 \chi_b \rightarrow \phi_c^0 \chi_d] = 2\gamma_{ca,bd} \\ \mathcal{M}[\phi_a^+ \chi_b \rightarrow \phi_c^{0*} \phi_d^+] &= \mathcal{M}[\phi_a^0 \chi_b \rightarrow \phi_c^- \phi_d^+] = 2i\kappa_{ca,db} \\ \mathcal{M}[\phi_a^{0*} \phi_b^+ \rightarrow \phi_c^{0*} \phi_d^+] &= \mathcal{M}[\phi_a^- \phi_b^+ \rightarrow \phi_c^- \phi_d^+] = \delta_{ac,bd} \end{aligned}$$

- Not all scattering amplitudes are independent \Rightarrow perturbative unitarity bounds from scattering involving $|\mathbf{1}, \frac{3}{2}\rangle$, $|\mathbf{0}, \mathbf{1}\rangle$ and $|\mathbf{0}, \frac{1}{2}\rangle$ are redundant, since they are identical to the ones derived for $|\mathbf{2}, \frac{3}{2}\rangle$, $|\mathbf{2}, \mathbf{1}\rangle$ and $|\mathbf{1}, \frac{1}{2}\rangle$

The Model

Basis of States, using Q, Y and T

- T is also conserved
- $T = 0, 1$ in this model

$$\begin{aligned} |1,1\rangle \rightarrow |1,1,0\rangle : \phi_{[i}^+ \phi_j^0] &\equiv \frac{1}{\sqrt{2}} (\phi_i^+ \phi_j^0 - \phi_j^+ \phi_i^0) \\ |1,1,1\rangle : \phi_{(i}^+ \phi_j^0) &\equiv \frac{1}{\sqrt{2}} (\phi_i^+ \phi_j^0 + \phi_j^+ \phi_i^0) \end{aligned}$$

$$\begin{aligned} |0,0\rangle \rightarrow |0,0,0\rangle : \Phi_i \Phi_j^* &\equiv \frac{1}{\sqrt{2}} (\phi_i^+ \phi_j^- - \phi_i^0 \phi_j^{0*}) \\ |0,0,1\rangle : \overline{\Phi_i \Phi_j^*} &\equiv \frac{1}{\sqrt{2}} (\phi_i^+ \phi_j^- + \phi_i^0 \phi_j^{0*}) \end{aligned}$$

$$\mathcal{M}[\phi_a^+ \phi_b^+ \rightarrow \phi_c^+ \phi_d^+] = \mathcal{M}[\phi_{(a}^+ \phi_{b)}^0 \rightarrow \phi_{(c}^+ \phi_{d)}^0] = 2\lambda_{ca,db} + 2\lambda_{da,cb}$$

$$\mathcal{M}[\phi_a^+ \phi_b^{0*} \rightarrow \phi_c^+ \phi_d^{0*}] = \mathcal{M}[\overline{\Phi_a \Phi_b^*} \rightarrow \overline{\Phi_c \Phi_d^*}] = 2\lambda_{ca,bd}$$

The Model

Basis of States, using Q, Y and T

It is sufficient to apply partial-wave unitarity bounds to the scattering matrices constructed from the states listed in this table:

$ Q, Y, T\rangle$	State	Conditions	Dimensionality
$ 2, 2, 0\rangle$	$\phi_i^+ \phi_j^+$	$i \leq j$	$n_c(n_c + 1)/2$
$\left 2, \frac{3}{2}, \frac{1}{2}\right\rangle$	$\phi_i^+ \phi_j^+$	-	$n_D n_c$
$ 2, 1, 1\rangle$	$\phi_i^+ \phi_j^+$	$i \leq j$	$n_D(n_D + 1)/2$
$ 1, 1, 0\rangle$	$\{\phi_{[i}^+ \phi_j^0, \phi_i^+ \chi_j\}$	$\{i < j, -\}$	$n_D(n_D - 1)/2 + n_n n_c$
$\left 1, \frac{1}{2}, \frac{1}{2}\right\rangle$	$\{\phi_i^+ \chi_j, \phi_i^{0*} \phi_j^+\}$	-	$n_D(n_n + n_c)$
$ 1, 0, 1\rangle$	$\phi_i^+ \phi_j^{0*}$	-	n_D^2
$ 0, 0, 0\rangle$	$\{\Phi_i \Phi_j^*, \phi_i^+ \phi_j^-, \chi_i \chi_j\}$	$\{-, -, i \leq j\}$	$n_D^2 + n_c^2 + n_n(n_n + 1)/2$

The Model

Q, Y and T vs Q, Y

Ex. SM

$ Q, Y\rangle$	State	Dimensionality
$ 2, 1\rangle$	$\phi_1^+ \phi_1^+$	1
$ 1, 1\rangle$	$\phi_1^+ \phi_1^0$	1
$ 1, 0\rangle$	$\phi_1^+ \phi_1^{0*}$	1
$ 0, 1\rangle$	$\phi_1^0 \phi_1^0$	1
$ 0, 0\rangle$	$\phi_1^+ \phi_1^-, \phi_1^0 \phi_1^{0*}$	2

$ Q, Y, T\rangle$	State	Dimensionality
$ 2, 1, 1\rangle$	$\phi_1^+ \phi_1^+$	1
$ 1, 1, 1\rangle$	$\phi_{(1)}^+ \phi_1^0$	1
$ 1, 0, 1\rangle$	$\phi_1^+ \phi_1^{0*}$	1
$ 0, 1, 1\rangle$	$\phi_1^0 \phi_1^0$	1
$ 0, 0, 0\rangle$	$\Phi_1 \Phi_1^*$	1
$ 0, 0, 1\rangle$	$\Phi_1^- \Phi_1^*$	1

