TÉCNICO LISBOA

Superradiance in Einstein-Gauss-Bonnet gravity with Born-Infeld electrodynamics

2nd Cycle Integrated Project in Engineering Physics

Author
Oskar Borgvall González

Advisors
Filipe Moura
Richard Pires Brito

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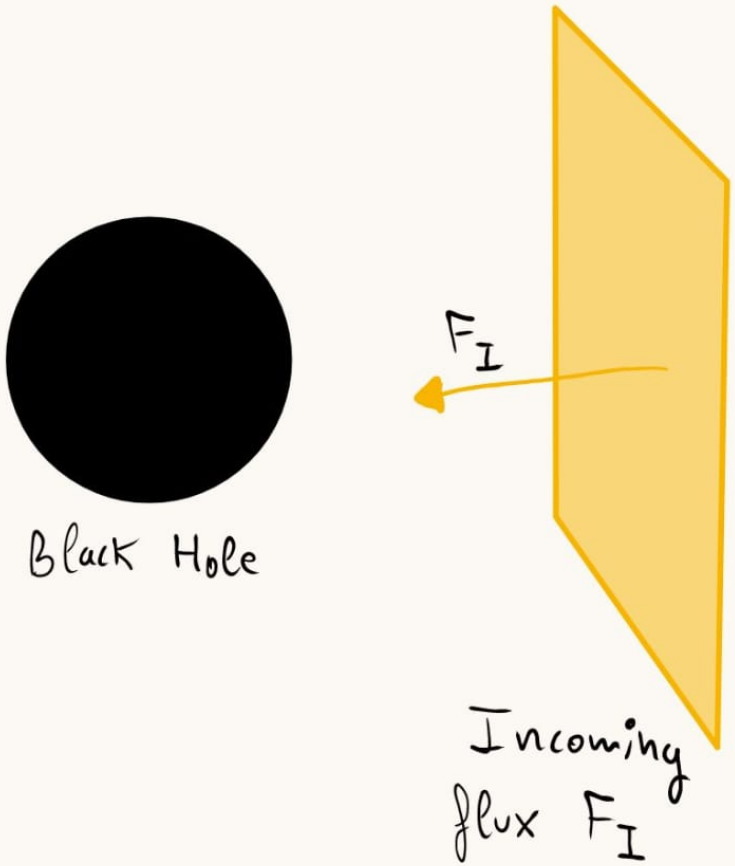
Instituto Superior Técnico, Lisbon

Black Hole Superradiance



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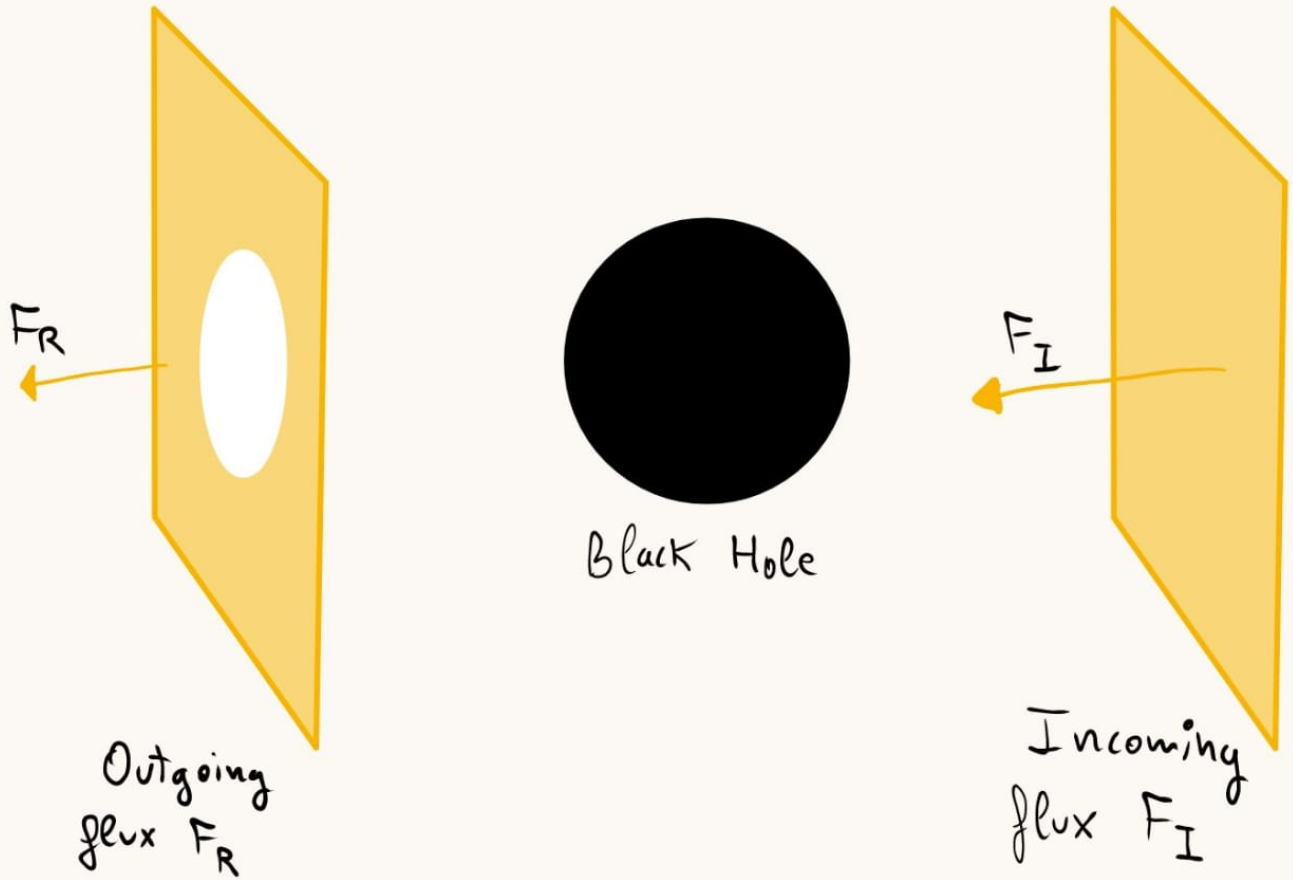

