The quantun Newton's bucket

Some considerations and a proposal

G.Torrieri

UNICAMP

FAPESP
FUNDAÇÃO DE AMPARO À PESQUISA DO ESTADO DE SÃO PAULO

Based on 2502.21298 with K.Jhosef,A.Facundes
and also 2504.17152 with G.M. Sampaio,G.Rabelo-Soares

Synopsis

Newton's bucket a much discussed passage in Principia

Modern relevance why that discussion was modern research

Active and passive rotations what does that mean?

Practice examples from quantum mechanics

Principles it's about symmetries

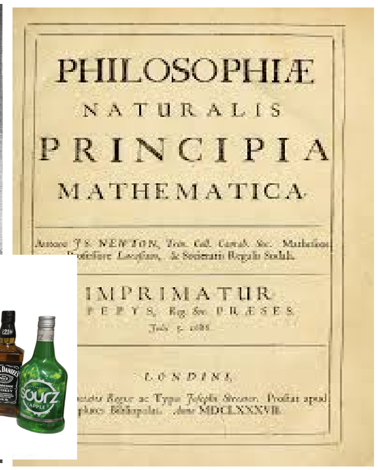
Fields and fluids many particles and equilibrium

A proposal how "topological states" could help

Коперник целый век трудился,
Чтоб доказать Земли вращенье
Дурак, он лучше бы напился,
тогда бы не было сомненья

Copernicus worked for a century
Proving the earth spins
Fool, why didn‘t he just get drunk
then there would be no doubt

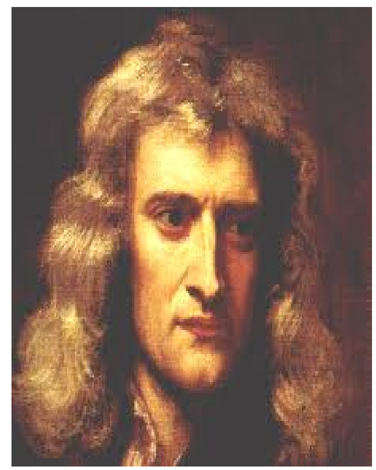
Russian folk song... proved wrong in Newton‘s principia!

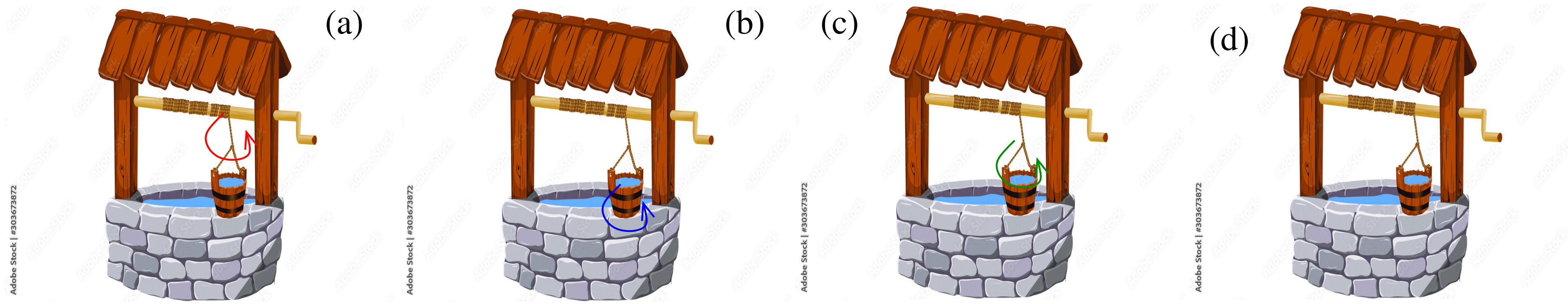

PHILOSOPHIÆ
NATURALIS
PRINCIPIA
MATHEMATICA.

Autore J S. NEWTON, Trin. Coll. Cantab. Soc. Matheseos Professore Lucasiano, & Societatis Regalis Sodali.

