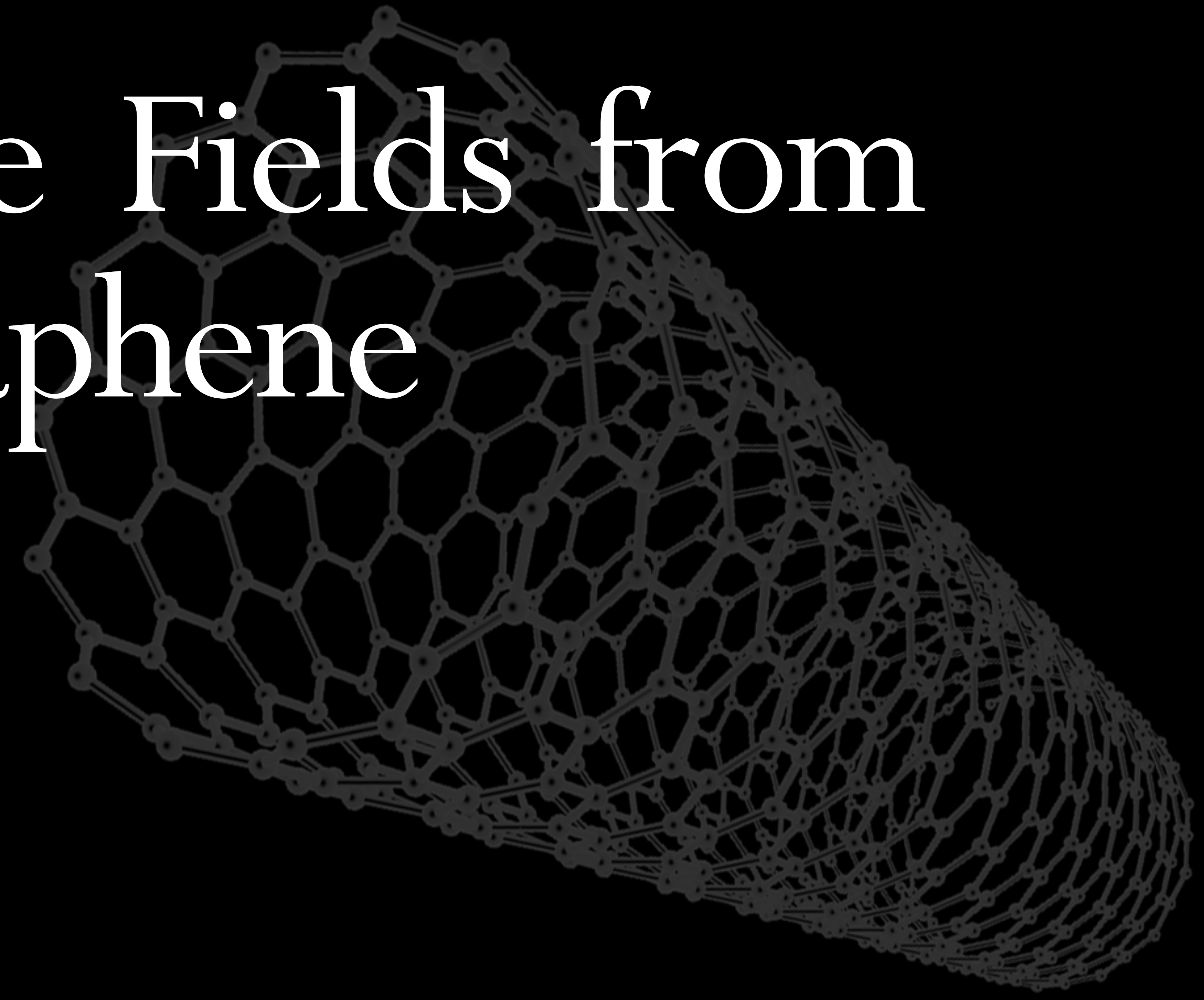
# Emergence of Pseudo-Gauge Fields from Evolving Geometries in Graphene

—Weyl & Dirac Semimetals as a Laboratory for HEP—



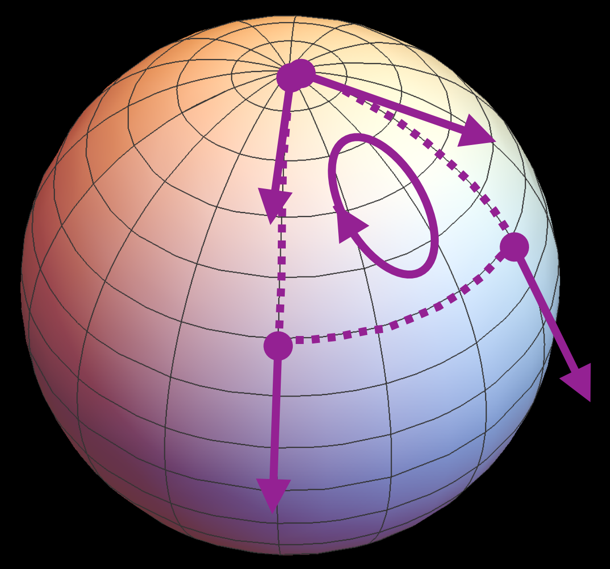
Pablo A. Morales

ARAYA RESEARCH

Imperial College London

# Fermions on Curved Spacetime

$$\Gamma^{\mu}{}_{\nu\rho} \longrightarrow \omega_{\mu}{}^{ab}$$



- Considering change under local Lorentz transformations

$$\psi' = S[L(x)]\psi(x)$$

Infinitesimally $\downarrow$

$$1 + i\delta\epsilon^{ab}\Sigma_{ab}$$

$$[\nabla_\mu, \nabla_\nu]\psi = \frac{1}{4}\underline{\gamma}^{\lambda}\underline{\gamma}^{\sigma} R_{\mu\nu\lambda\sigma}\psi$$

$$(\underline{\gamma}^{\mu}\nabla_\mu)^2 = \Box - (1/4)R$$

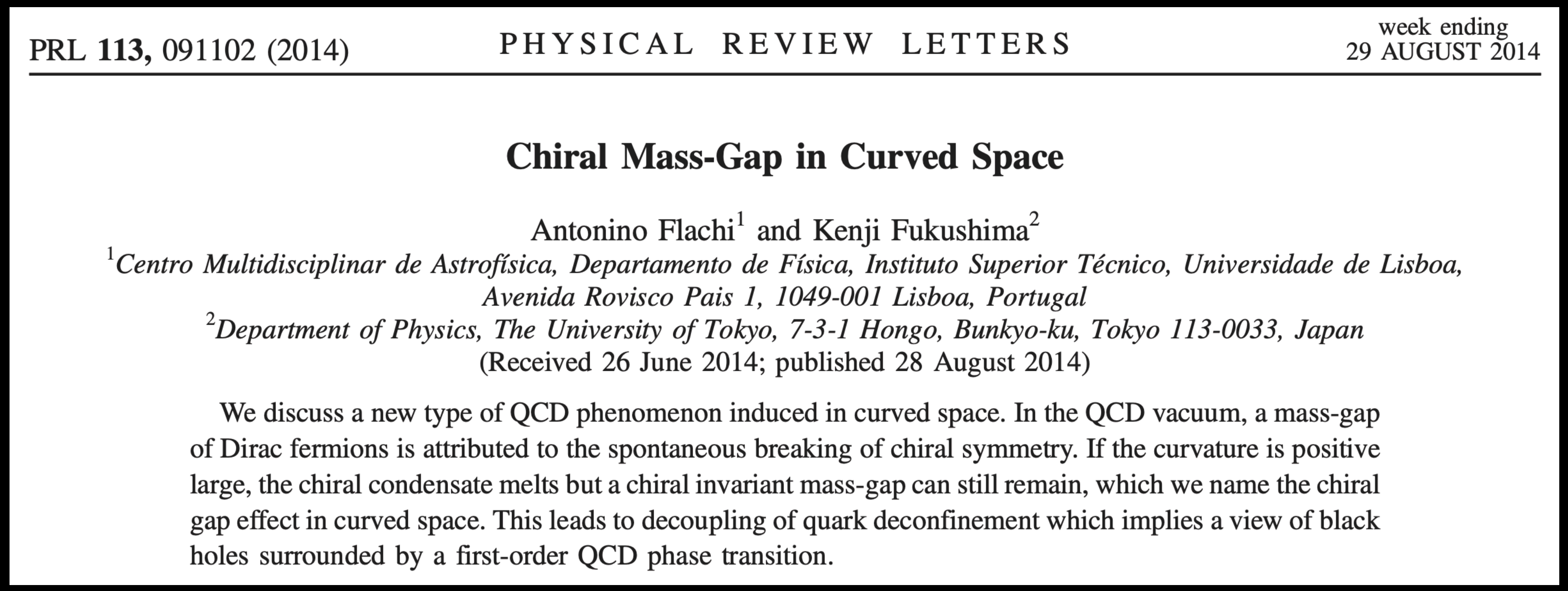
- This observation has profound consequences

Grand potential $\quad \log \mathrm{Det}\left[\Box + M_{\rm eff}^2 + \frac{R}{4}\right]$

contain R → resummation

$$M_{\rm eff} = G\langle\bar{\psi}\psi\rangle \qquad M_{\rm eff}^2 \to M_{\rm eff}^2 + \frac{R}{12}$$

[Fukushima, Flachi, PRL 113, 091102 (2014)]



PRL **113,** 091102 (2014) PHYSICAL REVIEW LETTERS week ending 29 AUGUST 2014

**Chiral Mass-Gap in Curved Space**

Antonino Flachi[1] and Kenji Fukushima[2]

[1] *Centro Multidisciplinar de Astrofísica, Departamento de Física, Instituto Superior Técnico, Universidade de Lisboa, Avenida Rovisco Pais 1, 1049-001 Lisboa, Portugal*

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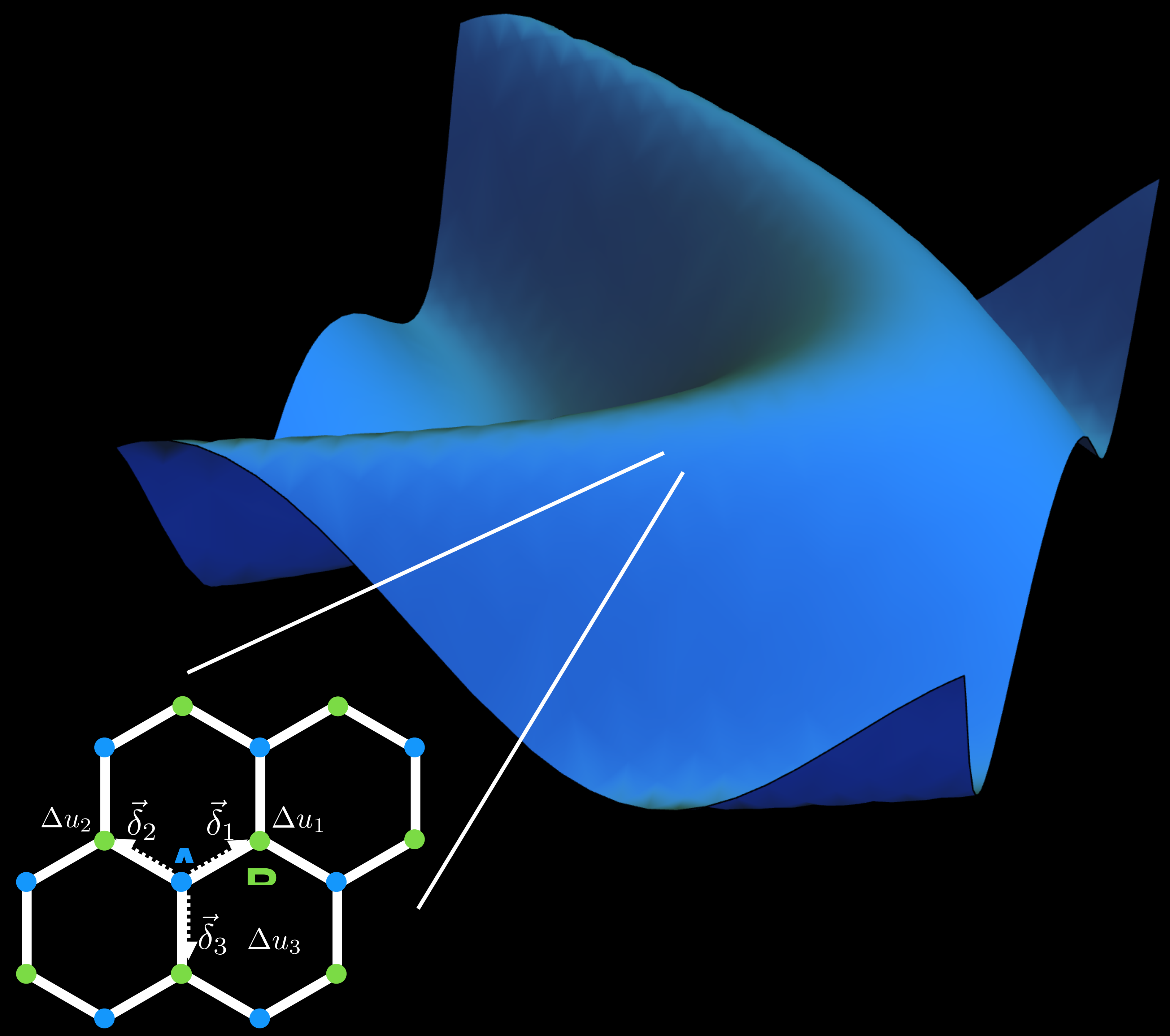
(Received 26 June 2014; published 28 August 2014)