The Model

Q, Y and T vs Q, Y

Ex. SM

Once redundancies are removed:

$ Q, Y\rangle$	State	Dimensionality
$ 2, 1\rangle$	$\phi_1^+ \phi_1^+$	1
$ 1, 1\rangle$	$\phi_1^+ \phi_1^0$	1
$ 1, 0\rangle$	$\phi_1^+ \phi_1^{0*}$	1
$ 0, 0\rangle$	$\phi_1^+ \phi_1^-, \phi_1^0 \phi_1^{0*}$	2

$ Q, Y, T\rangle$	State	Dimensionality
$ 2, 1, 1\rangle$	$\phi_1^+ \phi_1^+$	1
$ 1, 0, 1\rangle$	$\phi_1^+ \phi_1^{0*}$	1
$ 0, 0, 0\rangle$	$\Phi_1 \Phi_1^*$	1

Scattering Matrices and Eigenvalues

Recall:

$$16\pi |a_0| = \left| N_{ab} N_{cd} \frac{\partial^4 V_4}{\partial A_a \partial B_b \partial C_c^* \partial D_d^*} \right| \leq 8\pi$$

Matrix Element

$$\left| 2, \frac{3}{2}, \frac{1}{2} \right\rangle$$

$$16\pi a_0 [\phi_a^+ \phi_b^+ \rightarrow \phi_c^+ \phi_d^+] = \delta_{ca,db}$$

$$|1, 0, 1\rangle$$

$$16\pi a_0 [\phi_a^+ \phi_b^{0*} \rightarrow \phi_c^+ \phi_d^{0*}] = 2\lambda_{ca,bd}$$

$$|2, 2, 0\rangle$$

$$16\pi a_0 [\phi_a^+ \phi_b^+ \rightarrow \phi_c^+ \phi_d^+] = 4 N_{ab} N_{cd} \alpha_{ca,db}$$

$$|2, 1, 1\rangle$$

$$16\pi a_0 [\phi_a^+ \phi_b^+ \rightarrow \phi_c^+ \phi_d^+] = 2 N_{ab} N_{cd} (\lambda_{ca,db} + \lambda_{da,cb})$$

$$\left| 1, \frac{1}{2}, \frac{1}{2} \right\rangle$$

$$16\pi a_0 [\phi_a^{0*} \phi_b^+ \rightarrow \phi_c^{0*} \phi_d^+] = \delta_{ac,db}$$

$$16\pi a_0 [\phi_a^+ \chi_b \rightarrow \phi_c^+ \chi_d] = 2 \gamma_{ca,bd}$$

$$16\pi a_0 [\phi_a^+ \chi_b \rightarrow \phi_c^{0*} \phi_d^+] = 2i \kappa_{ca,db}$$

$$|0, 0, 0\rangle$$

$$16\pi a_0 [\varphi_a^+ \varphi_b^- \rightarrow \varphi_c^+ \varphi_d^-] = 4 \alpha_{ba,cd}$$

$$16\pi a_0 [\chi_a \chi_b \rightarrow \chi_c \chi_d] = 24 N_{ab} N_{cd} \beta_{ab,cd}$$

$$16\pi a_0 [\varphi_a^+ \varphi_b^- \rightarrow \chi_c \chi_d] = 2 N_{cd} \zeta_{ba,cd}$$

$$16\pi a_0 [\Phi_a \Phi_b^* \rightarrow \varphi_c^+ \varphi_d^-] = \sqrt{2} \delta_{ba,cd}$$

$$16\pi a_0 [\Phi_a \Phi_b^* \rightarrow \chi_c \chi_d] = 2\sqrt{2} N_{cd} \gamma_{ba,cd}$$

$$16\pi a_0 [\Phi_a \Phi_b^* \rightarrow \Phi_c \Phi_d^*] = 4\lambda_{ba,cd} + 2\lambda_{ca,bd}$$

$$|1, 1, 0\rangle$$

$$16\pi a_0 [\phi_{[a}^+ \phi_{b]}^0 \rightarrow \phi_{[c}^+ \phi_{d]}^0] = 2\lambda_{ca,db} - 2\lambda_{dacb}$$

$$16\pi a_0 [\varphi_a^+ \chi_b \rightarrow \varphi_c^+ \chi_d] = 2 \zeta_{ca,bd}$$

$$16\pi a_0 [\phi_{[a}^+ \phi_{b]}^0 \rightarrow \varphi_c^+ \chi_d] = 2\sqrt{2} i \kappa_{ba,cd}$$

Mathematica Notebook

- To automate the generation of scattering matrices, compute their eigenvalues, and ensure these values remain below 8π , we have developed a Mathematica notebook, **BounDS**
- The user simply needs to specify the values of n_D , n_C , and n_n , and, if necessary, the transformation properties of the fields under [additional flavour and/or generalized CP symmetries \(abelian and non-abelian\)](#).

BounDS then performs the following steps:

1

Computes the full set of linearly independent quartic couplings allowed by the specified symmetries

2

Calculates the seven independent scattering matrices $|Q, Y, T\rangle$

3

Block-diagonalizes the scattering matrices by appropriately swapping rows and columns

4

Outputs the quartic part of the scalar potential and the scattering matrices

5

Provides closed-form expressions for the eigenvalues of the scattering matrices whenever possible

<https://github.com/andremilagre/BounDS.git>