High Energy Physics

Static Quark Model

Hugo Fernandes

Some Particle Physics History

In Physics, and especially in Particle Physics, it is important to look up to historical review of concepts, discoveries and achievements from early theories.

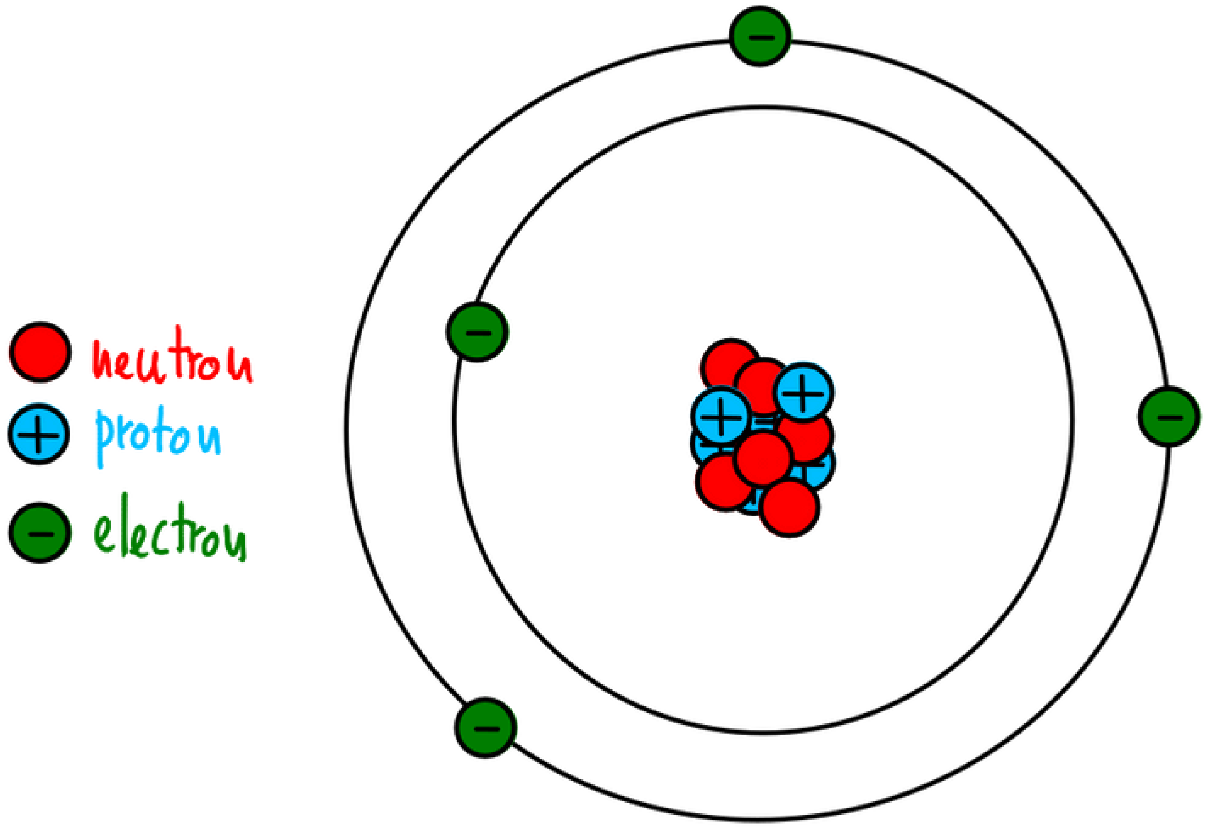
Examining the stages of their approach including the confusions and proposals along the way offers valuable insight into our ongoing quest for even more refined theories, like the Quark Model.

Some Particle Physics History

The discovery of the neutron (by J.Chadwick) indicated that the nuclei was composed of neutrons and protons and the existence of a force that counteract the electromagnetic repulsion holding nucleons together- the strong force .

That force has an range similar to size of the nucleus.