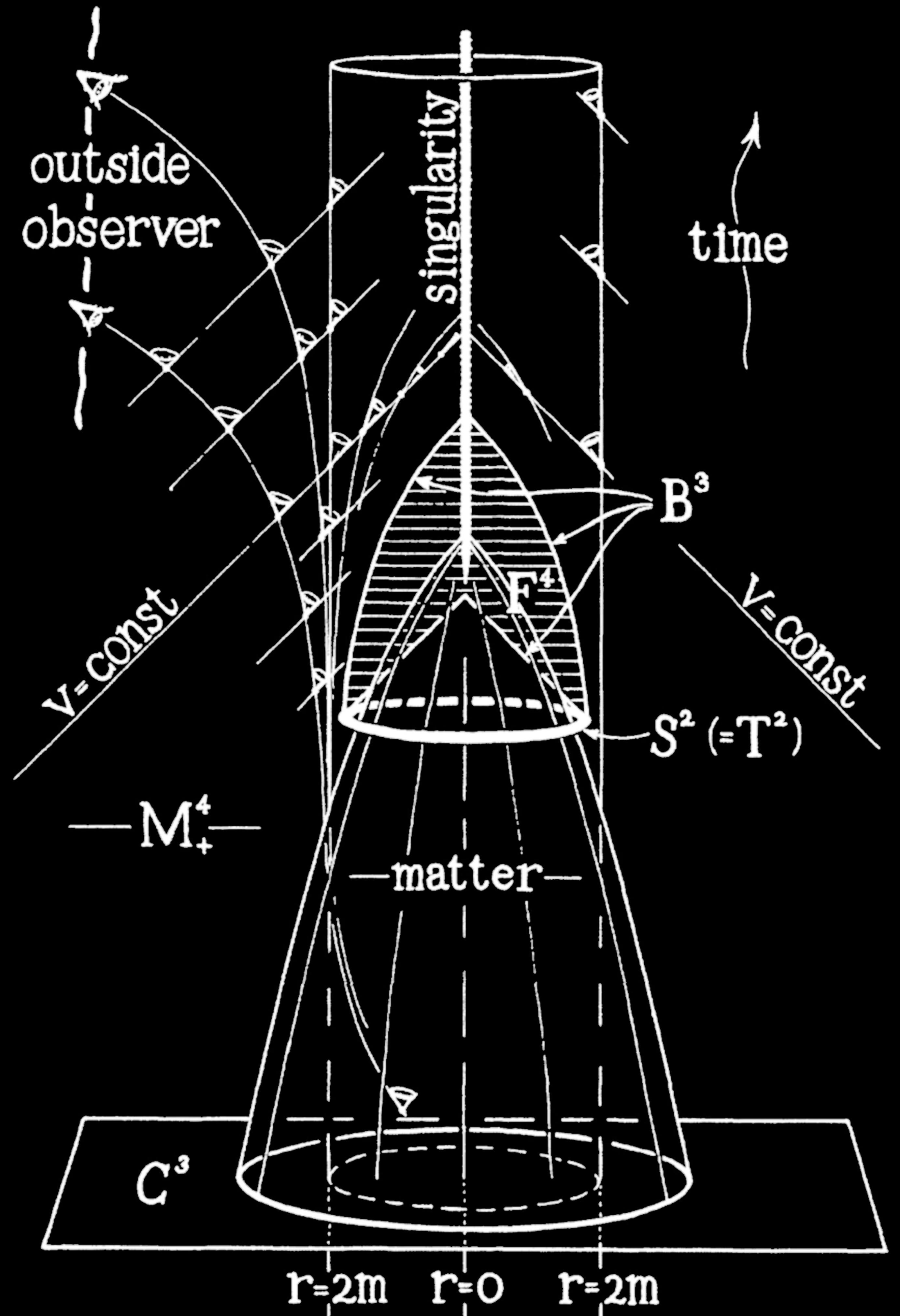
We discuss a new type of QCD phenomenon induced in curved space. In the QCD vacuum, a mass-gap of Dirac fermions is attributed to the spontaneous breaking of chiral symmetry. If the curvature is positive large, the chiral condensate melts but a chiral invariant mass-gap can still remain, which we name the chiral gap effect in curved space. This leads to decoupling of quark deconfinement which implies a view of black holes surrounded by a first-order QCD phase transition.

# Two-fold motivation

Morphological Deformations




Space-Time Geometry



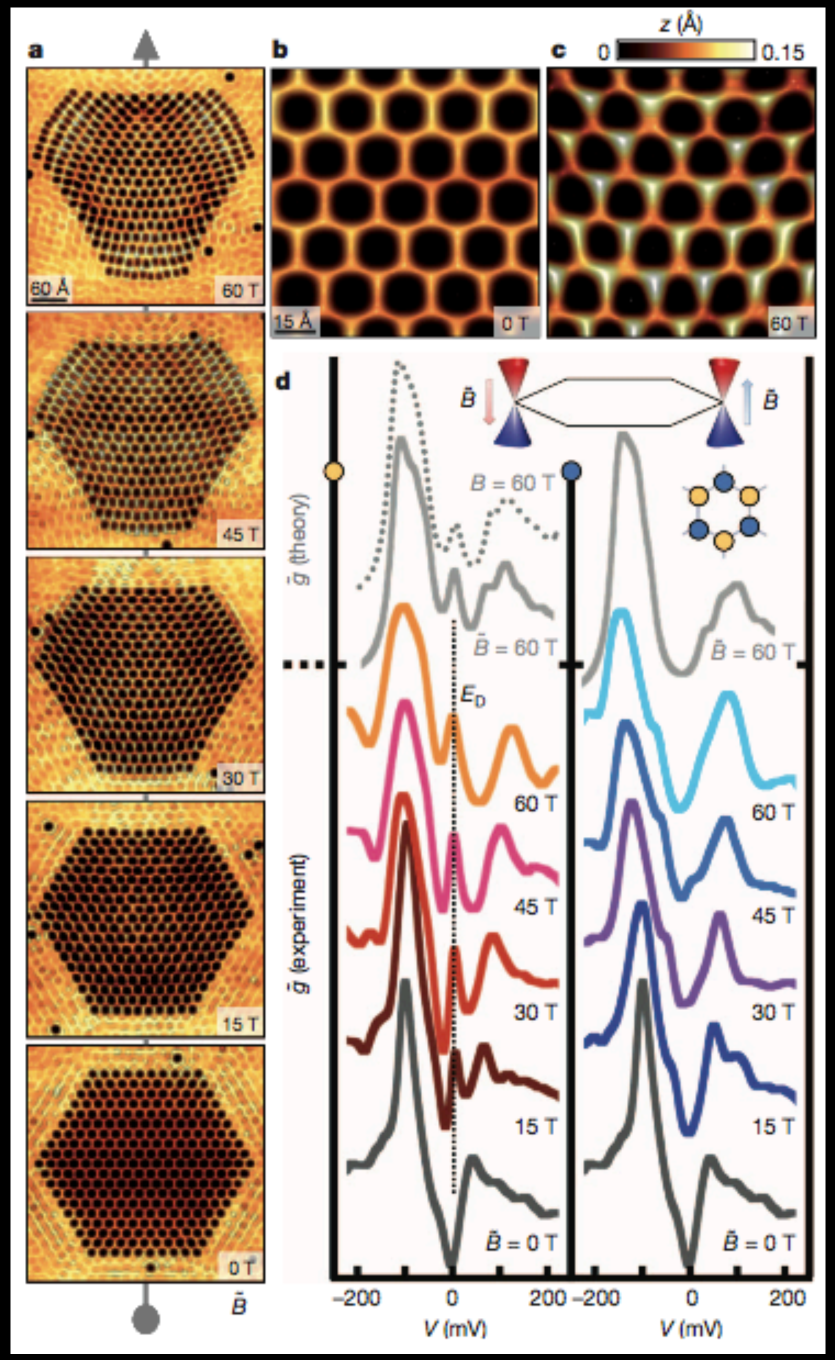
# A thus Pandora’s box is opened

LETTER

doi:10.1038/nature10941

**Designer Dirac fermions and topological phases in molecular graphene**

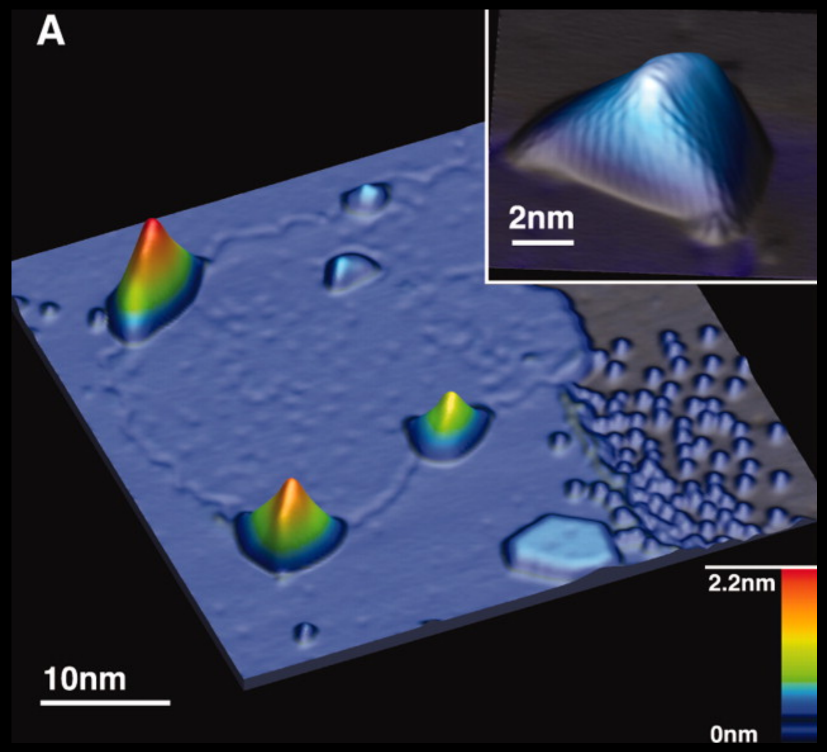
Kenjiro K. Gomes[1,2]*, Warren Mar[2,3]*, Wonhee Ko[2,4]*, Francisco Guinea[5] & Hari C. Manoharan[1,2]



(2012)

**Strain-Induced Pseudo–Magnetic Fields Greater Than 300 Tesla in Graphene Nanobubbles**

N. Levy,[1,2]*† S. A. Burke,[1]*‡ K. L. Meaker,[1] M. Panlasigui,[1] A. Zettl,[1,2] F. Guinea,[3] A. H. Castro Neto,[4] M. F. Crommie[1,2]§



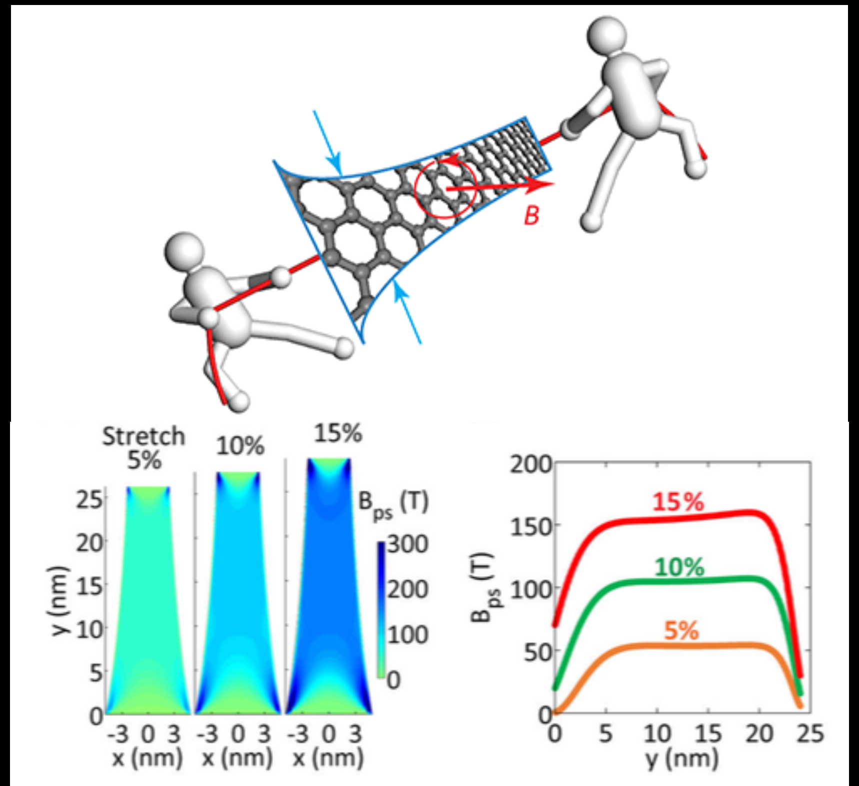

(2010)

(2015)

PRL 115, 245501 (2015) PHYSICAL REVIEW LETTERS week ending 11 DECEMBER 2015

**Programmable Extreme Pseudomagnetic Fields in Graphene by a Uniaxial Stretch**

Shuze Zhu,[1] Joseph A. Stroscio,[2] and Teng Li[1,*]
[1]Department of Mechanical Engineering, University of Maryland, College Park, Maryland 20742, USA
[2]Center for Nanoscale Science and Technology, NIST, Gaithersburg, Maryland 20899, USA
(Received 24 September 2015; published 8 December 2015)

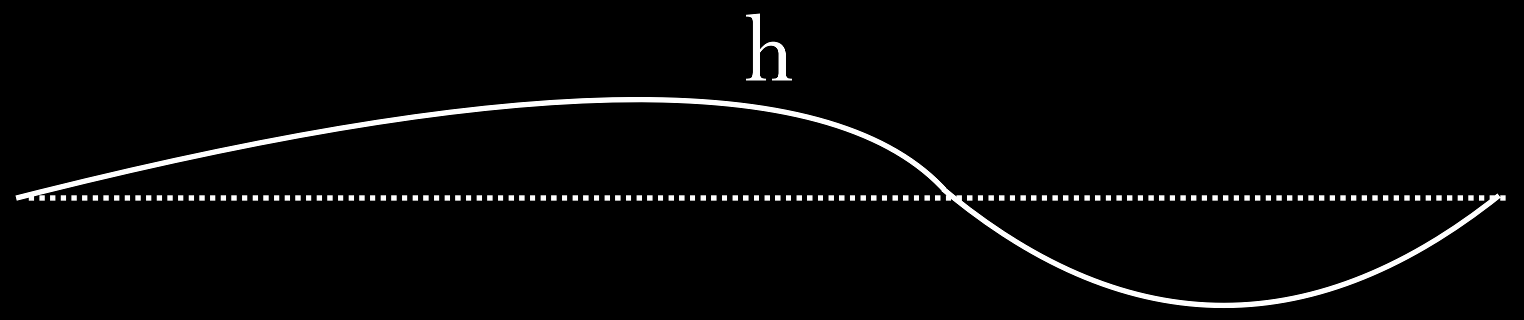


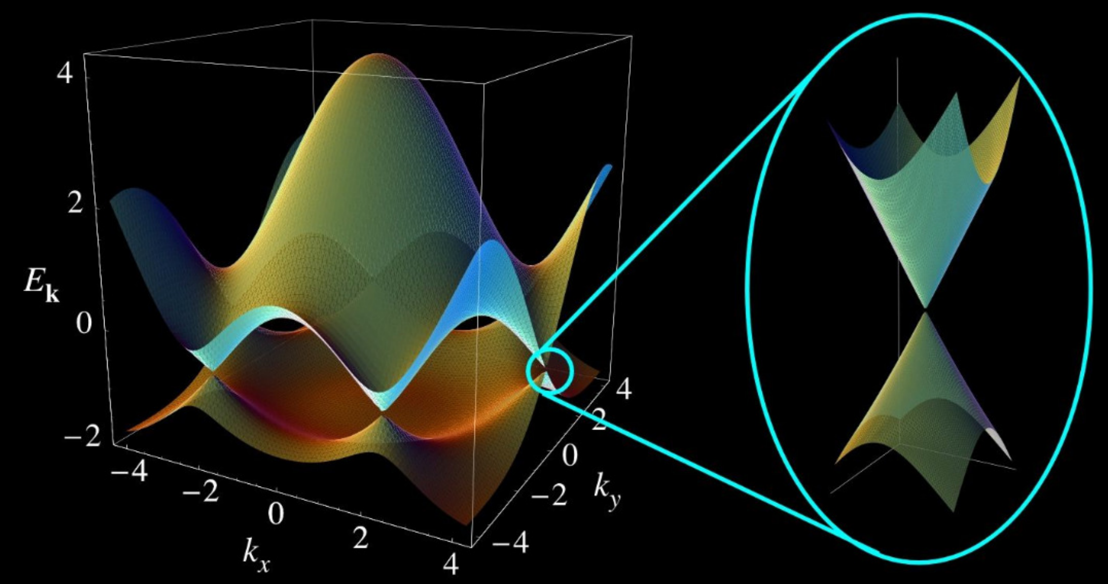
# A curved QFT description of graphene membranes and elastic theory

