Examining the quantum nature of the chiral effects

Rémy Larue - ShanghaiTech University

In collaboration with Jérémie Quevillon, Diego Saviot } LAPTh Annecy

Weyl and Dirac semi-metals as a laboratory for high-energy physics

Examining the quantum nature of the chiral effects

TO APPEAR SOON (NEXT WEEK)

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Outline

* Chiral effects (CSE & CVE, CME not discussed)
* Finite T & μ QFT in curved + electromagnetic background
* Axial current and anomaly
* Anomaly at global equilibrium
* Summary

Chiral effects

* Massless fermionic fluid with 4-velocity $u^\mu = \frac{1}{\sqrt{1-v^2}}\begin{pmatrix}1\\ -\vec{v}\end{pmatrix}$
  Chemical potential μ_0, temperature T_0

$$\langle j_5^\mu \rangle = \langle \bar{\Psi}\gamma^\mu\gamma_5\Psi\rangle \supset \overbrace{\frac{\mu_0}{2\pi^2}B^\mu}^{\text{CSE}} + \overbrace{\left(\frac{\mu_0^2}{2\pi^2}+\frac{T_0^2}{6}\right)\Omega^\mu}^{\text{CVE}}$$

$B^\mu = \frac{1}{2}\varepsilon^{\mu\nu\rho\sigma}u_\nu F_{\rho\sigma}$ magnetic field

$\Omega^\mu = \frac{1}{2}\varepsilon^{\mu\nu\rho\sigma}u_\nu \partial_\rho u_\sigma$ vorticity

Chiral effects

* Question : Are both CVE & CSE anomaly induced ?

* Our goal : Compute chiral anomaly at local & global equilibrium $(\mu(x), T(x))$, with gravitational & electromagnetic background, with fluid 4-velocity.

Finite μ & T QFT

Flat spacetime

* Massless fermion $\mathcal{L}[V_\mu] = \bar{\Psi}(i\not{\partial} - \not{V})\Psi$ $\qquad \mu \in \{t, i\}$

* Imaginary-time formalism:

$$Z = \mathrm{Tr}\, e^{-\beta_0 \left(H - \int d^3x\, \mu_0(x)\, j_V^t\right)}$$

$$j_V^\mu = \bar{\Psi}\gamma^\mu\Psi$$

Finite μ & T QFT

Flat spacetime

* Massless fermion $\mathcal{L}[V_\mu] = \bar{\Psi}(i\not{\partial} - \not{V})\Psi$ $\quad \mu \in \{t, i\}$

* Imaginary-time formalism:

$$Z = \mathrm{Tr}\, e^{-\beta_0 \left(H - \int d^3x\, \mu_0(x)\, j_V^t\right)} = \int_{b.c} \mathcal{D}\bar{\Psi}\mathcal{D}\Psi\, e^{-\int_0^{\beta_0} dt \int d^3x\, \mathcal{L}[v]}$$

$$j_V^\mu = \bar{\Psi}\gamma^\mu\Psi$$

↳ $v_\mu = V_\mu - \delta_\mu^t \mu_0(x)$

↳ b.c : $\{\Psi(i\beta_0) = -\Psi(0),\ \bar{\Psi}(i\beta_0) = -\bar{\Psi}(0)\}$

Finite μ & T QFT

* Electric field + chemical potential $\Rightarrow$ electro-chemical potential

$$\mu_0 \Rightarrow \mathcal{L}[\vartheta] \text{ with } \vartheta_t = V_t + \mu_0 = \mu_{ec}$$

Arbitrary separation

eg: J. Newman, N.P. Balsara
Electrochemical systems
2021

Finite μ & T QFT

* Electric field + chemical potential $\Rightarrow$ electro-chemical potential

$$\mu_0 \Rightarrow \mathcal{L}[\boxed{v}] \text{ with } v_t = V_t + \mu_0 = \mu_{ec}$$

eg: J. Newman, N.P. Balsara
Electrochemical systems
2021

Arbitrary separation

Reference point : eg $V_{elec} = V_t - V_t(\vec{E}=\vec{0})$, $\vec{E} = \text{grad}\, V_{elec}$

$$\mu = \mu_0 + V_t(\vec{E}=\vec{0})$$

$$\Rightarrow \boxed{\mu_{ec} = V_{elec} + \mu}$$

Finite μ & T QFT

* Electric field + chemical potential $\Rightarrow$ electro-chemical potential

$$\mu_0 \Rightarrow \mathcal{L}[v] \text{ with } v_t = V_t + \mu_0 = \mu_{ec}$$

eg: J. Newman, N.P. Balsara
Electrochemical systems
2021

* $$Z = \int_{b.c} \mathcal{D}\bar{\Psi}\mathcal{D}\Psi \, e^{-\int_0^{\beta_0} dt \int d^3x \, \bar{\Psi}\left[i\gamma^\mu(\partial_\mu + i\delta^i_\mu V_i) - \gamma^t \mu_{ec}\right]\Psi}$$