James Chadwick (1891- 1974)

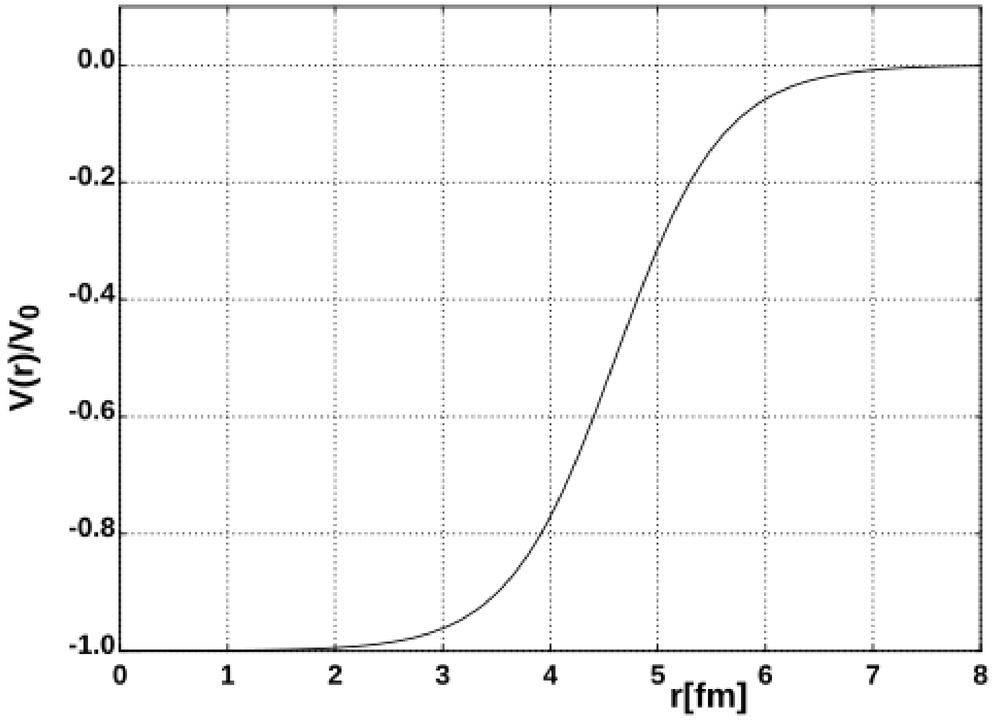


Some Particle Physics History

In 1935, Yukawa proposed that the nuclear force could be described by the exchange of a spinless particle between nucleons: a meson.

Following Yukawa, different non-relativistic potential models were developed, like the Woods Saxon-potential, which is a mean field potential for the nucleons inside the atom.

Hideki Yukawa (1907-1981)



Woods–Saxon potential for A = 50, relative to V_0 with a = 0.5 fm and R=4.6fm

$$\Rightarrow V(r) = -\frac{V_0}{1+\exp(\frac{r-R}{a})}$$

Some Particle Physics History

Experimentally, it had become possible for collisions at relativistic regimes, but the use of non-relativistic potentials was inappropriate to cover these.

Some Particle Physics History

Experimentally, it had become possible for collisions at relativistic regimes, but the use of non-relativistic potentials was inappropriate to cover these.

The search for a classification scheme for the increasing number of stable hadrons begin in the 1950’s . Experimental results showed that many hadrons shared similar masses, charges, and other quantum numbers, suggesting an underlying structure.

Name	Model	Isotopic Spin	Strangeness	Ordinary Spin
$\mathfrak{N}$		1/2	0	1/2
$\bar{\mathfrak{N}}$		1/2	0	1/2
Λ		0	−1	1/2?
$\bar{\Lambda}$		0	1	1/2?
π	$\mathfrak{N}+\bar{\mathfrak{N}}$	1	0	0
$\theta(\tau)$	$\mathfrak{N}+\bar{\Lambda}$	1/2	1	0?
$\bar{\theta}(\bar{\tau})$	$\bar{\mathfrak{N}}+\Lambda$	1/2	−1	0?
Σ	$\mathfrak{N}+\bar{\mathfrak{N}}+\Lambda$	1	−1	1/2?
Ξ	$\bar{\mathfrak{N}}+\Lambda+\Lambda$	1/2	−2	1/2?