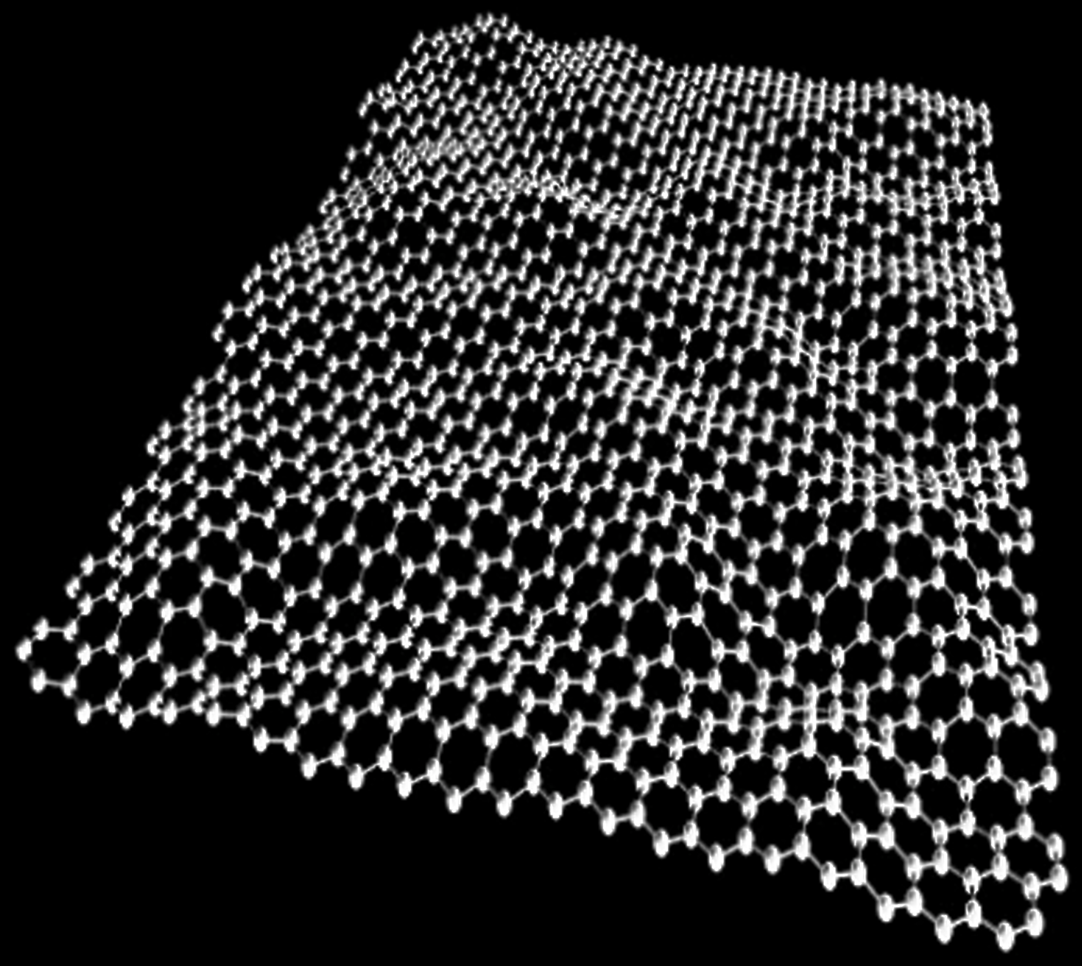
- QFT Hamiltonian in a curved space arising from a metric related to the strain tensor

$$g_{ij} = \eta_{ij} + u_{ij} \longrightarrow \begin{cases} e^i_a = \delta^i_a - \delta_{aj} u^{ij} \\ e_{ia} = \delta_{ia} + \delta^j_a u_{ij} \end{cases}$$

Strain tensor $$u_{ij} = \frac{1}{2}(\partial_i u_j + \partial_j u_i + \partial_i h \partial_j h)$$



Hamiltonian $$H = i \int d^2x \sqrt{g} \bar{\psi} \gamma^a e^i_a (\partial_i + \Omega_i) \psi$$

*[de Juan, Sturla, Vozmediano, PRL **108**, 227205 (2012)]*

# In the quest of a low-energy description

## —an ongoing story—

[Roberts, Wiseman, PRB **105**, 195412 (2022)]

Dirac dynamics need correction: $\gamma^a v_a^{ij}\partial_i\partial_j\Psi$

[A. Iorio and P. Pais, PRB **106**, 157401 (2022)]

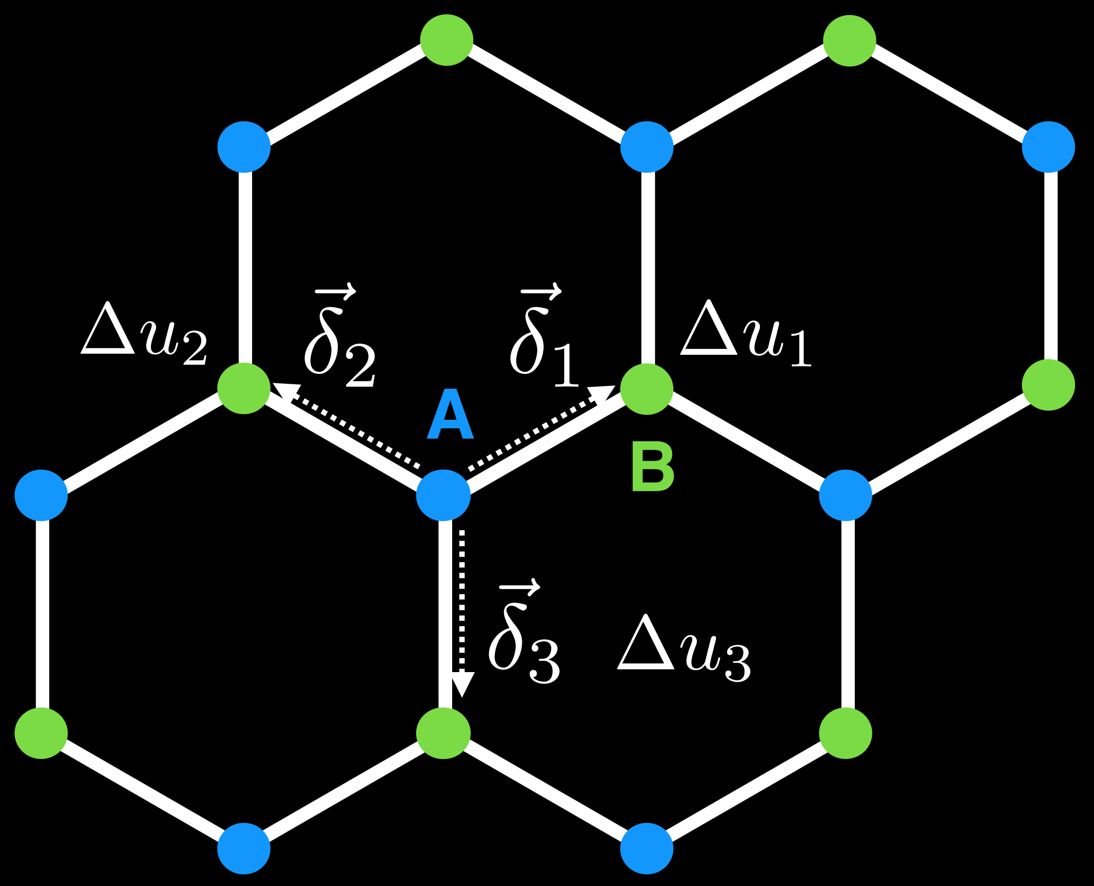
Really?! Might be the case for elastic def.

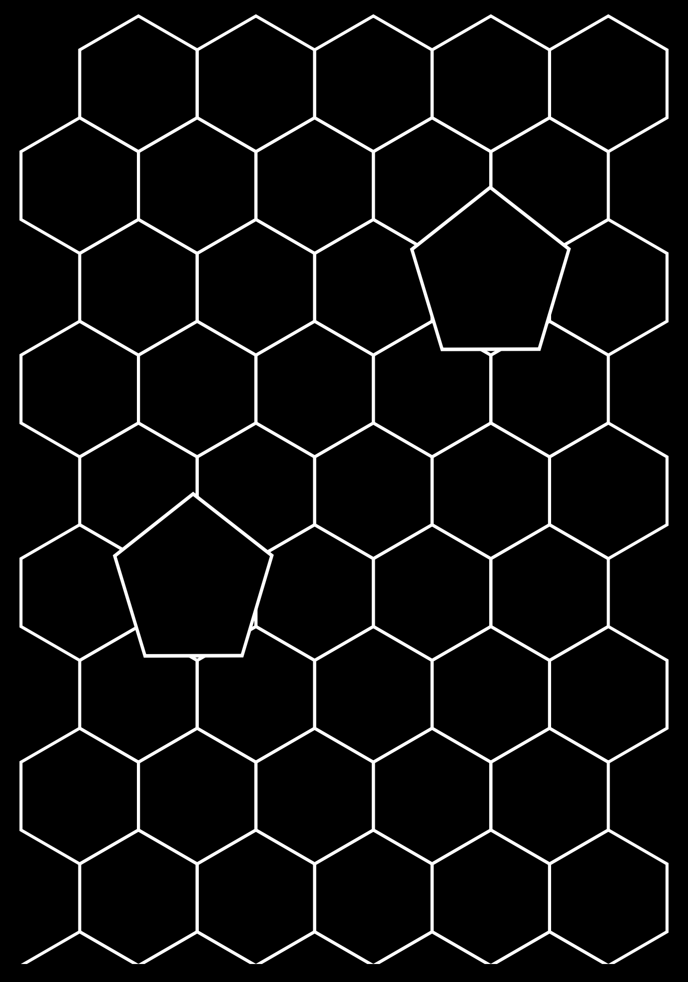
[Roberts, Wiseman, PRB **106**, 157402 (2022)]

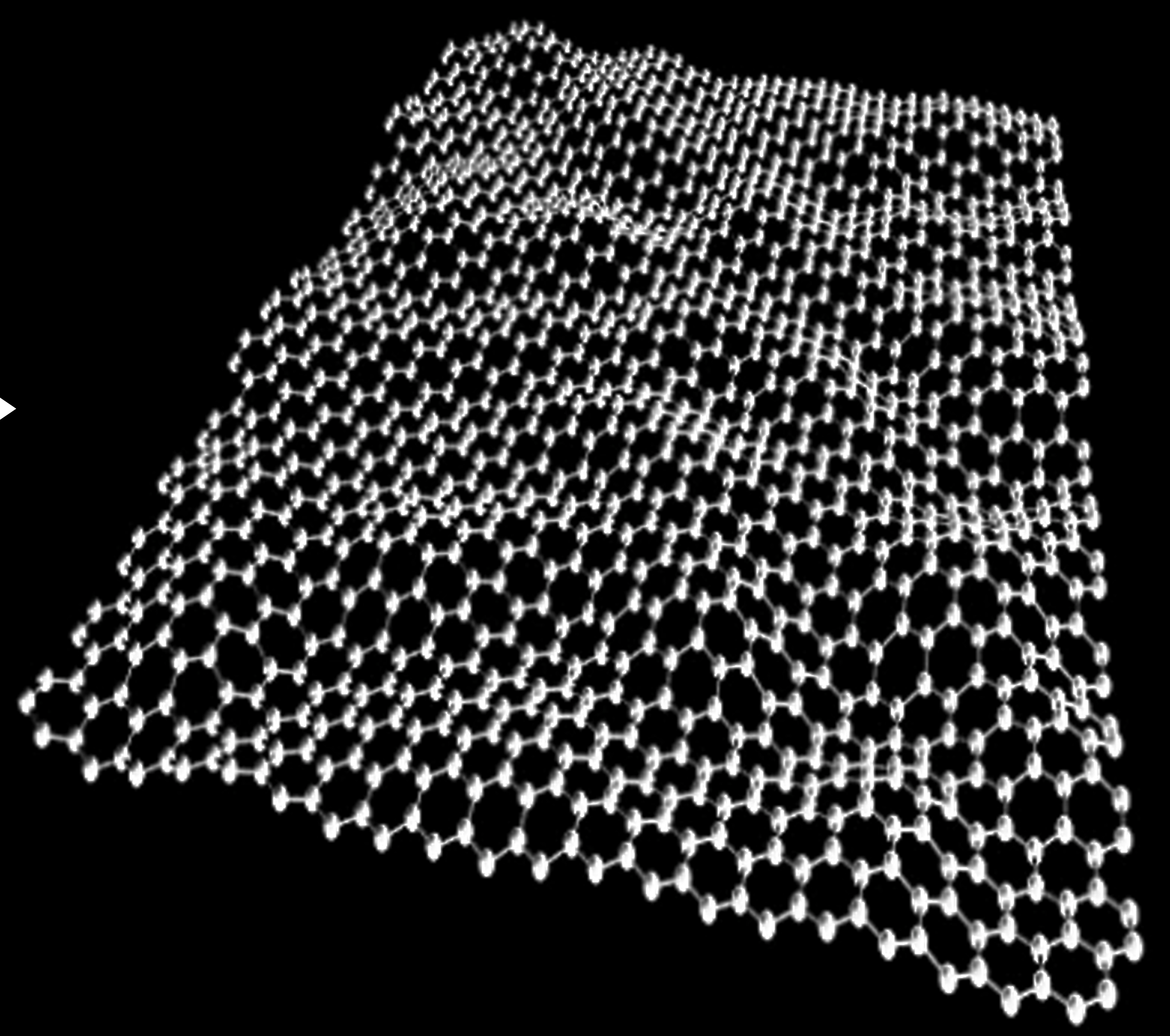
No! Both elastic and curved backgrounds

[Morales, Copinger, PRB **107**, 195412 (2023)]