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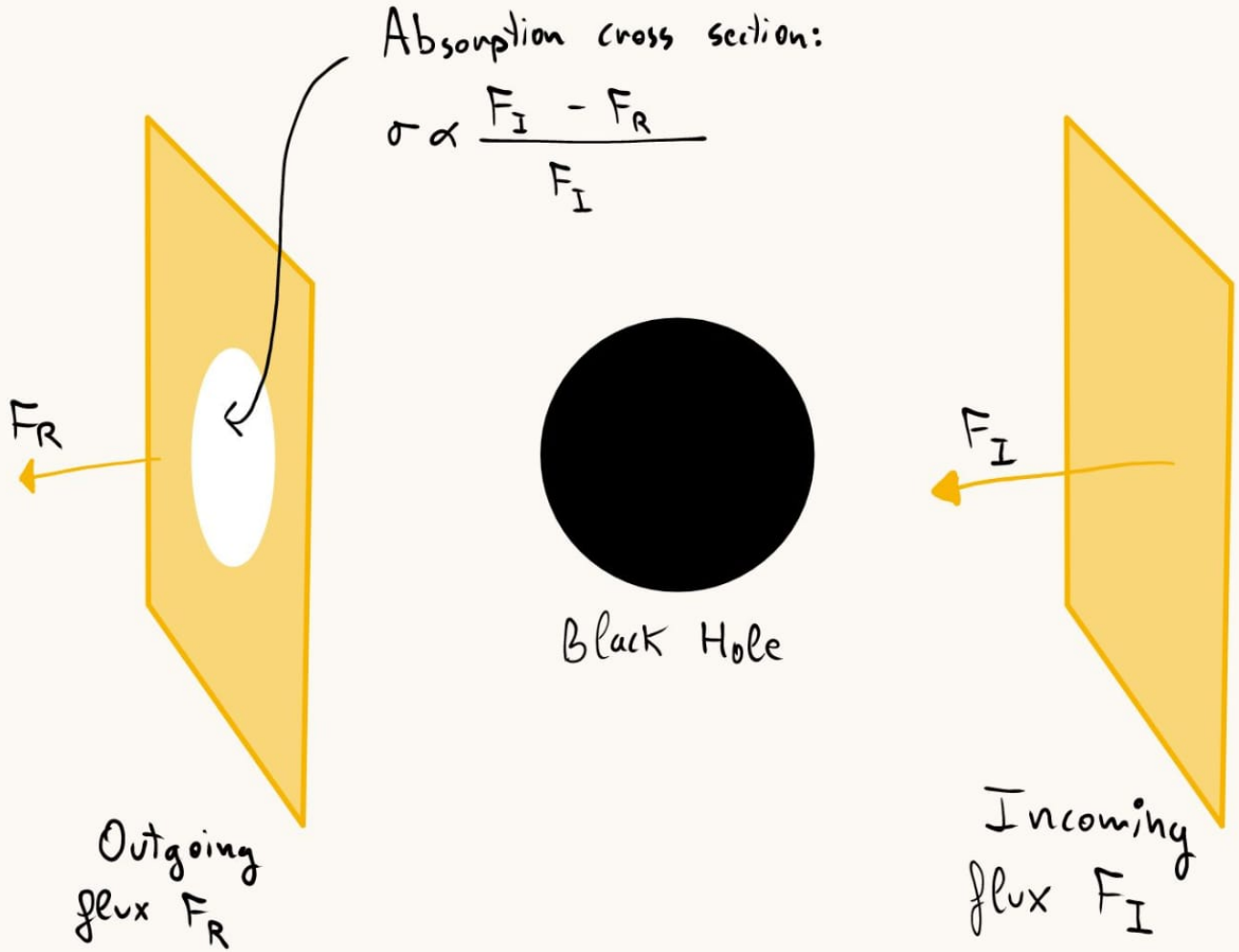

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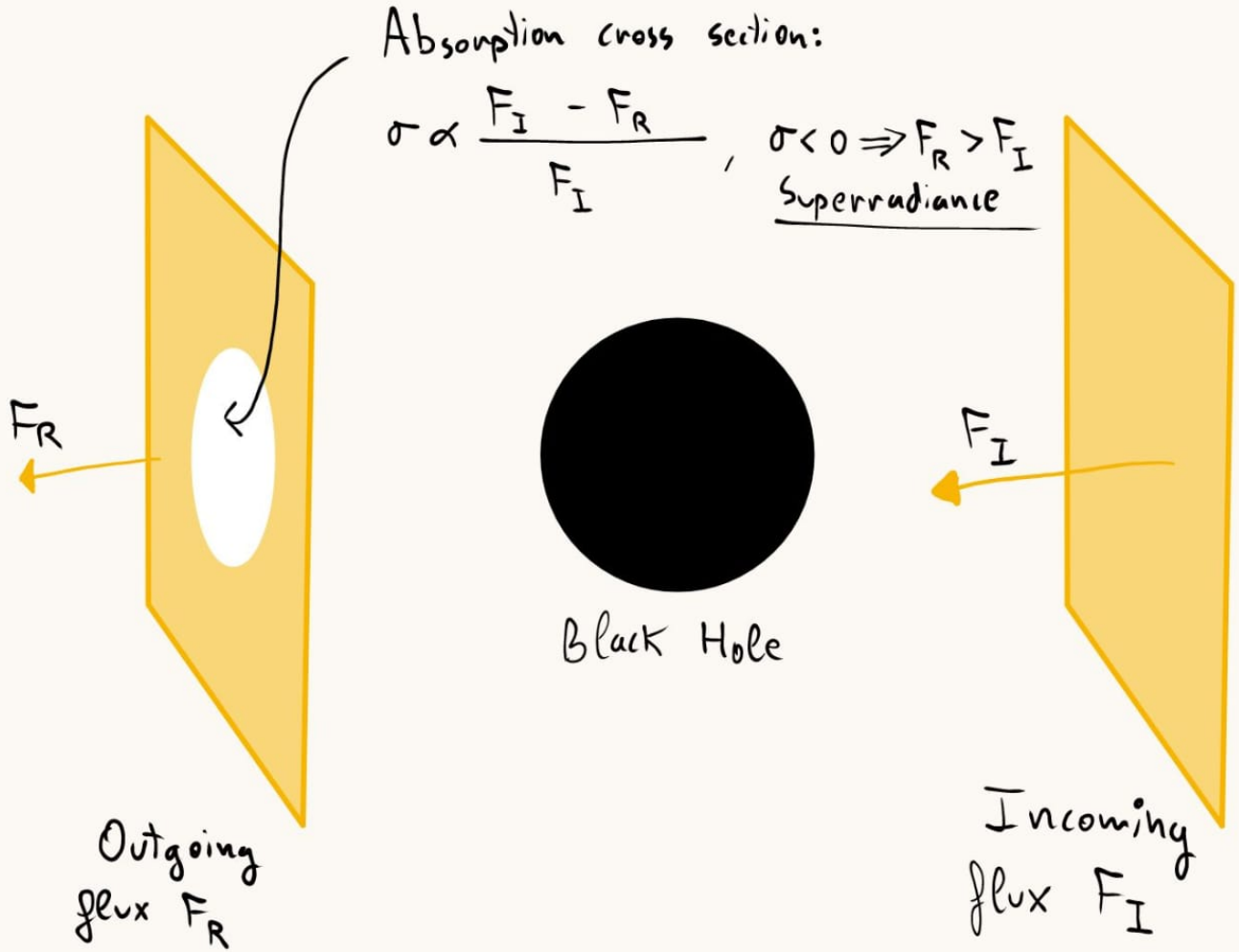