S. Sakata, “On a composite model for the new particles*,”

A Possible Symmetry in Sakata’s Model for Bosons-Baryons System

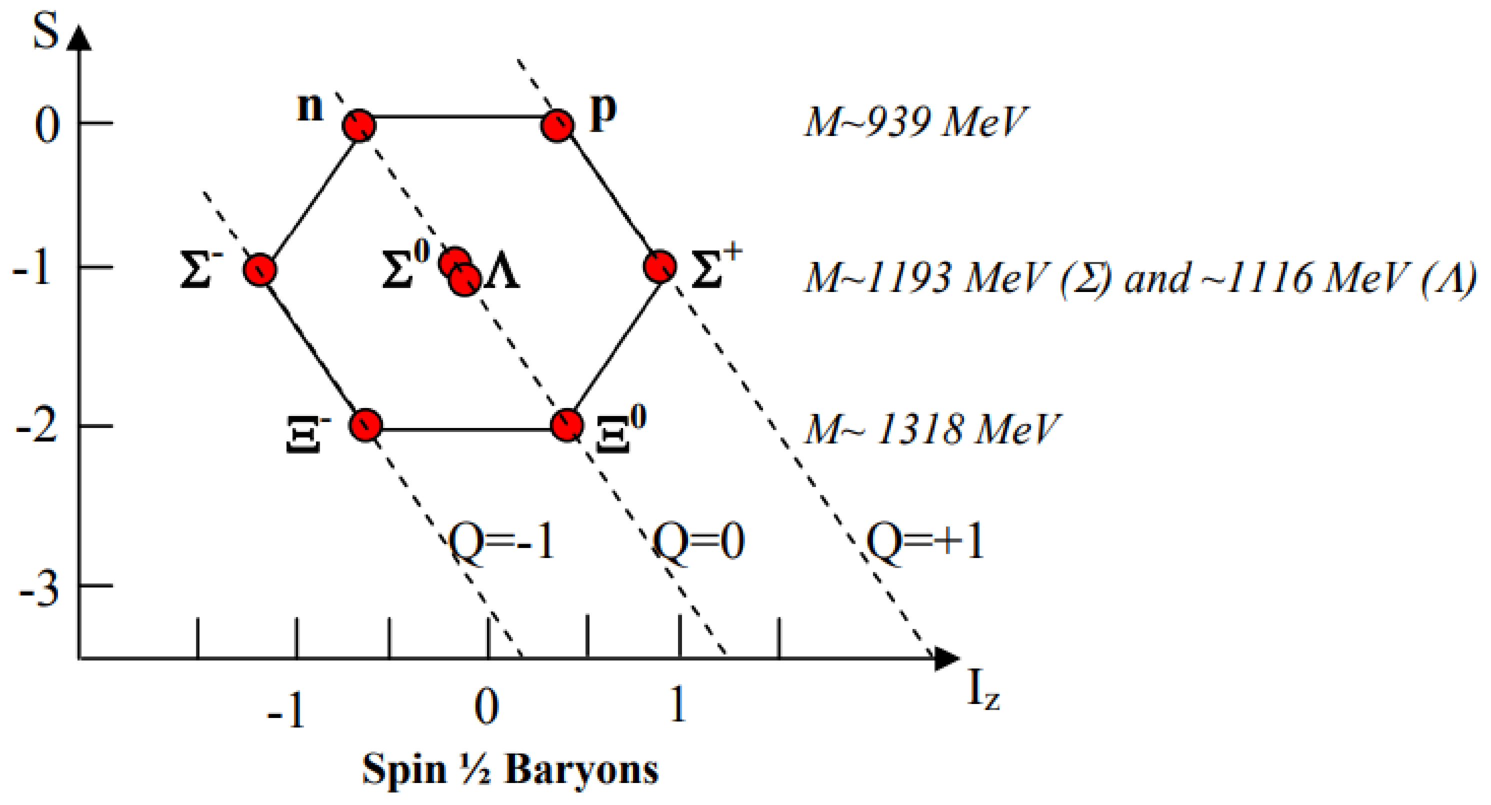
Mineo IKEDA* and Shuzo OGAWA**

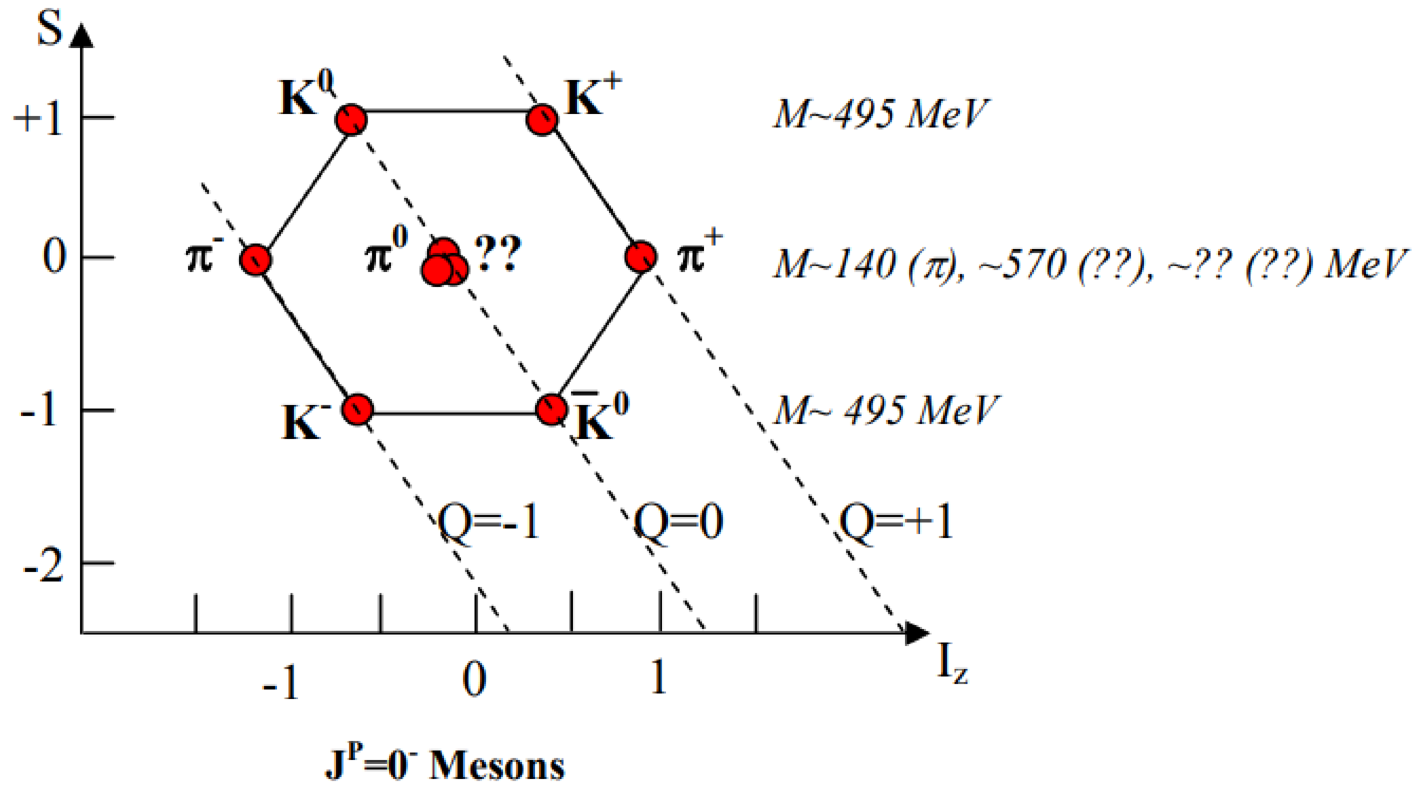
Research Institute for Theoretical Physics and Department of Physics***
Hiroshima University, Hiroshima

In this paper we study a possible symmetry in Sakata's model for the strongly interacting particles. In the limiting case in which the basic particles, proton, p, neutron, n and Λ-particle, Λ, have an equal mass, our theory holds the invariance under the exchange of p and Λ or n and Λ in addition to the usual charge independence and the conservation of electrical and hyperonic charge.