Finite μ & T QFT

Curved spacetime

Gravity

Finite μ & T QFT

Curved spacetime

Gravity

Fluid velocity $u^\mu = \frac{1}{\sqrt{1-v^2}}\begin{pmatrix}1\\ -\vec{v}\end{pmatrix}$, $\eta_{\mu\nu} = \begin{pmatrix}1 & & & \\ & -1 & & \\ & & -1 & \\ & & & -1\end{pmatrix}$

Finite μ & T QFT

Curved spacetime

Gravity

Fluid velocity

$$u^\mu = \frac{1}{\sqrt{1-v^2}}\begin{pmatrix}1\\ -\vec{v}\end{pmatrix}, \quad \eta_{\mu\nu} = \begin{pmatrix}1 & & & \\ & -1 & & \\ & & -1 & \\ & & & -1\end{pmatrix}$$

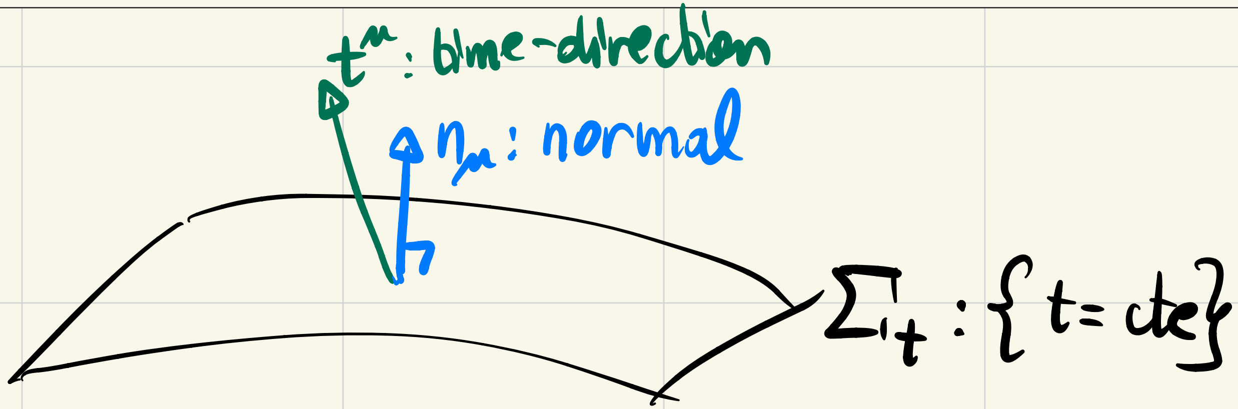
$$\downarrow \quad \begin{aligned} t' &= t \\ \vec{x}' &= \vec{x} + \vec{v}t \end{aligned}$$

$$u^\mu = \frac{1}{\sqrt{1-v^2}}\begin{pmatrix}1\\ \vec{0}\end{pmatrix}, \quad g_{\mu\nu} = \begin{pmatrix}1-v^2 & \vec{v}\\ \vec{v} & -1_3\end{pmatrix}$$

Fluid rest frame

Finite μ & T QFT

* Foliation:

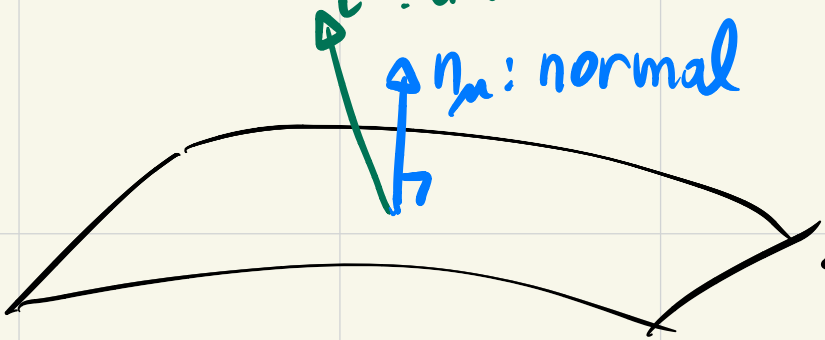


* $$Z = \mathrm{Tr}\, e^{-\int d\Sigma_4 n_\mu \left(T^{\mu\nu}\beta_\nu - \frac{\mu_0}{T} j^\mu_V\right)}$$

Landau, Lifshits

Finite μ & T QFT

* Foliation:



* $$Z = \mathrm{Tr}\, e^{-\int d\Sigma_t\, n_\mu \left(T^{\mu\nu} \beta_\nu - \frac{\mu_0}{T} j^\mu_V \right)}$$

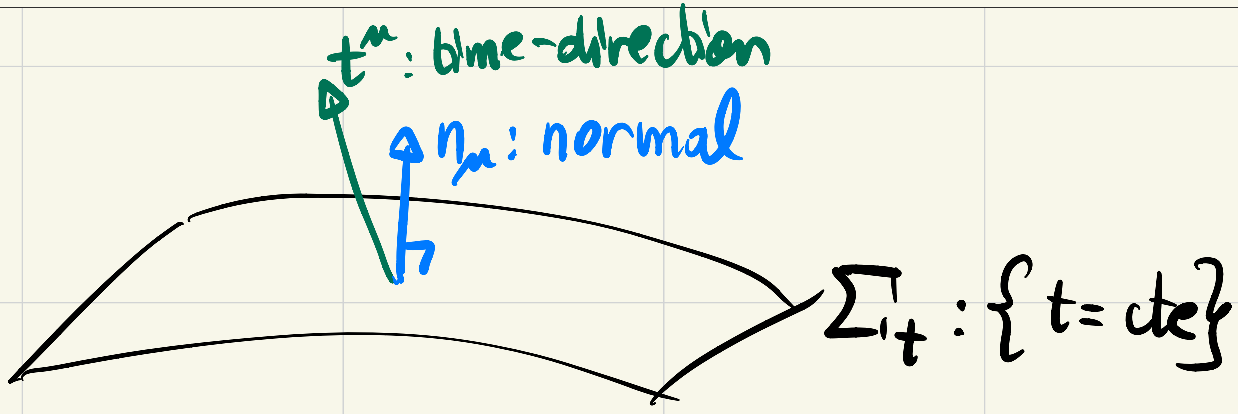
Energy-momentum tensor

$\bar{\psi} \gamma^\mu \psi$

Landau, Lifshits

Finite μ & T QFT

* Foliation:



* $$Z = \mathrm{Tr}\, e^{-\int d\Sigma_t\, n_\mu \left(T^{\mu\nu} \beta_\nu - \frac{\mu_0}{T} j^\mu \right)}$$

Landau, Lifshits

Energy-momentum tensor

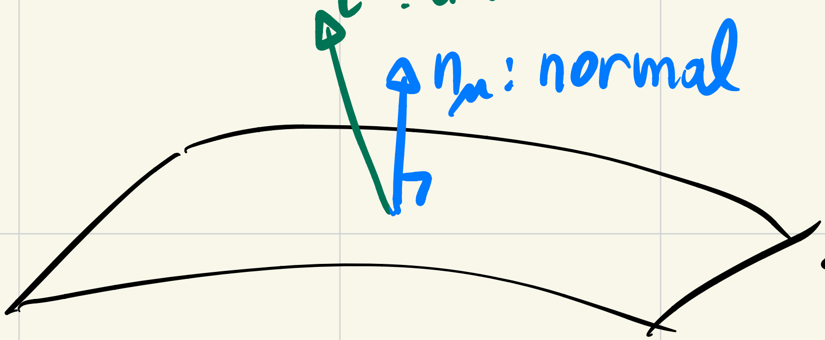
$\bar{\Psi} \gamma^\mu \Psi$

4-temperature

$$\beta_\mu = \frac{u_\mu}{T_0}, \quad \beta^2 = \frac{1}{T(x)^2}$$

Finite μ & T QFT

* Foliation:



* $$Z = \operatorname{Tr} e^{-\int d\Sigma_\mu n_\mu \left(T^{\mu\nu}\beta_\nu - \frac{\mu_0}{T} j^\mu_V \right)}$$

