1290 FACULDADE DE CIÊNCIAS E TECNOLOGIA UNIVERSIDADE D COIMBRA

Transversal beam dynamics

A LOOK INTO THE MOVEMENT OF SINGLE PARTICLES UNDER THE INFLUENCE OF EXTERNAL TRANSVERSE BENDING AND FOCUSING FIELDS IN A SYNCHROTRON OR STORAGE RING.

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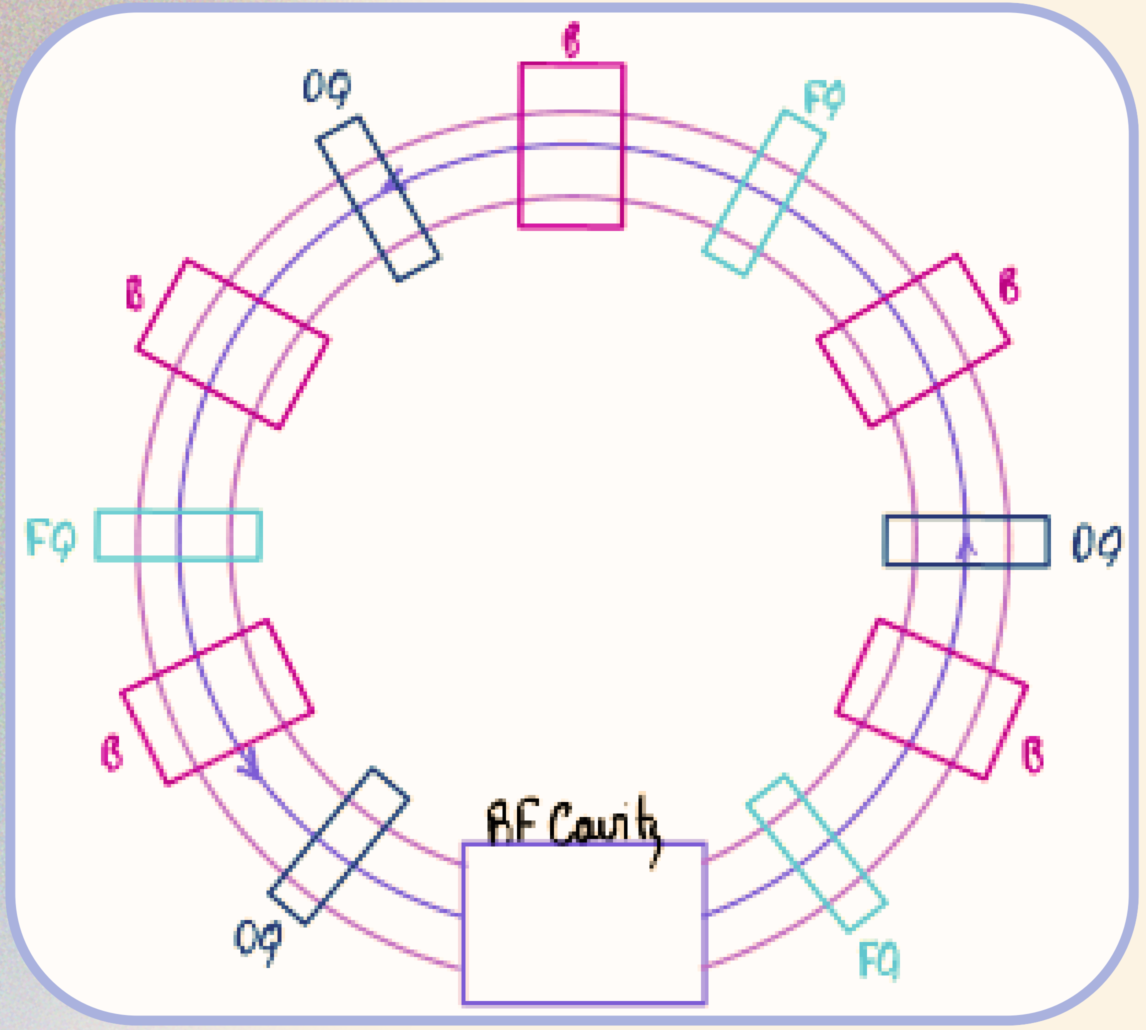
SYNCHROTRON

A synchrotron is a type of circular particle accelerator, that uses a varying magnetic field, to curve the particles path and a synchronized radiofrequency (RF) electric field, to accelerate charged particles to very high energies.

The synchrotron accelerates the beam as a series of discrete pulses or "bunches" as they are called. Each short pulse is injected at low field and then the field rises in proportion to the momentum of particles as they are accelerated.