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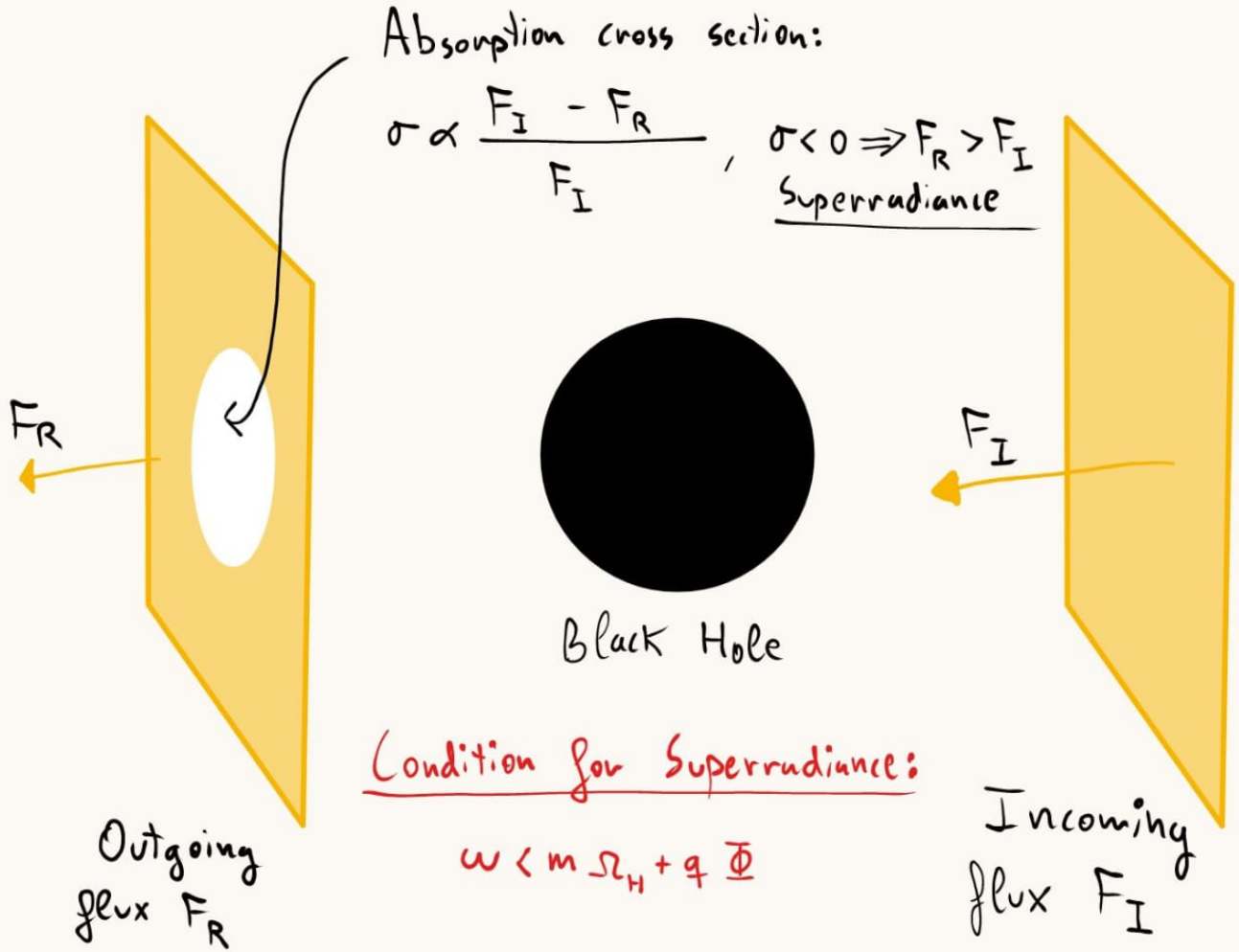


Einstein-Gauss-Bonnet+Born-Infeld Gravity

Action for Einstein's GR with Maxwell electrodynamics:

$$S = \int d^d x \Big[\frac{-\sqrt{g}R}{4} - \frac{\sqrt{g}}{4} F_{AB} F^{AB} \Big]. \tag{1}$$

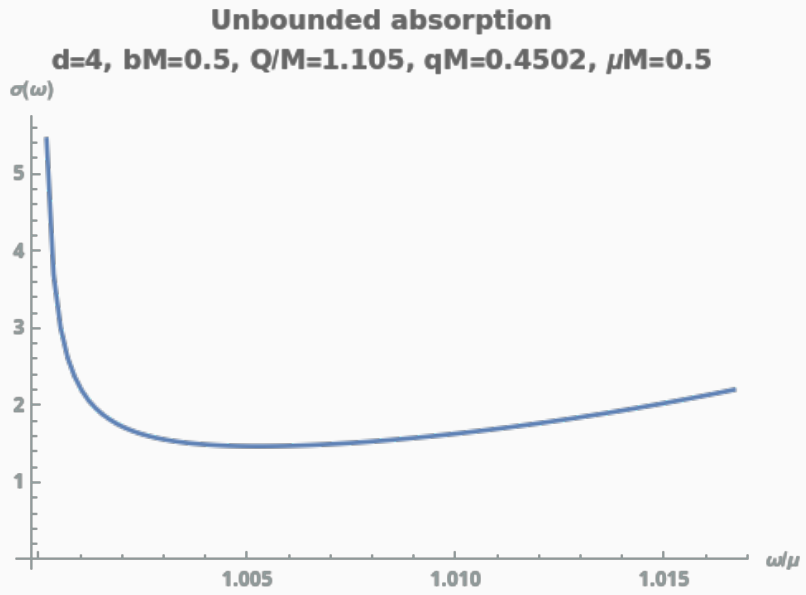
However we will use a String-inspired modified theory: Einstein-Gauss-Bonnet gravity with Born-Infeld electrodynamics **[Wiltshire(1988)]**

