Quantum control &
Open Quantum Systems

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Café com Fisica, Out 2025

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Motivation

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Classical half-adder using trapped-ion quantum bits: Toward energy-efficient computation

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Simulating Logic

Landauer's principle

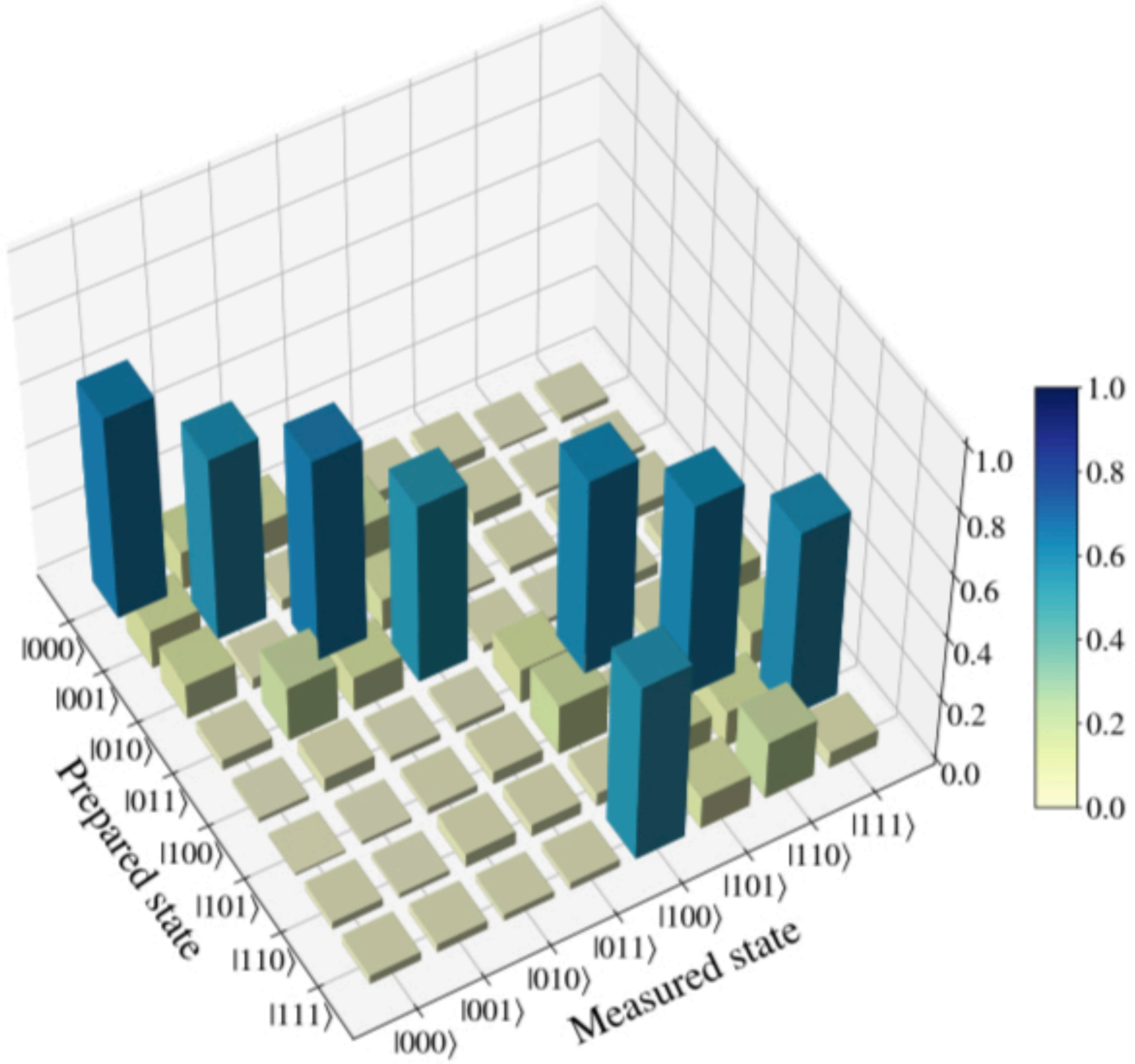
Irreversible bit operation ⇒ energy dissipation

$$E \geq k_B T \ln 2$$

Simulating Logic

$$H \approx \frac{\hbar J}{2}\left(Z_1 Z_2 + Z_2 Z_3\right) + \frac{\hbar\delta}{2} Z_2 + \frac{\hbar\Omega}{2} X_2$$

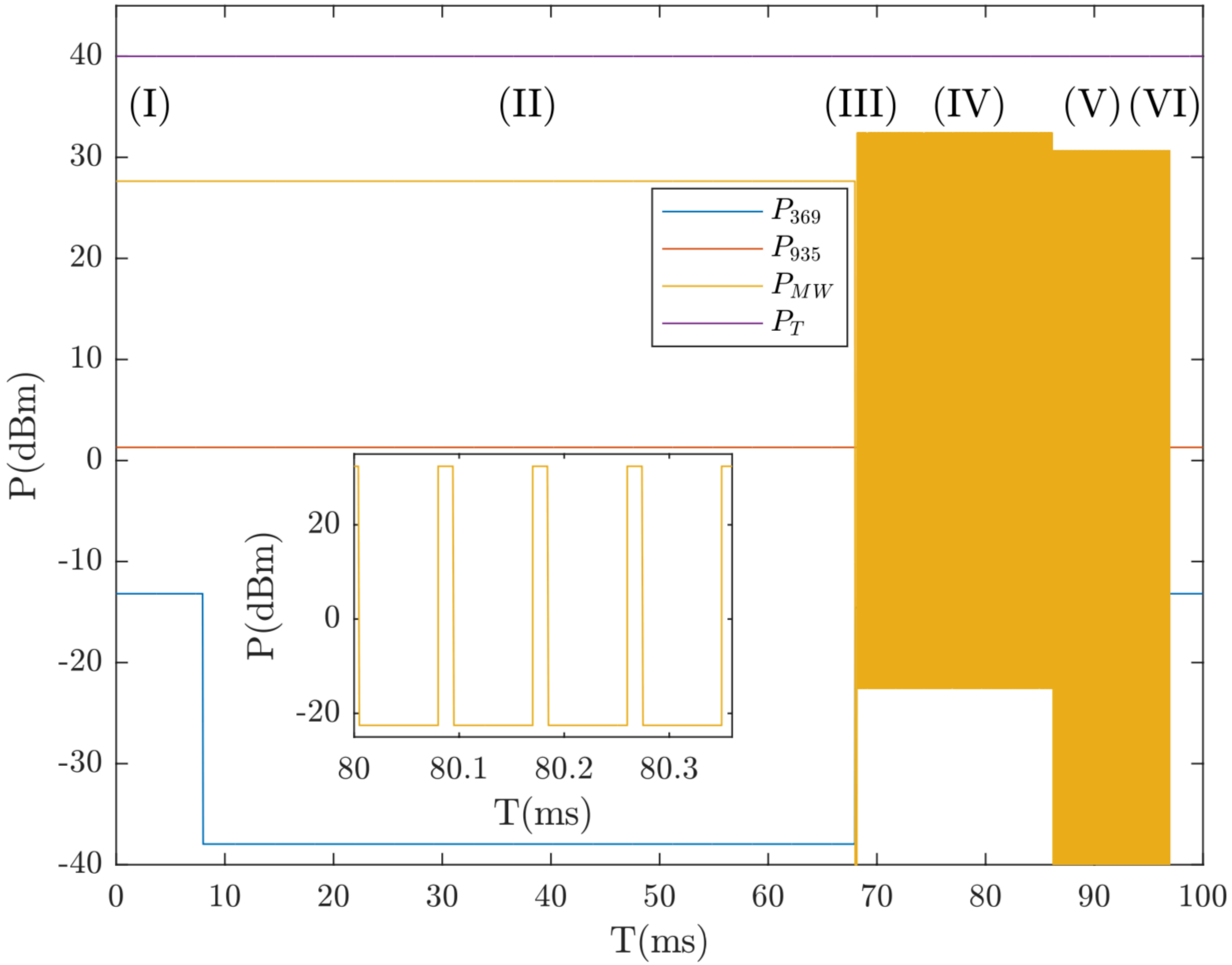
Simulating Logic



Half-Adder classical fidelity: 60.6%
Equivalent two-qubit fidelity: 95.5%

Simulating Logic

Operation	Laser 369 nm	Laser 935 nm	Microwave: Pulse	Microwave: Dyn. decoupling # π-pulses	Microwave: Dyn. decoupling Cost	Total
I. Doppler c.	380 nJ	11 µJ	4.6 mJ	—	—	4.6 mJ
II. Sideband c.	10 nJ	81 µJ	35 mJ	—	—	35 mJ
III. State prep.	7 nJ	0.3 µJ	—	—	—	0.3 µJ
NOT	—	—	8.8 µJ	—	—	8.8 µJ
IV. Toffoli	—	—	9.2 nJ	3×200	1.1 mJ	1.1 mJ
V. CNOT	—	—	8.8 µJ	2×120	0.44 mJ	0.44 mJ
Half-Adder	—	—	8.8 µJ	840	1.5 mJ	1.5 mJ
VI. Readout	140 nJ	4 µJ	—	—	—	4.2 µJ



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Limitations of Quantum Control

Limitations of Quantum Control

PHYSICAL REVIEW RESEARCH **6**, 023296 (2024)

Competition of decoherence and quantum speed limits for quantum-gate fidelity in the Jaynes-Cummings model