B - Bending magnet (dipole magnet)
FQ - Focussing quadrupole magnet
DQ - Defocusing quadrupole magnet

Circulation frequency of the synchronous particle:

$$\omega_0 = \frac{\omega_{RF}}{h} = \frac{v_\theta}{\rho} = \frac{qB_y}{m} = \frac{qB_y}{\gamma m_0}$$

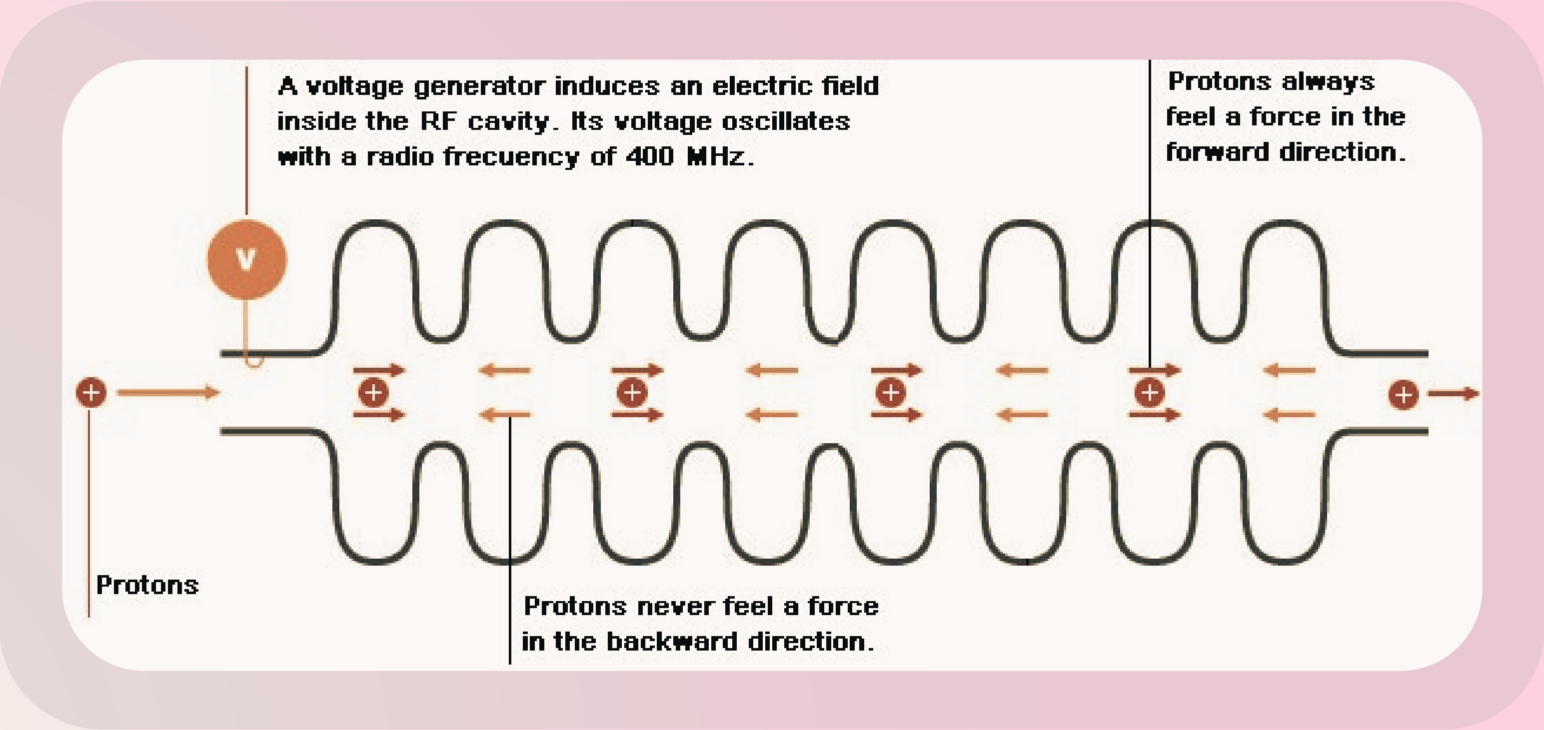
Radius of the particle trajectory is constant.

$$\rho = \frac{p}{qB_y} = \text{const.}$$

Magnetic field increases with beam momentum.

RF CAVITY

A RF cavity is a resonant metallic structure (usually cylindrical or pillbox-shaped) that confines an oscillating electric field at a specific radiofrequency. This field interacts with charged particles, accelerating them along the desired trajectory.