Eightfold Way

In 1961 Gell-Mann proposed a scheme that would put the jungle of particles in some sort of order, a la Mendeleyev 's Periodic Table.





Before we move on to the Quark Model, we must review the machinery of symmetries and underlying structures the group theory

Symmetries and Group Theory

Symmetries play a central role in particle physics and one aim of particle physics is to discover the fundamental symmetries of our universe.

But first of all, what is a group? Since most of us had group theory, I will rush the introduction.

Symmetries and Group Theory

Symmetries play a central role in particle physics and one aim of particle physics is to discover the fundamental symmetries of our universe.

But first of all, what is a group? Since most of us had group theory, I will rush the introduction.

A group G is a set with a map $G \times G \to G$ known as group multiplication satisfying the following properties:

- Associativity $(g \cdot h) \cdot l = g \cdot (h \cdot l), \quad \forall g, h, l \in G$
- Inverse $\forall g \in G \, \exists g' \in G \quad g' \cdot g = g \cdot g' = e$
- Identity $\forall g \in G \, , \exists e \in G, \quad g \cdot e = g$

Symmetries and Group Theory

We will work with continous groups, this is

$$g = g(\alpha), \quad \alpha = \{\alpha_a\}, \quad a = 1, \ldots, N.$$

Now, suppose that out system is invariant under

$$\psi \to \psi' = U\psi \tag{1}$$

We will work with continous groups, this is

$$g = g(\alpha), \quad \alpha = \{\alpha_a\}, \quad a = 1, \ldots, N.$$

Now, suppose that out system is invariant under

$$\psi \to \psi' = U\psi \qquad (1)$$

To conserve the norm under this transformation, it requires that is unitary, this is

$$U^\dagger U = 1 \qquad (2)$$

If we Taylor expand $U(\alpha)$ in Taylor series close enough of the identity element

$$U(d\alpha) = 1 + i(G_a)d\alpha_a \qquad (3)$$

$$\Downarrow$$

$$G_a \equiv -i \frac{\partial}{\partial \alpha_a} U(\alpha)\Big|_{\alpha=0}$$

Symmetries and Group Theory

For U to be unitary

$$UU^\dagger = 1 \Rightarrow \; G^\dagger = G$$ is hermitian and thus, it corresponds to an observable quantity

In particle physics we are mostly interested in representations of a group, this is, a map that associates to each group element a linear transformation acting on a particular vector space, V

$$R: \quad G \to GL(V)$$

Symmetries and Group Theory

For U to be unitary

$$UU^{\dagger} = 1 \Rightarrow \; G^{\dagger} = G$$ is hermitian and thus, it corresponds to an observable quantity

In particle physics we are mostly interested in representations of a group, this is, a map that associates to each group element a linear transformation acting on a particular vector space, V

$$R: \quad G \rightarrow GL(V)$$

To get a better grasp and remember about representations and Lie algebra, lets see an well known example: the angular momentum in quantum mechanics

In tensorial notation, $\vec{L}$ is given by

$$L_i = \sum_{j,k=1}^{3} \epsilon_{ijk} q_j p_k \quad (4)$$

The componentes of $\vec{L}$ have the following commutation relation

$$[L_i, L_j] = i \sum_{k=1}^{3} \epsilon_{ijk} L_k \quad (5)$$

And with a linear combination of L_1 and L_2 we can write the well known ladder operators

$$L_{\pm} = L_1 \pm i L_2 \quad (6)$$

Symmetries and Group Theory

Relating this example to group theory

Hilbert space $\mathcal{H}$	$\Longleftrightarrow$	Vector space V
$L_i = \sum_{j,k=1}^{3} \epsilon_{ijk} q_j p_k$	$\Longleftrightarrow$	Generators
L_3	$\Longleftrightarrow$	Diagonal Generator
$[L_i, L_j] = i\sum_{k=1}^{3} \epsilon_{ijk} L_k$	$\Longleftrightarrow$	$[T_a, T_b] = i f_{abc} T_c$ (Lie Algebra of a group)
$L_\pm = L_1 \pm i L_2$	$\Longleftrightarrow$	Ladder operators of our theory

SU(3)

The special unitary $SU(N)$ groups play a special role in particle physics

$$SU(N) = \{U \in U(N) : \det(U) = 1\} \tag{7}$$

Due to our lack of time, I will focus only on some representations of $SU(3)$

Representations of $SU(3)$ are given in terms of tensor fields that transform under $SU(3)$ in diferent ways.