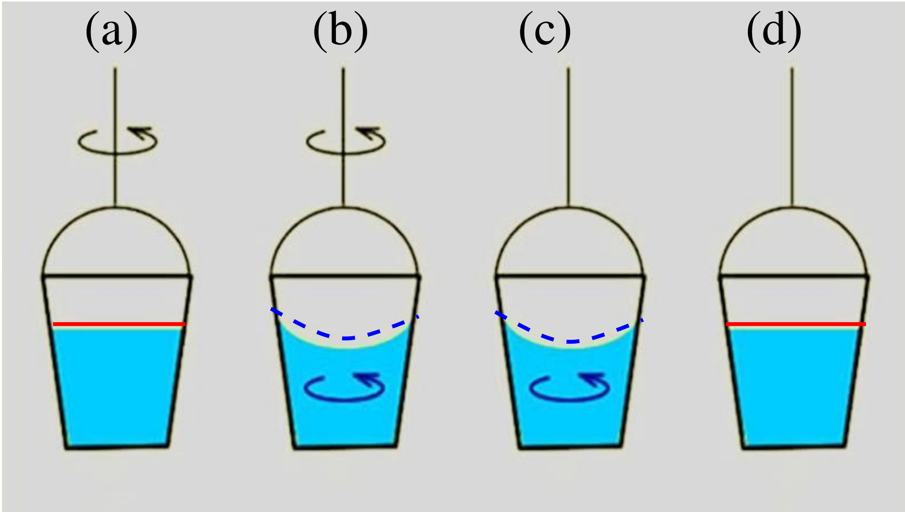
IMPRIMATUR
S. PEPYS, Reg. Soc. PRÆSES.
Julii 5. 1686.

LONDINI,
Jussu Societatis Regiæ ac Typis Josephi Streater. Prostat apud plures Bibliopolas. Anno MDCLXXXVII.





If a vessel, hung by a long cord, is so often turned about that the cord is strongly twisted, then filled with water, and held at rest together with the water; after, by the sudden action of another force, it is whirled about in the contrary way, and while the cord is untwisting itself, the vessel continues for some time this motion; the surface of the water will at first be plain, as before the vessel began to move; but the vessel by gradually communicating its motion to the water, will make it begin sensibly to revolve, and recede by little and little, and ascend to the sides of the vessel, forming itself into a concave figure... I.Newton, principia



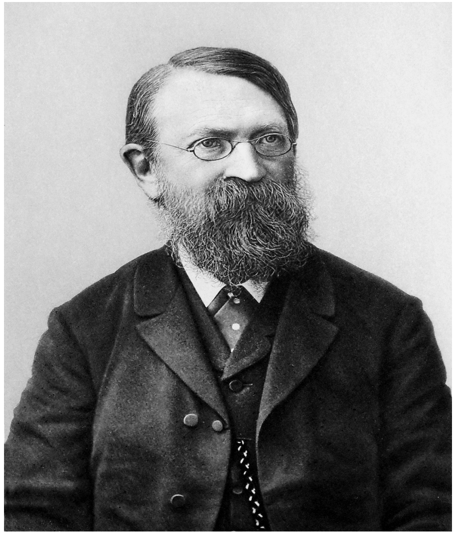
This ascent of the water shows its endeavour to recede from the axis of its motion; and the true and absolute circular motion of the water, which is here directly contrary to the relative, discovers itself, and may be measured by this endeavour. ... And therefore, this endeavour does not depend upon any translation of the water in respect to ambient bodies, nor can true circular motion be defined by such translation. ...; but relative motions...are altogether destitute of any real effect. I.Newton, principia

And of course, as a consequence...

An observer just needs to look at the water level to see if the rotation is active or passive!