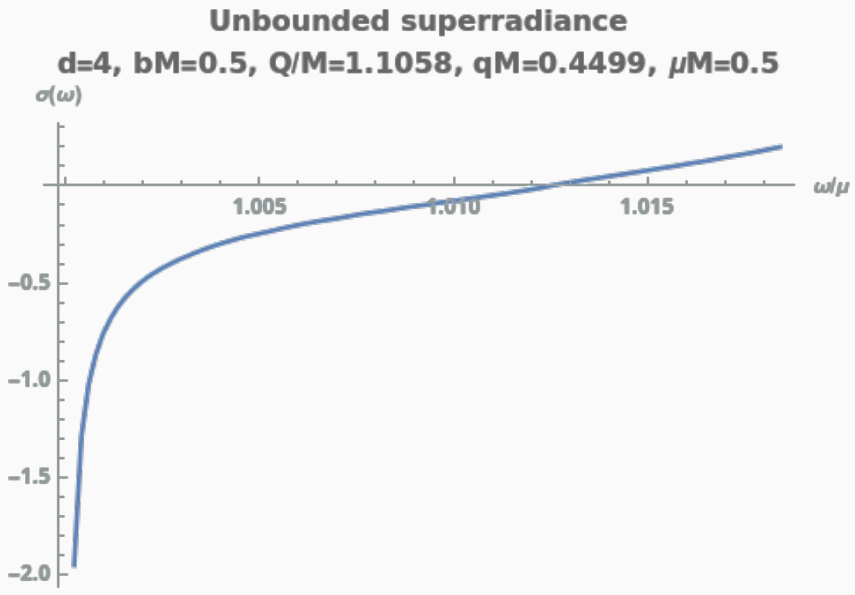
$$S = \int d^d x \Big[\frac{-\sqrt{g}R}{4} + \alpha\sqrt{g} \left(R_{ABCD} R^{ABCD} - 4 R_{AB} R^{AB} + R^2 \right) + \\ + b^2 \left(\sqrt{g} - \sqrt{\det \left(g_{EF} + \frac{F_{EF}}{b} \right)} \right) \Big]. \tag{2}$$

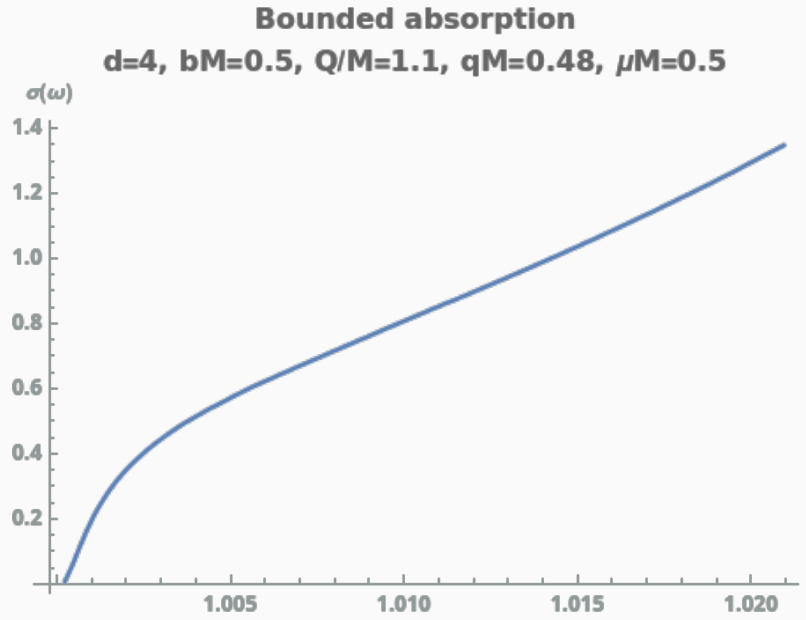
In the limit $\alpha \to 0$, $b \to +\infty$, Eq. 2 reduces to Eq. 1.

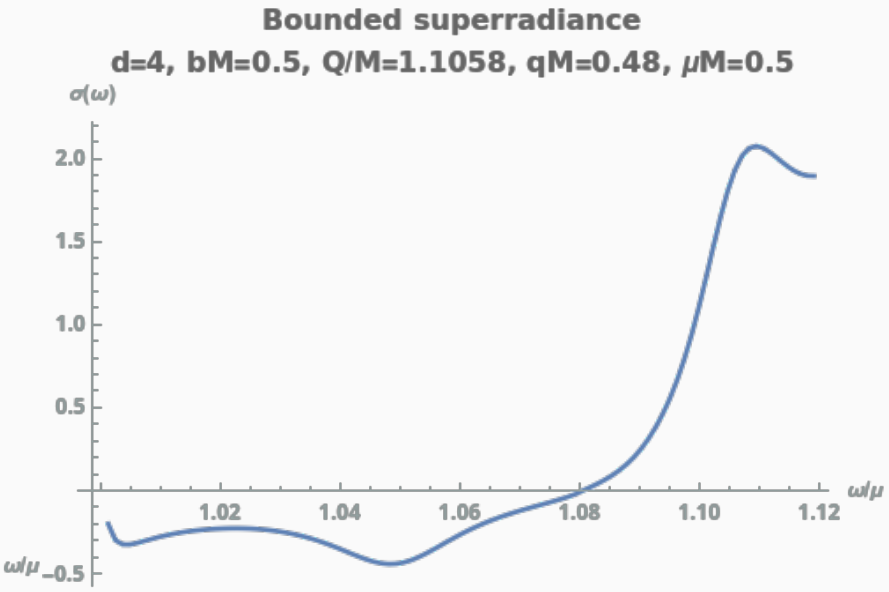
We consider a charged, spherically symmetric black hole solution from this modified theory.