SU(3)

The special unitary $SU(N)$ groups play a special role in particle physics

$$SU(N) = \{U \in U(N) : \det(U) = 1\} \tag{7}$$

Due to our lack of time, I will focus only on some representations of $SU(3)$

Representations of $SU(3)$ are given in terms of tensor fields that transform under $SU(3)$ in diferent ways.

Fundamental representation $\quad \mathbf{3}: \; \psi_a \to U_a{}^b \psi_b, \quad a, b = 1, 2, 3$

SU(3)

The special unitary $SU(N)$ groups play a special role in particle physics

$$SU(N) = \{U \in U(N) : \det(U) = 1\} \qquad (7)$$

Due to our lack of time, I will focus only on some representations of $SU(3)$

Representations of $SU(3)$ are given in terms of tensor fields that transform under $SU(3)$ in diferent ways.

Fundamental representation $\mathbf{3}: \ \psi_a \to U_a{}^b \psi_b, \quad a, b = 1, 2, 3$

Anti-fundamental representation $\bar{\mathbf{3}}: \ \psi^a \to U^{*a}{}_b \psi^b, \quad a, b = 1, 2, 3$

SU(3)

The special unitary $SU(N)$ groups play a special role in particle physics

$$SU(N) = \{U \in U(N) : \det(U) = 1\} \qquad (7)$$

Due to our lack of time, I will focus only on some representations of $SU(3)$

Representations of $SU(3)$ are given in terms of tensor fields that transform under $SU(3)$ in diferent ways.

Fundamental representation $\mathbf{3}: \ \psi_a \to U_a{}^b \psi_b, \quad a, b = 1, 2, 3$

Anti-fundamental representation $\bar{\mathbf{3}}: \ \psi^a \to U^{*a}{}_b \psi^b, \quad a, b = 1, 2, 3$

Tensor representation $\psi^{a_1 \dots a_q}{}_{b_1 \dots b_p} \to U^{*a_1}{}_{c_1} \dots U^{*a_q}{}_{c_q} U_{b_1}{}^{d_1} \dots U_{b_p}{}^{d_p} \psi^{c_1 \dots c_q}{}_{d_1 \dots d_p}$

Quark Model

In 1964 Gell-Mann proposed a model of 3 sub-particles that would explain the organization of hadrons in octets, decuplets, and singlets.

The similarity between the masses of certain groups of mesons and baryons suggests an underlying symmetry between the quark constituents, and for the case of the three lightest quarks this is naturally $SU(3)$

Quark Model

In 1964 Gell-Mann proposed a model of 3 sub-particles that would explain the organization of hadrons in octets, decuplets, and singlets.

The similarity between the masses of certain groups of mesons and baryons suggests an underlying symmetry between the quark constituents, and for the case of the three lightest quarks this is naturally $SU(3)$

Starting by discussing the SU(3) Lie algebra. The most widely used basis for this algebra is given by the Gell-Mann matrices.

$$\lambda_1 = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad \lambda_2 = \begin{pmatrix} 0 & -i & 0 \\ i & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad \lambda_3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad \lambda_4 = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix},$$

$$\lambda_5 = \begin{pmatrix} 0 & 0 & -i \\ 0 & 0 & 0 \\ i & 0 & 0 \end{pmatrix}, \quad \lambda_6 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}, \quad \lambda_7 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -i \\ 0 & i & 0 \end{pmatrix}, \quad \lambda_8 = \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \end{pmatrix}$$

Quark Model

Due to the lack of time, I can´t show the full demonstration, but the big picture is

Lets choose a basis $T_i = \lambda_i/2$

And also define

$$Q \equiv T_3 + \frac{Y}{2}$$

$\swarrow$ electric charge; $\Downarrow$ isospin; $\searrow$ hypercharge

The fundamental representation of $SU(3)$, $\mathbf{3}$ correspond to the quark states