So accelleration, unlike velocity, is absolute! Even in vacuum . but then, what is it with respect of?

vs



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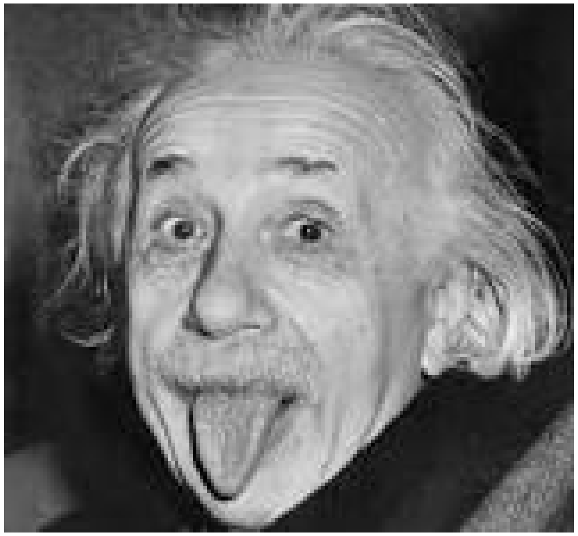

Ostrogradski, Mach and Einstein each had a possible answer (each still surprisingly relevant and discussed)

Ostrogradski Hamiltonian depending on accelleration inherently unstable
No energy minimum ("ghosts "), no topological mixing, only dissipative

Mach "distant stars in the universe" (i.e. boundary conditions!)

Einstein Set of "freely falling frame" determined by localish gravitational field. ($G_{\mu\nu}|_{local} \rightarrow R_{\mu\nu\alpha\beta}|_{nonlocal} \rightarrow \Gamma_{\alpha\mu\nu}|_{min}$) Local equivalence of active and passive transformations in gravitational motion cornerstone of general relativity (evades Ostrogradski because of local symmetries!)

Einstein most likely correct . but arguments <u>classical</u> as is acceleration

$$\frac{1}{m}\frac{d\hat{p}}{dt} = \frac{1}{m}\vec{F} = \frac{d^2\left[\hat{A}_0\hat{x}\right]}{d\hat{x}dt} \quad is \quad not \quad \left|\langle i|\, U_{i\rightarrow f}\, |f\rangle\right|^2!$$