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Quantum Speed Limits

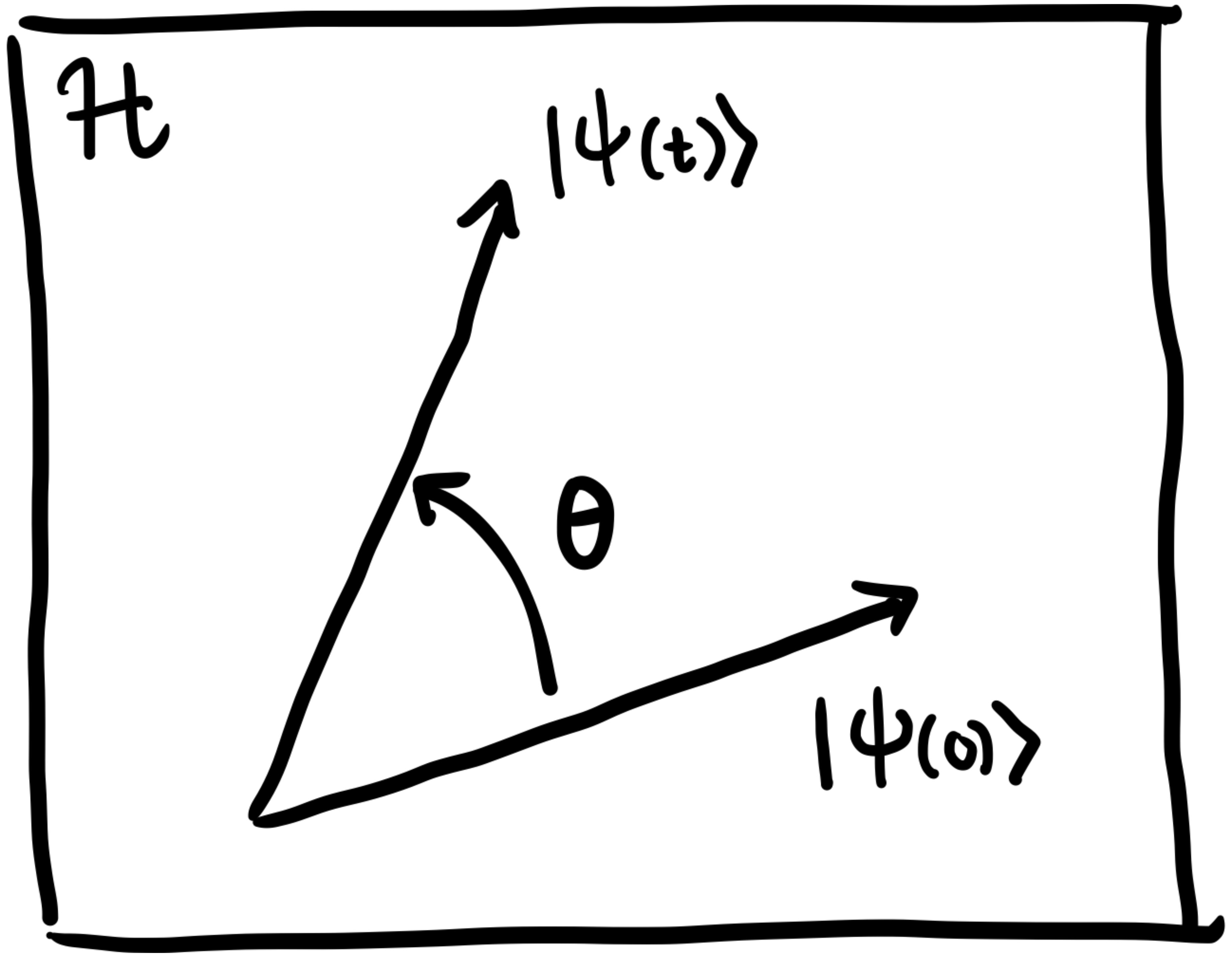
- Mandelstam and Tamm:

$$\Delta H \, \Delta t \geq \hbar\theta$$

- Margolus and Levitin:

$$\langle H - E_0 \rangle \, \Delta t \geq \hbar\theta$$

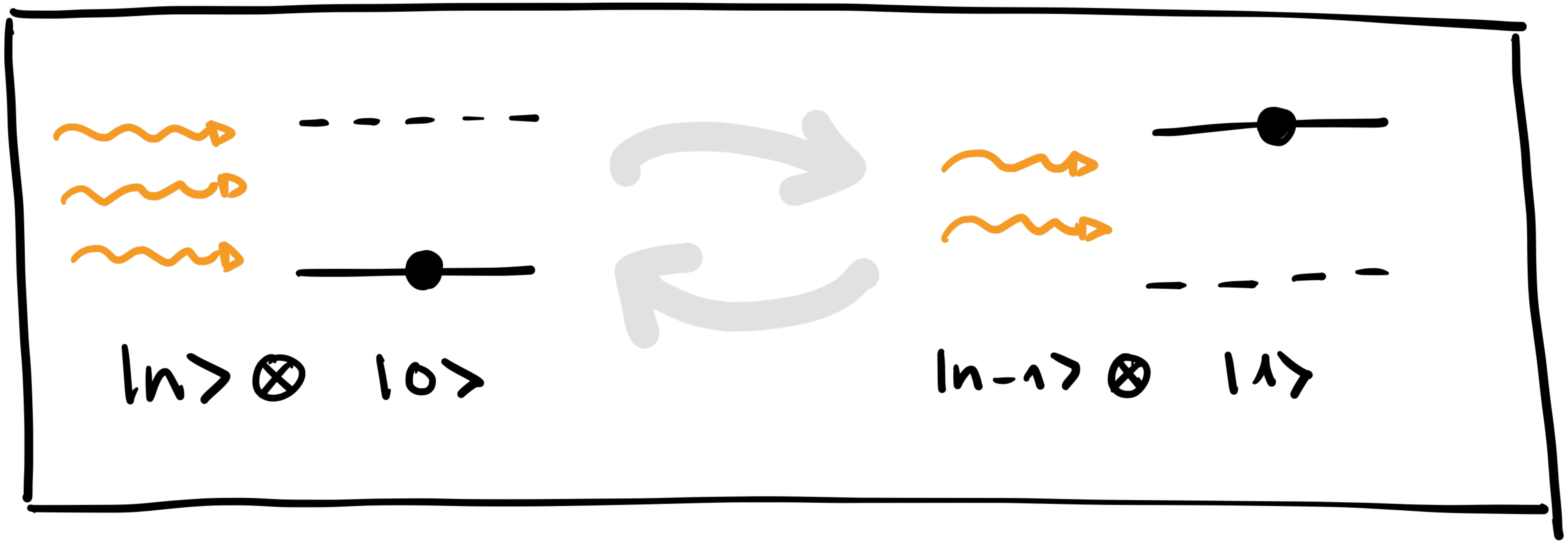
Fast operations need more energy



Limitations of Quantum Control

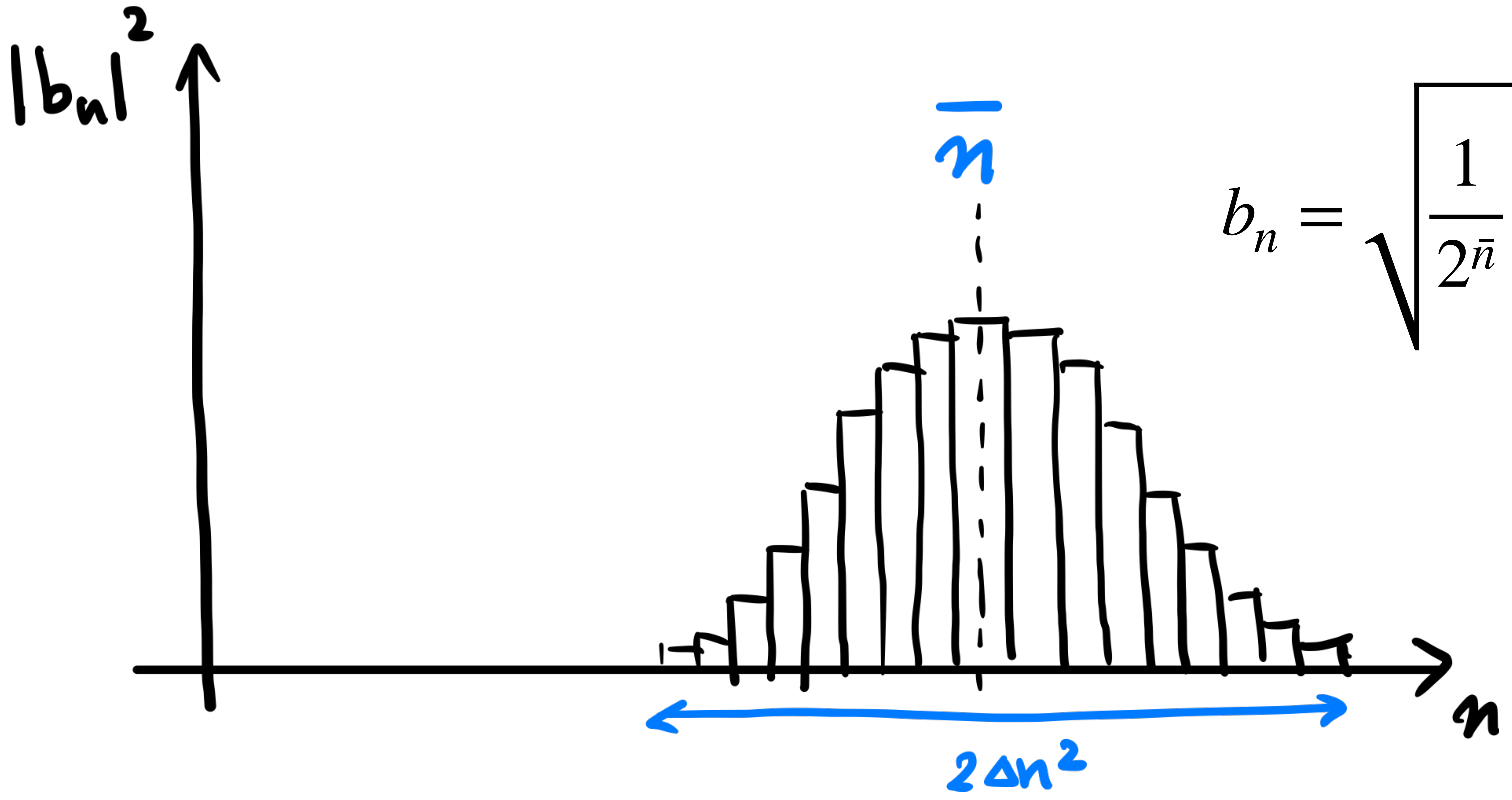
- Cavity system: a QHO drive and logical qubit
- Interacting via the Jaynes-Cummings (JC) Hamiltonian

$$H = i\hbar g \left(b\ell^\dagger - b^\dagger \ell \right)$$



Energy and Fidelity in Driven QS

- As in Quantum Speed Limits, we also want to control $\langle H \rangle$ and ΔH, so we pick a binomial distribution state for the drive



$$b_n = \sqrt{\frac{1}{2^{\bar{n}}} \binom{N_{\Delta n}}{k_{\bar{n},\Delta n}}}$$