Absorption Cross Section









Parameter space in classical GR

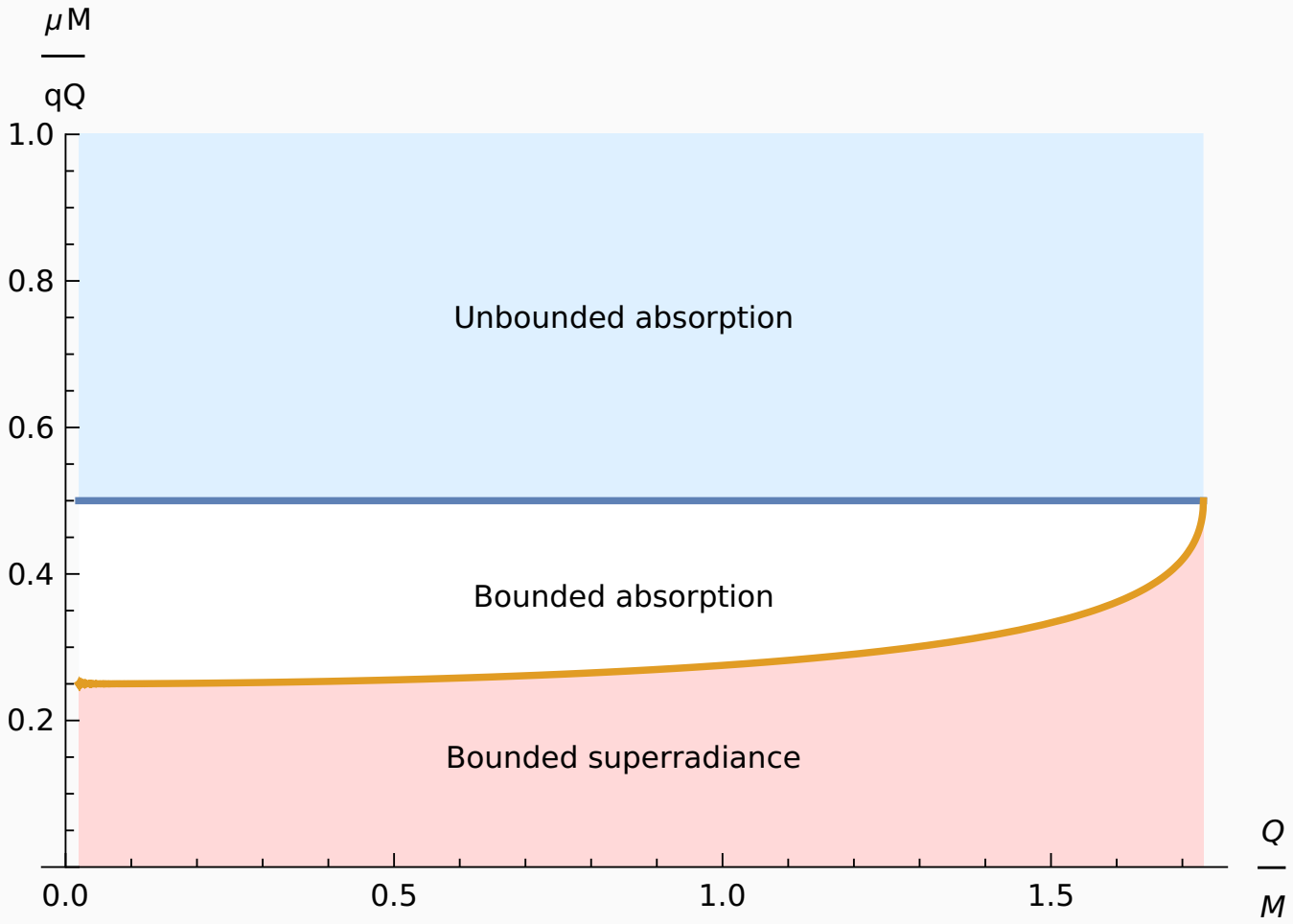


Figure 1: Parameter space for a charged, massive scalar field and a charged black hole in classical GR [de Paula et al.(2024)de Paula, Leite, Dolan, and Crispino]. In this case, no unbounded superradiant regime is found.