Cannot be defined if commutators are non-negligible

STAR
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1701.06657

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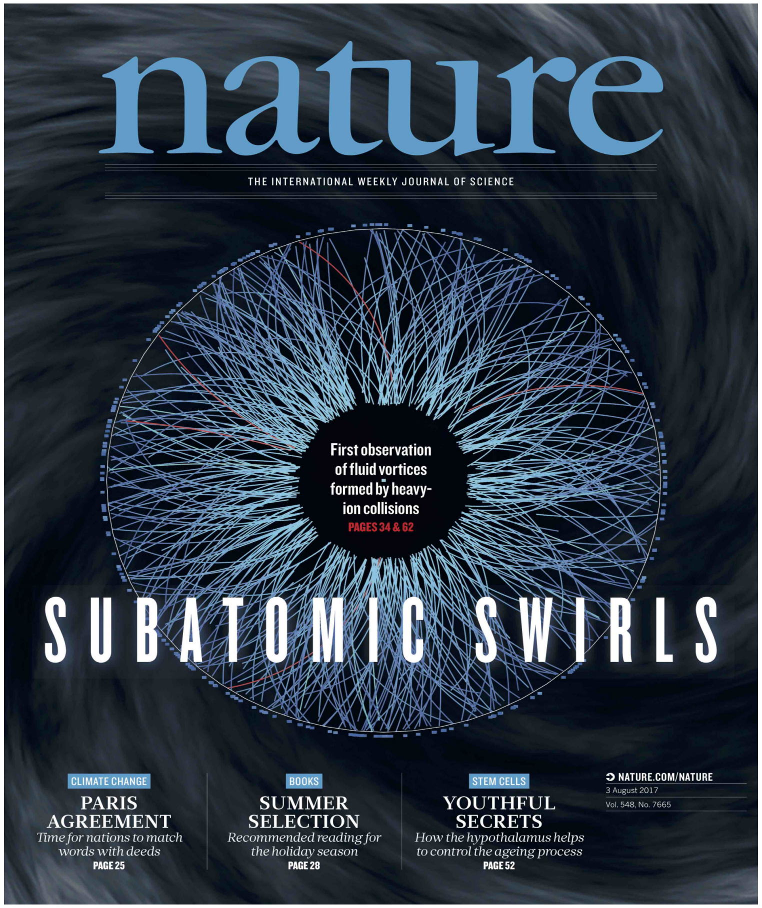
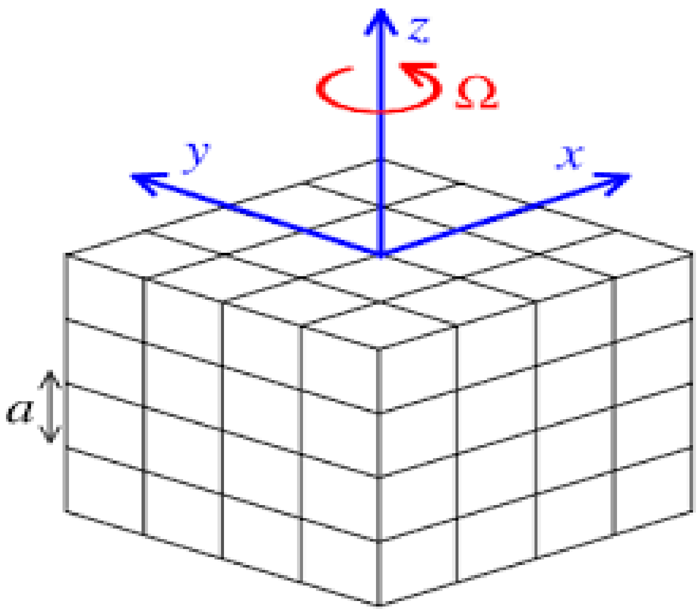
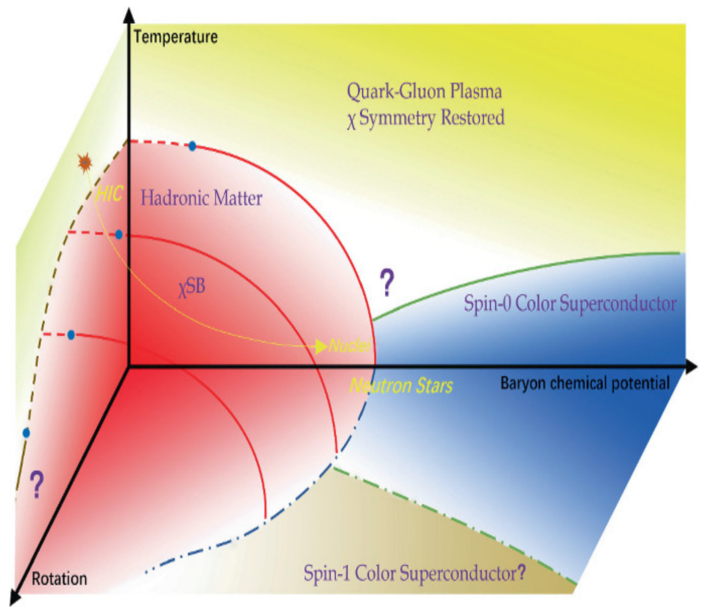
Polarization by vorticity
in heavy ion collisions



Experiment (heavy ions, ultracold atoms) ignited study of rotating QCD,QFT

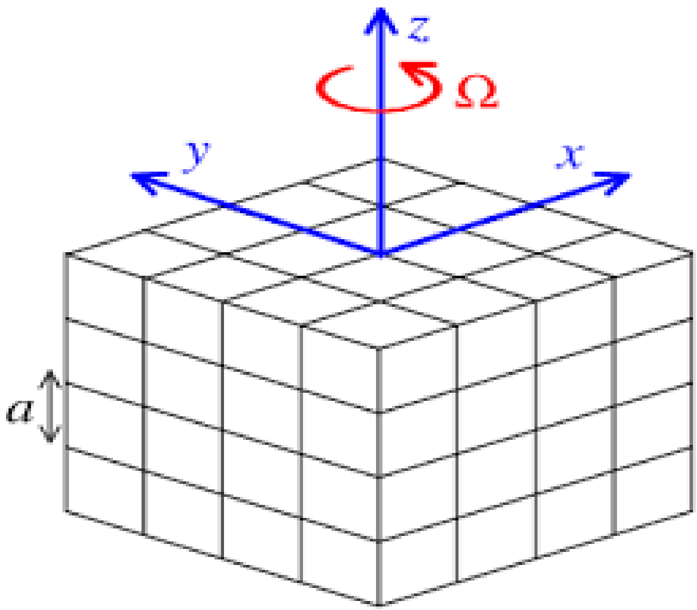
Modern relevance χ-phase diagrams in rotating frames