Well, time distortions are disease free:)



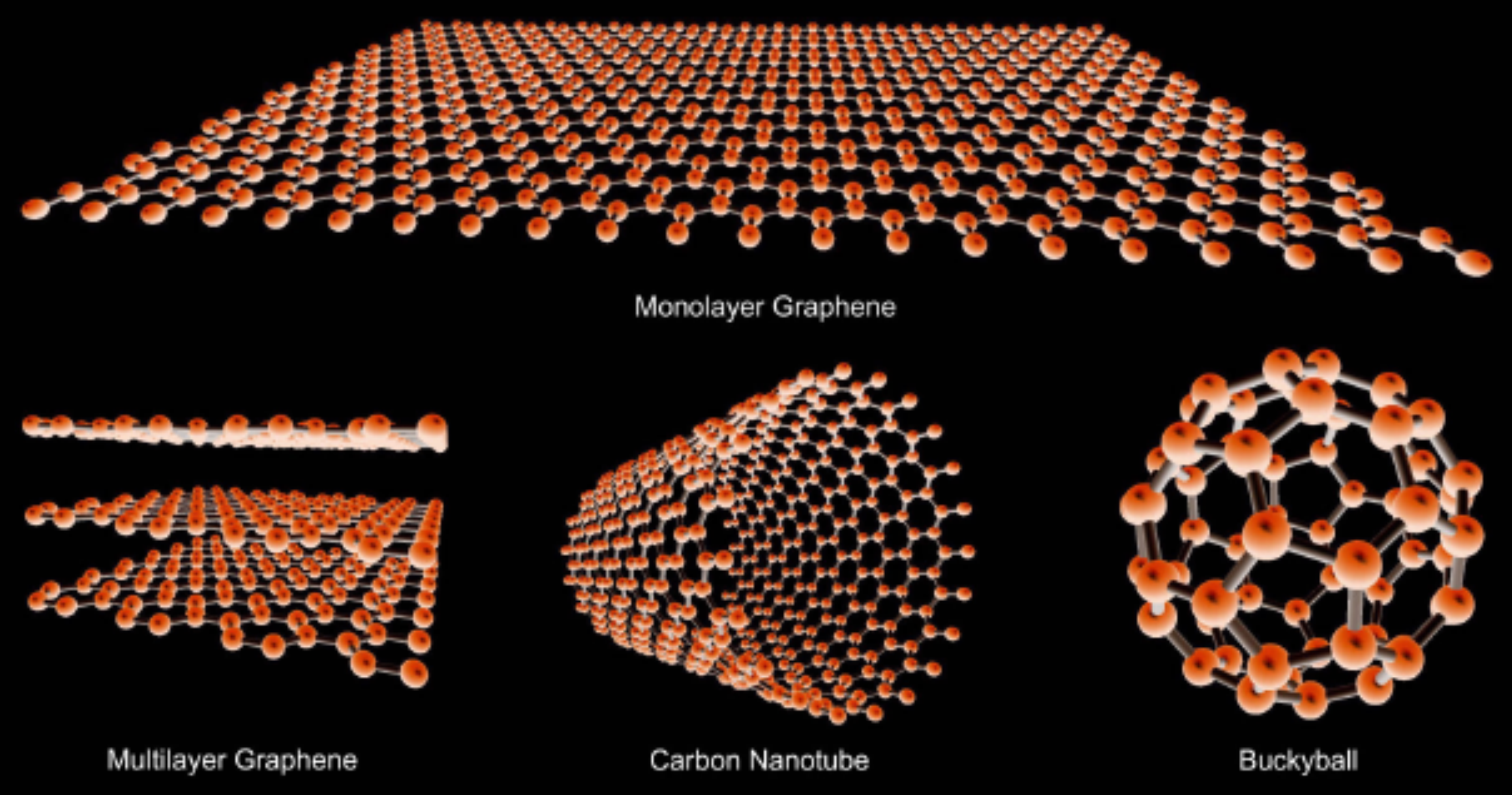


Types of deformation of graphene structures

- Disclinations, Stresses, topological defects

Deformations might be tricky: $\gamma^a v_a^{ij} \partial_i \partial_j \Psi$

[Roberts, Wiseman, PRB **105**, 195412 (2022)]




Is there a map from time-dependent curved space geometries to their corresponding quantum mechanical system.

Mapping to an effective quantum mechanical system, emergent pseudo-gauge fields may be recognized.

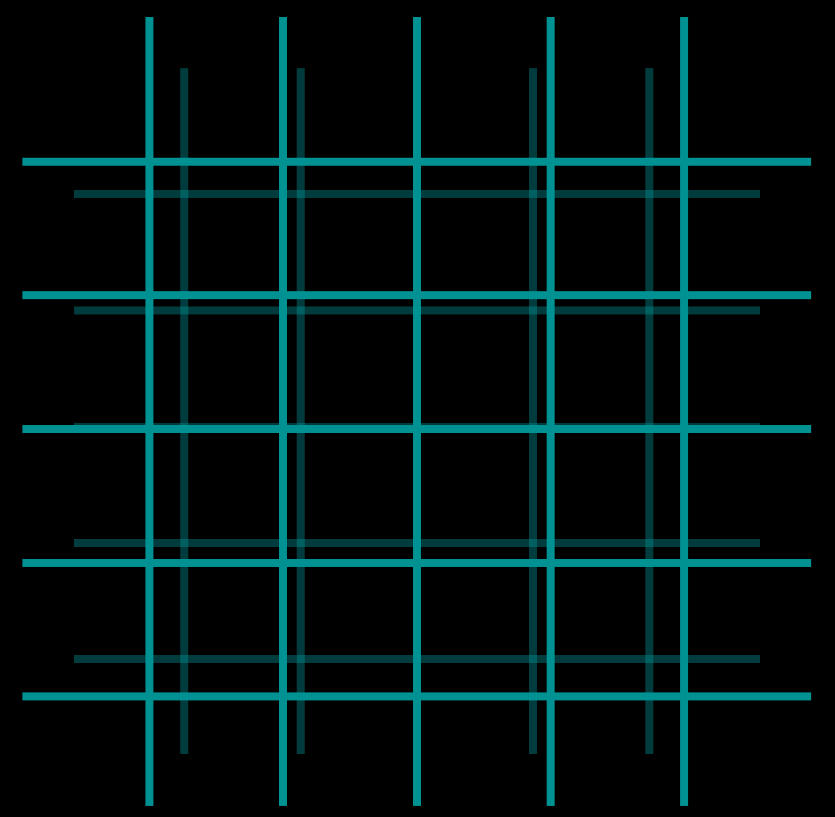
Two cases: 1. Adiabatic case
2. Non-Adiabatic high-frequency case

[Morales, Copinger, PRB **107**, 075432 (2023)]

# Time dependent backgrounds & FNC

Is it possible to generalize the curved spacetime description to include time dependent distortions of the metric? To address this, we specialize on the FLRW background.

$$ds^2 = -(dx^0)^2 + a(x^0)^2 \delta_{ij} dx^i dx^j$$



Fermions × expanding geometry

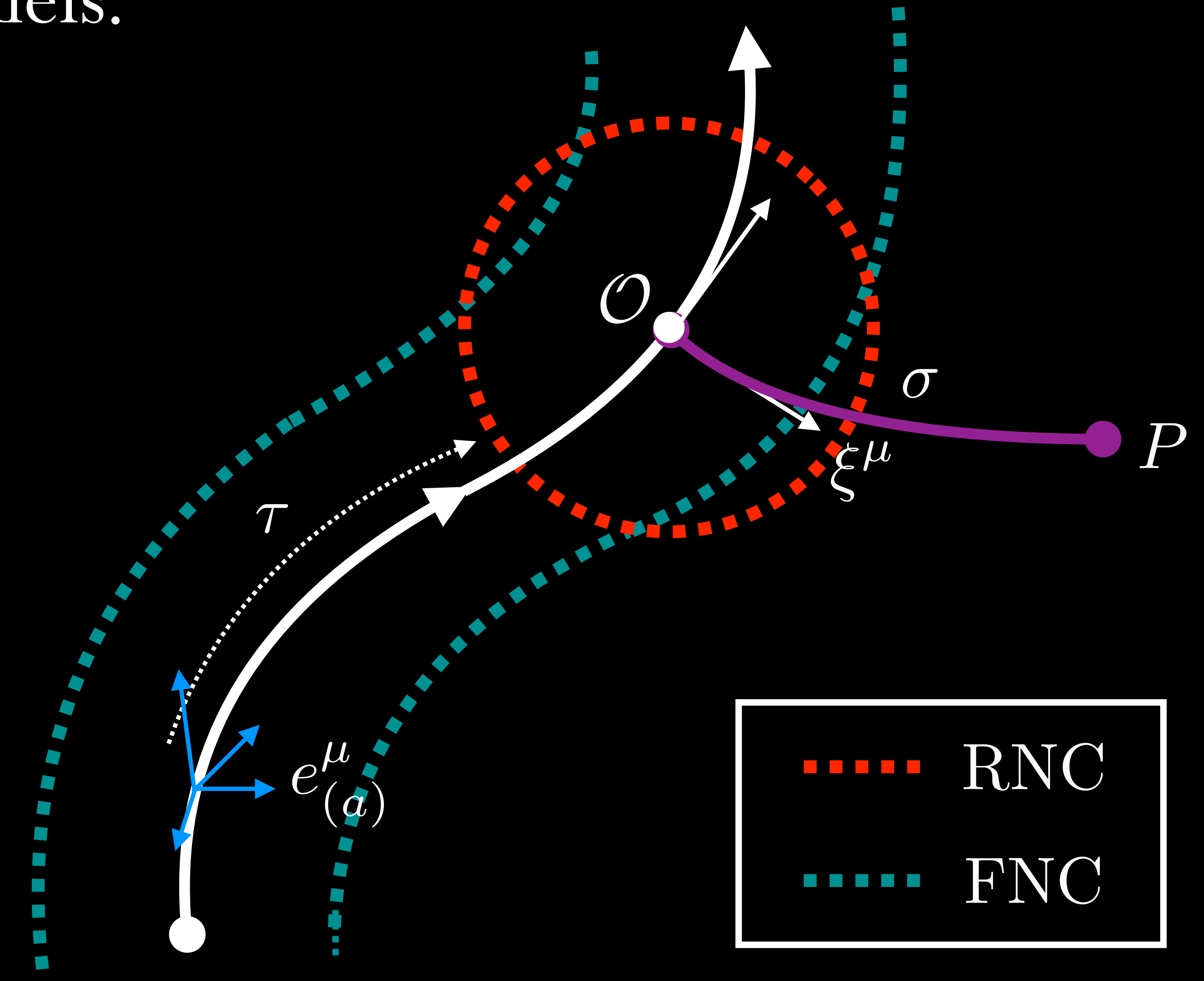
Benefits…

1. No higher derivative corrections.
2. Time dependent condensed matter setups.
3. Probe for cosmological models.

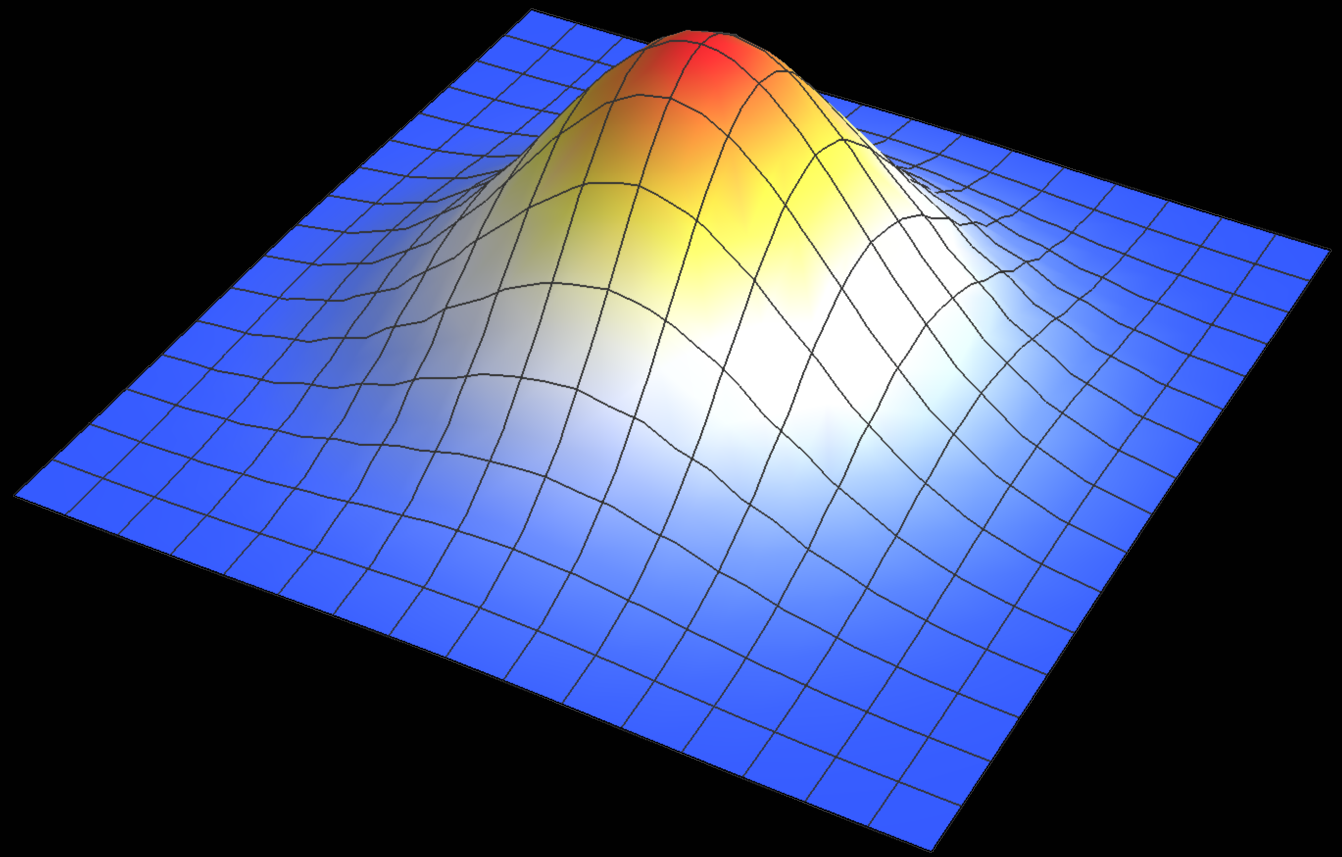
$$g_{00} = -1 - R_{0l0m} y^l y^m + \cdots ,$$

$$g_{0i} = -\frac{2}{3} R_{0lim} y^l y^m + \cdots ,$$

$$g_{ij} = \delta_{ij} - \frac{1}{3} R_{iljm} y^l y^m + \cdots ,$$



$$\mathcal{H} = -i\gamma_0\gamma^i(\partial_i - \Omega_i) + i\Omega_0 - \frac{i}{2}y^l y^m(R_{0l0m}\gamma_0\gamma^i\partial_i + R_{0lim}\partial^i) - \frac{i}{6}y^l y^m(R_{iljm}\gamma_0\gamma^j\partial^i + R_{0ljm}\gamma^j\gamma^i\partial_i)$$