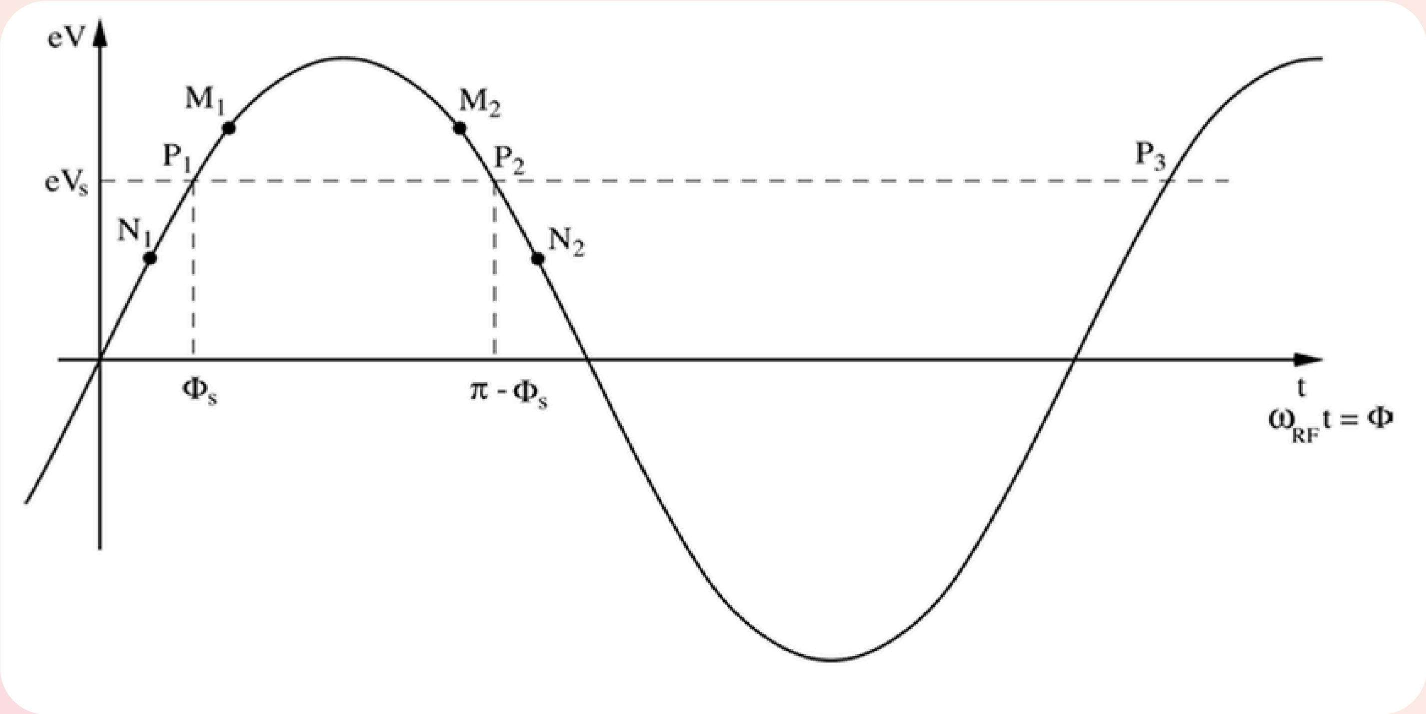


Multi-cell Cavity

The RF cavities also maintain the particle bunches tightly grouped, ensuring high luminosity at the collision points and thereby maximizing the number of particle collisions.

PHASE FOCUSING

$$E_z = \hat{E}_z \sin \omega_{RF} t = \hat{E}_z \sin \phi(t)$$



- M1 and N1 move toward the synchronism ⇒ STABLE
- M2 and N2 move away from synchronism ⇒ UNSTABLE

The phase stability occurs in synchronous accelerators where the acceleration is made by using radiofrequency electric fields.

If successive accelerating gaps (radio-frequency cavities) are arranged such that a given particle always sees the same RF phase and gets the same energy gain, that particle is called synchronous particle.

TYPES OF MAGNETS

In a constant transverse magnetic field B, a particle will see a constant deflecting force and the trajectory will be part of a circle, whose bending radius ρ is determined by the particle momentum p = mv and the external B field:

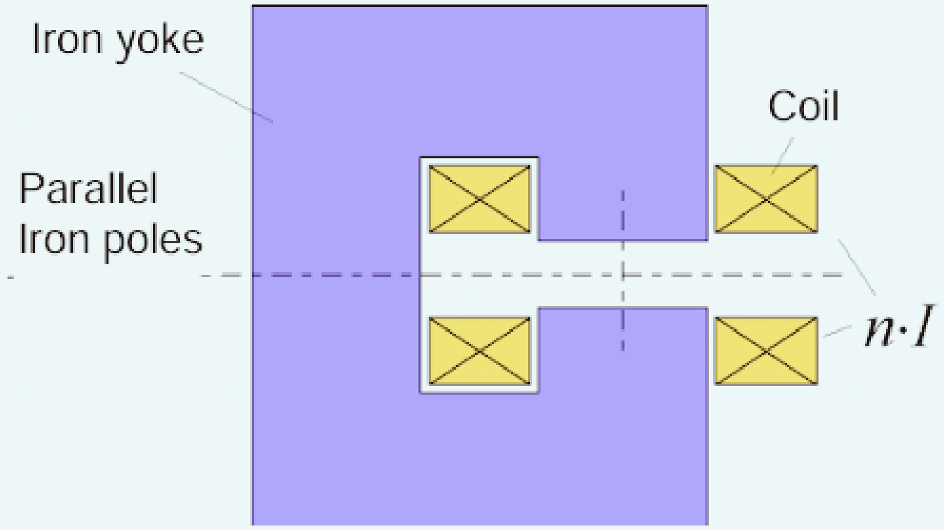
$$\frac{1}{\rho(x, y, s)} = \frac{q}{p} B_y(x, y, s)$$

Expansion of the magnetic field along the reference radius:

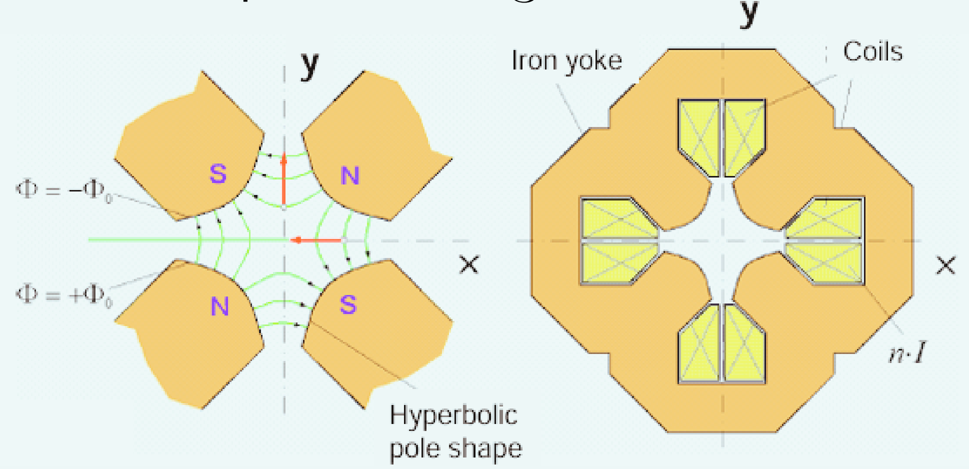
$$\frac{q}{p} B_y(x) = \frac{1}{\rho} + kx + \frac{1}{2!} mx^2 + \dots$$

Dipole	Quadrupole	Sextupole
Bending	Focusing	Correction

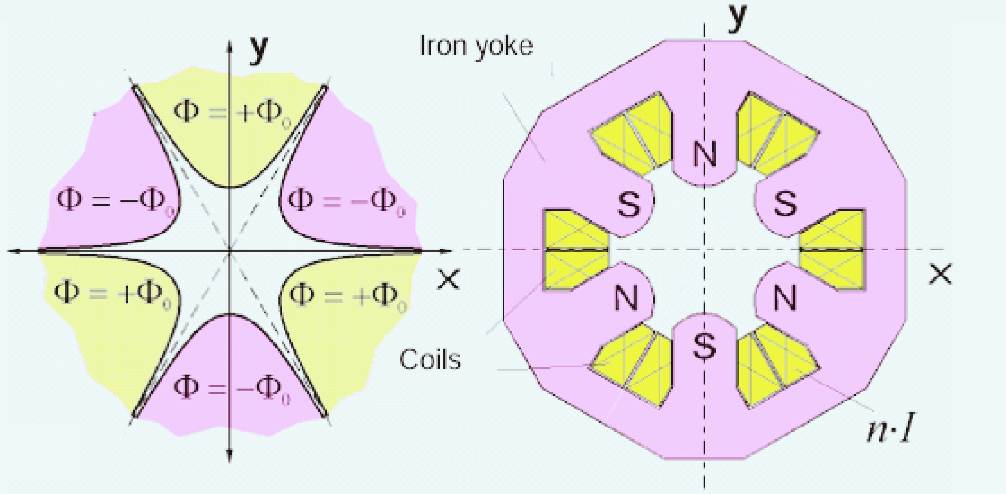
Dipole magnet



Quadrupole magnet



Sextupole magnet



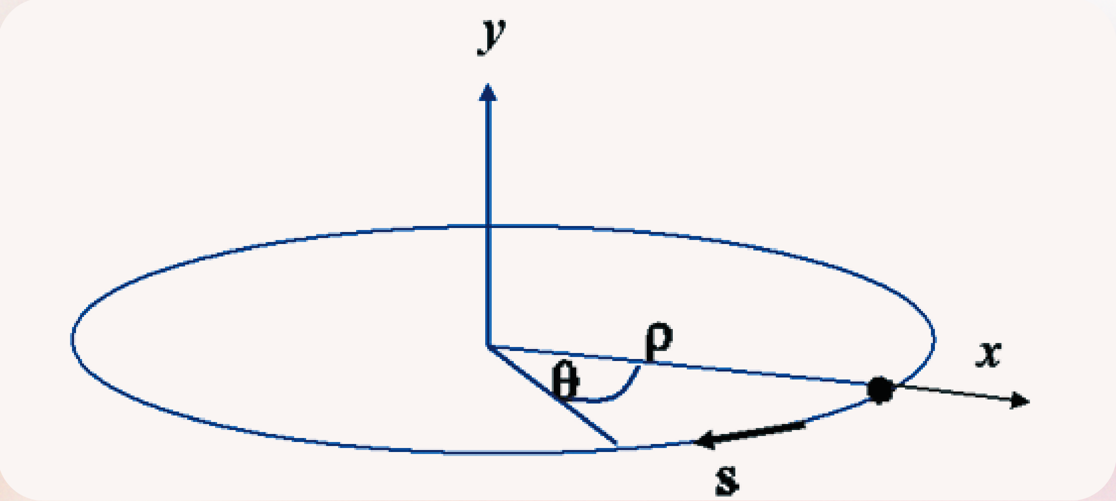
EQUATIONS OF MOTION

Magnetic fields are used in general in circular accelerators to provide the bending force and to focus the particle beam.