A wealth of work, with lattice and EFTs, on the phase diagram of rotating matter, phase diagram results but what makes the QGP rotate?

A simple answer: rigidity



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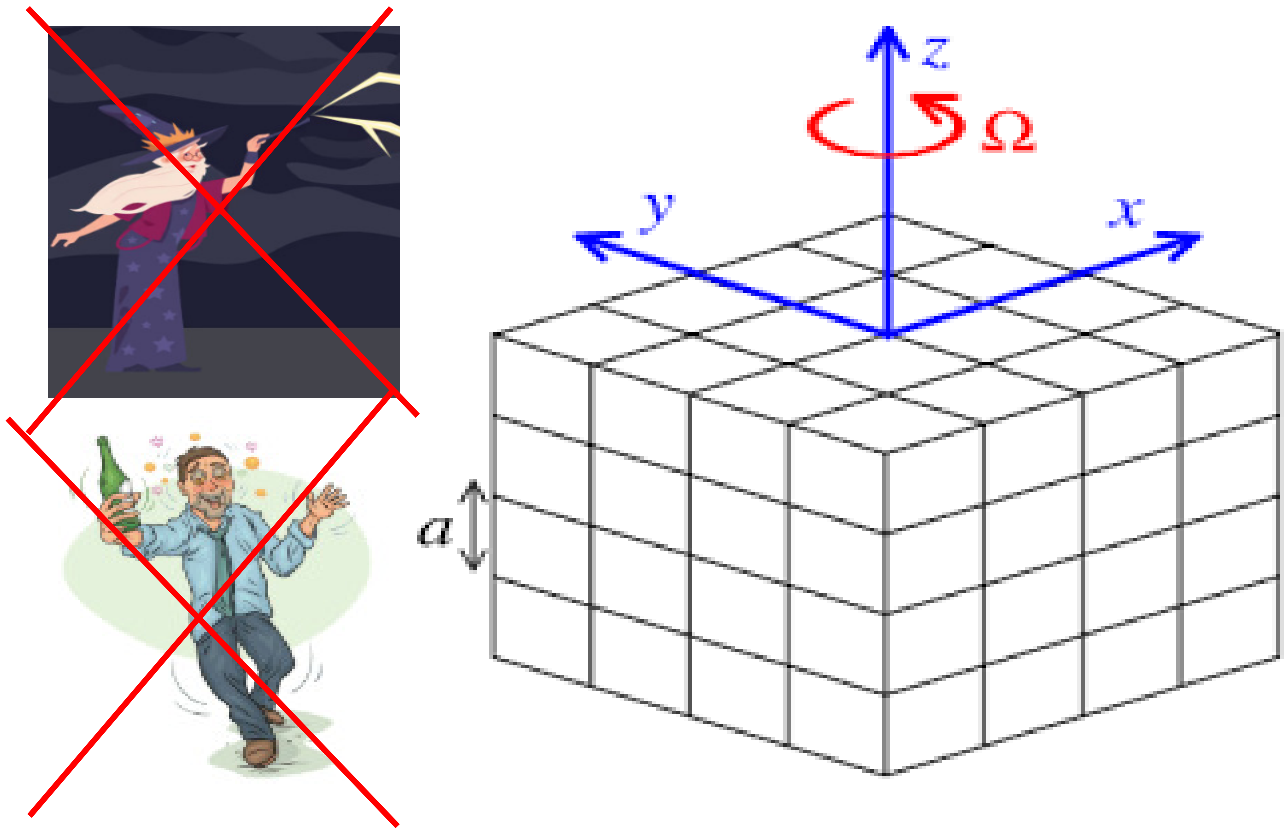
For rigid body angular momentum consrvation and Galilean relativity ensure equivalence of passive and active rotation