An immediate drawback can be seen in that non-Hermitian terms are present, making comparison to an emergent quantum mechanical setting challenging

Particle number conservation in a time-dependent metric

Problems may be bypassed through careful consideration of the curved inner product

$$h_{\text{eff}} := -\sqrt{-g}\gamma^0\underline{\gamma}^0(x)\mathscr{H}$$

$$\left(\Box - \frac{1}{4}R + \bar{\Delta}\right)\bar{\mathscr{G}}(y,0) = -\delta(y)\mathbb{1}_s\,,$$

$$\mathscr{D}_2 := (\partial_a - \Omega_a)^2 - \eta^{ab}\Gamma^c_{ab}\partial_c - \frac{1}{4}R + \bar{\Delta}$$
$$+\, y^l y^m \left[R_{0l0m}\partial^i + \frac{1}{3}R^{i\ j}_{\ l\ m}\partial_j - \frac{4}{3}R_{0lim}\partial_0\right]\partial_i$$

[Morales, Copinger, PRB **107**, 075432 (2023)]

$$\mathscr{D}_2 \xrightarrow{\text{T.I.}} -\partial_0^2 + (\partial_i + i\sigma^3\mathcal{A}_i^{\mathrm{Ad}})^2 - \frac{1}{6}RL^2 - \frac{1}{12}R$$

$$B_{\mathrm{ps}} := \hbar R/4e$$

[Castro-Villareal, Ruiz-Sanchez, PRB **95**, 125432 (2017)]

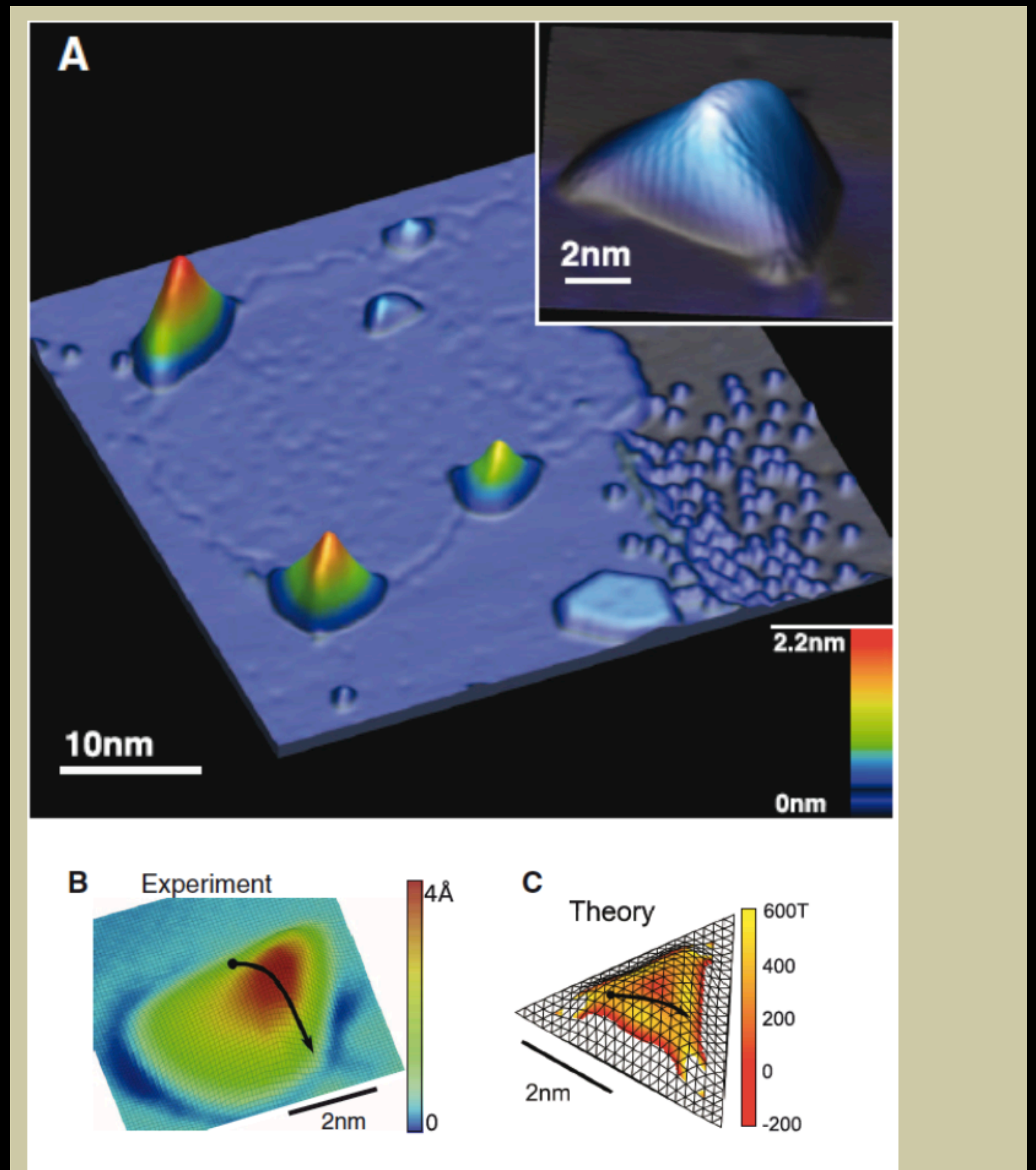



**Fig. 3.** (**A**) Experimental topographic line scan and experimentally determined $B_s$ profile over the tip trajectory shown by black line in (B). (**B**) STM topography of graphene nanobubble. (**C**) Topography of theoretically simulated graphene nanobubble with calculated $B_s$ color map. (**D**) Simulated topographic line scan and $B_s$ profiles extracted from line shown in (C).

**Strain-Induced Pseudo–Magnetic Fields Greater Than 300 Tesla in Graphene Nanobubbles**

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30 JULY 2010 VOL 329 SCIENCE www.sciencemag.org

$$\mathcal{A}_i^{\mathrm{Ad}} = \frac{1}{8}\left(R\epsilon_{im} + 4R^j{}_{[i}\epsilon_{m]j}\right)y^m \qquad \mathcal{A}_i^{\mathrm{F}} = \frac{1}{2}\left[\left(R_{00} + \frac{3}{4}R\right)\epsilon_{im} + R^j{}_{[i}\epsilon_{m]j} + R_m{}^j\epsilon_{ji}\right]y^m$$

[Morales, Copinger, PRB **107**, 075432 (2023)]

$$\mathscr{D}_2 = \mathscr{D}_2^{\rm EM} + \mathscr{D}_2^{\rm int}\,, \qquad \begin{aligned} \mathscr{D}_2^{\rm EM} &= -\partial_0^2 + (\partial_i + i\sigma^3 \mathcal{A}_i^{\rm Ad})^2 - \frac{1}{12}R\,, \\ \mathscr{D}_2^{\rm int} &= -H^2 y^i \sigma_i \partial_0 - H^2 y^i y_i \partial^j \partial_j - \frac{1}{3}H^2 L^2 \end{aligned}$$

DeWitt/Schwinger proper time integral $\bar{\mathscr{G}}(y,0) = \lim_{\epsilon\to 0} \int_0^\infty ds\, i\langle y| e^{-i[\mathscr{D}_2 + \epsilon(1-i)]s}|0\rangle$

Analogies with electromagnetic setup

$$P_\pm e^{-i\mathscr{D}_2^{\rm EM}s} = P_\pm e^{-i[-\partial_0^2 + (\partial_i \pm i\mathcal{A}_i^{\rm Ad})^2 - \frac{1}{12}R]s}$$

$$P_\pm e^{-i\not{\Pi}^2 s} = P_\pm e^{-i[-\partial_0^2 + (\partial_i + ieA_i)^2 \mp \epsilon^{ij} e F_{ij}]s}$$

Then comparing the above two one can see that for a projection of P+ of a curved space system leads to an equivalent electromagnetic system with positive coupling, e. Likewise for projection P_

HOW DO YOU MAKE SENSE OUT THE DISPERSION RELATION WITH TIME DEPENDANCE?! → Floquet/Magnus expansion

# Floquet for graphene

Similar to Bloch Theorem but for time

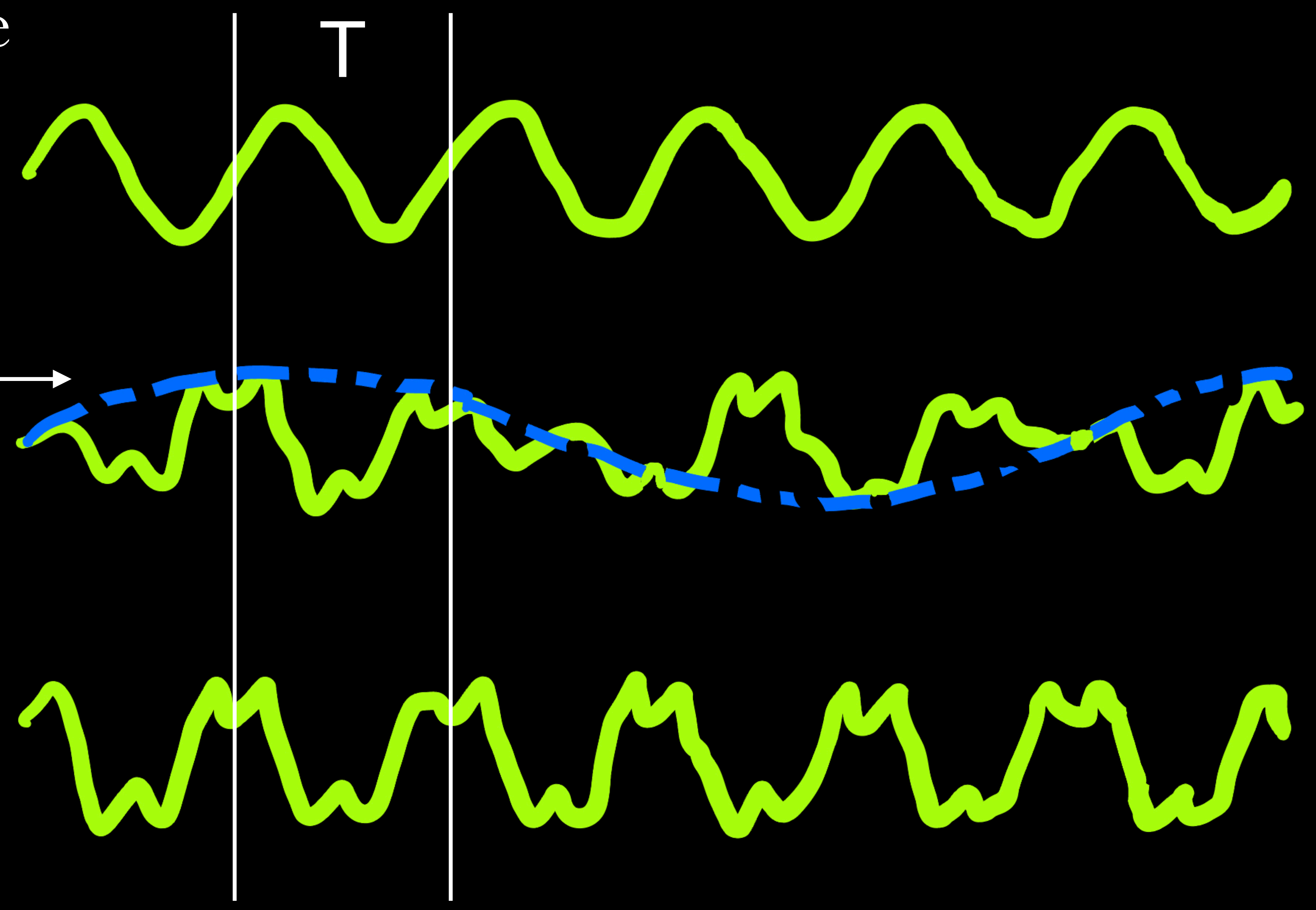
$$\mathcal{H}(\tau) = \mathcal{H}(\tau + T)$$

Slow varying phase

$$|\psi_n(\tau)\rangle = e^{-i\epsilon_n \tau/\hbar}|u_n(\tau)\rangle$$

Periodic Floquet state

$$|u_n(\tau)\rangle = |u_n(\tau + T)\rangle$$




Floquet engineering is the manipulation of quantum systems through the use of driven external fields periodic in time

Under a high-frequency Hamiltonian, an expansion is possible whereby one averages over the period giving way to a time-independent effective Floquet Hamiltonian

## effective Floquet Hamiltonian

$$h^{\mathrm{F}}_{\mathrm{eff}} = \frac{i}{T}\ln U(T,0)$$

Evolution operator $U(y^0,0) = \mathcal{T}\exp\left(-i\int_0^{y^0} dt'\, h_{\mathrm{eff}}\right)$

Hermitian

Magnus expansion

$$h^{\mathrm{F}}_{\mathrm{eff}} \simeq \mathcal{H}_0 + iT^{-1}\int_0^T dy^0 \int_0^{y^0} dy'^0 [h_{\mathrm{int}}, \mathcal{H}_0]$$

Let us illustrate the FLRW case

$$h_{\mathrm{int}} = \frac{i}{2}(\dot{H} + \tfrac{3}{2}H^2)y^i\sigma_i + h_{\mathrm{L}}$$

A merit of the Floquet approach, in addition to providing a static formulation, is that the commutator is now diagonal

Comparison with EM fields setup

$A_i\sigma^3 \to \mathcal{A}^{\mathrm{F}}_i$

$$h^{\mathrm{F}}_{\mathrm{eff}} \simeq \mathscr{H}_0 + [h^{\mathrm{F}}_{\mathcal{F}}]_T + i[h^{\mathrm{C}}_{\mathrm{L}}]_T\,,$$
$$h^{\mathrm{F}}_{\mathcal{F}} := -2(\mathcal{A}^{\mathrm{F}}_1 p_2 - \mathcal{A}^{\mathrm{F}}_2 p_1) - i\mathcal{F}^{\mathrm{F}}_{12}$$

$$[f]_T := \frac{v_F^2}{T}\int_0^T dy^0 \int_0^{y^0} dy'^0 f(y'^0) \qquad \mathcal{A}^{\mathrm{F}}_i = \frac{1}{2}\left(\dot{H} + \frac{3}{2}H^2\right)\epsilon_{ji}y^j\,, \quad \mathcal{F}^{\mathrm{F}}_{12} = \dot{H} + \frac{3}{2}H^2$$

*[Morales, Copinger, PRB **107**, 075432 (2023)]*

1. We demonstrated the emergence of a flat spacetime Hermitian Hamiltonian from its curved spacetime counterpart by virtue of FNC, which fully describes the time-dependent dynamics.

