High Energy Physics

Hadron Structure and DIS

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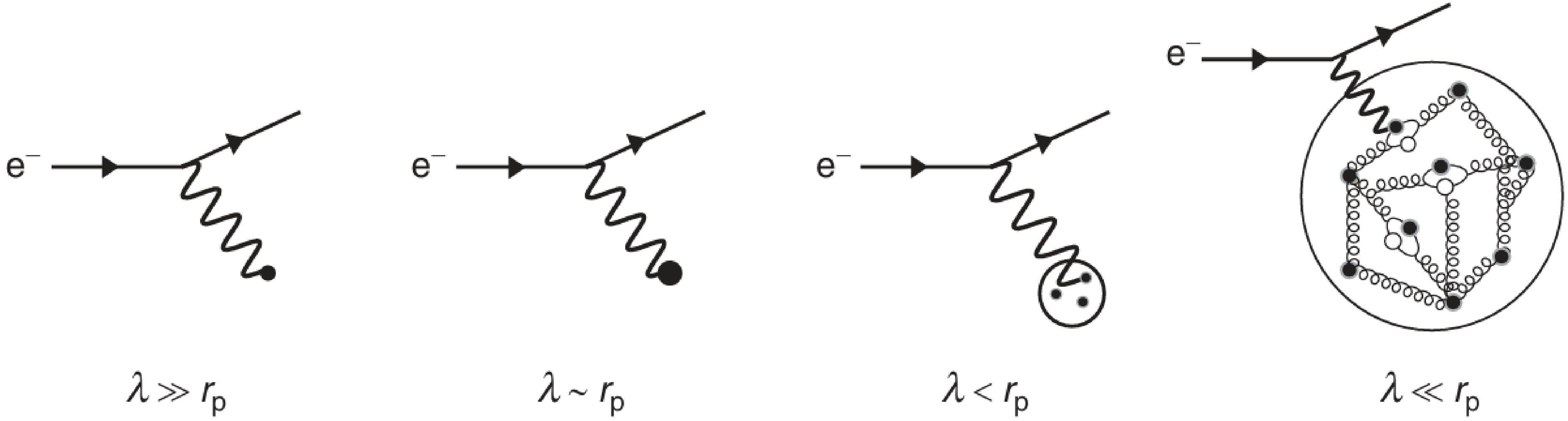
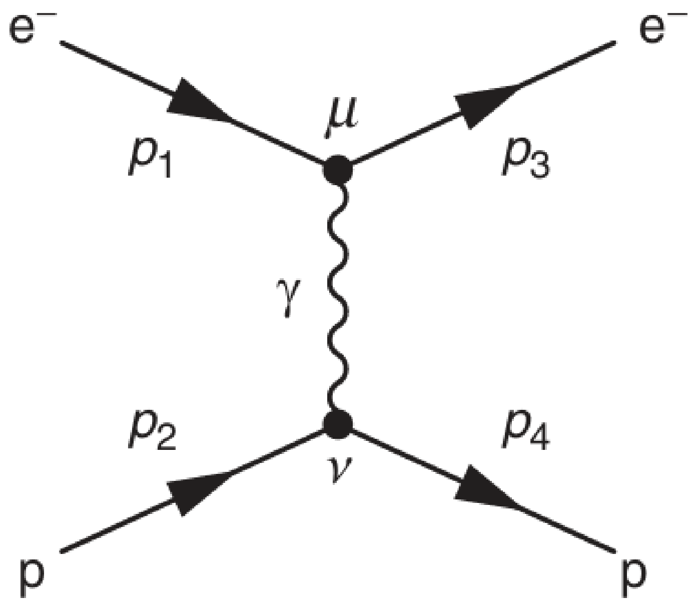
Scattering of an electron off a proton