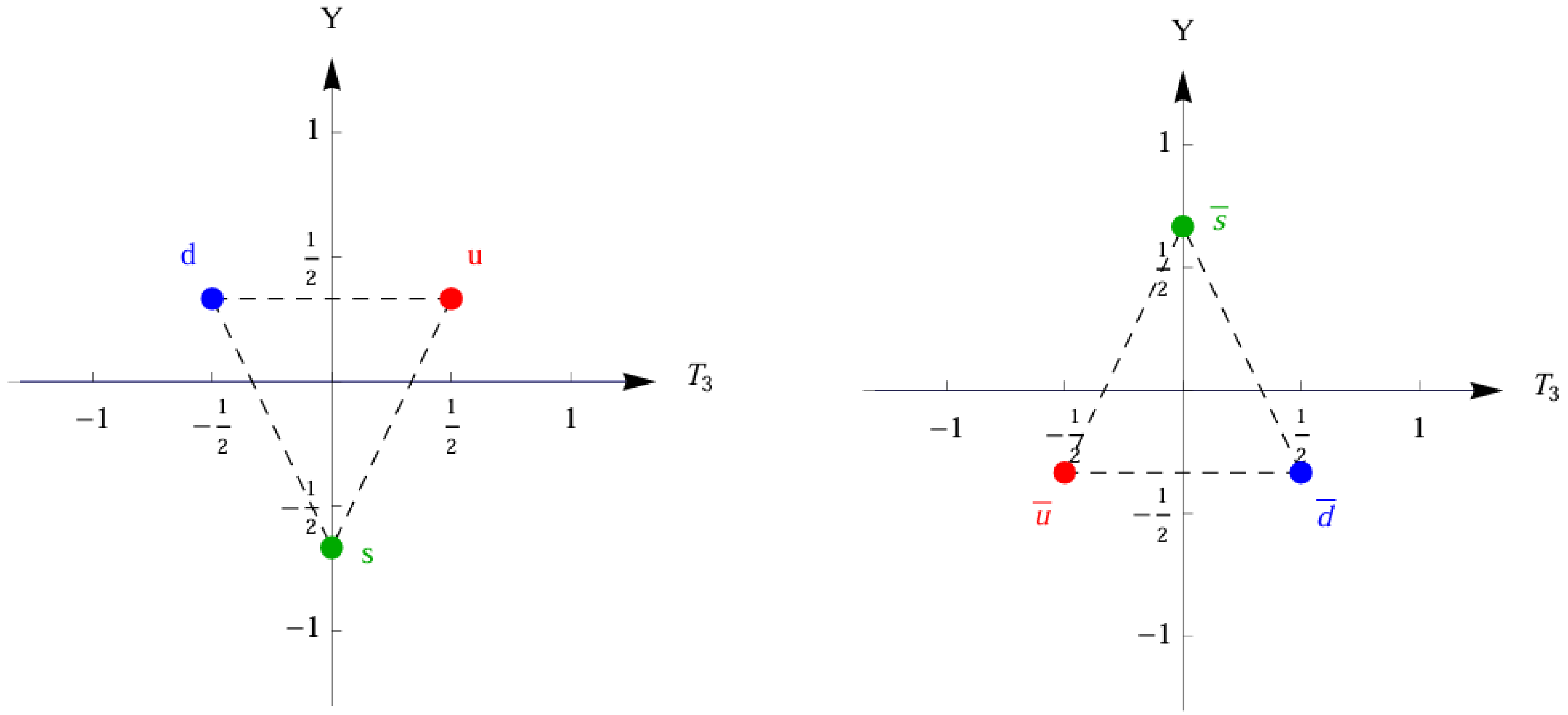
$$\psi = \begin{pmatrix} u \\ d \\ s \end{pmatrix}$$