2. We determined an entirely new class of pseudo-gauge field existing at high-frequency, which differs from the adiabatic one through contributions coming from the temporal part of the Hermitic corrected spin connection $\bar{\Omega}_0$

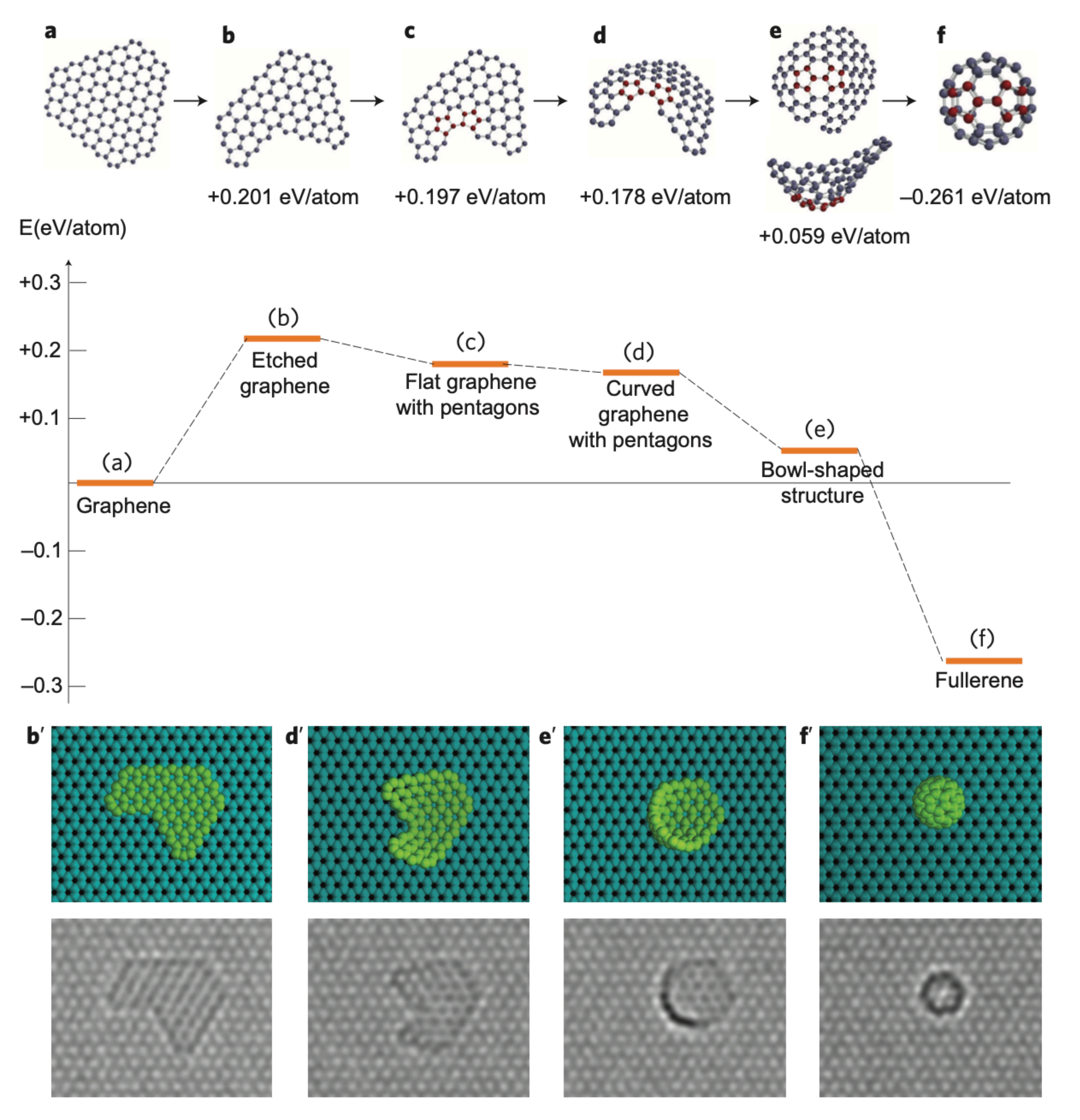
3. We extended our understanding of such emergent pseudo-gauge fields to encompass small variations in FNC time for an FLRW metric such that $\dot{H} \ll H$ , for Hubble parameter H. It was shown the emergent temporally inhomogeneous pseudo-magnetic fields agree with their static counterparts.

‘Inverse’ problem—How about shape?

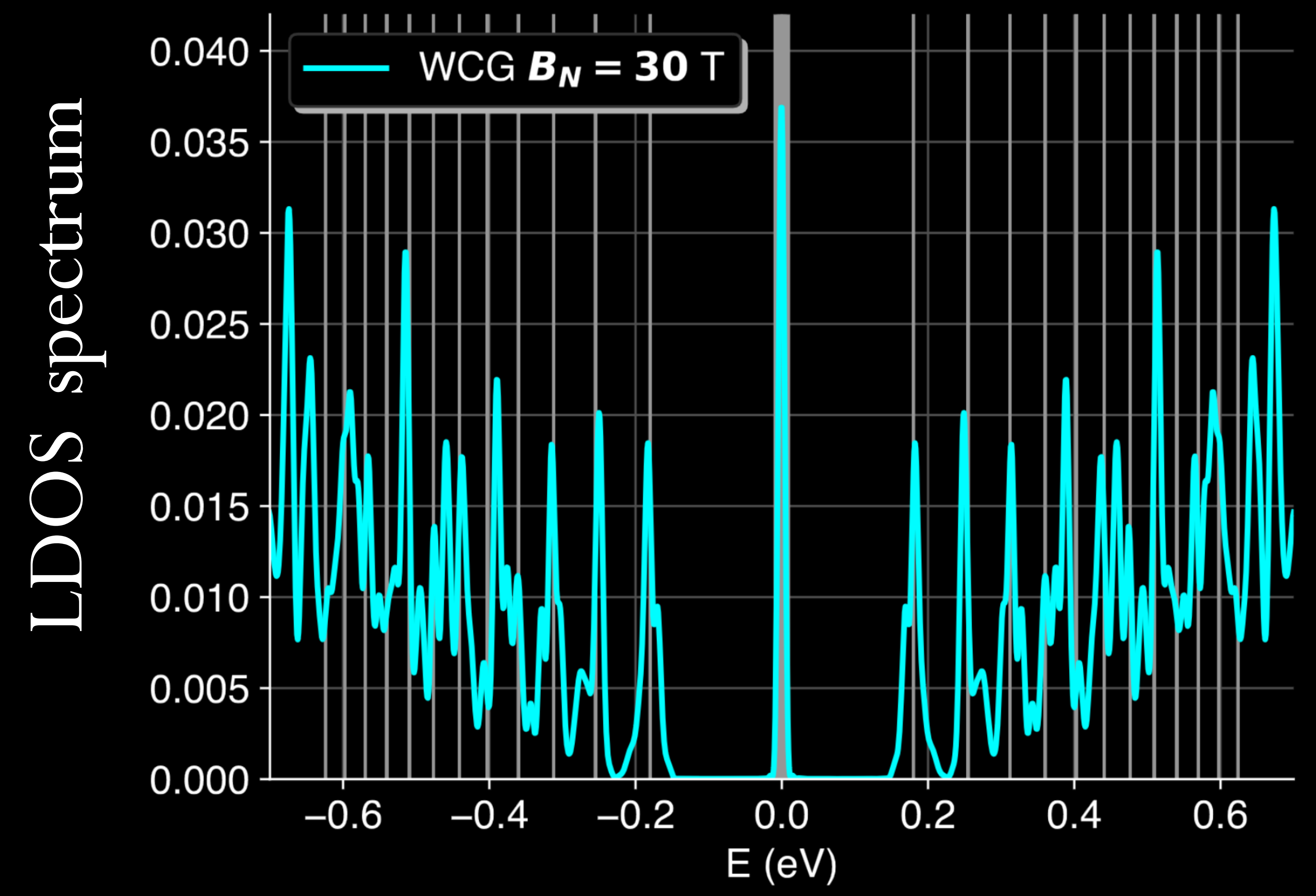
[Morales, Castro-Villareal **110**, 195430 (2024)]

# Spontaneous shapes

# Producing C-structures at the lab




[Chuvilin, Nature (2010)]




[Espinosa-Champo, Naumis, Castro-Villareal, ***PRB accepted***, (2024)]

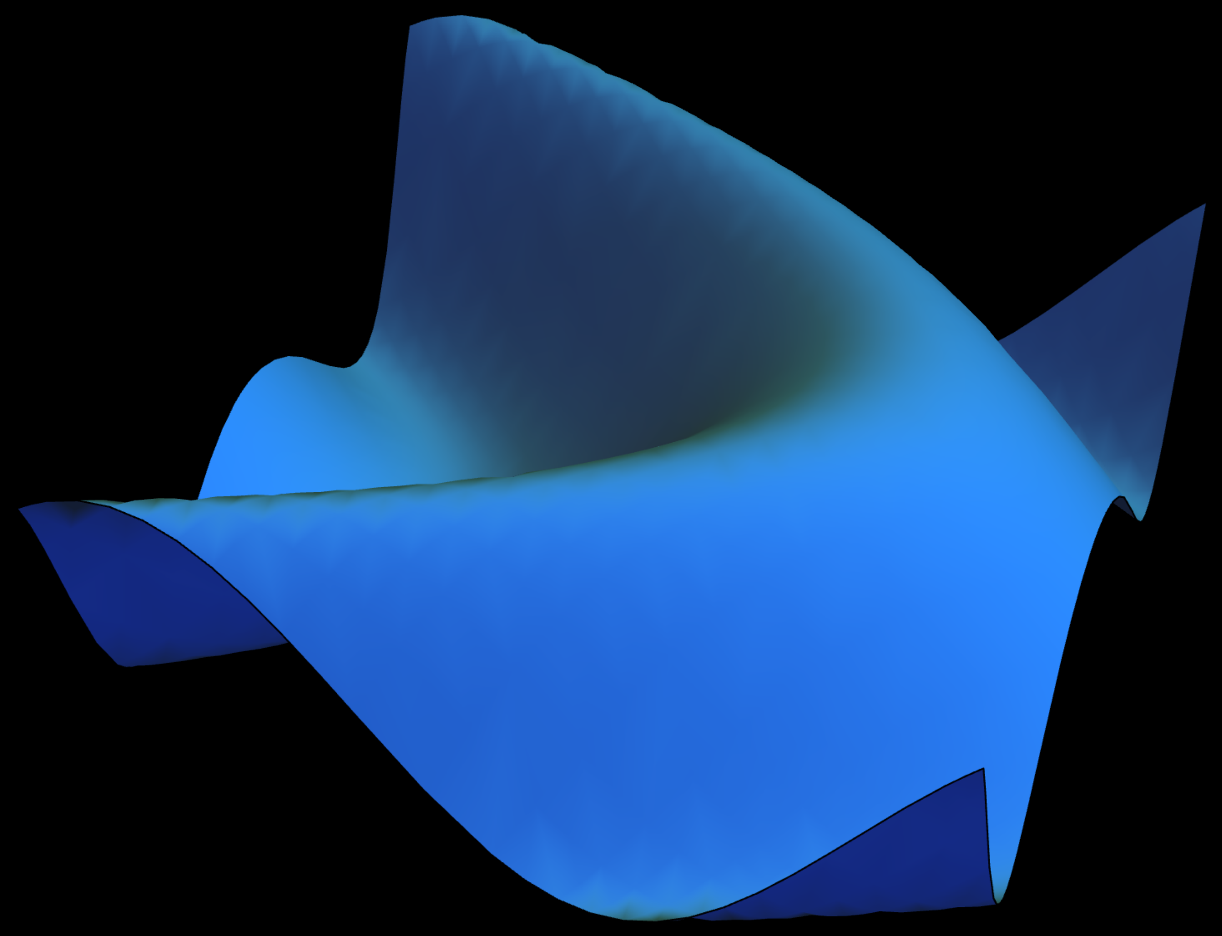
We can arrive at a low energy description from the TB description IF we treat topological defects with care.

Care = considering the emergence of non-abelian Wilson line

The debate is fierce. But ultimately, we need to resort experiments to settle this.