But relativity and quantum mechanics inevitably break this,“rigid EFT“?



either we must include in the calculation what makes the system rotate or we must make sure passive and active rotations produce the same physics

A related issue: Can accelleration "melt" an order parameter?

Yes! geometric Unruh effect intuition

No! actual Unruh paper, zero modes

What happens to an accellerated system in equilibrium?

Heating order parameter decreases

Refrigeration order parameter increases

While it is uncontroversial that an accellerated observer sees a thermal state, when the phase transitions present "thermality" quickly becomes confusing. Accelleration also suggested to "thermalize" a semi-classical field . Also is there a rotational Unruh effect? (No horizon but Sokolov-Ternov!)

Now for quantum mechanics: What is an active and passive rotation?

Active the Hamiltonian makes the system rotate

$$H \to H + H_{rotation}$$

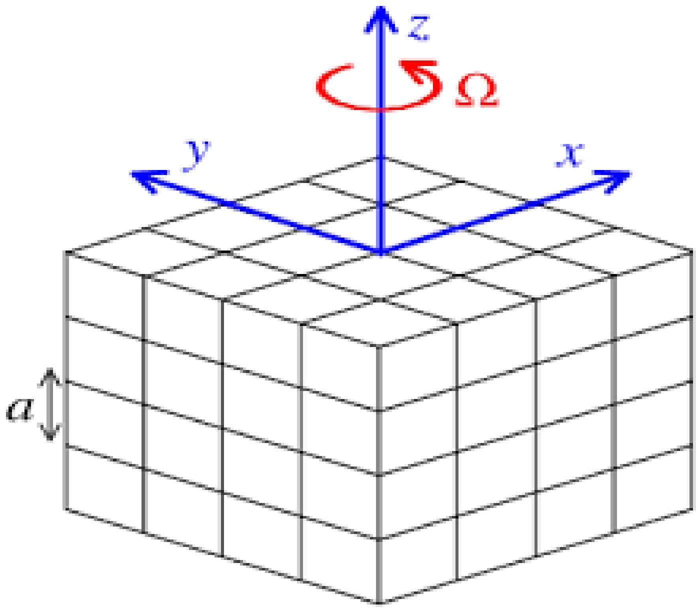
for $E_{n \gg 1}$ reduces to semiclassical motion (?)

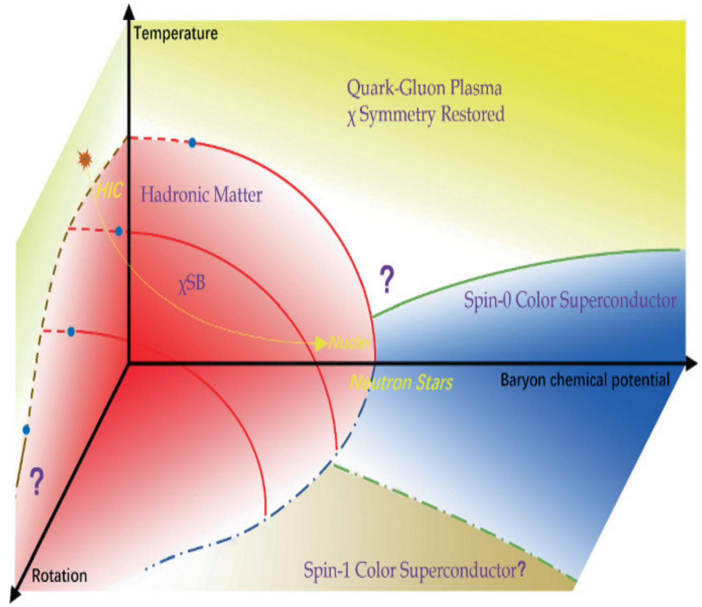
Passive we are observing the system with a rotating detector

$$\hat{\rho} \to U(t)^{-1} \hat{\rho} U$$

Schrodinger and Heisenberg pictures?