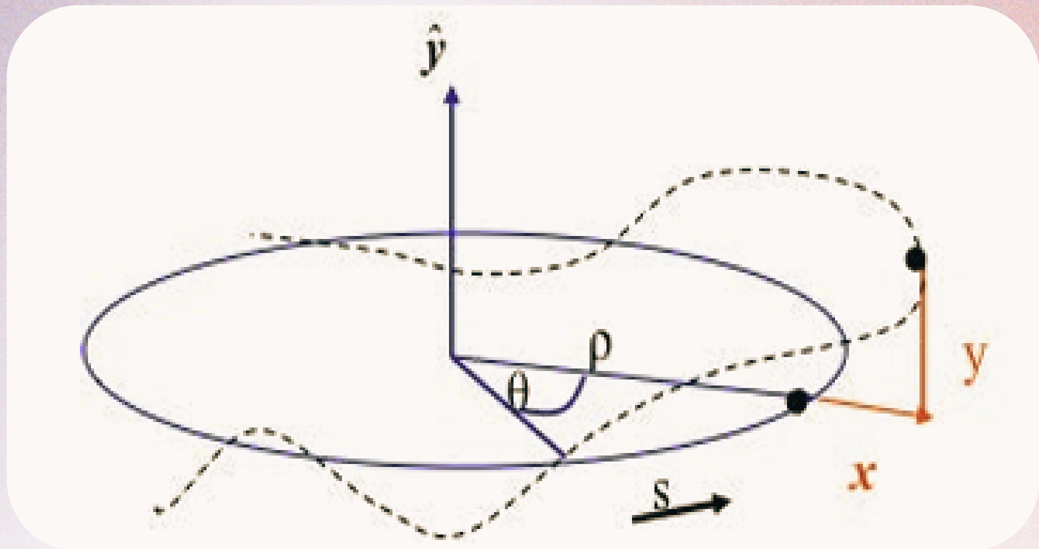
Gradient fields generated by quadrupole lenses are used to do this job. These lenses generate a magnetic field that increases linearly as a function of the distance from the magnet centre:

$$B_y = -g \cdot x, \quad B_x = -g \cdot y$$

$$g = \frac{\partial B_y}{\partial x}$$



Coordinate system used and orbit of an idealized particle



Ideal circular orbit and real particle trajectory with its transverse coordinates x and y

k along the particle trajectory is a periodic function and a measure for focusing and defocussing forces from quadrupoles.

$$k = \frac{g}{B \cdot \rho}$$

The following differential equation describes the transverse motion of a particle with respect to the design orbit.

Horizontal motion:

$$x'' + \left(\frac{1}{\rho^2} + k\right)x = 0$$

$K = 1/\rho^2 + k$

Vertical motion:

$$y'' - ky = 0$$

$K = -k$

SOLUTION OF THE EQUATION OF MOTION

The general solution for the position and angle of the trajectory can be derived as a function of the initial conditions x_0 and x_0′. In the case of a quadrupole, thin lenses, dipole and drift, we obtain:

$$x(s) = x_0 \cosh\sqrt{k}s + \frac{x_0'}{\sqrt{k}} \sinh\sqrt{k}s$$

$$x'(s) = x_0\sqrt{k}\sinh\sqrt{k}s + x_0'\cosh\sqrt{k}s$$

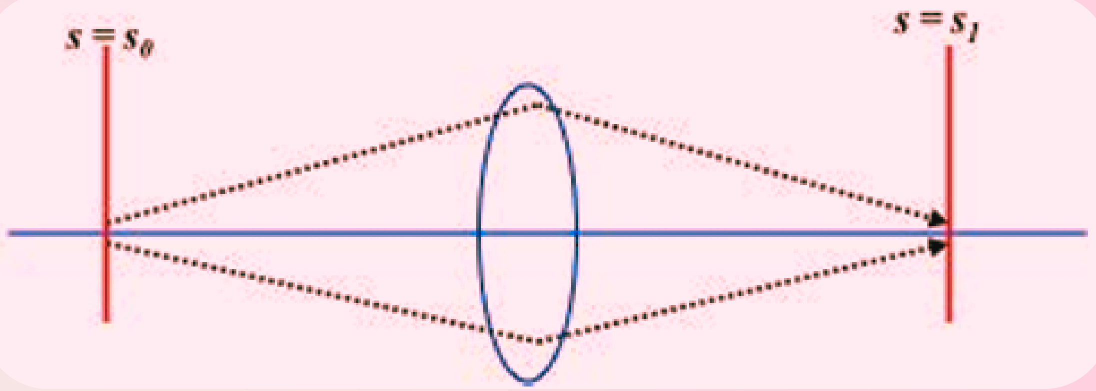
or, written in a more convenient matrix form,

$$\begin{pmatrix} x \\ x' \end{pmatrix}_s = M \cdot \begin{pmatrix} x \\ x' \end{pmatrix}_0$$

Given the particle amplitude and angle in front of the lattice element, x_0 and x_0′, we obtain their values after the element by a simple matrix multiplication. The matrix M depends on the properties of the magnet, and we obtain the following expressions for three typical lattice elements.