Limitations of Quantum Control

Proposition 1

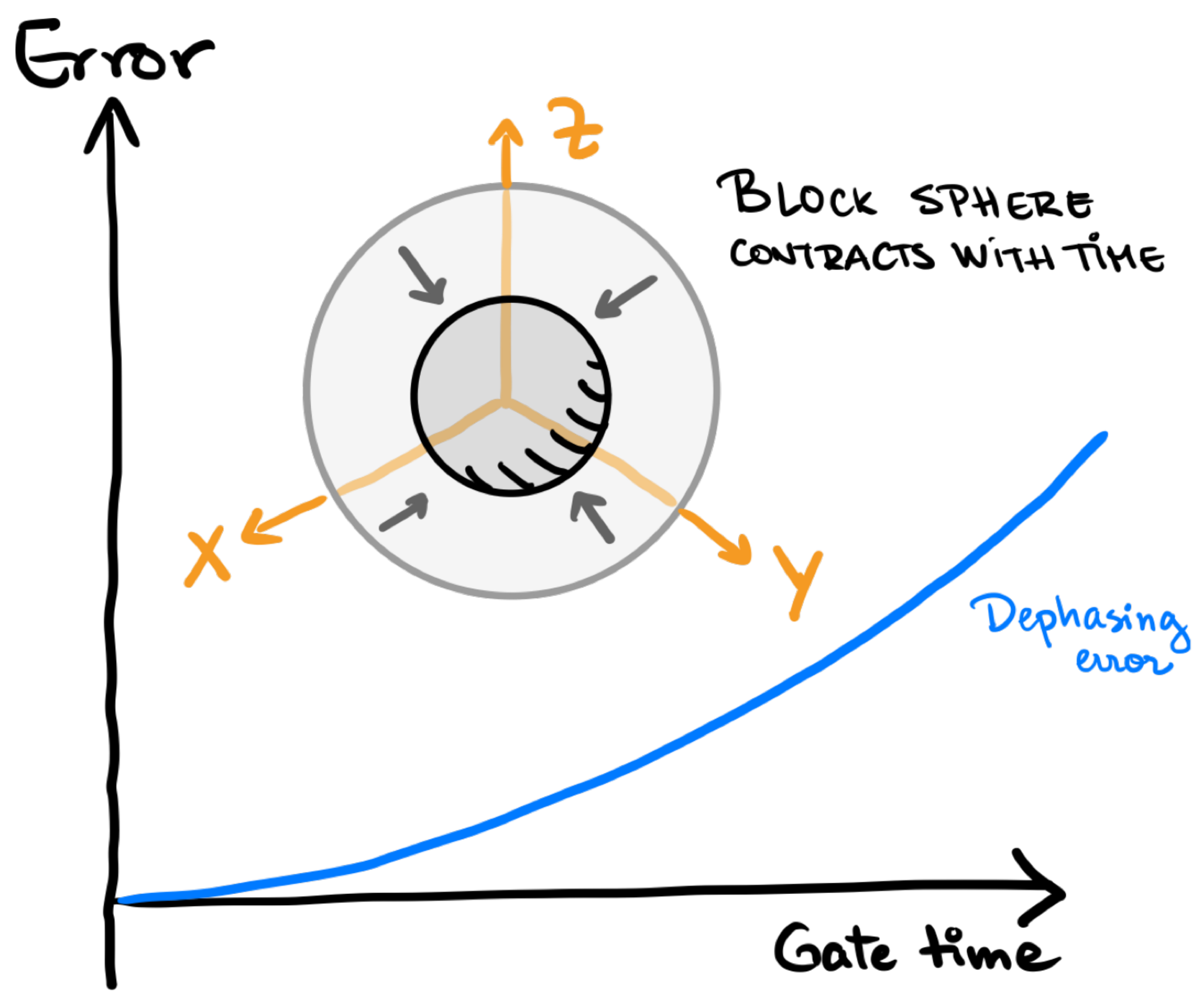
- $\rho_t = \mathrm{tr}_{\mathrm{drive}}|\Psi(t)\rangle\langle\Psi(t)|$
- We can quantify the error by the purity

$$\epsilon_t = 1 - \mathrm{tr}(\rho_t^2)$$

- The average error grows as

$$\bar{\epsilon}_t \sim \frac{g^2 s}{6} t^2 + \frac{\sin\left(g\sqrt{\bar{n}}t\right)^2}{6s\bar{n}}$$

and $s = \Delta n^2/\bar{n}$



Energy and Fidelity in Driven QS

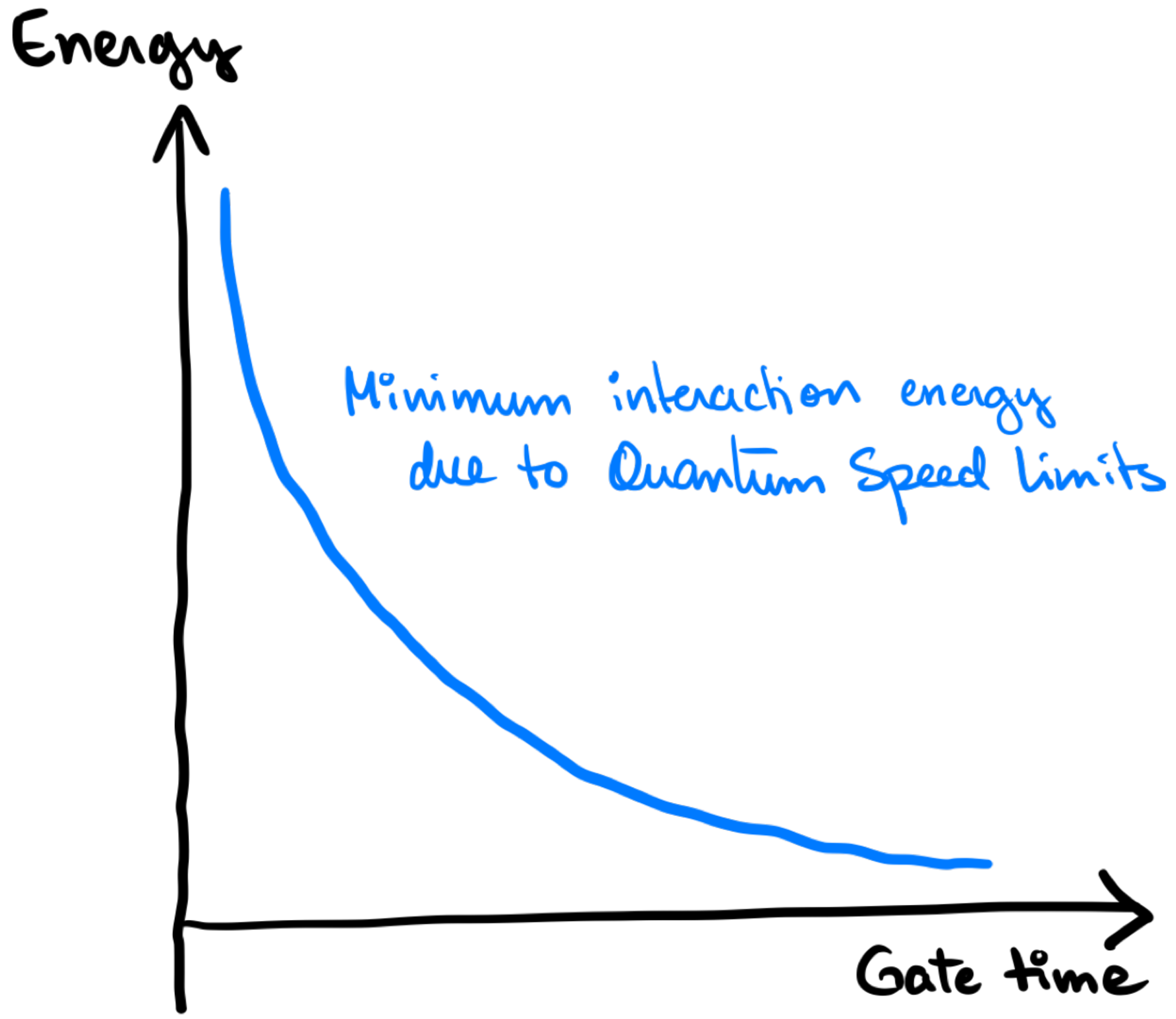
Proposition 2

- Now, suppose we want to rotate the qubit by θ in its Hilbert space
- In the JC case, have