$$e^- p \rightarrow e^- p$$

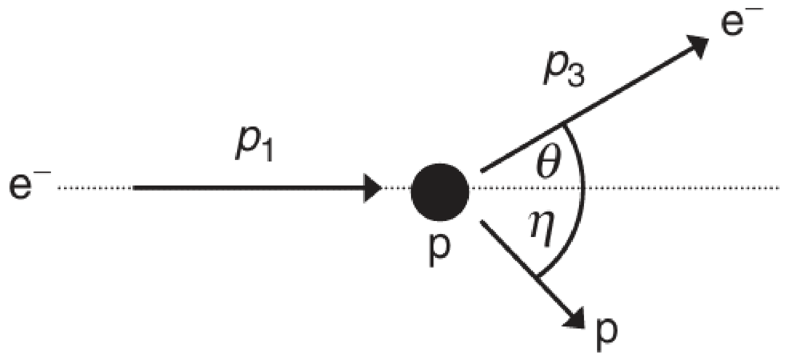


Rutherford and Mott scattering



$$\mathcal{M}_{fi} = \frac{Q_q e^2}{q^2}\,[\overline{u}(p_3)\gamma^\mu u(p_1)]\,g_{\mu\nu}\,[\overline{u}(p_4)\gamma^\nu u(p_2)]$$

$$= \frac{e^2}{q^2} j_e \cdot j_p$$



Rutherford and Mott scattering

electron current

$$u_{\uparrow}(p_1) = N_e \begin{pmatrix} 1 \\ 0 \\ \kappa \\ 0 \end{pmatrix}, \ u_{\downarrow}(p_1) = N_e \begin{pmatrix} 0 \\ 1 \\ 0 \\ -\kappa \end{pmatrix} \text{ and } u_{\uparrow}(p_3) = N_e \begin{pmatrix} c \\ s \\ \kappa c \\ \kappa s \end{pmatrix}, \ u_{\downarrow}(p_3) = N_e \begin{pmatrix} -s \\ c \\ \kappa s \\ -\kappa c \end{pmatrix}.$$

$$j_{e\uparrow\uparrow} = \overline{u}_{\uparrow}(p_3)\gamma^{\mu}u_{\uparrow}(p_1) = (E + m_e)\left[(\kappa^2 + 1)c, 2\kappa s, +2i\kappa s, 2\kappa c\right],$$

$$j_{e\downarrow\downarrow} = \overline{u}_{\downarrow}(p_3)\gamma^{\mu}u_{\downarrow}(p_1) = (E + m_e)\left[(\kappa^2 + 1)c, 2\kappa s, -2i\kappa s, 2\kappa c\right],$$

$$j_{e\downarrow\uparrow} = \overline{u}_{\uparrow}(p_3)\gamma^{\mu}u_{\downarrow}(p_1) = (E + m_e)\left[(1 - \kappa^2)s, 0, 0, 0\right],$$

$$j_{e\uparrow\downarrow} = \overline{u}_{\downarrow}(p_3)\gamma^{\mu}u_{\uparrow}(p_1) = (E + m_e)\left[(\kappa^2 - 1)s, 0, 0, 0\right].$$

$$N_e = \sqrt{E + m_e}$$

$$s = \sin(\theta/2)$$

$$c = \cos(\theta/2)$$

$$\kappa = \frac{\mathrm{p}}{E + m_e} \equiv \frac{\beta_e \gamma_e}{\gamma_e + 1}$$

Rutherford and Mott scattering

proton current

$$u_{\uparrow}(p_2) = \sqrt{2m_\mathrm{p}}\begin{pmatrix}1\\0\\0\\0\end{pmatrix} \qquad u_{\downarrow}(p_2) = \sqrt{2m_\mathrm{p}}\begin{pmatrix}0\\1\\0\\0\end{pmatrix} \qquad u_{\uparrow}(p_4) \approx \sqrt{2m_\mathrm{p}}\begin{pmatrix}c_\eta\\-s_\eta\\0\\0\end{pmatrix} \qquad u_{\downarrow}(p_4) \approx \sqrt{2m_\mathrm{p}}\begin{pmatrix}-s_\eta\\-c_\eta\\0\\0\end{pmatrix}$$

$$j_{\mathrm{p}\uparrow\uparrow} = -j_{\mathrm{p}\downarrow\downarrow} = 2m_\mathrm{p}\left[c_\eta, 0, 0, 0\right]$$

$$j_{\mathrm{p}\uparrow\downarrow} = j_{\mathrm{p}\downarrow\uparrow} = -2m_\mathrm{p}\left[s_\eta, 0, 0, 0\right]$$