Meaningful comparison in semiclassical limit, $E_{n>>1}$ where "wavepacket rotates"





By this definition, this is a "passive" rotation, since no mean field included, just Christoffel symbols. But detectors do not rotate around the QGP!

Consider rotating wavepacket by electric field

Cylindrical box, in n, l, m basis w.r.t. ϕ

$$\langle r, \phi, z| \, |n, m, l\rangle = \psi(\varrho, \phi, z) = A_{n,\ell,m} \left(\frac{\varrho}{na}\right)^m \left(\frac{z}{na}\right)^{\ell-m} e^{-\frac{\sqrt{\varrho^2+z^2}}{na}} e^{im\phi}$$

Active rotation expand the wavepacket into Eigenstates and evolve with $\hat{H} = \frac{\hat{\mathbf{p}}^2}{2m} - \frac{\alpha}{r} - \omega \cdot \hat{\mathbf{J}}$,

$$\hat{\rho}(t) = e^{iH\frac{t}{\hbar}} \left(\sum_{m_\ell, m_s} C^{J,M_1}_{s,m_\ell,\ell,m_1} C^{J,M_2}_{s,m_\ell,\ell,m_2} \rho_{m_1,m_2} |M_1\rangle\langle M_2| \right) e^{-iH\frac{t}{\hbar}}$$

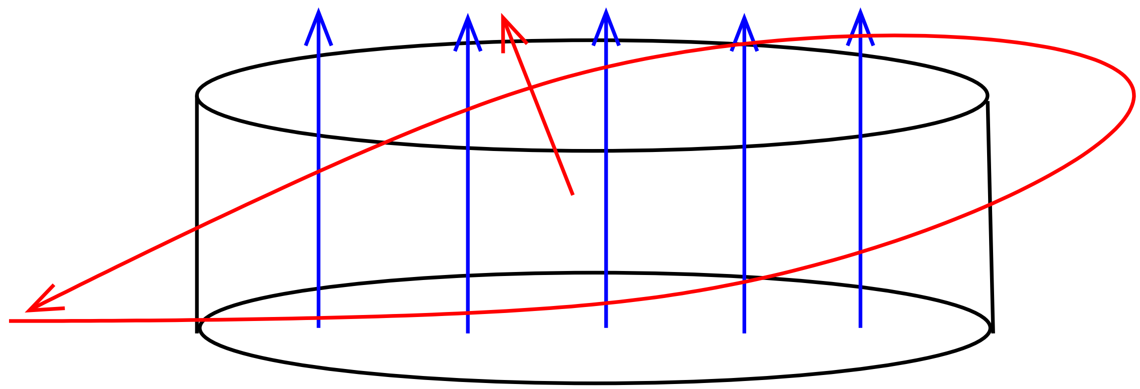
Passive rotation rotate the density matrix

$$\hat{\rho}(\phi,t) = U^{\dagger}(\phi,t)\hat{\rho}(0,0)U(\phi,t)\hat{\rho}(\phi,t) = e^{iJ_z\frac{\phi}{\hbar}}\hat{\rho}(0,0)e^{-iJ_z\frac{\phi}{\hbar}} =$$

$$= \sum_{m_\ell,m_s} C^{J,M_1}_{s,m_\ell,\ell,m_1} C^{J,M_2}_{s,m_\ell,\ell,m_2} \rho_{m_1,m_2} e^{i(M_1-M_2)\omega_z t}$$

So they match Somehow unsurprising, since action of Hamiltonian and of rotation the same . but...