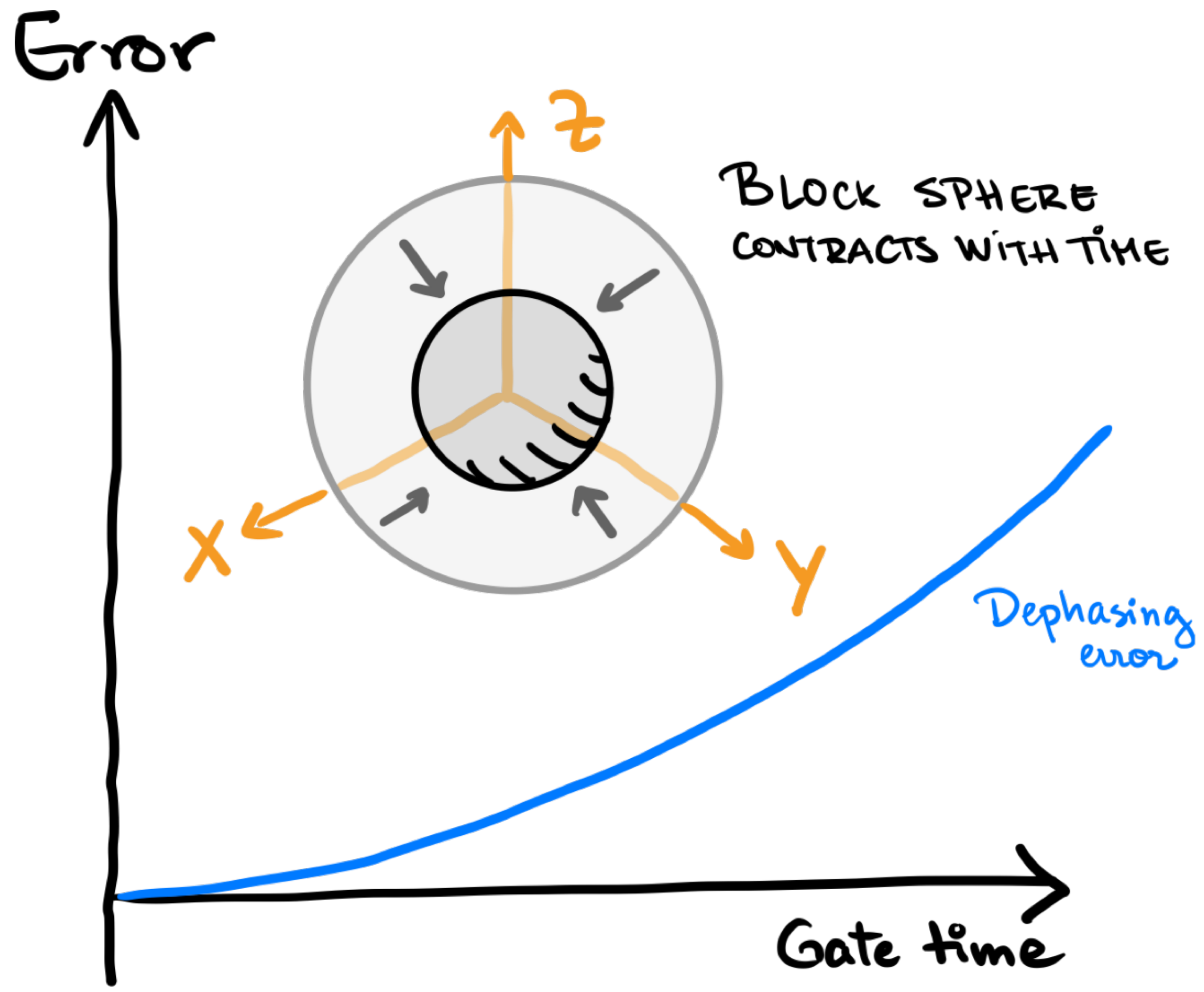
$$\Delta H \approx \langle H \rangle \sim \hbar g \sqrt{\bar{n}},$$

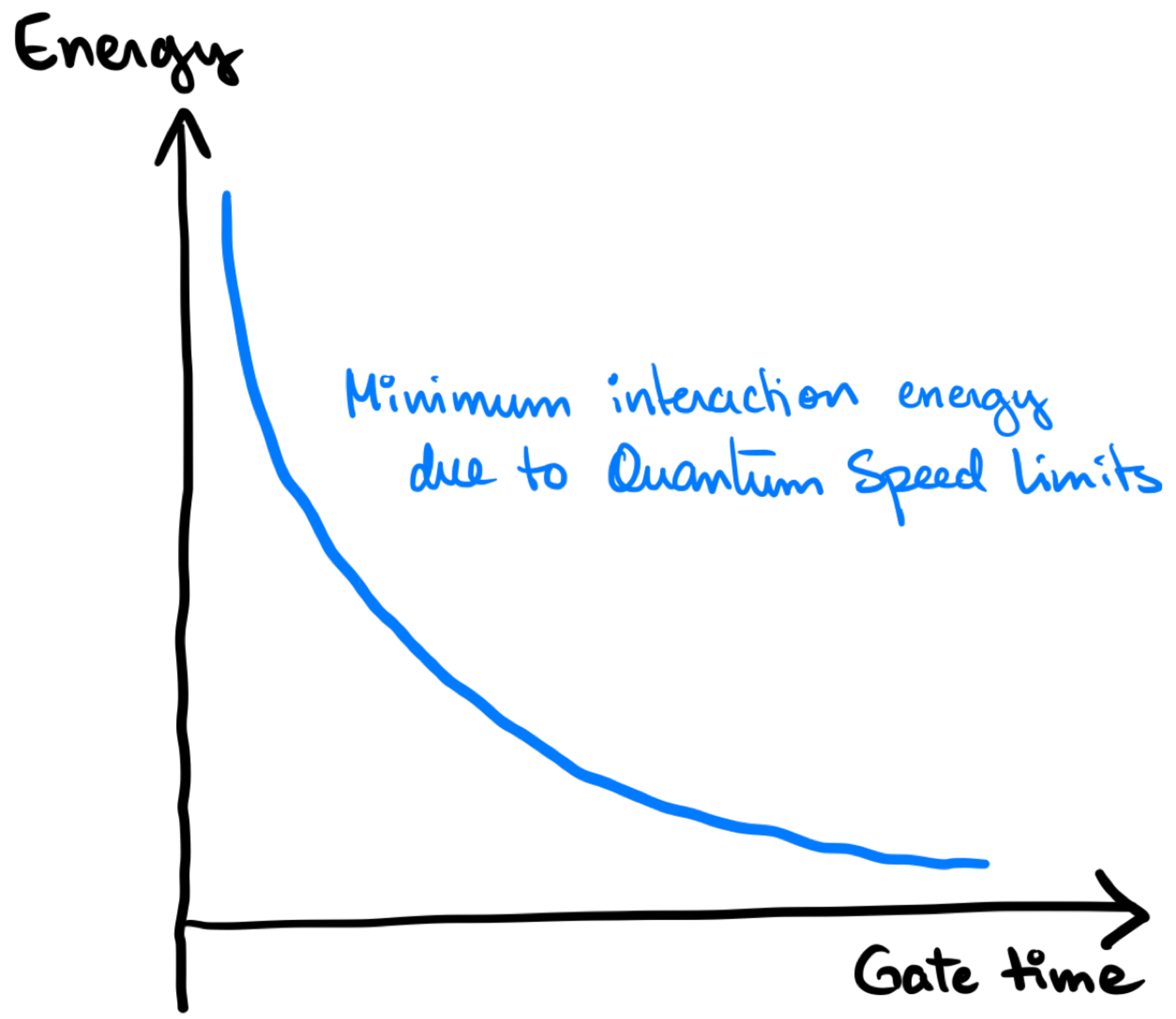
- The QSLs imply that

$$\Delta t_{\mathrm{gate}} \gtrsim \frac{\theta}{g\sqrt{\bar{n}}}.$$



Energy and Fidelity in Driven QS

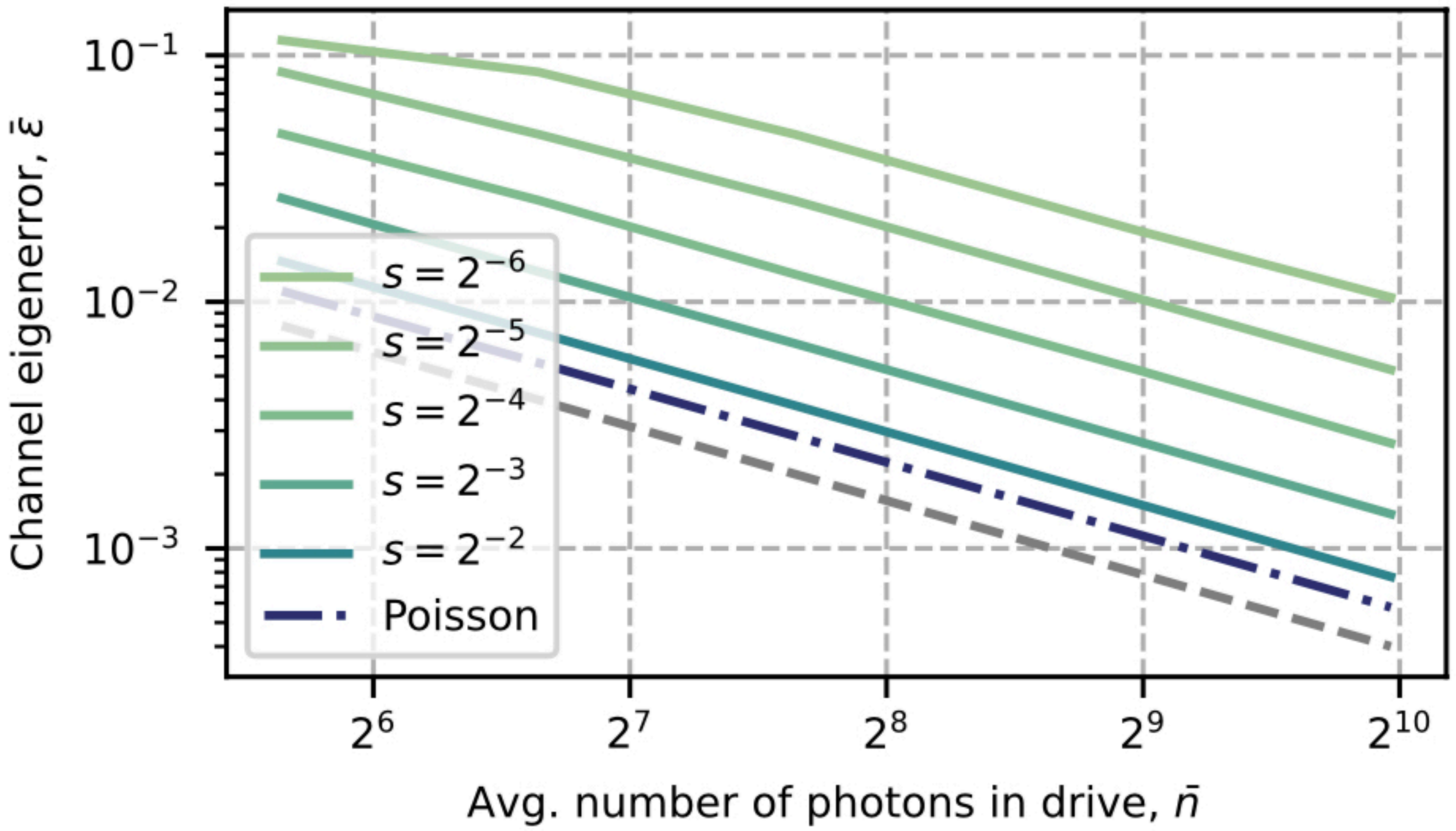




$$\implies \varepsilon \sim \frac{1}{\langle H_{\text{drive}} \rangle}$$

- Therefore, the energy-fidelity trade-off is seen as a balance between
  1. Depolarising-like noise that grows with time; and
  2. Minimum gate time due to a Quantum Speed Limit

Limitations of Quantum Control



Energy and Fidelity in Driven QS

Driven quantum systems may suffer from a minimal energy requirement

$$\varepsilon \gtrsim 1/E$$

We connected this bound to Quantum Speed Limits (QSLs).