$$c_\eta = \cos(\eta/2)$$

$$s_\eta = \sin(\eta/2)$$

Rutherford and Mott scattering

matrix element

$$
\begin{aligned}
\langle|\mathcal{M}_{fi}^2|\rangle &= \frac{1}{4}\sum|\mathcal{M}_{fi}^2| \\
&= \frac{1}{4}\frac{e^4}{q^4}\times 4m_\mathrm{p}^2(E+m_\mathrm{e})^2\cdot\left[c_\eta^2+s_\eta^2\right]\cdot\left[4(1+\kappa^2)^2c^2+4(1-\kappa^2)^2s^2\right] \\
&= \frac{4m_\mathrm{p}^2m_\mathrm{e}^2e^4(\gamma_\mathrm{e}+1)^2}{q^4}\left[(1-\kappa^2)^2+4\kappa^2c^2\right] \\
&= \frac{16m_\mathrm{p}^2m_\mathrm{e}^2e^4}{q^4}\left[1+\beta_\mathrm{e}^2\gamma_\mathrm{e}^2\cos^2\frac{\theta}{2}\right]
\end{aligned}
$$

$$
\langle|\mathcal{M}_{fi}^2|\rangle = \frac{m_\mathrm{p}^2m_\mathrm{e}^2e^4}{\mathrm{p}^4\sin^4(\theta/2)}\left[1+\beta_\mathrm{e}^2\gamma_\mathrm{e}^2\cos^2\frac{\theta}{2}\right]
$$

Rutherford and Mott scattering

$$\frac{d\sigma}{d\Omega} = \frac{1}{64\pi^2}\left(\frac{1}{m_p + E_1 - E_1\cos\theta}\right)^2 \langle|\mathcal{M}_{fi}|^2\rangle$$

$$\left(\frac{d\sigma}{d\Omega}\right)_{\text{Rutherford}} = \frac{\alpha^2}{16E_K^2\sin^4(\theta/2)}$$

$\beta_e\gamma_e \ll 1$ $\qquad E_1 \sim m_e \ll m_p$

$$\left(\frac{d\sigma}{d\Omega}\right)_{\text{Mott}} = \frac{\alpha^2}{4E^2\sin^4(\theta/2)}\cos^2\frac{\theta}{2}$$

$\beta_e\gamma_e \gg 1$

Form Factors

$$V(\mathbf{r}) = \int \frac{Q\rho(\mathbf{r}')}{4\pi|\mathbf{r} - \mathbf{r}'|} \mathrm{d}^3\mathbf{r}'$$

$$\mathcal{M}_{fi} = \langle\psi_f|V(\mathbf{r})|\psi_i\rangle = \int e^{-i\mathbf{p}_3\cdot\mathbf{r}} V(\mathbf{r}) e^{i\mathbf{p}_1\cdot\mathbf{r}} \, \mathrm{d}^3\mathbf{r} \;\; = \iint e^{i\mathbf{q}\cdot\mathbf{r}} \frac{Q\rho(\mathbf{r}')}{4\pi|\mathbf{r} - \mathbf{r}'|} \mathrm{d}^3\mathbf{r}' \mathrm{d}^3\mathbf{r}$$

$$= \iint e^{i\mathbf{q}\cdot(\mathbf{r}-\mathbf{r}')} e^{i\mathbf{q}\cdot\mathbf{r}'} \frac{Q\rho(\mathbf{r}')}{4\pi|\mathbf{r} - \mathbf{r}'|} \mathrm{d}^3\mathbf{r}' \mathrm{d}^3\mathbf{r} \;\; = \int e^{i\mathbf{q}\cdot\mathbf{R}} \frac{Q}{4\pi|\mathbf{R}|} \mathrm{d}^3\mathbf{R} \int \rho(\mathbf{r}') e^{i\mathbf{q}\cdot\mathbf{r}'} \, \mathrm{d}^3\mathbf{r}'$$

$$F(\mathbf{q}^2) = \int \rho(\mathbf{r}) e^{i\mathbf{q}\cdot\mathbf{r}} \, \mathrm{d}^3\mathbf{r}$$

$$\left(\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega}\right)_{\mathrm{Mott}} \rightarrow \frac{\alpha^2}{4E^2 \sin^4(\theta/2)} \cos^2\left(\frac{\theta}{2}\right) \left|F(\mathbf{q}^2)\right|^2$$