Landau, Lifshits

$T^{\mu\nu}$: Energy-momentum tensor

β_ν: 4-temperature

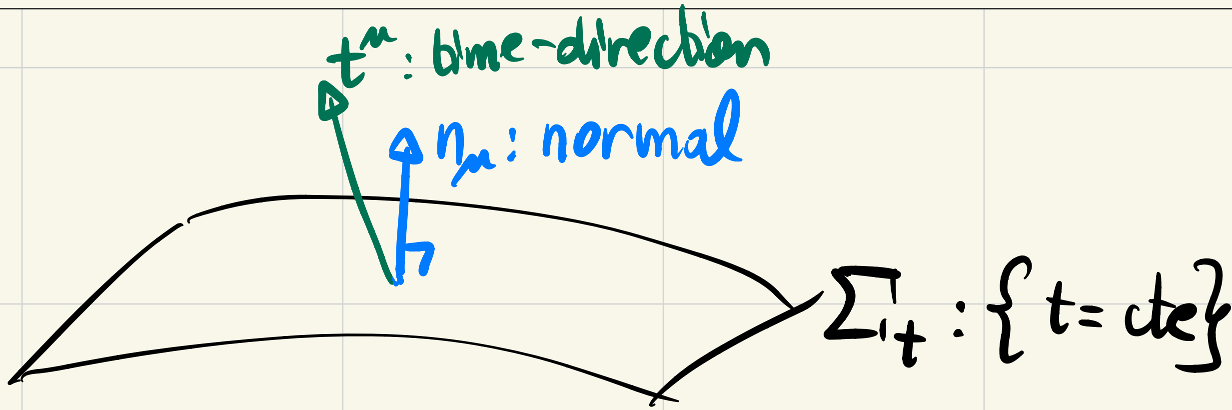
j^μ_V: $\bar{\psi}\gamma^\mu\psi$

$$\beta_\mu = \frac{u_\mu}{T_0}, \quad \beta^2 = \frac{1}{T(x)^2}$$

T_0: Reference temperature

Finite μ & T QFT

* Foliation:



* Hydrostatic gauge:

Choose coordinate s.t. $\begin{cases} t^\mu \doteq \begin{pmatrix} 1 \\ \vec{0} \end{pmatrix} \\ \beta^\mu \propto t^\mu \end{cases}$ where β^μ 4-temperature

$$\beta^2 = \frac{t^\mu}{T_0} = \frac{1}{T(x)^2}$$

Rk: $T(x) = \frac{T_0}{\sqrt{g_{tt}}}$ but not global equilibrium

M. Hongo 2016

Finite μ & T QFT

$$* \quad Z = \mathrm{Tr}\, e^{-\int d\Sigma_i\, n_\mu \left(T^{\mu\nu}\beta_\nu - \frac{\mu_0}{T} j^\mu_V\right)}$$

$$= \int_{b.c} \mathcal{D}\bar{\Psi}\mathcal{D}\Psi\, e^{-\int_0^{\beta_0} dt \int d^3x\, \bar{\Psi}\left(i\nabla\!\!\!/ + i\not{\omega} - \gamma^i V_i - \gamma^t \sqrt{g_{tt}}\,\mu_{ec}\right)\Psi}$$

Finite μ & T QFT

* $$Z = \mathrm{Tr}\, e^{-\int d\Sigma_i\, n_\mu \left(T^{\mu\nu}\beta_\nu - \frac{\mu_0}{T} j^\mu_V\right)}$$

$$= \int_{b.c} \mathcal{D}\bar{\Psi}\mathcal{D}\Psi\, e^{-\int_0^{\beta_0} dt \int d^3x\, \bar{\Psi}\left(i\partial\!\!\!/ + \underbrace{i\omega\!\!\!/}_{\text{spin-connection}} - \overbrace{\gamma^i V_i}^{\text{vector gauge field}} - \gamma^t \underbrace{\sqrt{g_{tt}}\,\mu_{ec}}_{\text{electro-chemical potential}}\right)\Psi}$$

Rk: Diffeo-invariant measure $\Rightarrow$ $\Psi = g^{1/4}\Psi_0$ Fujikawa 81'

$\bar{\Psi} = g^{1/4}\bar{\Psi}_0$ Toms 87'

Global equilibrium

* Global equilibrium: Z independent from the time-slice

↳ β_μ Killing: $\nabla_\mu \beta_\nu + \nabla_\nu \beta_\mu = 0 \iff \partial_t g_{\mu\nu} = 0$

↳ $\partial_\mu \left(\frac{\mu_{ec}}{T} \right) = 0 \iff T \partial_\mu \left(\frac{\mu}{T} \right) + E_\mu = 0$

Israel, Stewart 79'

Chiral anomaly

* $\mathcal{L}$ at finite T & μ_{ee} invariant under $\Psi' = e^{i\theta\gamma_5}\Psi$

$\bar{\Psi}' = \bar{\Psi} e^{i\theta\gamma_5}$

$$\Leftrightarrow \partial_\mu j_5^\mu = \partial_\mu\left(\bar{\Psi}\gamma^\mu\gamma_5\Psi\right) = 0$$

Chiral anomaly

* $\mathcal{L}$ at finite T & μ_{ee} invariant under $\psi' = e^{i\theta\gamma_5}\psi$
$\bar\psi' = \bar\psi e^{i\theta\gamma_5}$

$$\Leftrightarrow \partial_\mu j_5^\mu = \partial_\mu\left(\bar\psi \gamma^\mu \gamma_5 \psi\right) = 0$$