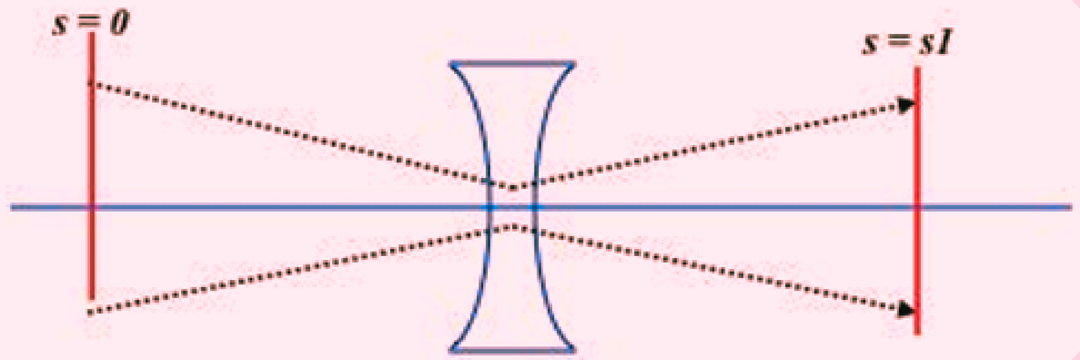
Focusing quadrupole (K>0):

$$M_{QF} = \begin{pmatrix} \cos(\sqrt{|K|}l & \frac{1}{\sqrt{|K|}}\sin(\sqrt{|K|}l \\ -\sqrt{|K|}\sin(\sqrt{|K|}l & \cos(\sqrt{|K|}l \end{pmatrix}$$



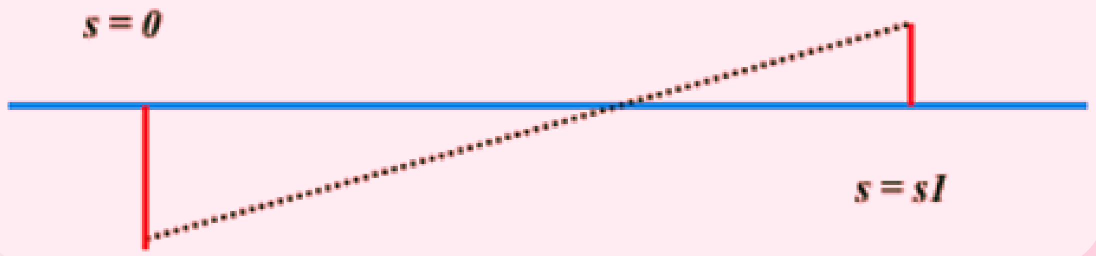
Defocusing quadrupole (K<0):

$$M_{QD} = \begin{pmatrix} \cosh(\sqrt{|K|}l & \frac{1}{\sqrt{|K|}}\sinh(\sqrt{|K|}l \\ \sqrt{|K|}\sinh(\sqrt{|K|}l & \cosh(\sqrt{|K|}l \end{pmatrix}$$

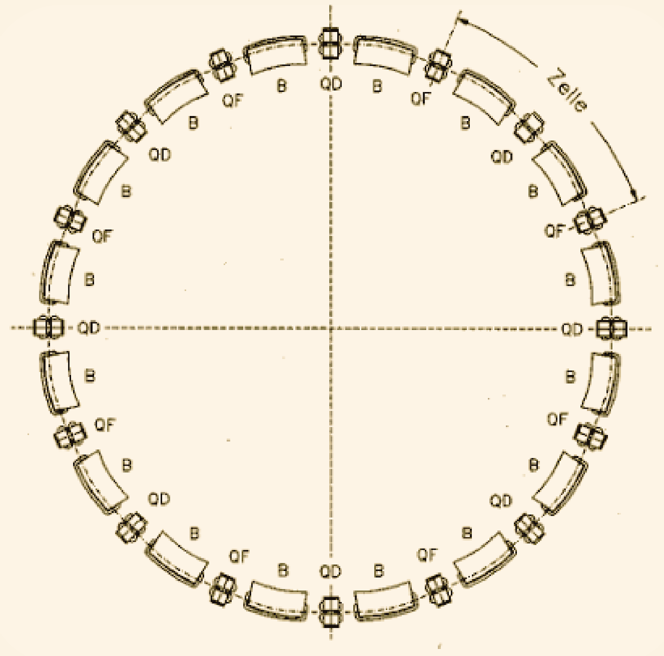


Drift space (K = 0):

$$M_{\text{drift}} = \begin{pmatrix} 1 & l \\ 0 & 1 \end{pmatrix}$$

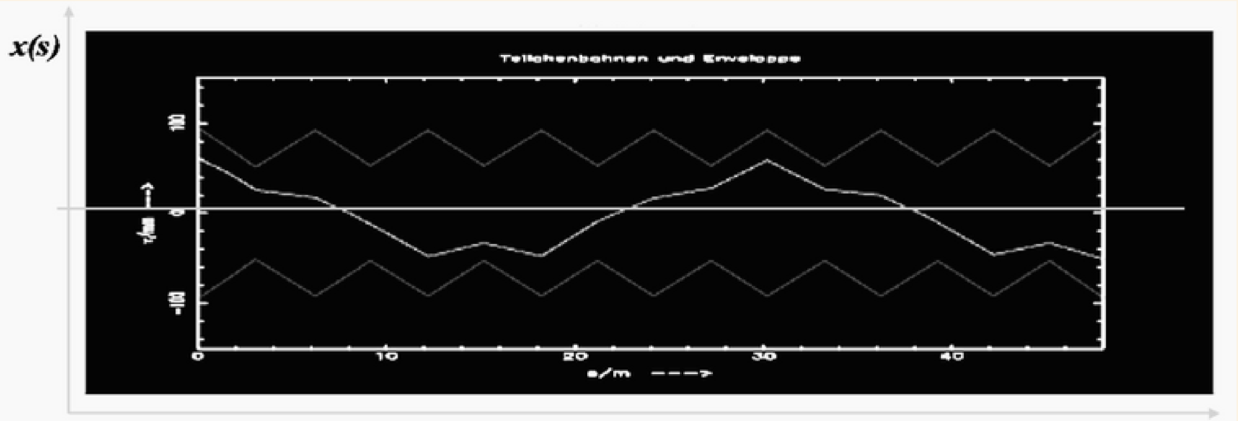


Consider a storage ring built only out of focusing and defocusing quadrupoles and dipole magnets in between.