Rosenbluth formula

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega} = \frac{\alpha^2}{4E_1^2 \sin^4(\theta/2)} \frac{E_3}{E_1} \left(\frac{G_E^2 + \tau G_M^2}{(1+\tau)} \cos^2 \frac{\theta}{2} + 2\tau G_M^2 \sin^2 \frac{\theta}{2} \right)$$

$$Q^2 \left(1 + \frac{Q^2}{4m_\mathrm{p}^2}\right) = \mathbf{q}^2 \qquad \tau = \frac{Q^2}{4m_\mathrm{p}^2}$$

In the limit: $Q^2 \ll 4m_\mathrm{p}^2$

$$G_E(Q^2) \approx G_E(\mathbf{q}^2) = \int e^{i\mathbf{q}\cdot\mathbf{r}} \rho(\mathbf{r}) \mathrm{d}^3\mathbf{r}$$

$$G_M(Q^2) \approx G_M(\mathbf{q}^2) = \int e^{i\mathbf{q}\cdot\mathbf{r}} \mu(\mathbf{r}) \mathrm{d}^3\mathbf{r}$$

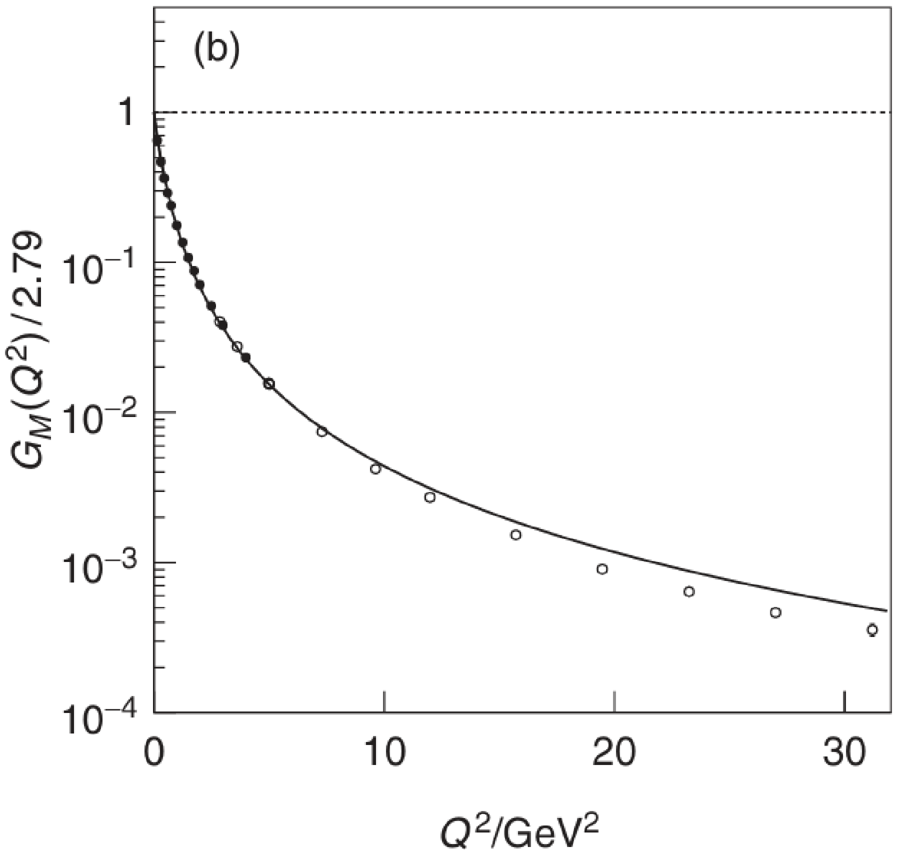
Rosenbluth formula

High Q

$$\left(\frac{d\sigma}{d\Omega}\right)_{\text{elastic}} \sim \frac{\alpha^2}{4E_1^2 \sin^4(\theta/2)} \frac{E_3}{E_1} \left[\frac{Q^2}{2m_p^2} G_M^2 \sin^2\frac{\theta}{2}\right]$$

