Instituto Superior Técnico

Projeto MEFT

On Flavour Invariants of the N Higgs Doublet Model

Author:
João C. Belas (ist1102455)

Supervisors:
Prof. João P. Silva
Prof. Jorge C. Romão

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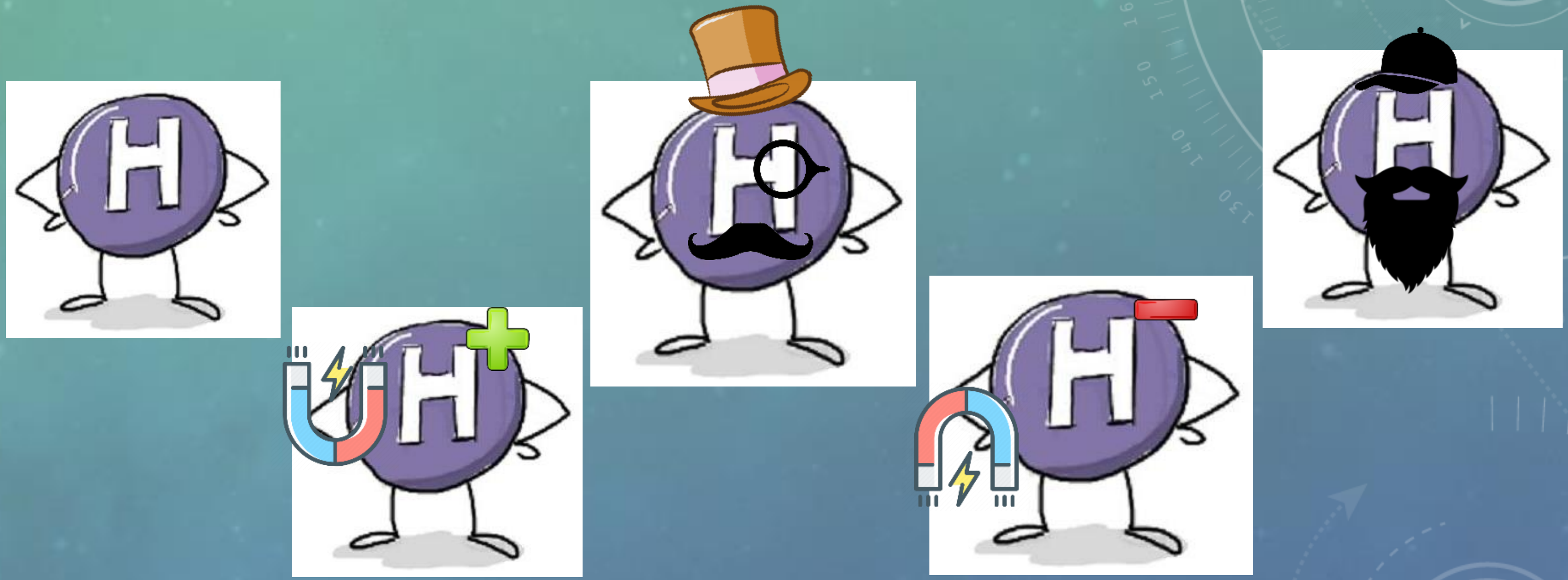
Multi Higgs Models

Standard Model

$$\Phi = \begin{pmatrix} G^+ \\ \frac{1}{\sqrt{2}}(v+h+iG^0) \end{pmatrix}$$

Multi Higgs (Beyond the Standard Model)



$$\mathcal{H}_0 = \begin{pmatrix} G^+ \\ \frac{1}{\sqrt{2}}(v+H^0+iG^0) \end{pmatrix} \quad \mathcal{H}_k = \begin{pmatrix} H_k^+ \\ \frac{1}{\sqrt{2}}(R_k+iI_k) \end{pmatrix}$$

Multi Higgs Models' Flavour Sector

The interaction between the quarks and the Higgs bosons in multi Higgs models is given by

$$
\begin{aligned}
-\mathcal{L}_Y = & \left(\overline{n_L} M_d n_R + \overline{p_L} M_u p_R\right) + \frac{H^0}{v}\left(\overline{n_L} M_d n_R + \overline{p_L} M_u p_R\right) \\
& + \frac{R_k}{v}\left(\overline{n_L} N_{dk} n_R + \overline{p_L} N_{uk} p_R\right) + \frac{i I_k}{v}\left(\overline{n_L} N_{dk} n_R - \overline{p_L} N_{uk} p_R\right) \\
& + \frac{\sqrt{2} H_k^+}{v}\left(\overline{p_L} N_{dk} n_R - \overline{p_R} N_{uk}^\dagger n_L\right) + h.c..
\end{aligned}
$$