Starting in front of a focusing magnet, a typical part of this structure, expressed in matrix form, would be:

$$M_{total} = \ldots \cdot M_{QF} \cdot M_B \cdot M_{QD} \cdot M_B \cdot M_{QF}$$



Single-particle trajectory in a storage ring (white). In grey, the pattern of the beta function is shown, which can be interpreted as the maximum beam size or aperture needed in the example.

TWISS PARAMETERS

Given the solution of Hill's equation:

$$x(s) = \sqrt{\varepsilon}\sqrt{\beta(s)}\cos(\psi(s) + \phi)$$

$$x'(s) = -\frac{\sqrt{\varepsilon}}{\sqrt{\beta(s)}}\{\alpha(s)\cos(\psi(s) + \phi) + \sin(\psi(s) + \phi)\}$$

And we get an expression for the integration constant **e**

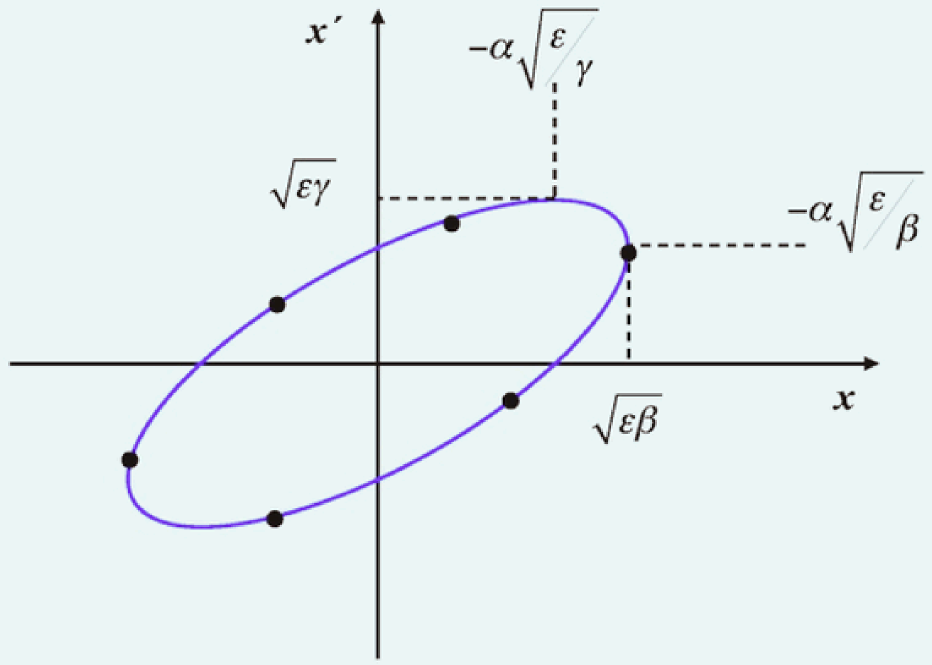
$$\varepsilon = \gamma(s)x^2(s) + 2\alpha(s)x(s)x'(s) + \beta(s)x'^2(s)$$

$$\alpha(s) = \frac{-1}{2}\beta'(s)$$

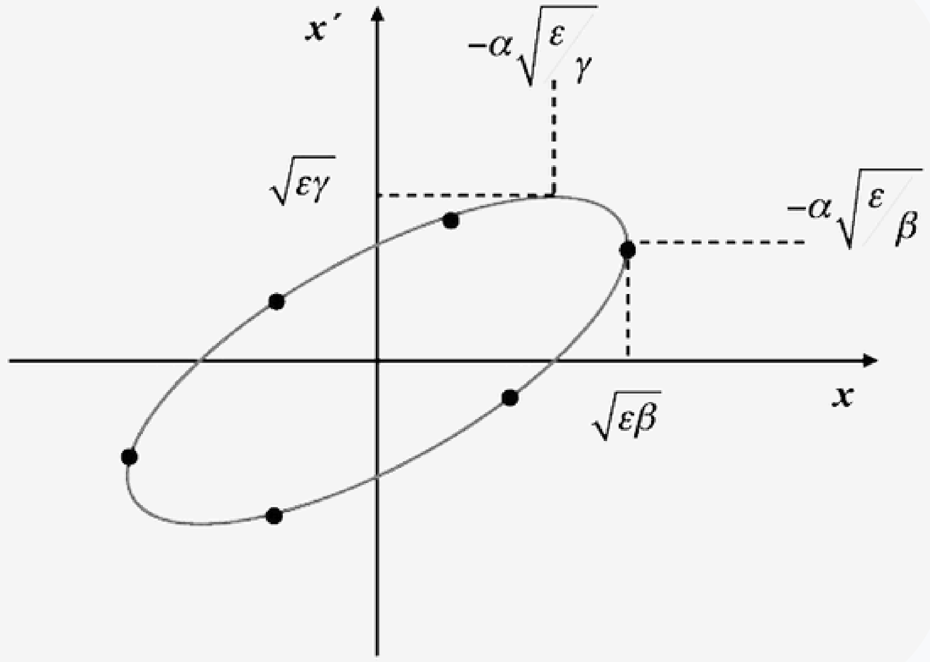
$$\gamma(s) = \frac{1 + \alpha(s)^2}{\beta(s)}$$