* Z is not invariant:

$$Z = \int_{b.c(\psi,\bar\psi)} \mathcal{D}\bar\psi \mathcal{D}\psi \, e^{-\int_0^{\beta_0} dt \int d^3x \, \mathcal{L}[\psi,\bar\psi]} = \int_{b.c(\psi',\bar\psi')} \mathcal{D}\bar\psi' \mathcal{D}\psi' \, e^{-\int_0^{\beta_0} dt \int d^3x \, \mathcal{L}[\psi',\bar\psi']}$$

$\psi' = e^{i\theta\gamma_5}\psi$
$\bar\psi' = \bar\psi e^{i\theta\gamma_5}$

Chiral anomaly

* $\mathcal{L}$ at finite T & μ_{ee} invariant under
$$\Psi' = e^{i\theta\gamma_5}\Psi$$
$$\bar{\Psi}' = \bar{\Psi} e^{i\theta\gamma_5}$$

$$\Leftrightarrow \partial_\mu j_5^\mu = \partial_\mu\left(\bar{\Psi}\gamma^\mu\gamma_5\Psi\right) = 0$$

* Z is not invariant:

$$Z = \int_{b.c} \mathcal{D}\bar{\Psi}\mathcal{D}\Psi \, e^{-\int_0^{\beta_0} dt \int d^3x \, \mathcal{L}[\Psi, \bar{\Psi}]} = \int_{b.c} J[\theta] \, \mathcal{D}\bar{\Psi}\mathcal{D}\Psi \, e^{-\int_0^{\beta_0} dt \int d^3x \left(\mathcal{L} - \bar{\Psi}(\not\partial\theta)\gamma_5\Psi\right)}$$

$$\theta(t = i\beta_0) = \theta(t = 0)$$

$\Rightarrow$ b.c unchanged

Chiral anomaly

$$* \quad \log J[\theta] = \int_0^{\beta_0} dt \int d^3x\, \theta(t,\vec{x})\, \mathcal{A} = \log \frac{\det_{\beta_0}\left[i\not{D} - \gamma^t \sqrt{g_{tt}}\mu_{ec}\right]}{\det_{\beta_0}\left[i\not{D} - \gamma^t \sqrt{g_{tt}}\mu_{ec} - (\not{\partial}\theta)\gamma_5\right]}$$

Filoche, Quevillon, Vuong, R.L 23'

$$= \mathrm{Tr}_{\beta_0}\left[(\partial_\mu \theta)\gamma^\mu \gamma_5 \frac{1}{i\not{D} - \gamma^t \sqrt{g_{tt}}\mu_{ec}}\right] + \mathcal{O}(\theta^2)$$

$$D_\mu = \partial_\mu + \omega_\mu + i\,\delta^i_\mu V_i$$

Chiral anomaly

$$* \quad \log J[\theta] = \int_0^{\beta_0} dt \int d^3x\, \theta(t,\vec{x})\, \mathcal{A} = \log \frac{\det_{\beta_0}\left[i\cancel{D} - \gamma^t \sqrt{g_{tt}}\mu_{ec}\right]}{\det_{\beta_0}\left[i\cancel{D} - \gamma^t \sqrt{g_{tt}}\mu_{ec} - (\cancel{\partial}\theta)\gamma_5\right]}$$

Filoche, Quevillon, Vuong, R.L 23'
Quevillon, R.L, 23'

$$= \mathrm{Tr}\left[(\partial_\mu\theta)\gamma^\mu\gamma_5 \frac{1}{i\cancel{D} - \gamma^t\sqrt{g_{tt}}\mu_{ec}}\right]$$