FIRST

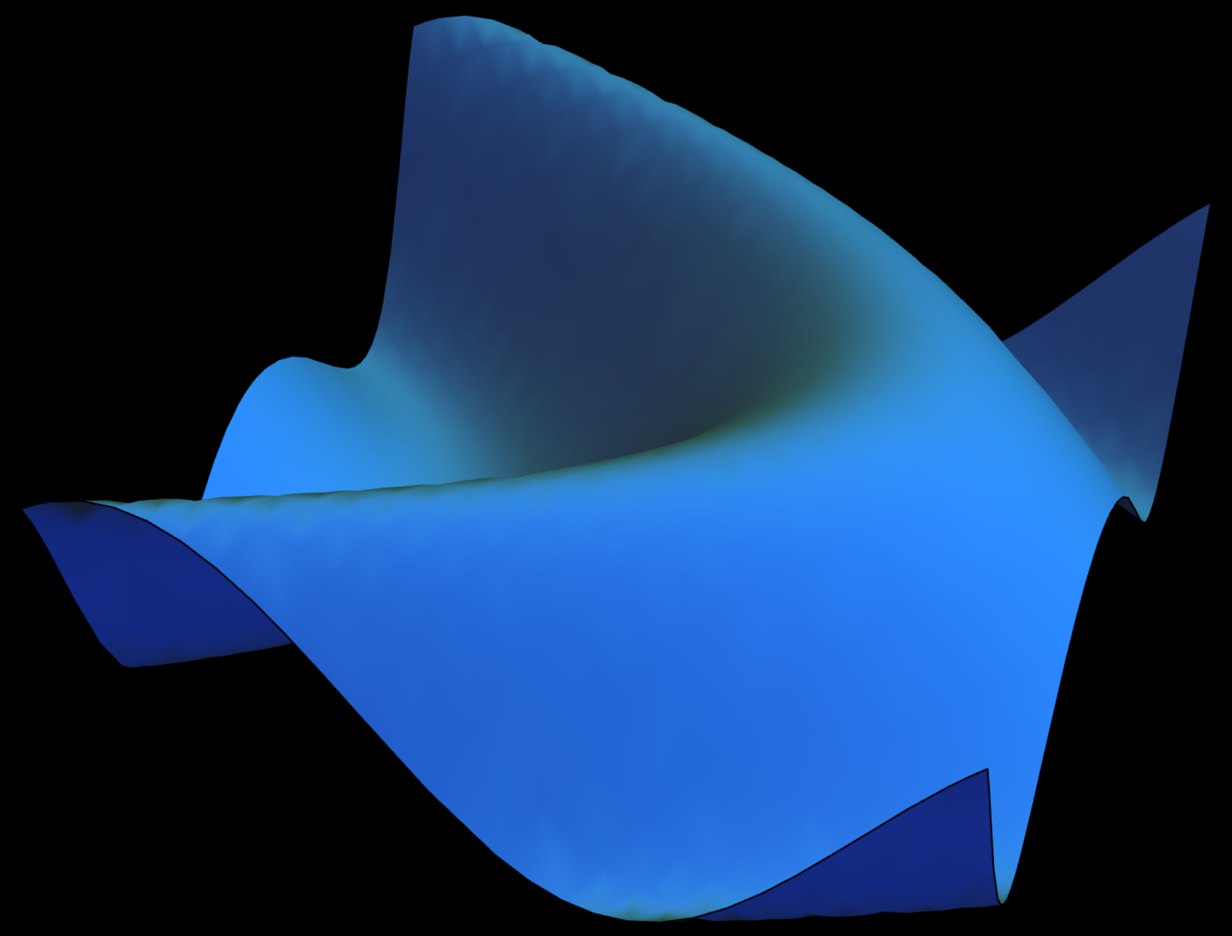
$$\mathbf{X} : \mathcal{D} \subset \mathbb{R}^2 \to \Sigma \subset \mathbb{R}^3$$



What Spatial Geometries do (2+1)-Dimensional Quantum Field Theory Vacua Prefer?

*[S. Fishetti, L- Wallis, and T. Wiseman, PRL, **120**, 261601 (2018)]*

# Emergent Elastic Surfaces from two-dimensional Dirac materials



$$\mathbf{X} : \mathcal{D} \subset \mathbb{R}^2 \to \Sigma \subset \mathbb{R}^3$$

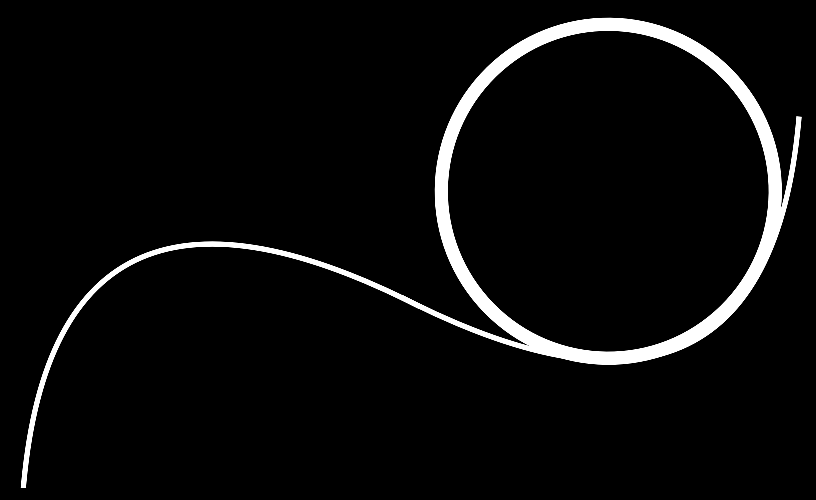
The few seconds review on differential geometry

Geometrical invariants

$$K_{ab} = \mathbf{e}_a \cdot \partial_b \mathbf{n}$$

$K = \mathrm{Tr}(K_{ab})$ Mean curvature

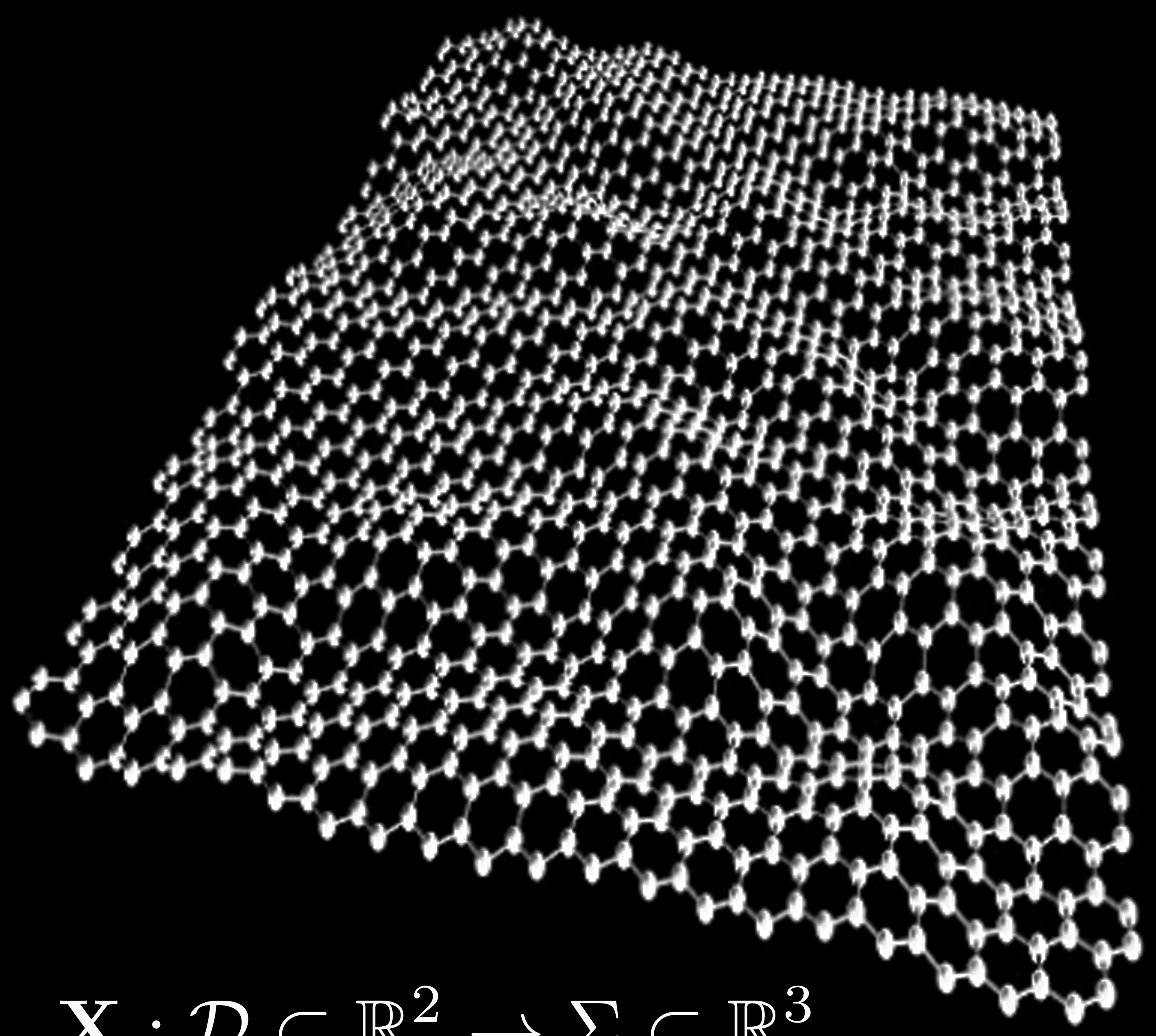
$R_g = \det(K_{ab})$ Intrinsic curvature



Classical part: Heinfrich-Calham

$$H[\mathbf{X}] = \int_\Sigma d^2x \sqrt{g} \left[\frac{\alpha}{2} K^2 + \sigma^{\text{eff}}\right]$$

# Understanding the Shape of two-dim Dirac materials



$$\mathbf{X} : \mathcal{D} \subset \mathbb{R}^2 \to \Sigma \subset \mathbb{R}^3$$

$$\delta H_{\text{eff}}[\mathbf{X}] = H[\mathbf{X}] + \delta H_\psi^{\text{ren}}[\mathbf{X}]$$

Classical part: Heinfrich-Calham

$$H[\mathbf{X}] = \int_\Sigma d^2x\sqrt{g}\left[\frac{\alpha}{2}K^2 + \sigma^{\text{eff}}\right]$$

Dirac field contribution, what is the membrane made of

$$\delta H_\psi^{\text{ren}}[\mathbf{X}] = \frac{1}{8\pi\beta}\sum_{k\geq 0} \mathrm{g}_k^{\text{ren}}\ell_T^{2k-2}\int_\Sigma \mathrm{d}^2x\sqrt{g}\,\mathrm{tr}(E_k)$$

$$E_0 = \mathbf{1}\,, \quad E_1 = -\frac{\mathbf{1}}{12}R\,,$$

$$E_2 = \frac{1}{12}\Lambda^{\mu\nu}\Lambda_{\mu\nu} + \frac{\mathbf{1}}{180}[R^{\mu\nu\rho\sigma}R_{\mu\nu\rho\sigma} - R^{\mu\nu}R_{\mu\nu}] - \frac{1}{6}\nabla_\mu\nabla^\mu\left(\tfrac{1}{5}R - X\right)\mathbf{1} + \frac{1}{2}(\tfrac{1}{6}R - X)^2\mathbf{1}$$

[Morales, Castro-Villareal (2024)]

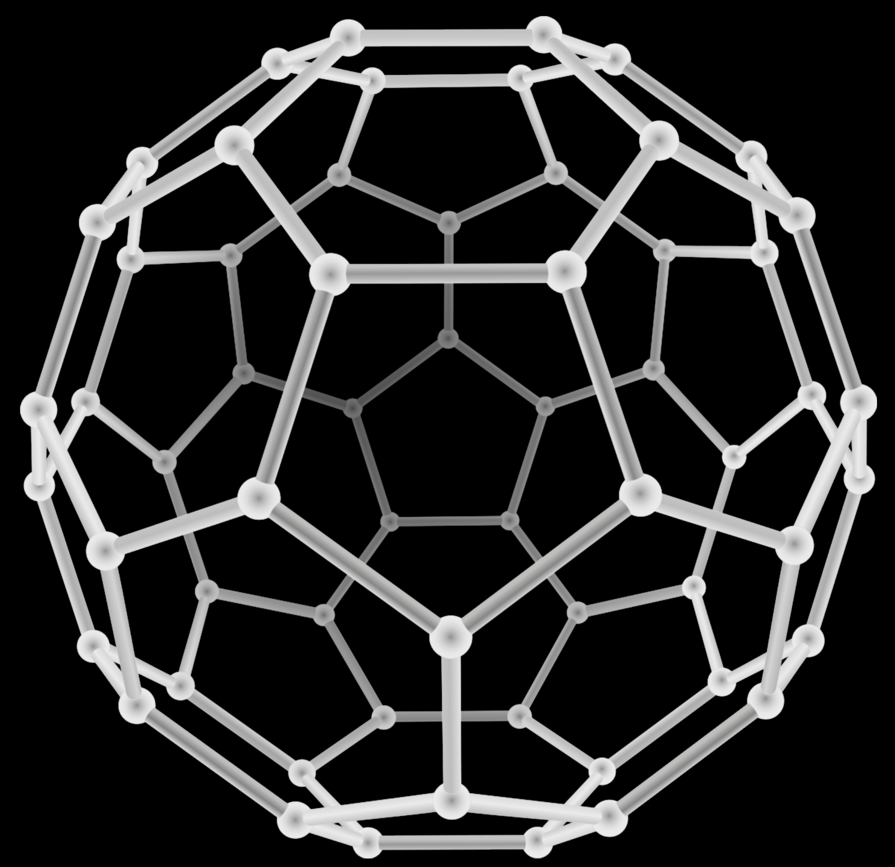
$$H_{\text{eff}}[\mathbf{X}] = \int_\Sigma d^2x\sqrt{g}\left[\frac{\alpha}{2}K^2 + \sigma^{\text{eff}} + \kappa_G^{\text{eff}} R + \frac{1}{2}\kappa_{(2)}^{\text{eff}} R^2\right]$$

$$r_{\text{sph}} = \left(\frac{2\kappa_{(2)}^{\text{eff}}}{\sigma_{\text{eff}}}\right)^{1/4} \approx 3.7803\,\text{nm}$$

With quantum corrected coefficients

$$\sigma^{\text{eff}} = 12\zeta(3)\frac{k_B T}{l_T^2} \qquad \kappa_{(2)}^{\text{eff}} = \frac{k_B T}{240\pi}\ell_{\text{T}}^2$$