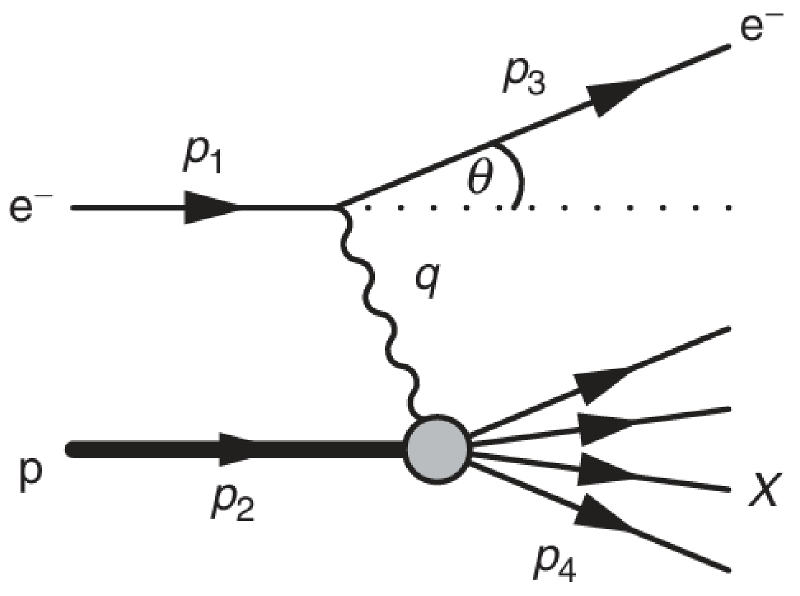
$$G_M(q^2) \propto q^{-4}$$

$$\left(\frac{d\sigma}{d\Omega}\right)_{elastic} \propto \frac{1}{Q^6} \left(\frac{d\sigma}{d\Omega}\right)_{Mott}$$



Deep Inelastic Scattering

Kinematic variables: x, y, ν, Q^2



$$Q^2 \equiv -q^2$$

$$x \equiv \frac{Q^2}{2p_2.q} \qquad 0 < x < 1$$

$$W^2 = p_4^2 = (p_2 + q)^2$$

$$y \equiv \frac{p_2.q}{p_2.p_1} \qquad 0 < y < 1$$

$$y = \left(\frac{2m_p}{s - m_p^2}\right)\nu$$

$$\nu \equiv \frac{p_2.q}{M}$$

$$Q^2 = (s - m_p^2)xy$$

Deep Inelastic Scattering

$$\frac{d\sigma}{d\Omega} = \frac{\alpha^2}{4E_1^2 \sin^4(\theta/2)} \frac{E_3}{E_1} \left(\frac{G_E^2 + \tau G_M^2}{(1+\tau)} \cos^2 \frac{\theta}{2} + 2\tau G_M^2 \sin^2 \frac{\theta}{2} \right)$$

$$\frac{d\sigma}{dQ^2} = \frac{4\pi\alpha^2}{Q^4} \left[\frac{G_E^2 + \tau G_M^2}{(1+\tau)} \left(1 - y - \frac{m_p^2 y^2}{Q^2} \right) + \frac{1}{2} y^2 G_M^2 \right]$$

$$\frac{d\sigma}{dQ^2} = \frac{4\pi\alpha^2}{Q^4} \left[\left(1 - y - \frac{m_p^2 y^2}{Q^2} \right) f_2(Q^2) + \frac{1}{2} y^2 f_1(Q^2) \right]$$

$$\frac{d^2\sigma}{dx\, dQ^2} = \frac{4\pi\alpha^2}{Q^4} \left[\left(1 - y - \frac{m_p^2 y^2}{Q^2} \right) \frac{F_2(x, Q^2)}{x} + y^2 F_1(x, Q^2) \right]$$