The quark fields can be rotated into the mass basis, where the mass matrices M_d and M_u are diagonal.

$$
n_L = V_{dL} d_L, \qquad n_R = V_{dR} d_R, \qquad p_L = V_{uL} u_L, \qquad p_R = V_{uR} u_R, \qquad V = V_{uL}^\dagger V_{dL}.
$$

$$
M_d = V_{dL} D_d V_{dR}^\dagger, \quad D_d = \mathrm{diag}\{m_d, m_s, m_b\}, \quad M_u = V_{uL} D_u V_{uR}^\dagger, \quad D_u = \mathrm{diag}\{m_u, m_c, m_t\},
$$

The CKM matrix, V, encodes the two relevant bases for the left chiral quarks.

Multi Higgs Models' Flavour Sector

The Lagrangian becomes

$$
\begin{aligned}
-\mathcal{L}_Y &= \left(\overline{d_L} D_d d_R + \overline{u_L} D_u u_R\right) + \frac{H^0}{v}\left(\overline{d_L} D_d d_R + \overline{u_L} D_u u_R\right) \\
&+ \frac{R_k}{v}\left(\overline{d_L} V_{dL}^\dagger N_{dk} V_{dR} d_R + \overline{u_L} V_{uL}^\dagger N_{uk} V_{uR} u_R\right) + \frac{iI_k}{v}\left(\overline{d_L} V_{dL}^\dagger N_{dk} V_{dR} d_R - \overline{u_L} V_{uL}^\dagger N_{uk} V_{uR} u_R\right) \\
&+ \frac{\sqrt{2} H_k^+}{v}\left(\overline{u_L} V_{uL}^\dagger N_{dk} V_{dR} d_R - \overline{u_R} V_{uR}^\dagger N_{uk}^\dagger V_{dL} d_L\right) + h.c.,
\end{aligned}
$$

By comparison,

$$
-\mathcal{L}_Y = \left(\overline{n_L} M_d n_R + \overline{p_L} M_u p_R\right) + \frac{H^0}{v}\left(\overline{n_L} M_d n_R + \overline{p_L} M_u p_R\right)
$$

A New Way to Look at Flavour Physics

There are bases where N_{uk} and N_{dk} are diagonal, achieved through some rotation matrices.

$$N_{dk} = U_{dLk} E_{dk} U^{\dagger}_{dRk}, \quad E_{dk} = \mathrm{diag}(n_{d_1k}, n_{d_2k}, n_{d_3k}),$$

$$N_{uk} = U_{uLk} E_{uk} U^{\dagger}_{uRk}, \quad E_{uk} = \mathrm{diag}(n_{u_1k}, n_{u_2k}, n_{u_3k}).$$

Computing the matrices that change from the mass basis to these new bases

$$C_{dLk} = V^{\dagger}_{dL} U_{dLk}, \quad C_{uLk} = V^{\dagger}_{uL} U_{uLk}, \quad C_{dRk} = V^{\dagger}_{dR} U_{dRk}, \quad C_{uRk} = V^{\dagger}_{uR} U_{uRk},$$

With the CKM matrix, they account for the different bases relevant for the quark fields.

A New Way to Look at Flavour Physics

The Lagrangian becomes

$$-\mathcal{L}_Y = \left(\overline{d_L} D_d d_R + \overline{u_L} D_u u_R\right) + \frac{H^0}{v}\left(\overline{d_L} D_d d_R + \overline{u_L} D_u u_R\right)$$

$$+\frac{R_k}{v}\left(\overline{d_L} C_{dLk} E_{dk} C^\dagger_{dRk} d_R + \overline{u_L} C_{uLk} E_{uk} C^\dagger_{uRk} u_R\right) + \frac{iI_k}{v}\left(\overline{d_L} C_{dLk} E_{dk} C^\dagger_{dRk} d_R - \overline{u_L} C_{uLk} E_{uk} C^\dagger_{uRk} u_R\right)$$

$$+\frac{\sqrt{2}H_k^+}{v}\left(\overline{u_L} V C_{dLk} E_{dk} C^\dagger_{dRk} d_R - \overline{u_R} C_{uRk} E_{uk} C^\dagger_{uLk} V d_L\right) + h.c..$$

By comparison,

$$+\frac{R_k}{v}\left(\overline{d_L} V^\dagger_{dL} N_{dk} V_{dR} d_R + \overline{u_L} V^\dagger_{uL} N_{uk} V_{uR} u_R\right) + \frac{iI_k}{v}\left(\overline{d_L} V^\dagger_{dL} N_{dk} V_{dR} d_R - \overline{u_L} V^\dagger_{uL} N_{uk} V_{uR} u_R\right)$$