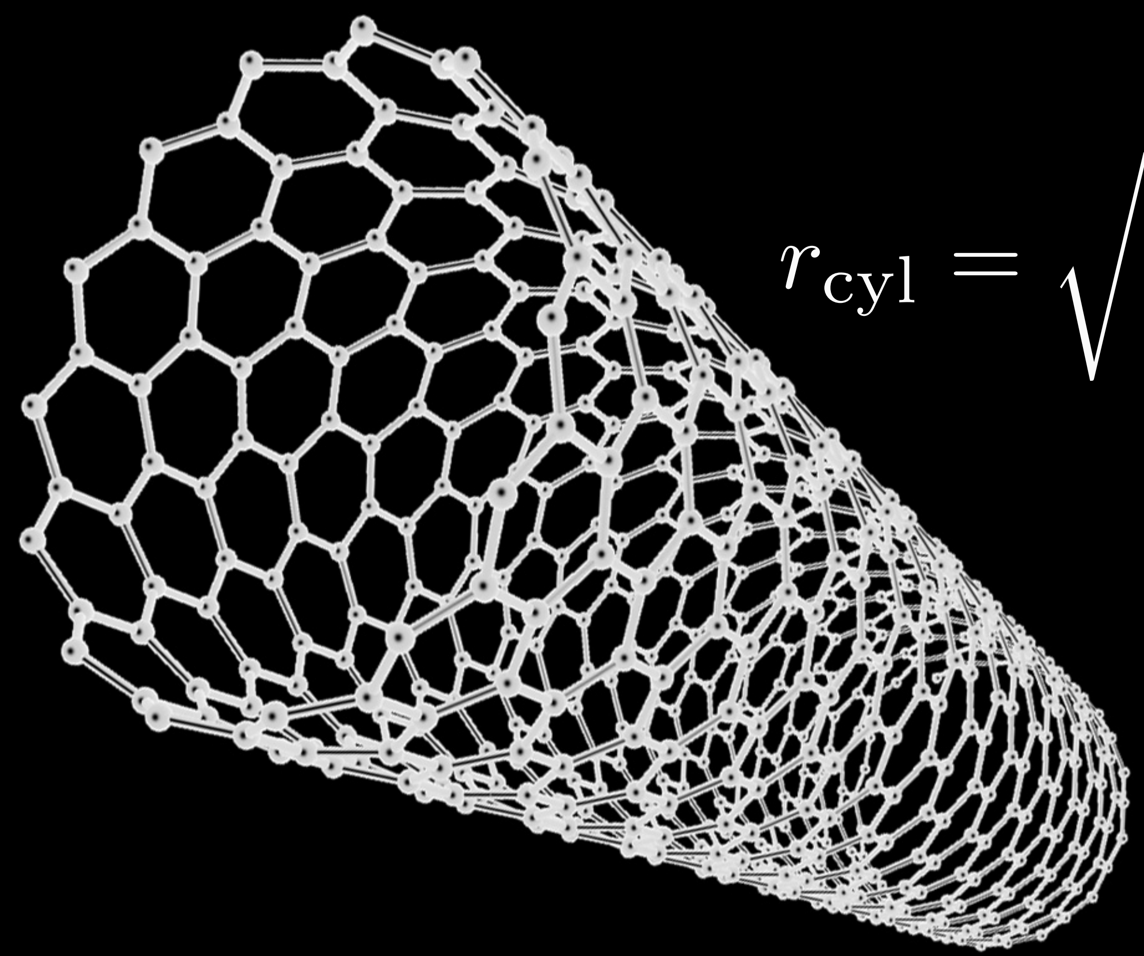
$$\kappa_G^{\text{eff}} = \frac{2}{3}\log(2) k_B T$$



Via Auxiliary Variable method [J. Guven (2004)]

Effective shape equation!

$$-\alpha\left[\Delta_g K + \frac{1}{2}K\left(K^2 - 2R\right)\right] + \sigma^{\text{eff}} K$$
$$+\kappa_{(2)}^{\text{eff}}\left(2K^{ab}\nabla_a\nabla_b R - 2K\Delta_g R - \frac{1}{2}R^2 K\right) = 0$$



$$r_{\text{cyl}} = \sqrt{\frac{\alpha}{2\sigma_{\text{eff}}}} \approx 6.90578\,\text{nm}$$

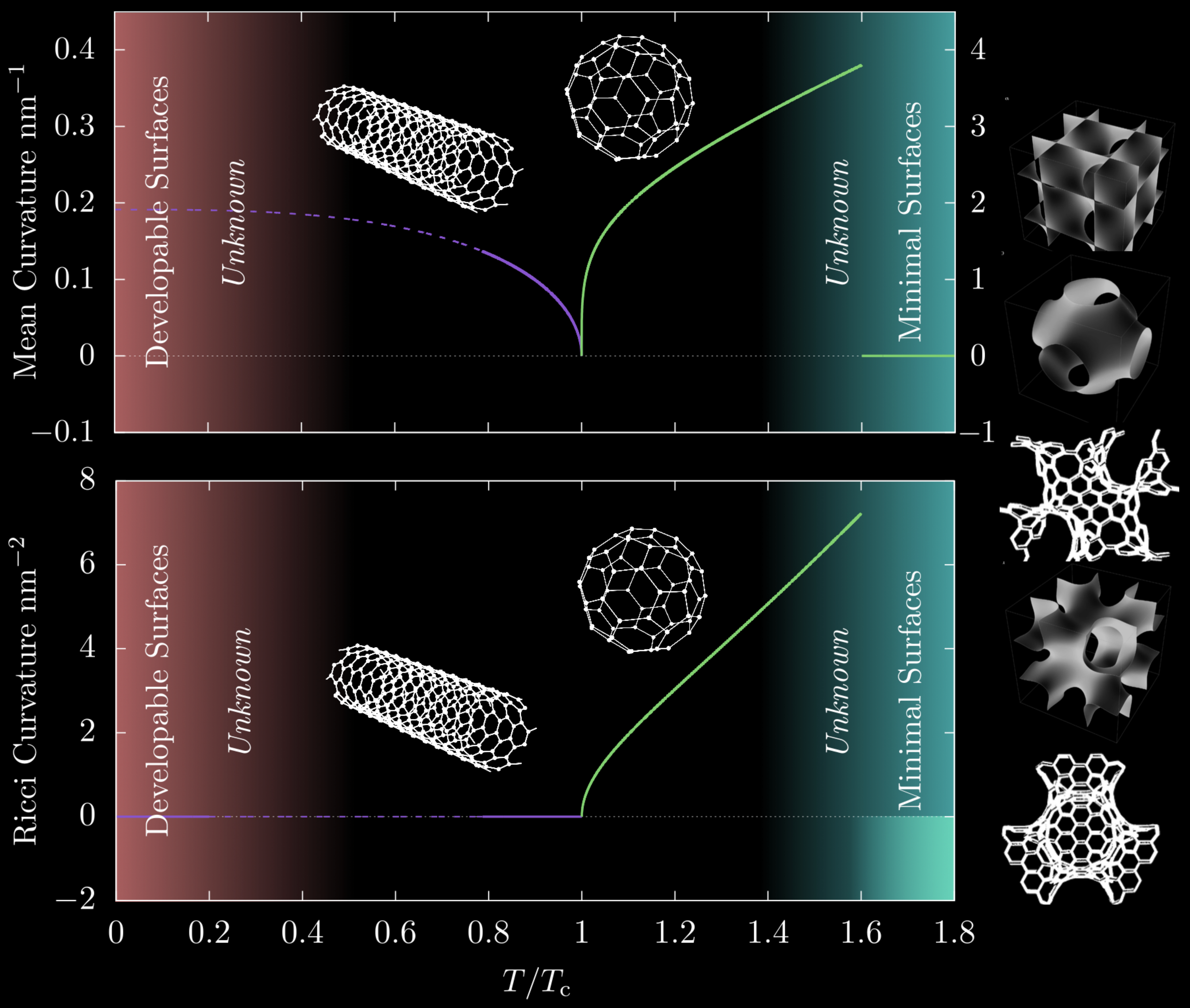
# A Phase Diagram For 2d-Dirac materials

Geometrical invariants from the shape equation

$$K_{\rm cyl} = \frac{1}{\ell_{T_{\rm c}}}\sqrt{\frac{6 g_v g_s \zeta(3) k_B T_{\rm c}}{\alpha}}\left[1-\left(\frac{T}{T_{\rm c}}\right)^3\right]^{\frac{1}{2}}$$

$$K_{\rm sph} = \frac{2(1440\pi\zeta(3))^{\frac{1}{4}}}{\ell_{T_{\rm c}}}\left(\frac{T}{T_{\rm c}}\right)^{\frac{1}{4}}\left[\left(\frac{T}{T_{\rm c}}\right)^3 - 1\right]^{\frac{1}{4}}$$

$$R_{\rm cyl} = 0 \qquad R_{\rm sph} = \frac{1}{2}K_{\rm sph}^2$$




[Morales, Castro-Villareal **110**, 195430 (2024)]

# Summary

New Phase diagram addressing spontaneous generation of 2d honeycomb lattice surfaces

Expect formation of negative curvature in the lab critical temperature to be measured!

Can we probe QFTs in Diral/Weyl semimetals? - Connection to Black Hole phenomena, Hawking radiation

Opens the door many interesting directions: how do boundary effects kick-in?

*[Morales, Castro-Villareal (2024)]*


感謝

# Fermions on Curved Spacetime

Curved Spacetime Dirac Equation

$$i\underline{\gamma}^{\mu}(x)\nabla_{\mu}\psi = 0$$

$$\nabla_{\mu}\psi = (\partial_{\mu} - \Omega_{\mu})\psi = \left(\partial_{\mu} - \frac{1}{8}\omega_{\mu}^{ab}\Sigma_{ab}\right)\psi$$

$$\Sigma^{ab} = [\gamma^{a}, \gamma^{b}],$$
$$\omega_{\mu}^{ab} = e_{\nu}^{a} g^{\nu\lambda}(\Gamma^{\sigma}_{\mu\lambda} e_{\sigma}^{b} - \partial_{\mu} e_{\lambda}^{b})$$

n-Vielbeins

$$ds^2 = g_{\mu\nu}(x)\mathrm{d}x^{\mu}\mathrm{d}x^{\nu} = \eta_{ab}\omega^{a}(x)\omega^{b}(x)$$

$$\{\mathrm{d}x^{\mu}\} \quad \{\omega^{a}(x)\} \quad \mathrm{span}\{T^{*}\mathcal{M}\} \longrightarrow \omega^{a}(x) = e_{\mu}^{a}(x)\mathrm{d}x^{\mu}$$

- Relation between vielbein & metric. However, the vielbeins are not uniquely determined

- Massless Dirac particles are nowadays known to emerge not only in high-energy physics in graphene, at the interface on the topological insulators, etc.

- In principle, deformed materials may realize a nonzero curvature in a controllable way.

From here we gather terms labeled in orders of u and q $H = H_q + H_u + H_{q,u}$

Dirac term $H_q = v_0 \sigma_i q_i \quad v_0 = \frac{3}{2} t_0 a$

Gauge fields $H_u = \frac{v_0}{2a} \beta \sigma_i K_{ijk} \epsilon_{kl} u_{jl}$

$$\mathcal{A}_x = \frac{\beta}{2a}(u_{xx} - u_{yy}), \qquad \mathcal{A}_y = \frac{\beta}{2a}(-2u_{xy})$$

A term induced in inhomogeneity $H_{q,u} = \frac{v_0}{4} \beta [2\sigma^i q_j u_{ij} + \sigma^i q_i u_{jj}]$

The total Hamiltonian can be written as

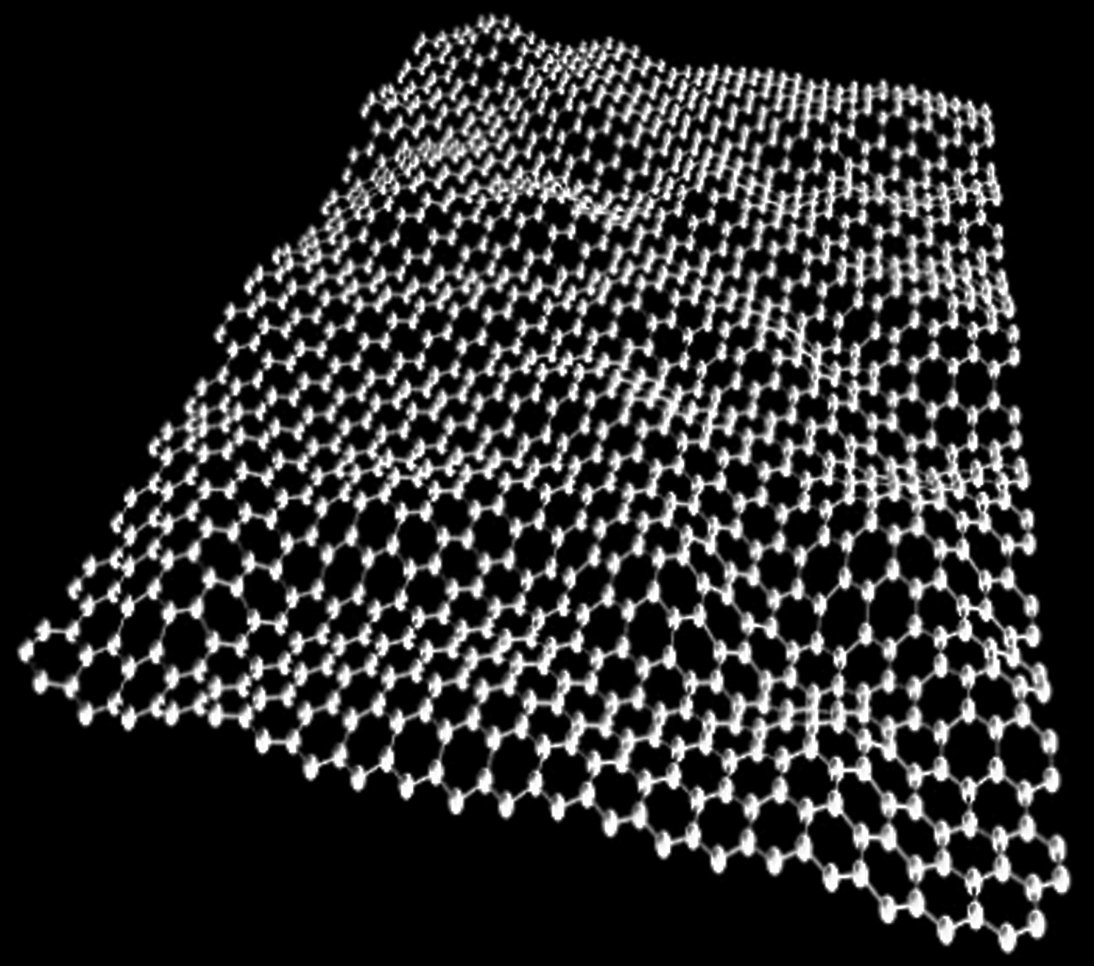
$$H = i v_{ij}(\vec{r}) \sigma_i \partial_j + i v_0 \sigma_i \Gamma_i + v_0 \sigma_i A_i$$

space depend Fermi velocity $v_{ij} = v_0 \left[ \eta_{ij} + \frac{\beta}{4}(2u_{ij} + \eta_{ij} u_{kk}) \right]$

'Geometrical' gauge field $\Gamma_i = \frac{\beta}{4} \left( \partial_j u_{ij} + \frac{1}{2} \partial_i u_{jj} \right)$

*[de Juan, Sturla, Vozmediano, PRL* ***108****, 227205 (2012)]*

- Although the Fermi velocity is approximately a hundredth of the speed of light, the masslessness of the quasiparticles brings the physics to the domain of relativistic QM



Both models are natural and predictive, one should expect that they will provide the same results when applied to curved graphene samples with given shapes.

$$H = \sum_{n,\boldsymbol{x}_A} \left( t_{n,\boldsymbol{x}_A + \frac{a\boldsymbol{l}_n}{2}} a^\dagger_{\boldsymbol{x}_A} b_{\boldsymbol{x}_A + a\boldsymbol{l}_n} + \mathrm{H.C.} \right)$$

$$\gamma^a v_a^{ij} \partial_i \partial_j \Psi$$

The Kinetic Dirac term is the same order as higher derivative terms, and it is therefore inconsistent to consider it in isolation.

*[Roberts, Wiseman, PRB **105**, 195412 (2022)]*