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Minimum Energy Requirements for Quantum Computation

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Conservative Quantum Computing

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On the classical character of control fields in quantum information processing

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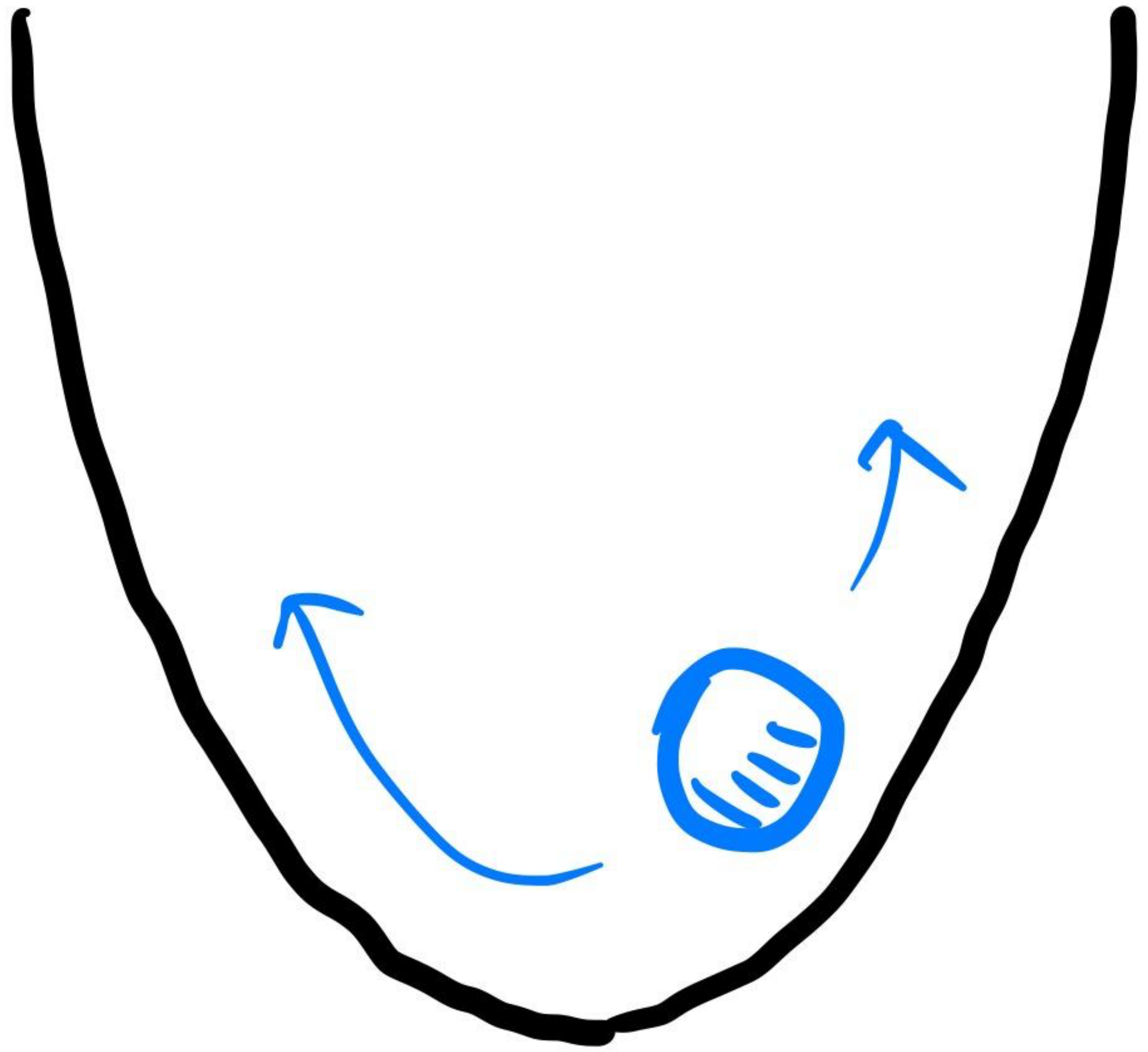
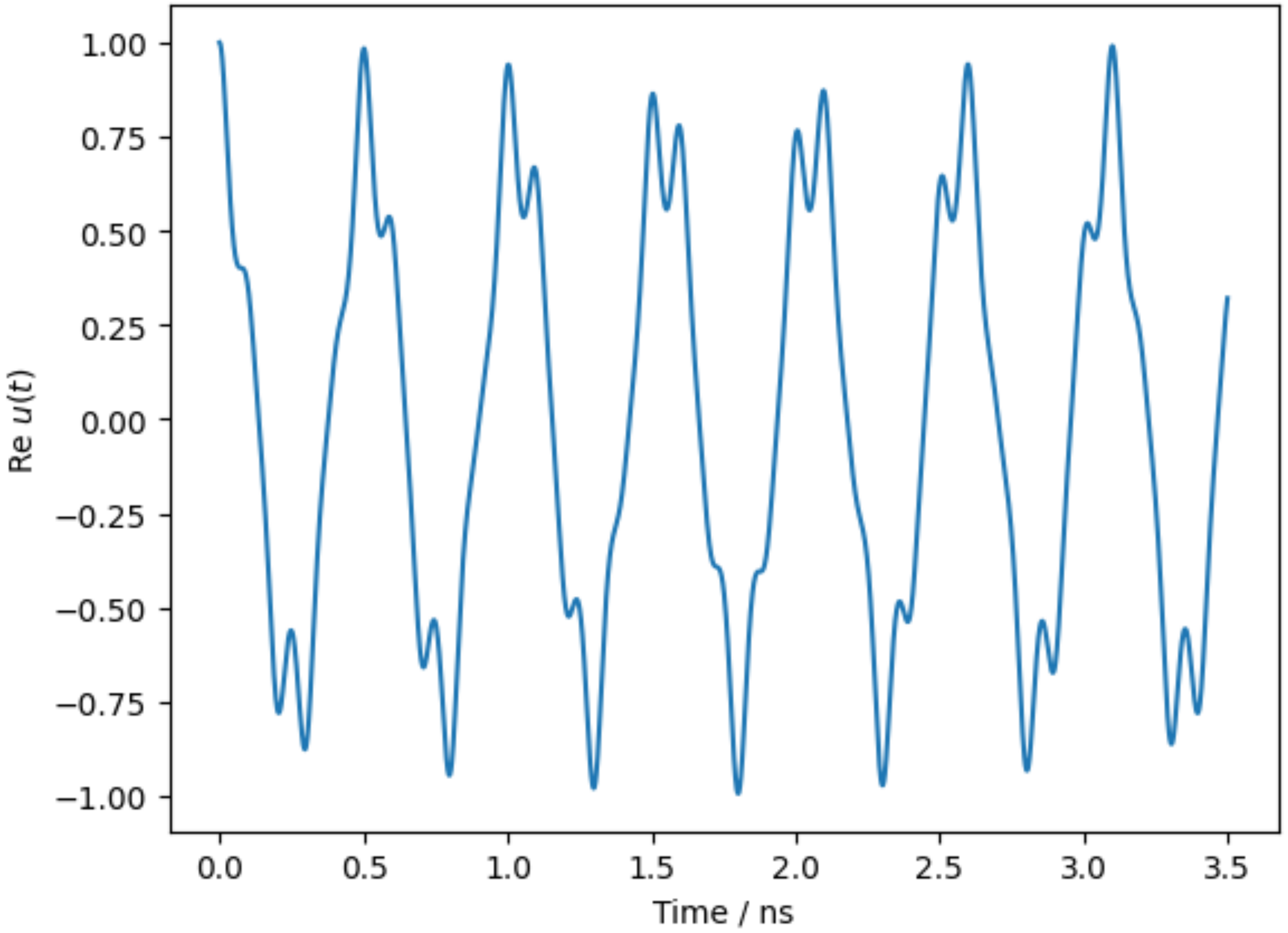
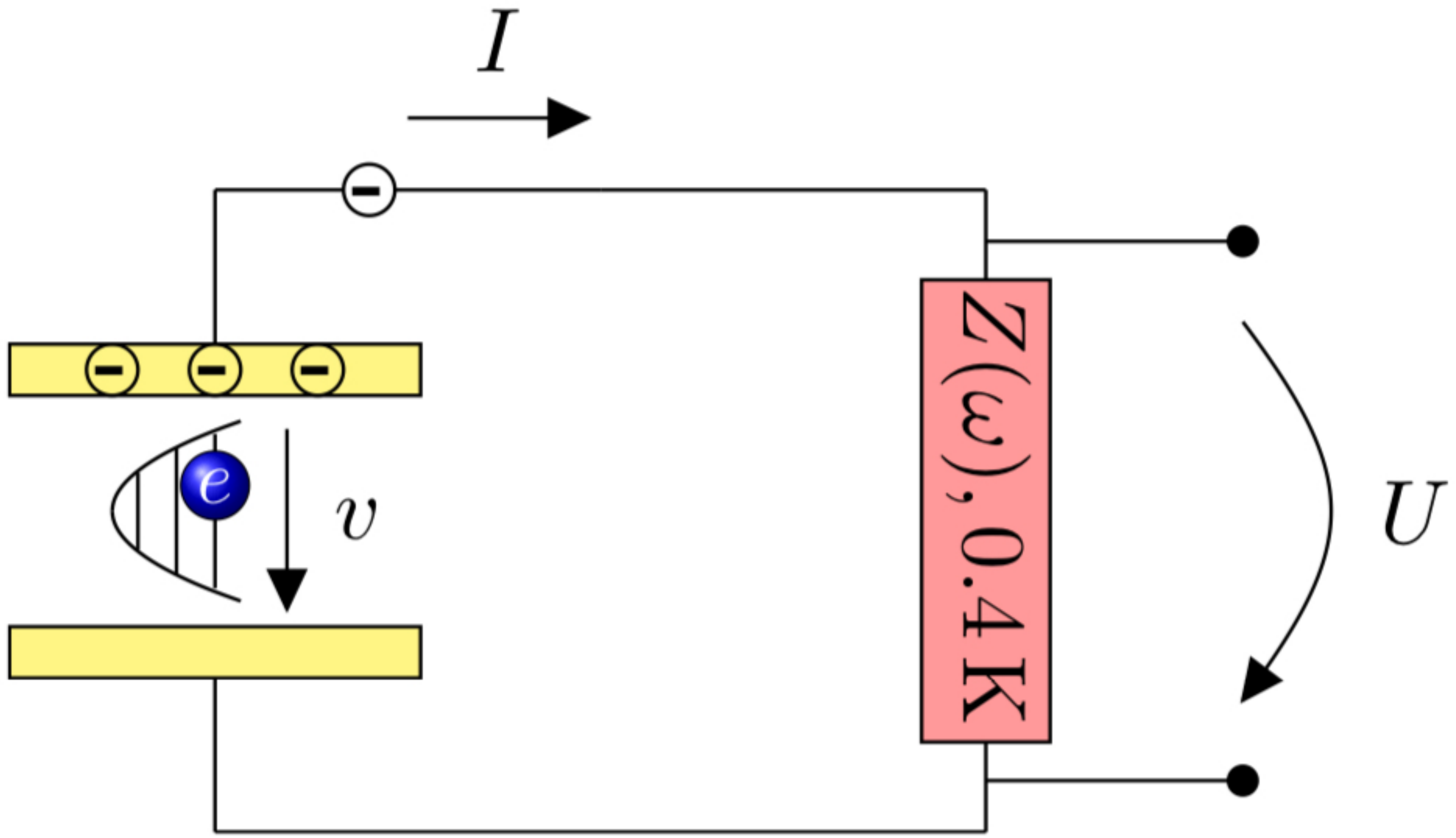
Efficient cooling of Trapped Electrons