Flavour Invariants

If we rotate the quarks among themselves, using a unitary transformation, the mass matrices become

$$M_d' = W_L^\dagger M_d W_n, \qquad M_u' = W_L^\dagger M_u W_p$$

If we construct Hermitian matrices

$$H_d = M_d M_d^\dagger, \qquad H_u = M_u M_u^\dagger,$$

they transform under the rotation of the quarks as

$$H_d' = W_L^\dagger M_d W_n W_n^\dagger M_d^\dagger W_L = W_L^\dagger M_d M_d^\dagger W_L, \qquad H_u' = W_L^\dagger M_u W_p W_p^\dagger M_u^\dagger W_L = W_L^\dagger M_u M_u^\dagger W_L$$

As such, it is possible to construct Flavour Invariants from the trace of these matrices, since

$$Tr(H_d') = Tr\left(W_L^\dagger M_d M_d^\dagger W_L\right) = Tr\left(W_L W_L^\dagger M_d M_d^\dagger\right) = Tr\left(M_d M_d^\dagger\right) = Tr(H_d),$$

$$Tr(H_u') = Tr\left(W_L^\dagger M_u M_u^\dagger W_L\right) = Tr\left(W_L W_L^\dagger M_u M_u^\dagger\right) = Tr\left(M_u M_u^\dagger\right) = Tr(H_u).$$

Flavour Invariants

The Standard Model's flavour sector can be completely described by 10 flavour invariants made from

$$H_d, \qquad H_u.$$

The quarks masses can be obtained from the following invariants,

$$I_1^0 = Tr(H_u), \quad I_2^0 = Tr(H_u^2), \quad I_3^0 = Tr(H_u^3), \quad I_4^0 = Tr(H_d), \quad I_5^0 = Tr(H_d^2), \quad I_6^0 = Tr(H_d^3),$$

and the CKM mixing angles and CP violating phase can be obtained from

$$I_7^0 = Tr(H_u H_d), \quad I_8^0 = Tr(H_u^2 H_d), \quad I_9^0 = Tr(H_u H_d^2), \quad I_{10}^0 = Tr(H_u^2 H_d^2).$$

The same reasoning can be applied for multi Higgs models, and extra flavour invariants created.

Thank You For Your Attention

Flavour Invariants

The SM flavour invariants can be simplified to

$$I_1^0 = Tr(D_u^2), \quad I_2^0 = Tr(D_u^4) \quad I_3^0 = Tr(D_u^6), \quad I_4^0 = Tr(D_d^2), \quad I_5^0 = Tr(D_d^4) \quad I_6^0 = Tr(D_d^6),$$
$$I_7^0 = Tr(D_u^2 V D_d^2 V^\dagger), \quad I_8^0 = Tr(D_u^4 V D_d^2 V^\dagger), \quad I_9^0 = Tr(D_u^2 V D_d^4 V^\dagger), \quad I_{10}^0 = Tr(D_u^4 V D_d^4 V^\dagger).$$

Then, for $D = diag(m_1, m_2, m_3)$, $\mathrm{E} = diag(n_1, n_2, n_3)$, and a unitary matrix U_{uni},

$$Tr(D^p) = m_1^p + m_2^p + m_3^p\,,$$

$$Tr(D^p U_{uni} E^q U_{uni}^\dagger) = m_1^p n_1^q c_{12}^2 c_{13}^2 + m_3^p n_3^q c_{13}^2 c_{23}^2 + m_1^p n_2^q c_{13}^2 s_{12}^2 + m_1^p n_3^q s_{13}^2 + m_2^p n_3^q c_{13}^2 s_{23}^2$$
$$+ m_3^p n_2^q \left(c_{12}^2 s_{23}^2 + 2c_{12}s_{12}s_{13}c_{23}s_{23}c_\delta + s_{12}^2 s_{13}^2 c_{23}^2\right) + m_3^p n_1^q \left(s_{12}^2 s_{23}^2 - 2c_{12}s_{12}s_{13}c_{23}s_{23}c_\delta + c_{12}^2 s_{13}^2 c_{23}^2\right)$$
$$+ m_2^p n_1^q \left(s_{12}^2 c_{23}^2 + 2c_{12}s_{12}s_{13}c_{23}s_{23}c_\delta + c_{12}^2 s_{13}^2 s_{23}^2\right) + m_2^p n_2^q \left(c_{12}^2 c_{23}^2 - 2c_{12}s_{12}s_{13}c_{23}s_{23}c_\delta + s_{12}^2 s_{13}^2 s_{23}^2\right),$$