Parameter space in EGB+BI gravity

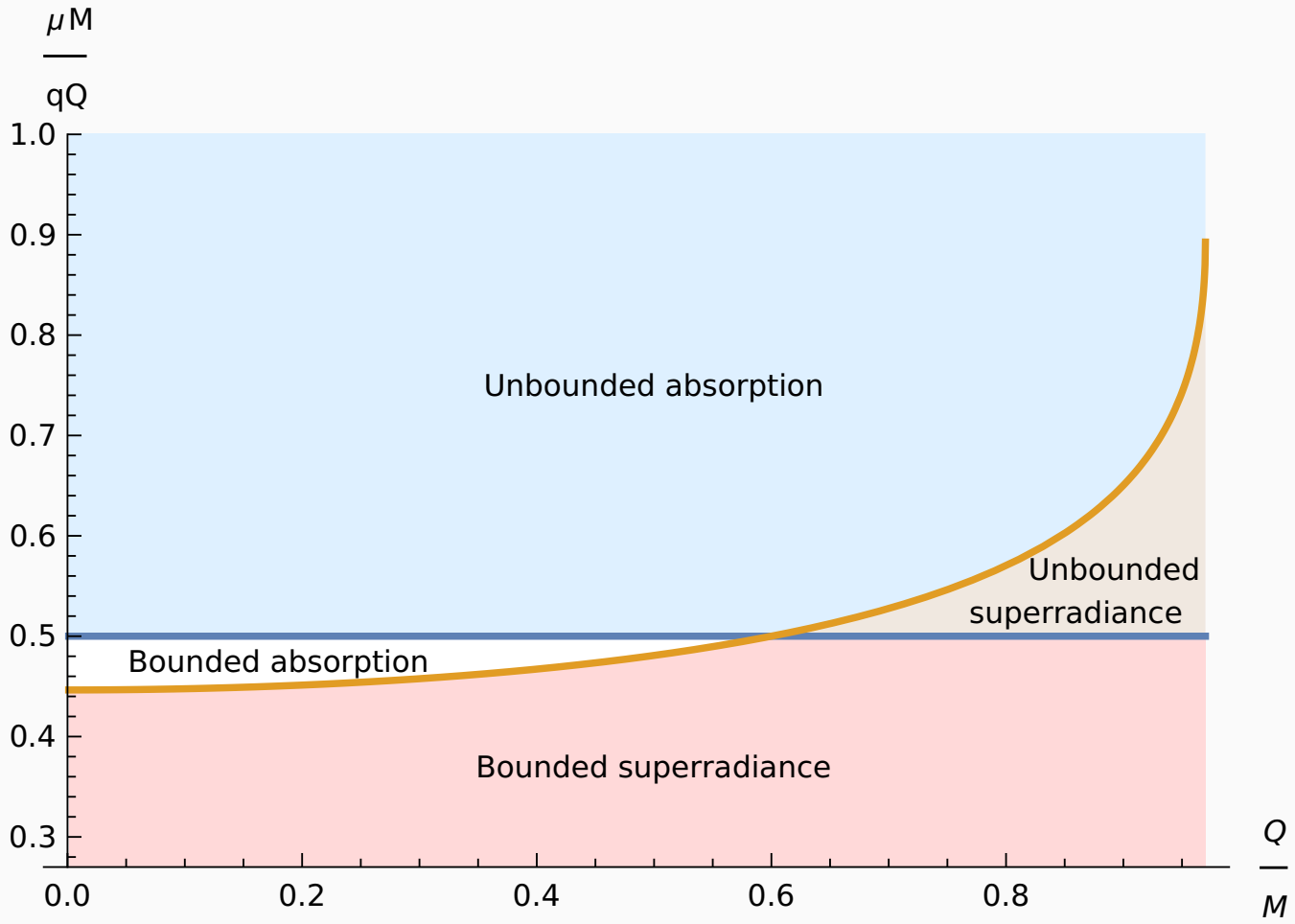


Figure 2: Parameter space for a charged, massive scalar field and a charged black hole in Einstein-Gauss-Bonnet gravity and Born-Infeld electrodynamics ($\alpha/M = 0.22, b\sqrt{M} = 5$).

Conclusions and future work

- We have characterized the presence of unbounded superradiance in an Einstein-Gauss-Bonnet black hole with Born-Infeld NED. Modifications of gravity or of electrodynamics can both enable an unbounded superradiant regime.
- Unbounded superradiance implies the BH is acting as an arbitrarily large wave amplifier. Taking into account backreaction on the metric could be necessary to fully characterize the dynamics.
- Open questions:
  - Relation between unbounded superradiance and superradiant instabilities?
  - Presence of unbounded superradiance in other models/rotational superradiance?

References i

M. A. A. de Paula, L. C. S. Leite, S. R. Dolan, and L. C. B. Crispino.
Absorption and unbounded superradiance in a static regular black hole spacetime.
Phys. Rev. D, 109(6):064053, 2024.
doi: 10.1103/PhysRevD.109.064053.

D. L. Wiltshire.
Black Holes in String Generated Gravity Models.
Phys. Rev. D, 38:2445, 1988.
doi: 10.1103/PhysRevD.38.2445.

Thank you for your attention!

Any questions?

Backup slides

Einstein-Gauss-Bonnet + Born-Infeld Black Hole

From our action, a spherically symmetric, charged black hole solution was found [Wiltshire(1988)], with metric given by:

$$ds^2 = -f(r)dt^2 + \frac{1}{f(r)}dr^2 + r^2 d\Omega_{d-2}^2, \tag{3}$$

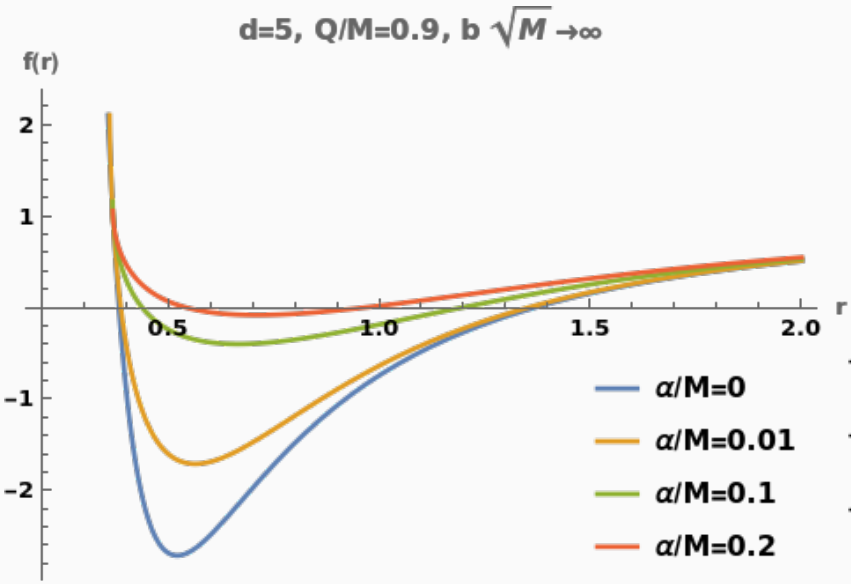


Figure 3: Metric function f(r) for $b \to \infty$ and different values of α.

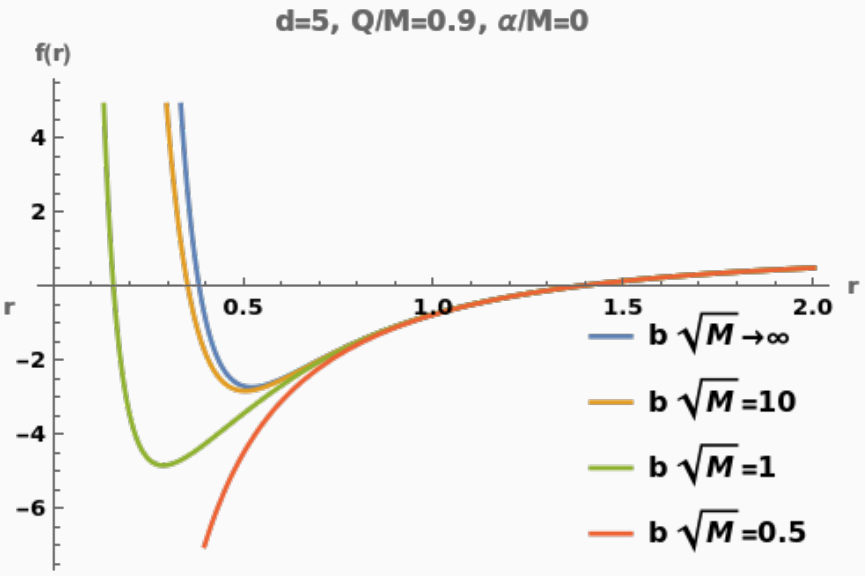


Figure 4: Metric function f(r) for $\alpha = 0$ and different values of b.