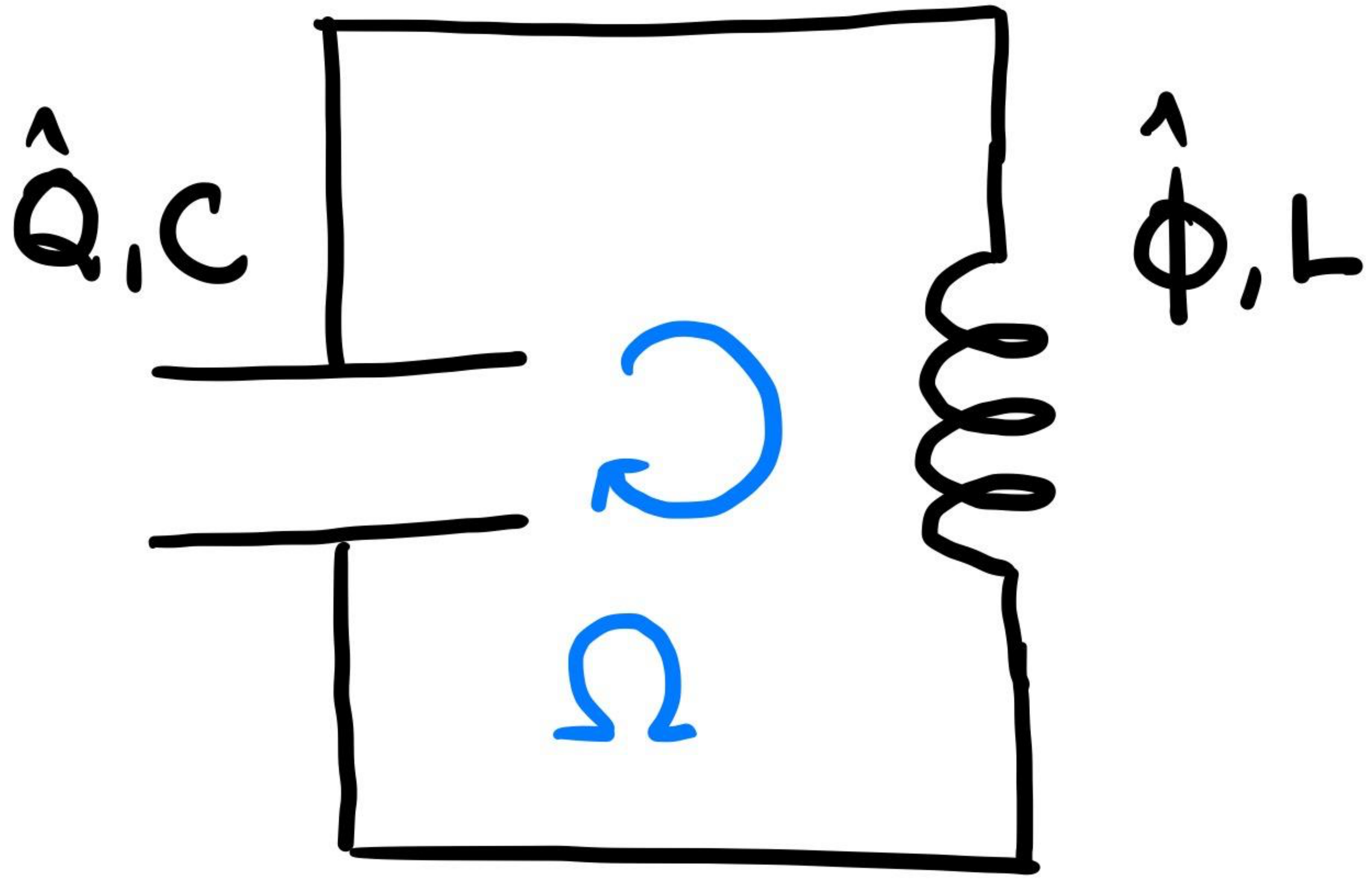
Efficient cooling of trapped electrons





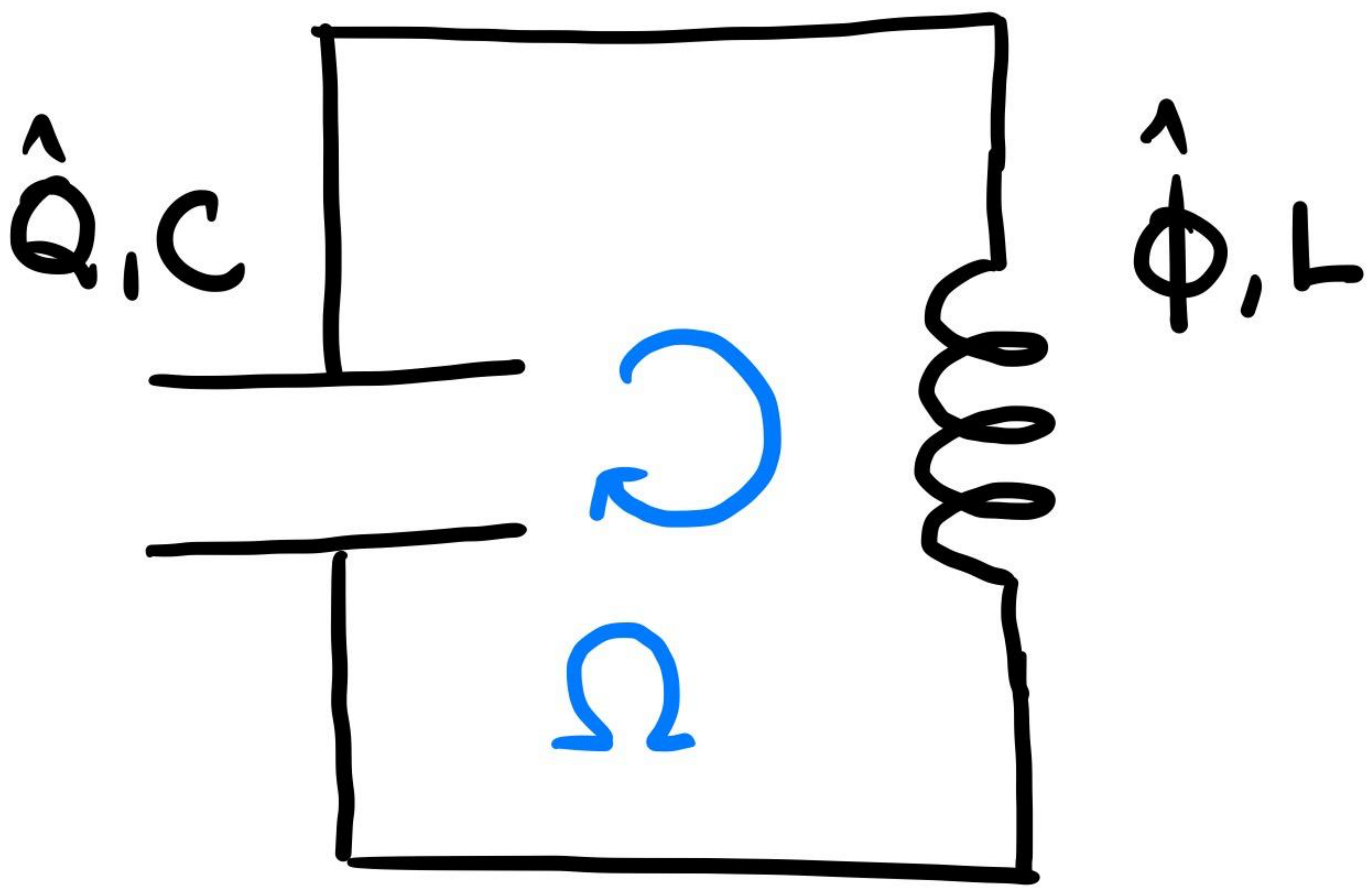
Efficient cooling of trapped electrons

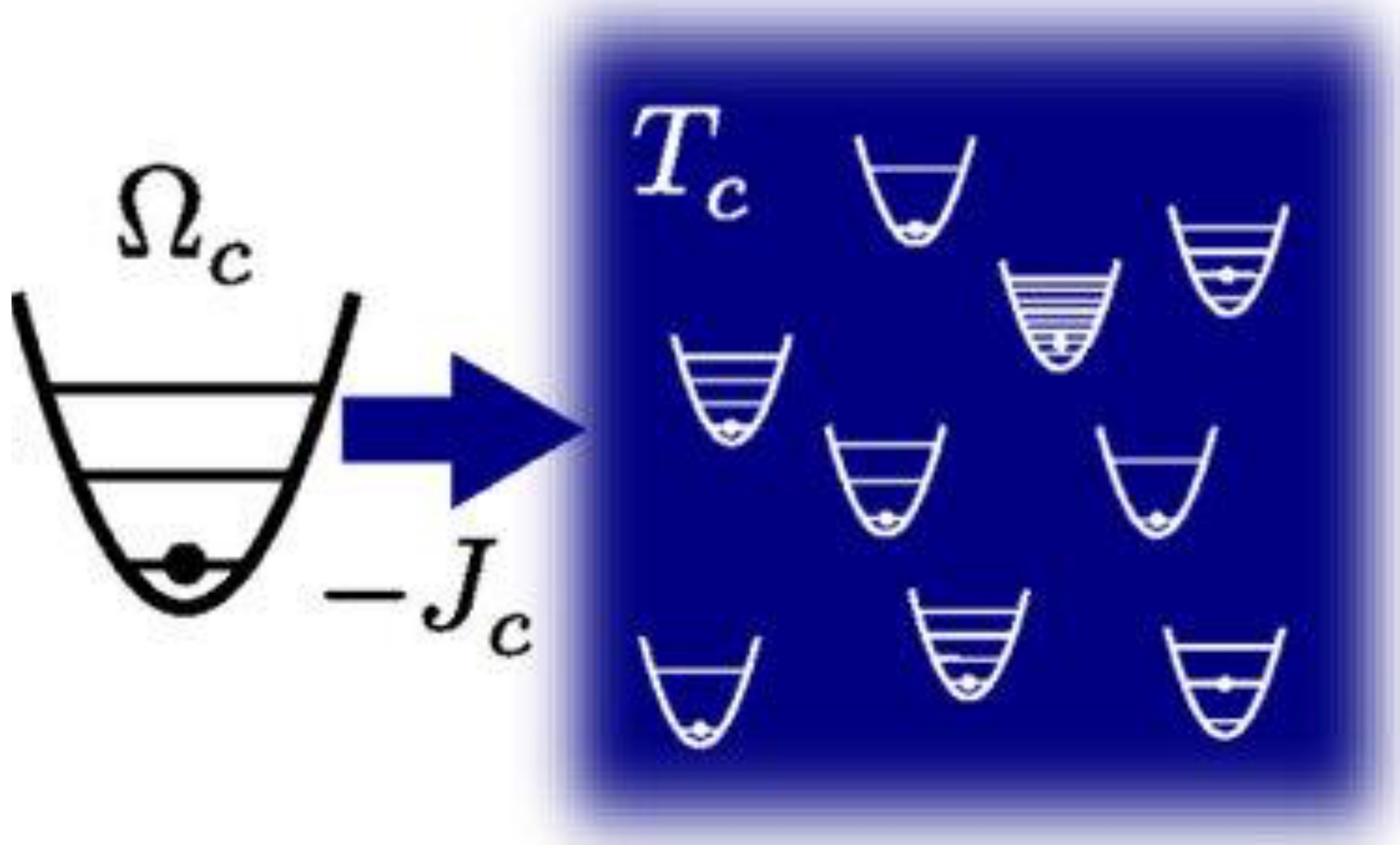




$$H' \approx ig(\Omega)\left(\hat{a}\hat{a}_\phi^\dagger - \hat{a}^\dagger\hat{a}_\phi\right)$$

Efficient cooling of trapped electrons





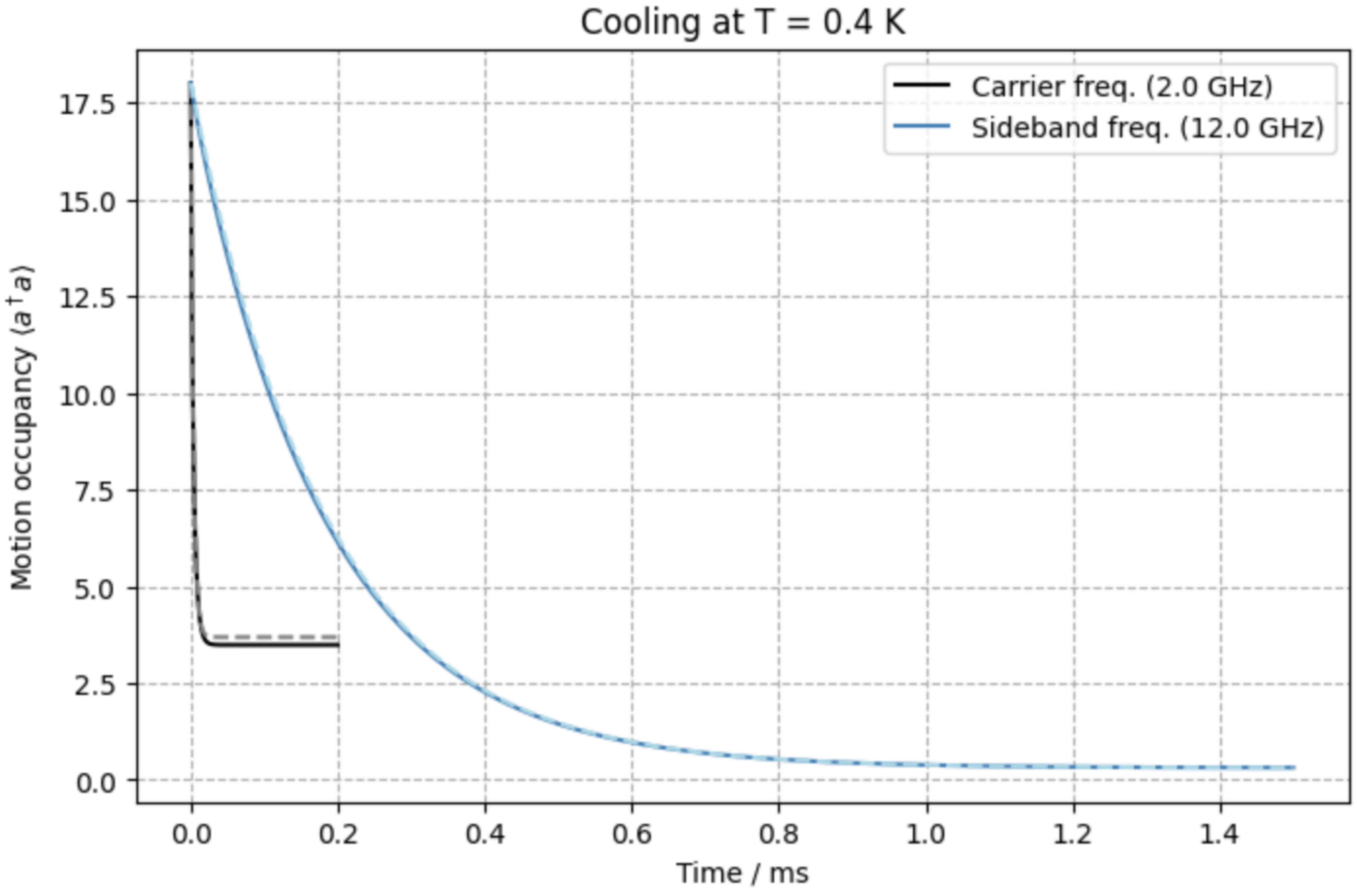
$$\langle a_\varphi^\dagger a_\varphi \rangle = \bar{n}(\Omega) \equiv \frac{1}{e^{\hbar\Omega/k_B T} - 1} \approx \frac{k_B T}{\hbar \Omega}$$

Efficient cooling of trapped electrons

Interaction picture

$$\frac{d}{dt}\begin{bmatrix}\hat{a}\\ \hat{a}_\phi\end{bmatrix} = \begin{bmatrix}0 & -g\\ g & \underbrace{-\kappa/2}_{\text{Dissipation}}\end{bmatrix}\begin{bmatrix}\hat{a}\\ \hat{a}_\phi\end{bmatrix} + \begin{bmatrix}0\\ \underbrace{\sqrt{\kappa}\,\hat{b}}_{\text{Fluctuation/ thermal noise}}\end{bmatrix}$$