Deep Inelastic Scattering

Bjorken scaling and the Callan-Gross relation

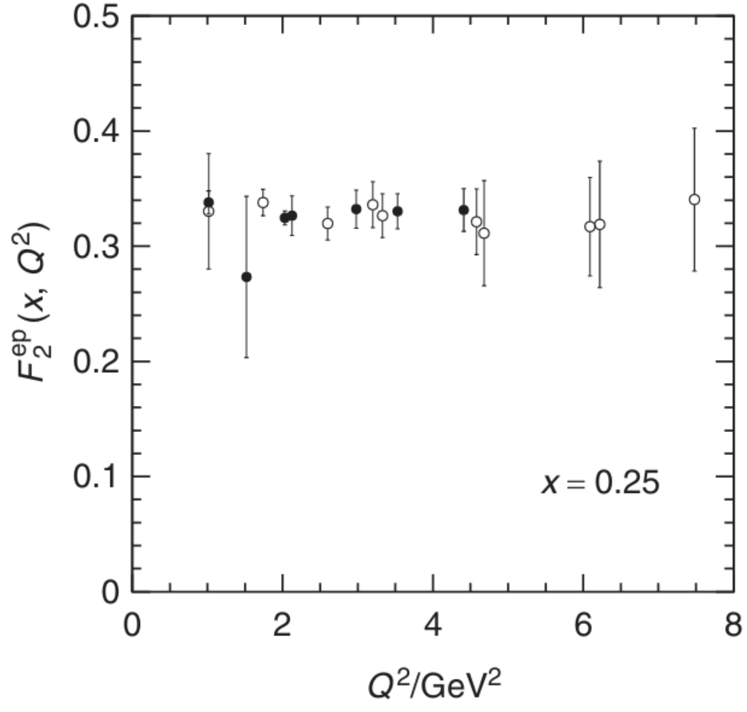
For deep inelastic scatering: $Q^2 \gg m_p^2 y^2$

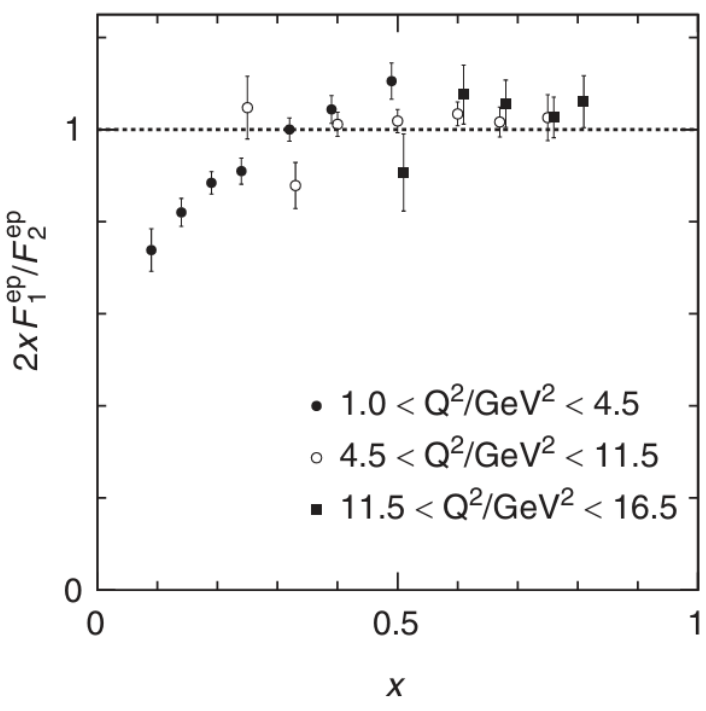
$$\frac{d^2\sigma}{dx\,dQ^2} \approx \frac{4\pi\alpha^2}{Q^4}\left[(1-y)\,\frac{F_2(x,Q^2)}{x} + y^2 F_1(x,Q^2)\right]$$