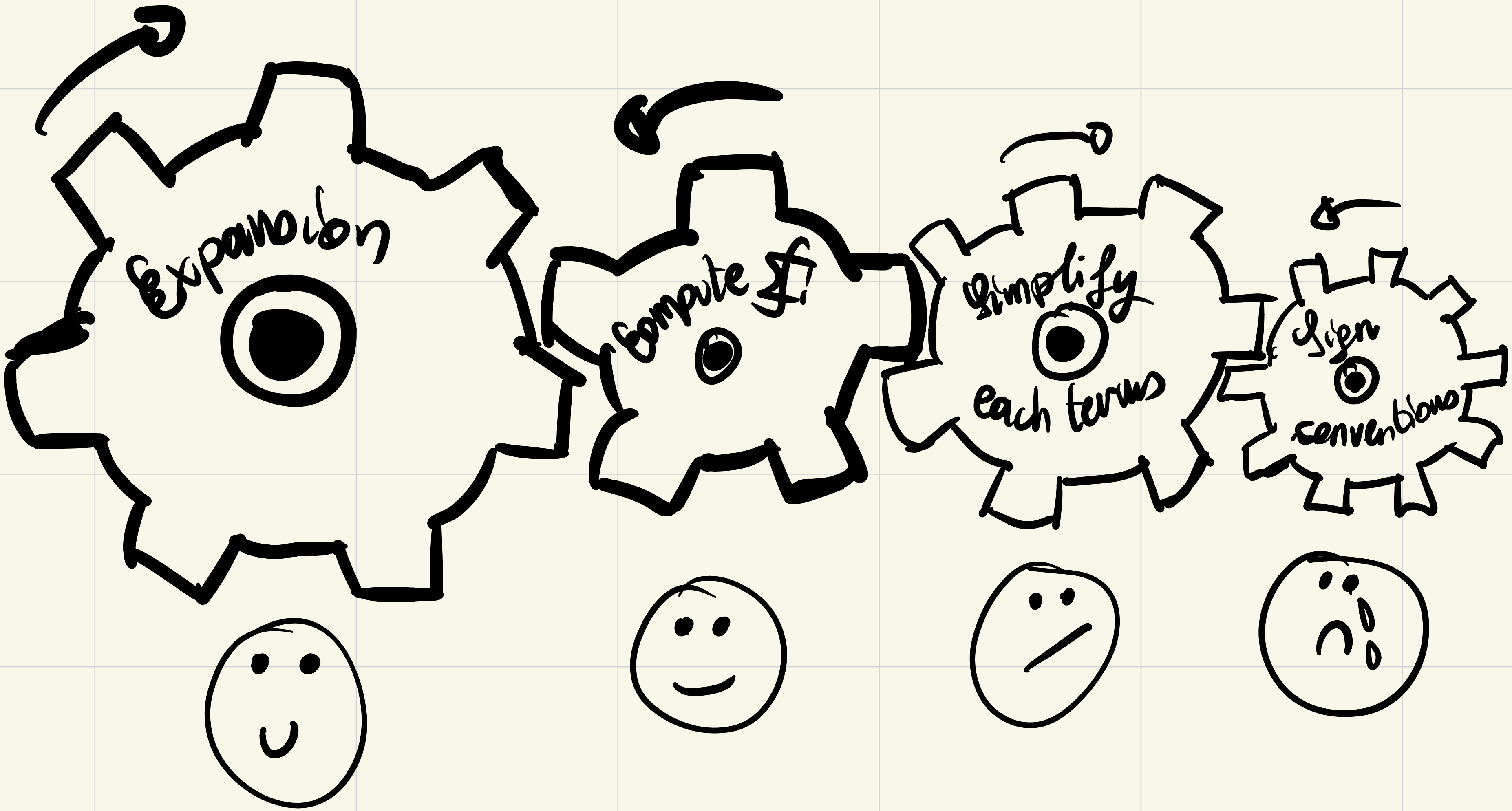
$$D_\mu = \partial_\mu + \omega_\mu + i\,\delta_\mu^i V_i$$

Covariant Derivative Expansion:

$$\boxed{\begin{gathered}\Rightarrow \log J[\theta] = -\int_0^{\beta_0} dt \int d^3x\, \frac{1}{\beta_0}\sum_{n\in\mathbb{Z}} \int \frac{d^3q}{(2\pi)^3}\, \mathrm{tr}(\cancel{\partial}\theta)\gamma_5 \sum_{k\geq 0}\left(\Delta_n(\vec{q})\, i\cancel{D}\right)^k \Delta_n(\vec{q}) \\ \Delta_n(\vec{q}) = \frac{\gamma^t\Omega_n + \gamma^i q_i}{g^{tt}\Omega_n^2 + 2g^{ti}q_i\Omega_n + g^{ij}q_iq_j}, \qquad \Omega_n = (2n+1)\pi T_0 - i\sqrt{g_{tt}}\mu_{ec}\end{gathered}}$$

Chiral anomaly

Compute....