Efficient cooling of trapped electrons



Other things

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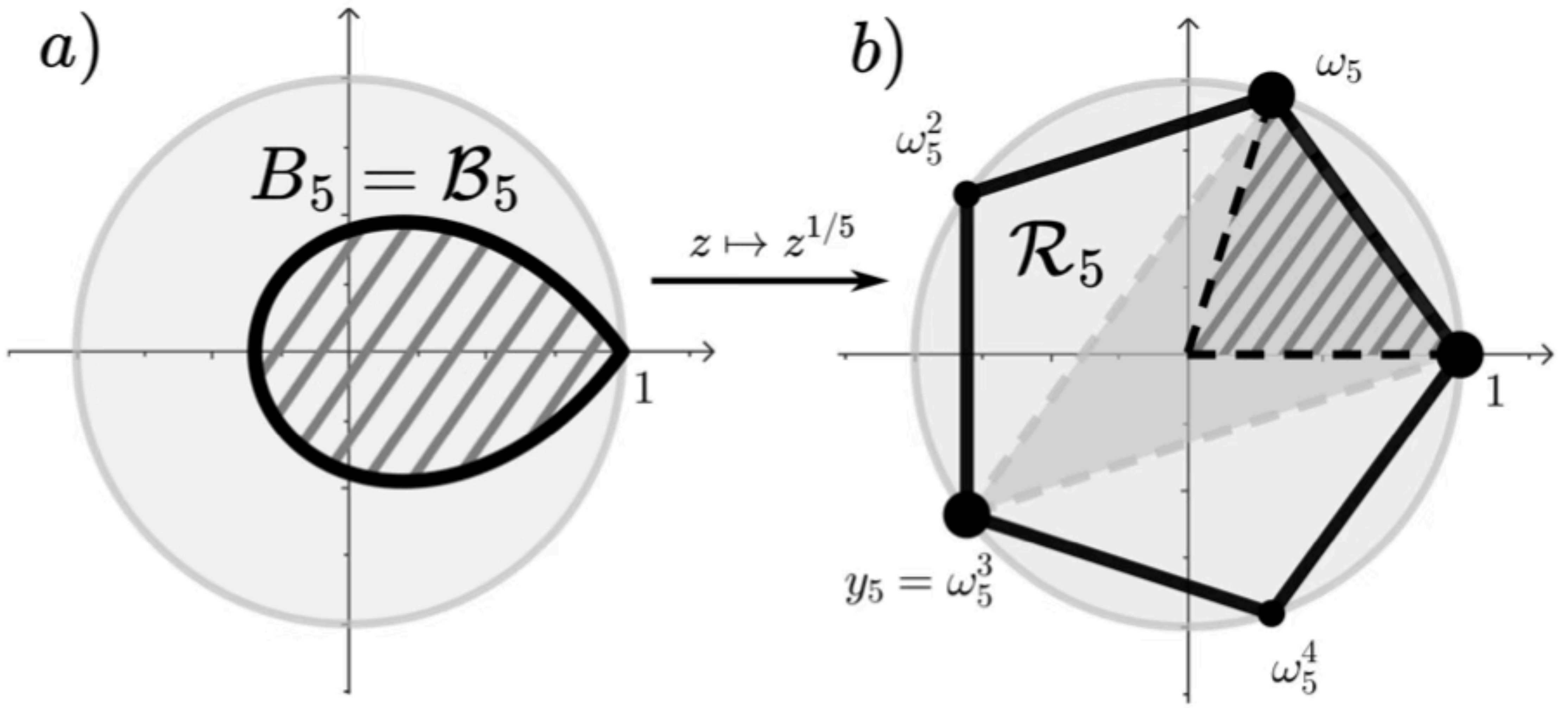
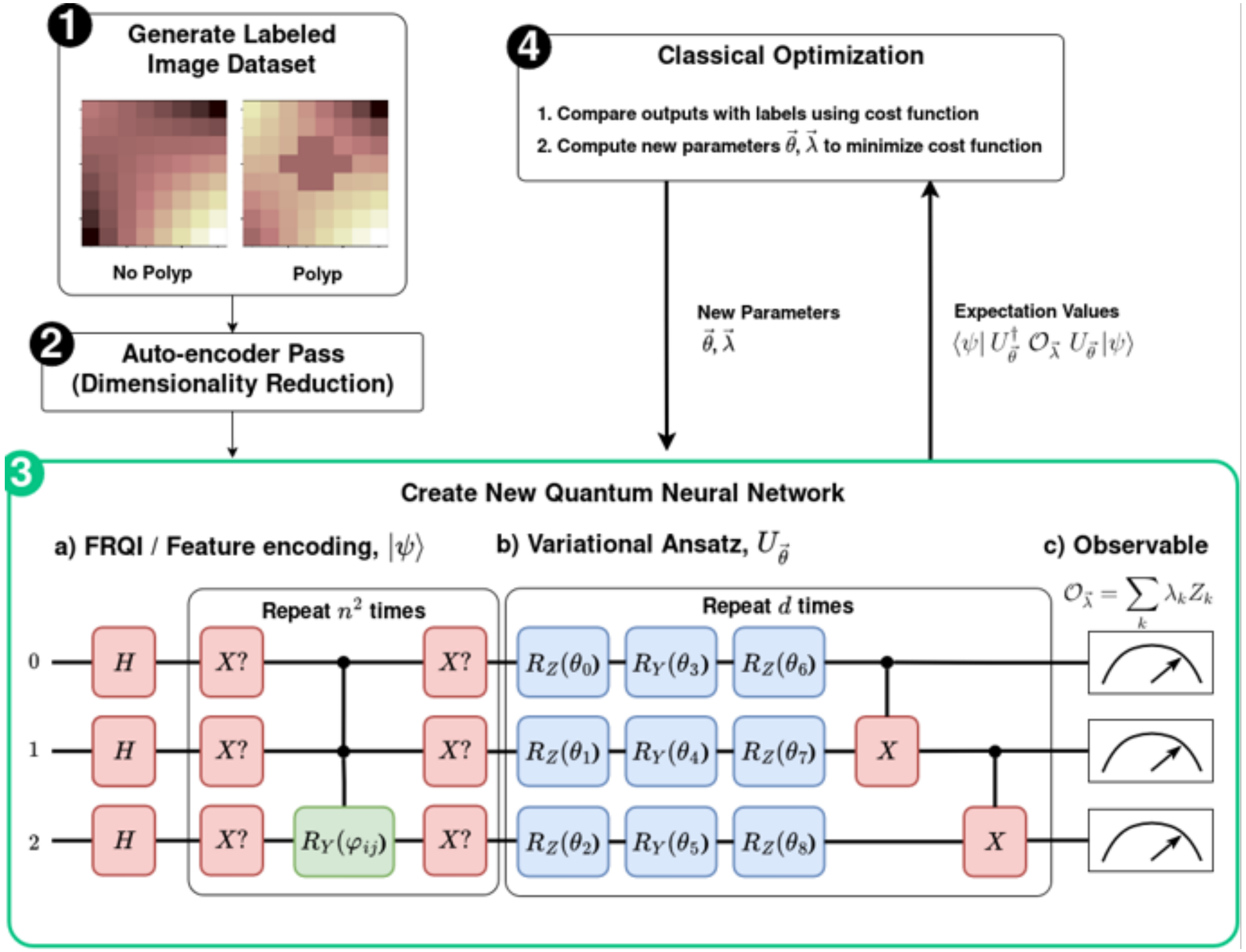


FIG. 1. Characterization of n-th order Bargmann invariants, illustrated here for $n = 5$, with the complex unit circle as reference. **a)** The range of values that pure Bargmann invariants

$$\mathrm{Tr}(\rho_1 \cdots \rho_n)$$

Other things



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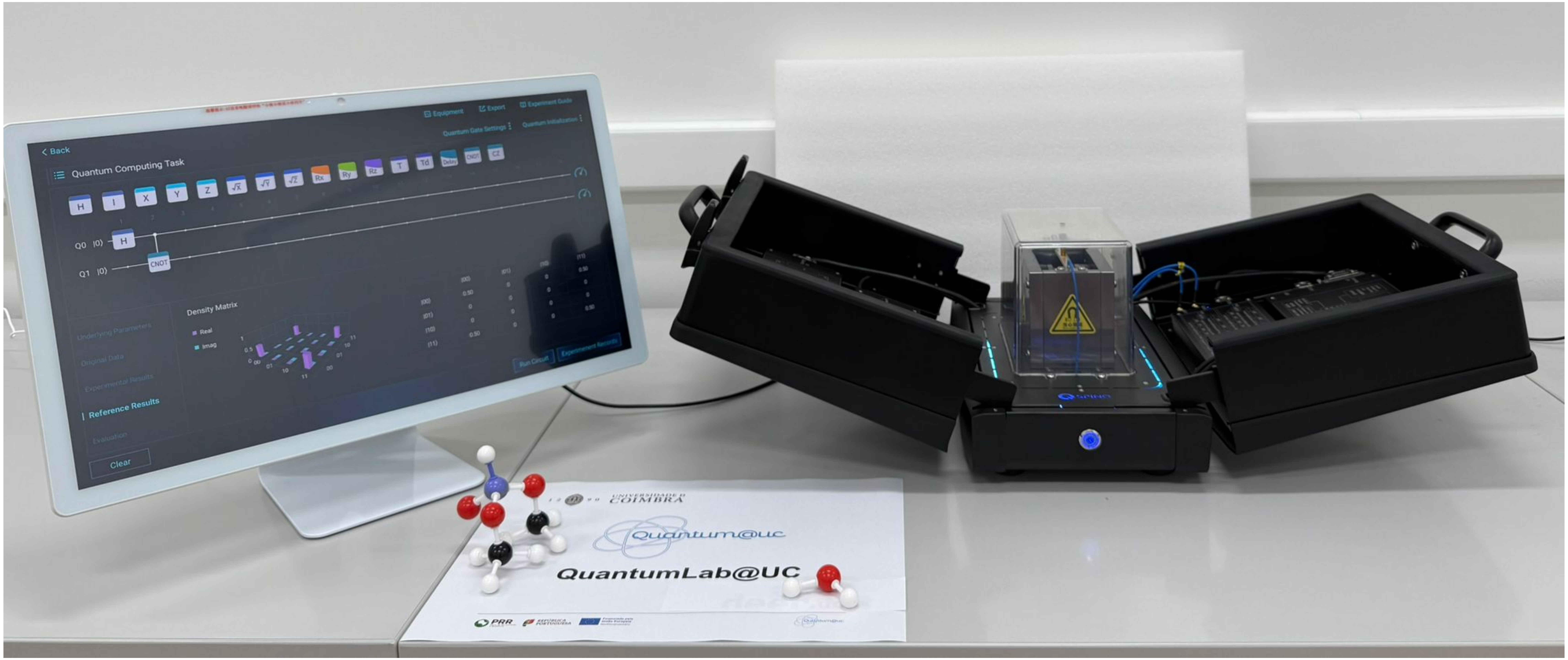

Bottom line