The anti-fundamental representation of $SU(3), \bar{3}$ correspond to the anti-quarks states

$$\bar{\psi} = \begin{pmatrix} \bar{u} \\ \bar{d} \\ \bar{s} \end{pmatrix}$$

Quark Model

The weight diagrams from 3 and $\bar{3}$ can be drawn as



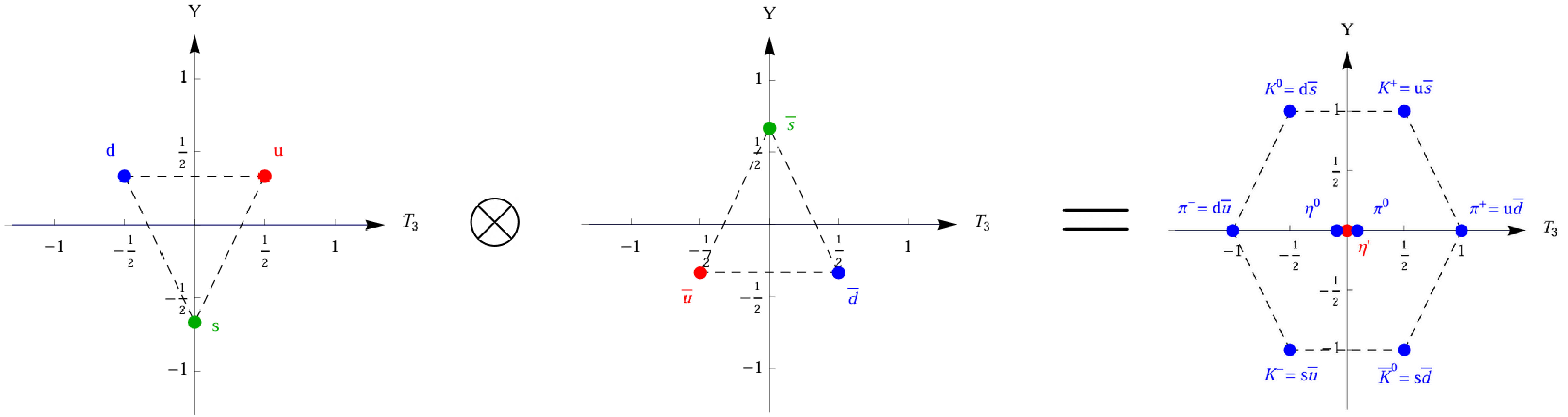
In SU(N) , the tensorial product

$$N \otimes \bar{N} = N^2 - 1 \oplus 1$$

Quark Model

As we know, mesons are made of a quark and a anti-quark.

If we made the tensorial product between 3 and $\bar{3}$



We got back one of Eightfold Way diagrams, but this time we built up the structure from an approximate symmetry, with the 3 lightest fundamental quarks

Successes and Problems with the Quark Model

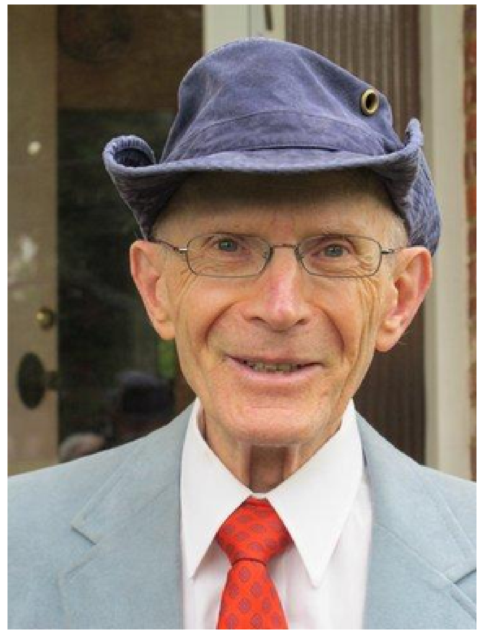
The model explained several new particles observed and it was supported by Deep Inelastic Scattering (DIS).

There were 2 clear problems:

- No free quarks had been observed
- Baryons Δ^{++} (uuu), Δ^{-} (ddd) and Ω^{-} (sss) seemed to violate Pauli´s exclusion principle.

In 1964, O.W. Greenberg proposed a new quantum number: colour

Quarks/Antiquarks have colour
Solves theoretical problem with Pauli´s exlusion principle
All naturally occurring particles are colourless



O.W. Greenberg

Questions?