Chiral anomaly

Local equilibrium axial current (analytic continuation to real-time)

$$\langle j_5^\mu \rangle = \underbrace{\left(\frac{\mu_{ec}^2(x)}{2\pi^2} + \frac{T^2(x)}{6}\right)\Theta^\mu}_{\text{CVE}} + \underbrace{\frac{\mu_{ec}(x)}{2\pi^2} B^\mu}_{\text{CSE}} + \mathcal{O}(\partial^3(V, \mu_{ec}, T, g))$$

Finite

Needs regularisation, then finite

Chiral anomaly

Local equilibrium axial current (analytic continuation to real-time)

$$\langle \hat{j}_5^\mu \rangle = \underbrace{\left(\frac{\mu_{ec}^2(x)}{2\pi^2} + \frac{T^2(x)}{6}\right) \Theta^\mu}_{\text{CVE}} + \underbrace{\frac{\mu_{ec}(x)}{2\pi^2} B^\mu}_{\text{CSE}} + \mathcal{O}(\partial^3(V, \mu_{ec}, T, g))$$

Finite

Needs regularisation, then finite

⚠ $$\Theta^\mu = -\frac{1}{2} \varepsilon^{\mu\nu\rho\sigma} u_\nu u^\lambda \underbrace{\omega_{\lambda,\rho\sigma}}_{\text{spin-connection}}$$

instead of $\Omega^\mu = \frac{1}{2} \varepsilon^{\mu\nu\rho\sigma} u_\nu \partial_\rho u_\sigma$

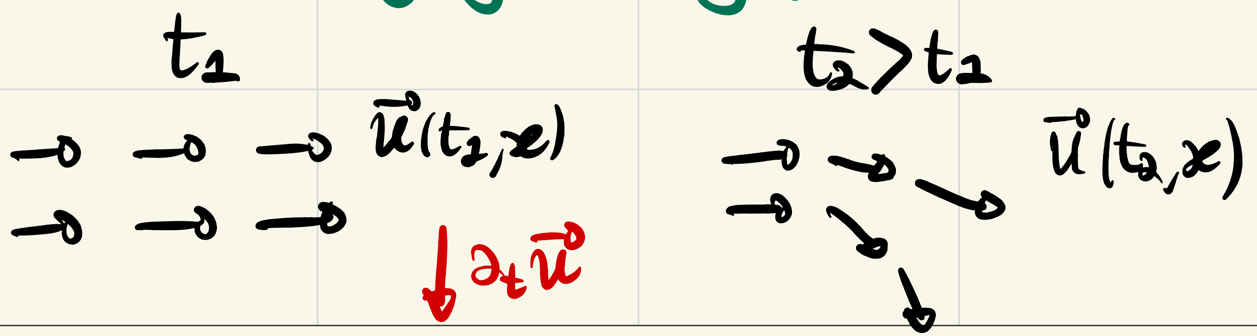
Chiral anomaly

Local equilibrium axial current: (analytic continuation to real-time)

$$\langle j_5^\mu \rangle = \underbrace{\left(\frac{\mu_{ec}^2(x)}{2\pi^2} + \frac{T^2(x)}{6} \right) \Theta^\mu}_{\text{CVE}} + \underbrace{\frac{\mu_{ec}(x)}{2\pi^2} B^\mu}_{\text{CSE}} + \mathcal{O}\left(\partial^3 (V, \mu_{ec}, T, g)\right)$$

Take eg $\partial_t g_{ij} = 0 \Rightarrow$ $\Theta^\mu = \Omega^\mu - \frac{1}{2} \delta^\mu_i (\vec{u} \wedge \partial_t \vec{u})^i$

Bending of velocity field



Chiral anomaly

Local equilibrium axial current: (analytic continuation to real-time)

$$\langle \hat{j}_5^\mu \rangle = \underbrace{\left(\frac{\mu_{ec}^2(x)}{2\pi^2} + \frac{T^2(x)}{6}\right) \Theta^\mu}_{\text{CVE}} + \underbrace{\frac{\mu_{ec}(x)}{2\pi^2} B^\mu}_{\text{CSE}} + \mathcal{O}(\partial^3(V, \mu_{ec}, T, g))$$

Take eg $\partial_t g_{\mu\nu} = 0 \Leftrightarrow \beta_\mu$ Killing $\Leftrightarrow$ Global equilibrium for T

$$\Rightarrow \Theta^\mu = \Omega^\mu$$

Chiral anomaly

Local equilibrium chiral anomaly:

$$\mathcal{A} = \frac{\mu_{ec}}{\pi^2} T \partial_\mu\left(\frac{\mu_{ec}}{T}\right) H^\mu + \left(\frac{\mu_{ec}^2}{2\pi^2} + \frac{T^2}{6}\right)\left(\nabla_\mu H^\mu + 2\frac{\partial_\mu T}{T} H^\mu\right) \Big] \text{ from CVE}$$

$$\text{from CSE } \Big[+ \frac{1}{2\pi^2} T \partial_\mu\left(\frac{\mu_{ec}}{T}\right) B^\mu - \frac{\mu_{ec}}{2\pi^2}\left(\frac{\partial_\mu T}{T} + a_\mu\right) B^\mu$$