$$F_1(x,Q^2) \to F_1(x)$$

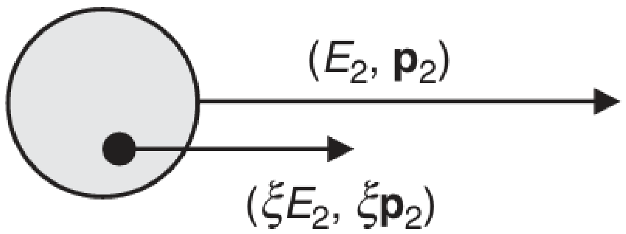
$$F_2(x,Q^2) \to F_2(x)$$

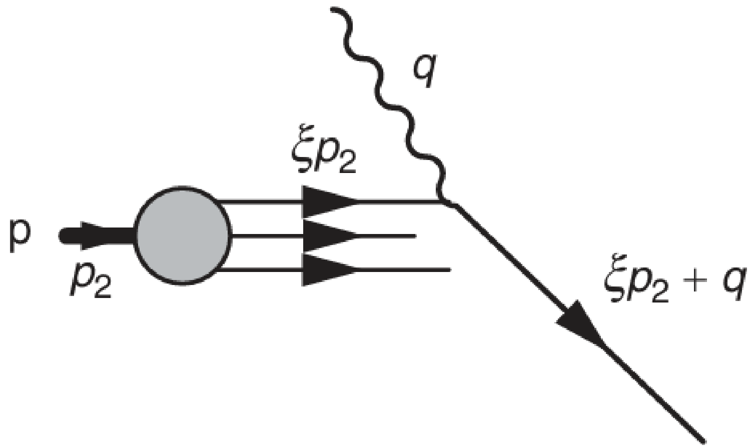
$$F_2(x) = 2xF_1(x)$$





Parton model





$$(\xi p_2 + q)^2 = \xi^2 p_2^2 + 2\xi p_2 \cdot q + q^2 = m_q^2$$

$$\xi = \frac{-q^2}{2p_2 \cdot q} = \frac{Q^2}{2p_2 \cdot q} \equiv x$$

In terms of the protón momentum

$$s = (p_1 + p_2)^2 \approx 2p_1 \cdot p_2 \qquad y = \frac{p_2 \cdot q}{p_2 \cdot p_1} \qquad x = \frac{Q^2}{2p_2 \cdot q}$$

For the quark

$$s_q = xs, \quad y_q = y \quad \text{and} \quad x_q = 1$$

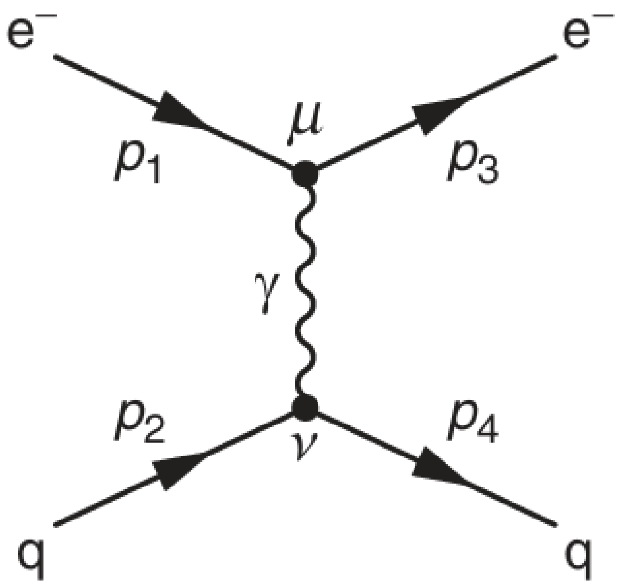
Parton model

$$\frac{d\sigma}{dq^2} = \frac{2\pi\alpha^2 Q_q^2}{q^4}\left[1 + \left(1 + \frac{q^2}{s}\right)^2\right]$$

$$q^2 = -Q^2 = -(s_q - m_q^2)x_q y_q$$

$$\frac{q^2}{s_q} = -x_q y_q = -y$$

$$\frac{d\sigma}{dQ^2} = \frac{4\pi\alpha^2 Q_q^2}{Q^4}\left[(1-y) + \frac{y^2}{2}\right]$$



Parton model

For a especific flavour we have

$$\frac{\mathrm{d}^2\sigma}{\mathrm{d}Q^2} = \frac{4\pi\alpha^2}{Q^4}\left[(1-y)+\frac{y^2}{2}\right] \times Q_i^2 q_i^{\mathrm{p}}(x)\,\delta x$$

Suming over all quark flavours

$$\frac{\mathrm{d}^2\sigma^{\mathrm{ep}}}{\mathrm{d}x\,\mathrm{d}Q^2} = \frac{4\pi\alpha^2}{Q^4}\left[(1-y)+\frac{y^2}{2}\right]\sum_i Q_i^2 q_i^{\mathrm{p}}(x)$$

$$F_2^{\mathrm{ep}}(x,Q^2) = 2xF_1^{\mathrm{ep}}(x,Q^2) = x\sum_i Q_i^2 q_i^{\mathrm{p}}(x)$$