The Tune is the number of oscillations per turn, which is nothing other than the overall phase advance of the transverse oscillation per revolution in units of 2π.

$$Q_y = \frac{1}{2\pi}\oint\frac{ds}{\beta(s)}$$



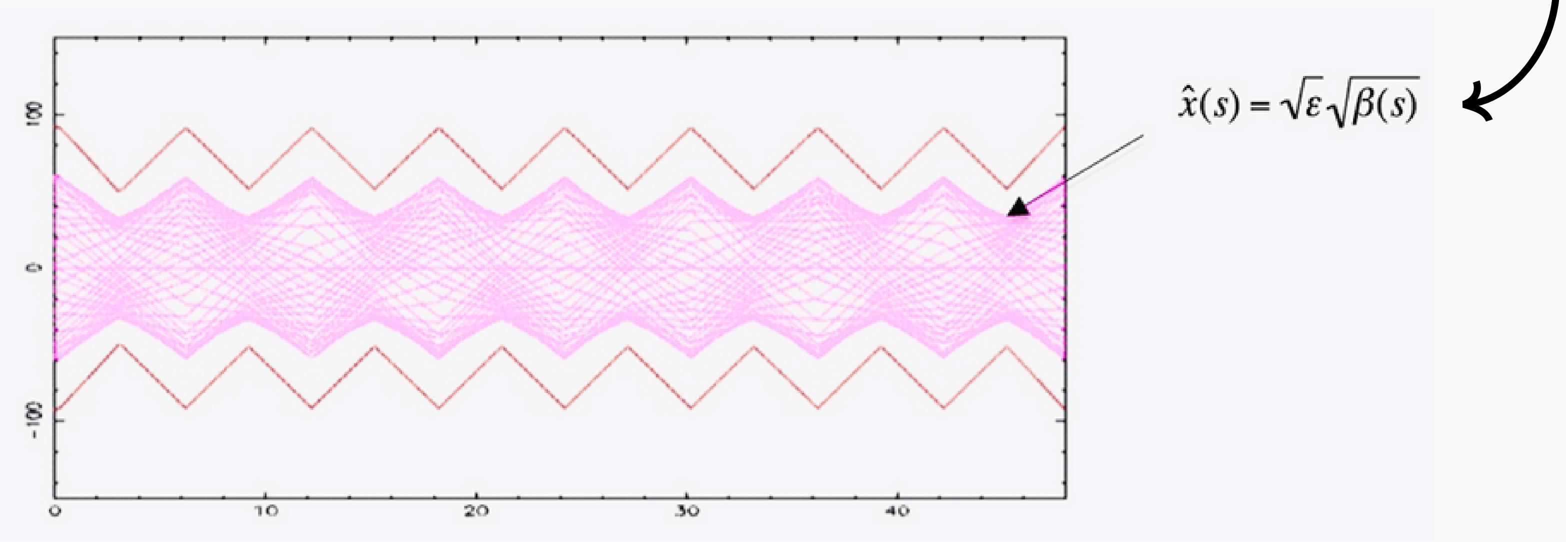
Liouville's theorem: all particles enclosed by an envelope ellipse will stay within that ellipse



The ellipse equation describes the position of particles in transverse phase space for a given emittance **e** with the initial phase **e** at the position s of the reference particle in the accelerator ring.

For a periodic accelerator (circular accelerator) or beam line with periodic magnetic structure the optical functions $\alpha(s), \gamma(s)$ *and* $\beta(s)$ are also periodic and a single particle is circulating on the phase-space ellipse at a given s.

maximum amplitude, or beam size, is obtained from



The ensemble of many single particles forms a pattern of overlapping trajectories that in the end we will observe as transverse intensity (or charge) distribution and that we will use to define the beam size.

DISPERSIVE EFFECTS

Until now we have treated the beam and the equation of motion as a mono-energetic problem. Unfortunately, in the case of a realistic beam, we have to deal with a considerable distribution of the particles in energy or momentum.

This momentum spread will lead to several effects concerning the bending of the dipole magnets and the focusing strength of the quadrupoles.

$$x'' + x\left(\frac{1}{\rho^2} - k\right) = \frac{\Delta p}{p} \cdot \frac{1}{\rho}$$

$$x(s) = x_h(s) + x_i(s)$$

xi is an additional contribution that has yet to be determined. For convenience, we usually normalize this second term and define a function, the so-called dispersion:

$$D(s) = \frac{x_i(s)}{\Delta p / p}$$

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