$$+ \mathcal{O}\left(\partial^3(g, V, \mu_{ec}, T)\right)$$

Acceleration $a_\mu = u^\lambda \nabla_\lambda u_\mu$

Chiral anomaly

Local equilibrium chiral anomaly:

$$\mathcal{A} = \frac{\mu_{ec}}{\pi^2} T \partial_\mu\left(\frac{\mu_{ec}}{T}\right) \Theta^\mu + \left(\frac{\mu_{ec}^2}{2\pi^2} + \frac{T^2}{6}\right)\left(\nabla_\mu \Theta^\mu + 2\frac{\partial_\mu T}{T}\Theta^\mu\right) \Big] \text{ from CVE}$$

from CSE

$$\Big[+ \frac{1}{2\pi^2} T \partial_\mu\left(\frac{\mu_{ec}}{T}\right) B^\mu - \frac{\mu_{ec}}{2\pi^2}\left(\frac{\partial_\mu T}{T} + a_\mu\right) B^\mu$$

$$+ \mathcal{O}\left(\partial^3(g, V, \mu_{ec}, T)\right)$$

$$T \partial_\mu\left(\frac{\mu}{T}\right) B^\mu + E_\mu B^\mu$$

Acceleration $a_\mu = u^\lambda \nabla_\lambda u_\mu$

Chiral anomaly

Local equilibrium chiral anomaly:

$$\mathcal{A} = \frac{\mu_{ec}}{\pi^2} T \partial_\mu\left(\frac{\mu_{ec}}{T}\right) \Theta^\mu + \left(\frac{\mu_{ec}^2}{2\pi^2} + \frac{T^2}{6}\right)\left(\nabla_\mu \Theta^\mu + 2\frac{a_\mu T}{T}\Theta^\mu\right) \Big] \text{ from CVE}$$

from CSE

$$\Big[+ \frac{1}{2\pi^2} T \partial_\mu\left(\frac{\mu_{ec}}{T}\right) B^\mu - \frac{\mu_{ec}}{2\pi^2}\left(\frac{a_\mu T}{T} + a_\mu\right) B^\mu$$

$$+ \mathcal{O}(\partial^3(g, V, \mu_{ec}, T))$$

Acceleration $a_\mu = u^\lambda \nabla_\lambda u_\mu$

Topological: arises from $\mathcal{F}^e \tilde{\mathcal{F}}^e$ with

$$\mathcal{F}^e_{\mu\nu} = \partial_\mu \mathcal{V}_\nu - \partial_\nu \mathcal{V}_\mu, \quad \mathcal{V}_\mu = V_\mu + S^t_\mu \sqrt{\beta_{tt}}\, \mu_{ec}$$

Chiral anomaly

Local equilibrium chiral anomaly:

$$\mathcal{A} = \frac{\mu_{ec}}{\pi^2} T \partial_\mu\left(\frac{\mu_{ec}}{T}\right) \Theta^\mu + \left(\frac{\mu_{ec}^2}{2\pi^2} + \frac{T^2}{6}\right)\left(\nabla_\mu \Theta^\mu + 2\frac{\partial_\mu T}{T}\Theta^\mu\right)$$ from CVE

from CSE $\left[+\frac{1}{2\pi^2} T \partial_\mu\left(\frac{\mu_{ec}}{T}\right) B^\mu - \frac{\mu_{ec}}{2\pi^2}\left(\frac{\partial_\mu T}{T} + \partial_\mu\right) B^\mu\right.$

$+ \mathcal{O}(\partial^3(g, V, \mu_{ec}, T))$

Not topological!

Not Lorentz invariant: $\Theta^\mu \propto$ spin-connection

UV (& IR) finite, no regularisation involved

Counter-terms? Fujikawa method?

Global equilibrium

* Sufficient conditions : β_μ Killing, $\partial_\mu\left(\frac{\mu_{ec}}{T}\right)=0$

$\Rightarrow$ $\boxed{\mathcal{A} = 0}$

⚠ Global equilibrium with electric field $\Rightarrow$ $\mathcal{A} \neq \# \vec{E}\cdot\vec{B}$

* Fujikawa, Leutwyler procedures $\Rightarrow$ same result $\mathcal{A}=0$

Summary

* First computation of chiral anomaly with:
  - generic background curvature & electromagnetic field
  - generic fluid velocity
  - local equilibrium $\mu(x)$ & $T(x)$
* Quantum + vacuum $\Rightarrow$ $\mathcal{A} \propto \vec{E}\cdot\vec{B}$

  Quantum + fluid $\Rightarrow$ $\mathcal{A} \neq \vec{E}\cdot\vec{B}$, $\partial T, \partial\mu, \partial v \ldots$
* Global equilibrium $\Rightarrow$ $\mathcal{A} = 0$