Born-Infeld electrodynamics

For a point-charge Q at r=0, the electrostatic potential is given by

$$\Phi(r) = -\int_{\infty}^{r} \frac{Q}{\sqrt{Q^2/b^2 + z^{2d-4}}} dz. \tag{4}$$

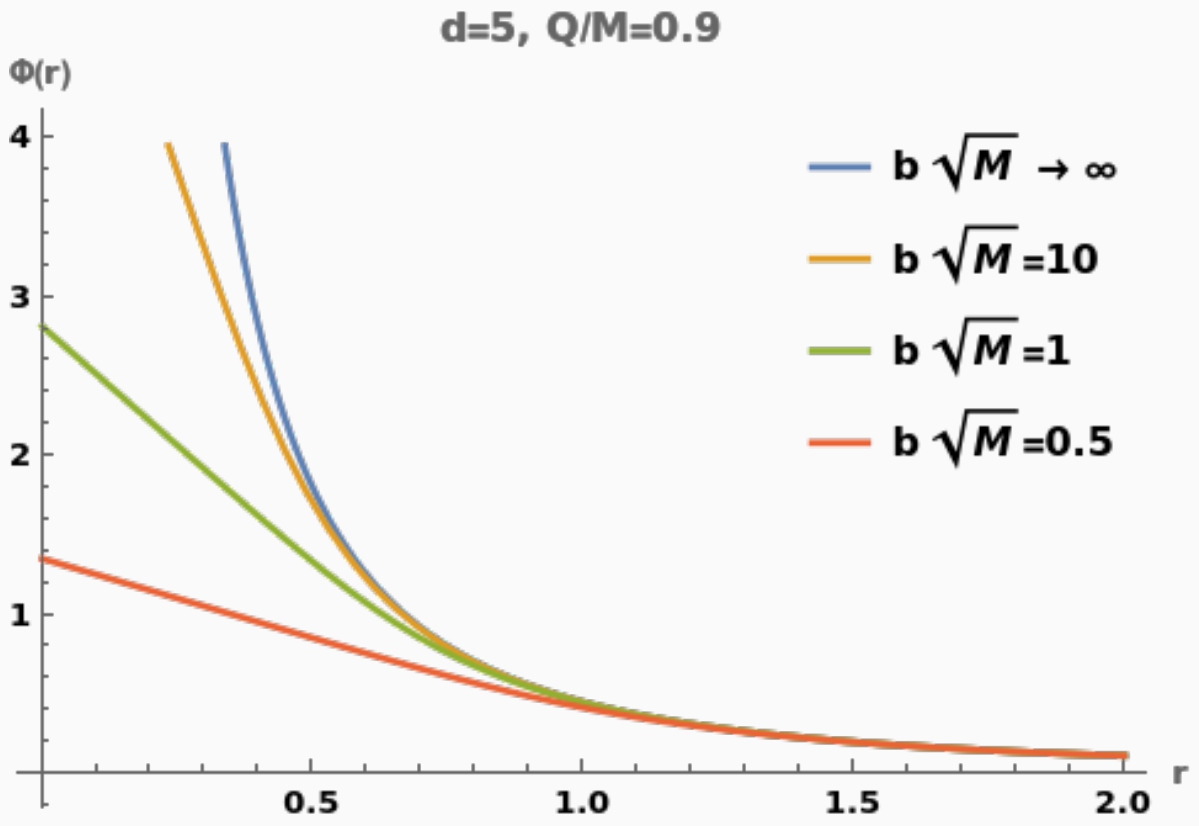


Figure 5: Electrostatic potential of a point charge for different values of b.

EGB + BI Black Hole solution

A spherically symmetric charged black hole is found, whose metric is given by [Wiltshire(1988)]:

$$ds^2 = -f(r)dt^2 + \frac{1}{f(r)}dr^2 + r^2 d\Omega_{d-2}^2, \tag{5}$$

$$f(r) = 1 + \frac{r^2}{8\tilde{\alpha}}\left(1 + \sqrt{1 + \frac{16\tilde{\alpha}}{r^{d-1}}\left(2M + U(r)\right)}\right), \qquad \tilde{\alpha} = (d-3)(d-4)\alpha, \tag{6}$$

$$U(r) = \frac{4Q}{d-1}\left(\frac{Qr^3}{(d-2)\left(r^d + \sqrt{\frac{Q^2 r^4}{b^2} + r^{2d}}\right)} + \int \frac{b^2 dr}{\sqrt{r^{2d-4} + Q^2}}\right). \tag{7}$$

Taking $\alpha \to 0$ and $b \to \infty$,

$$f(r) \to 1 - \frac{2M}{r^{d-3}} + \frac{2Q^2}{(d-3)(d-2)r^{2d-6}}. \tag{8}$$

Born-Infeld Electrodynamics

The electrostatic potential for a point charge Q at r=0 yields,

$$\Phi(r) = \frac{b^2}{Q}\left(\frac{1}{\sqrt{\pi}}\left(\frac{Q}{b}\right)^{1+\frac{1}{d-2}}\Gamma\left(1+\frac{1}{2d-4}\right)\Gamma\left(\frac{d-3}{2d-4}\right) - \right.$$
$$\left. - r\sqrt{\left(\frac{Q}{b}\right)^2 + r^{2d-4}}\,{}_2F_1\left(1, \frac{d-1}{2d-4}, 1+\frac{1}{2d-4}; -\frac{b^2 r^{2d-4}}{Q^2}\right)\right), \quad (9)$$

Taking $b \to \infty$,

$$\Phi(r) \to \frac{Q}{(d-3)r^{d-3}}. \quad (10)$$

Superradiance of a scalar field

Let us consider a scalar field with mass μ and charge q in the black hole background.

- Klein-Gordon equation $(\nabla_\nu - iqA_\nu)(\nabla^\nu - iqA^\nu)\Psi - \mu^2\Psi = 0$.
- Separation of variables. For a particular (l, m) mode:
  $\Psi = r^{-(d-2)/2} e^{-i\omega t} u_{\omega,l}(r) Y_{l,m}(\Omega) \longrightarrow \left(\frac{d^2}{dr_\star^2} - V(r)\right) u_{\omega,l}(r) = 0.$
- Boundary conditions

$$u_{\omega,l} \sim \begin{cases} T_{\omega l} e^{-i\kappa r_\star}, & r_\star \to -\infty \quad (r \to r_H) \\ I_{\omega l} e^{-i\tilde{\omega} r_\star} + R_{\omega l} e^{i\tilde{\omega} r_\star}, & r_\star \to +\infty \quad (r \to +\infty) \end{cases} \tag{11}$$

  where $\kappa = \lim_{r\to r_H} \sqrt{-V(r)} = \omega - q\Phi(r_H)$,
  $\tilde{\omega} = \lim_{r\to\infty} \sqrt{-V(r)} = \sqrt{\omega^2 - \mu^2}$.
- Absorption cross section given by

$$\sigma(\omega) = \sum_l \sigma_l(\omega), \qquad \sigma_l(\omega) = F_{d,l} \frac{1}{\tilde{\omega}^{(d-1)}} (\omega - q\Phi(r_H)) \frac{|T_{\omega l}|^2}{|I_{\omega l}|^2}. \tag{12}$$

- Superradiance occurs when $\omega < q\Phi(r_H)$: extracting electrostatic energy from the BH.

Superradiance and cross-section

$$\left(\frac{d^2}{dr_\star^2} - V_{\omega,l}(r)\right) u_{\omega,l}(r) = 0, \quad dr_\star = dr/f(r). \tag{13}$$

$$V_{\omega,l}(r) = f(r)\left(\frac{(d-2)(d-4)}{4}\frac{f(r)}{r^2} + \frac{d-2}{2r}\frac{df(r)}{dr} + \frac{l(l+d-3)}{r^2}\right) - (\omega - q\Phi(r))^2 . \tag{14}$$

Amplification factor:

$$Z_{\omega l} = \frac{|R_{\omega l}|^2}{|I_{\omega l}|^2} - 1 = -\frac{\omega - q\Phi(r_H)}{\kappa}\frac{|T_{\omega l}|^2}{|I_{\omega l}|^2}, \tag{15}$$

Cross section:

$$\sigma(\omega) = \sum_l \sigma_l(\omega), \tag{16}$$

$$\sigma_l(\omega) = \frac{\pi^{(d-2)/2}(l+d-4)!(2l+d-3)}{\Gamma\left(\frac{d-2}{2}\right)}\frac{1}{\kappa^{(d-1)}}(\omega - q\Phi(r_H))\frac{|T_{\omega l}|^2}{|I_{\omega l}|^2}. \tag{17}$$

Bounded/unbounded Cross-Section

Defining $U(r) = \lim_{\omega \to \mu} V_{\omega, l=0}(r)$ and expanding in powers of $1/r$:

- d=4 :

$$U(r) = 2\mu \frac{qQ - \mu M}{r} + \mathcal{O}(r^{-2}). \tag{18}$$

- d=5 :

$$U(r) = \frac{3/4 + qQ\mu - 2M\mu}{r^2} + \mathcal{O}(r^{-3}). \tag{19}$$

- d=6 :

$$U(r) = \frac{2}{r^2} + \mathcal{O}(r^{-3}). \tag{20}$$

- d=7 :

$$U(r) = \frac{15}{r^2} + \mathcal{O}(r^{-3}). \tag{21}$$

For $d \geq 6$, there is no propagative region extending to infinity in the limit $\omega \to \mu$.

Bounded/Unbounded Cross-Section

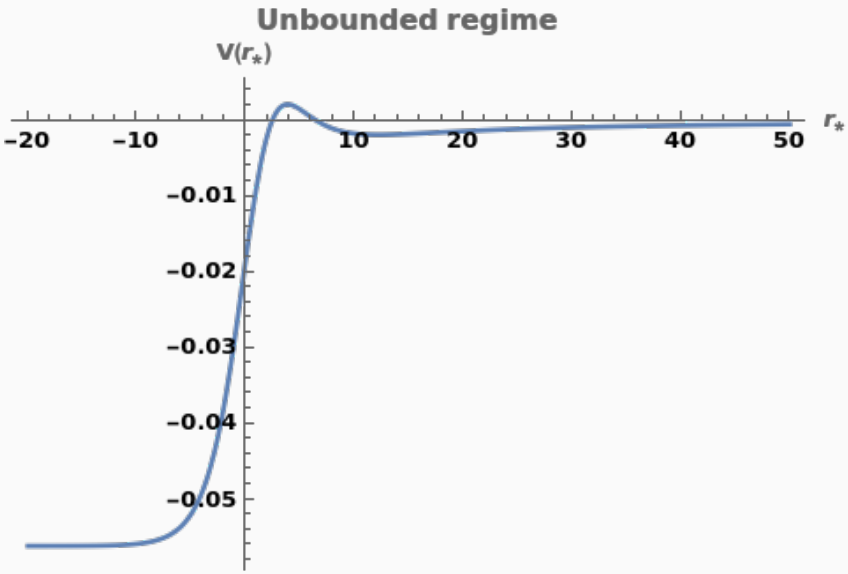


Figure 6: Effective potential $V(r)$ for $\omega = \mu$ in the unbounded region for the cross-section.

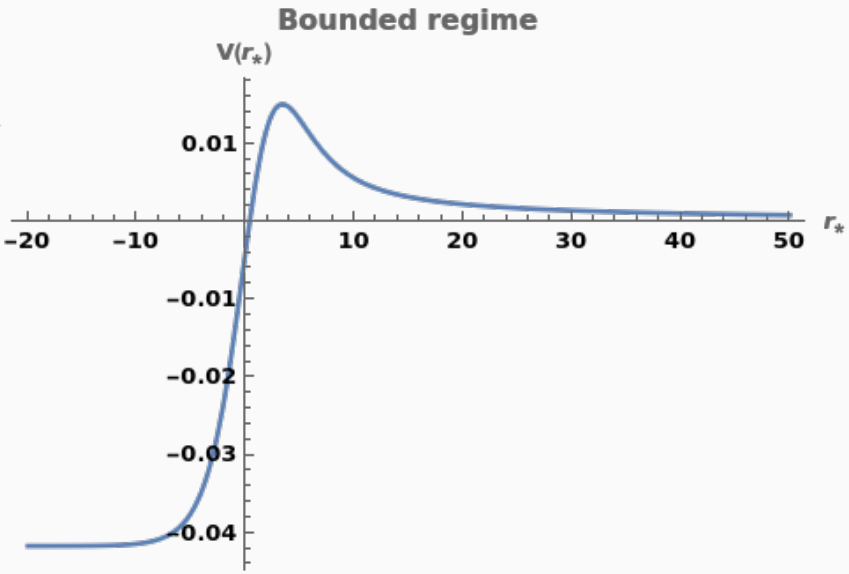


Figure 7: Effective potential $V(r)$ for $\omega = \mu$ in the bounded region for the cross-section.