Rotation by magnetic field



$$\hat{H} = \frac{\hat{\mathbf{p}}^2}{2m} - \frac{\alpha}{r} - \omega \cdot \hat{\mathbf{J}} - \hat{\mu} \cdot \hat{\mathbf{B}}(\mathbf{r})$$

where $\mathbf{B}(\mathbf{r}) = \left(\Omega_1 + \frac{\Omega_2}{r} + \frac{\Omega_3}{r^2}\right)\hat{\mathbf{z}}$ and $\hat{\mu} = \gamma \frac{q}{2m}\hat{\mathbf{S}}$ For this system

$$\langle r, \theta, \phi| \, |n, l, m\rangle = B_n r^{-\frac{H_1}{2\sqrt{H_0}}} e^{\sqrt{H_0} r} \left[-r^2 \frac{d}{dr}\right]^n \left[r^{\frac{2n-H_1}{\sqrt{H_0}}} e^{-2\frac{\sqrt{H_0}}{r}}\right] (-1)^m P_\ell^m(\cos\theta),$$

Active rotations become

$$\hat{\rho}(t) = \sum_{m_\ell, m_s} C^{J,M_1}_{s,m_\ell,\ell,m_1} C^{J,M_2}_{s,m_\ell,\ell,m_2} \rho_{m_1,m_2} \times$$

$$\exp\left\{ i \left[\left(\frac{-\alpha - \gamma \frac{q}{2m} M_1 \Omega_2}{(1+2n) + \sqrt{1 + 2\gamma \frac{q}{m} M_1}} \right)^2 - \left(\frac{-\alpha - \gamma \frac{q}{2m} M_2 \Omega_2}{(1+2n) + \sqrt{1 + 2\gamma \frac{q}{m} M_2}} \right)^2 + \right.\right.$$

$$\left.\left. M_2 \left(\omega_z + \gamma \frac{q}{2m} \right) - M_1 \left(\omega_z + \gamma \frac{q}{2m} \right) \right] t \right\}.$$

While passive rotation still $\sim \sum_{m_\ell, m_s} C^{J,M_1}_{s,m_\ell,\ell,m_1} C^{J,M_2}_{s,m_\ell,\ell,m_2} \rho_{m_1,m_2} e^{i(M_1 - M_2)\omega_z t}$
so the two not equivalent! but what happens if Maxwell's equations fully considered? See in a few slides!

But let's make it simpler for now: a cylindrical potential well

$$\hat{\rho}(t) = \sum_{m_\ell, m_s} C^{J,M_1}_{s,m_\ell,\ell,m_1} C^{J,M_2}_{s,m_\ell,\ell,m_2} \rho_{m_1,m_2} \times$$

$$\times \exp\left[\frac{i}{\hbar}\frac{1}{2m}\left(x^2_{M_1 a} - x^2_{M_2 a}\right) t\right] \exp\left[\frac{i}{\hbar}\left(M_2 - M_1\right)\omega t\right].$$

where x_{Ma}, $a = 1, 2, \ldots$, is the set of zeros of the M'th Bessel function. which again does not correspond to the passive case

$$\sim \sum_{m_\ell, m_s} C^{J,M_1}_{s,m_\ell,\ell,m_1} C^{J,M_2}_{s,m_\ell,\ell,m_2} \rho_{m_1,m_2} e^{i(M_1 - M_2)\omega_z t}$$

And now let us put a Coloumbic potential in a well Active rotation becomes

$$\hat{\rho}(t) = \sum_{m_\ell, m_s} C^{J,M_1}_{s,m_\ell,\ell,m_1} C^{J,M_2}_{s,m_\ell,\ell,m_2} \rho_{m_1,m_2} \times$$

$$\exp\left\{ i\alpha \left[\frac{1}{[n'_2(\omega) - 1/2]^2} - \frac{1}{[n'_1(\omega) - 1/2]^2} \right] \frac{t}{\hbar} \right\} \exp\left[\frac{i}{\hbar} (M_2 - M_1)\, \omega t \right]$$

where $E_{n',M} = -\frac{\alpha^2 m}{2[n'(\omega)-1/2]^2} - M\omega$. extra term absent in passive term

General principle... Angular momentum conservation

If

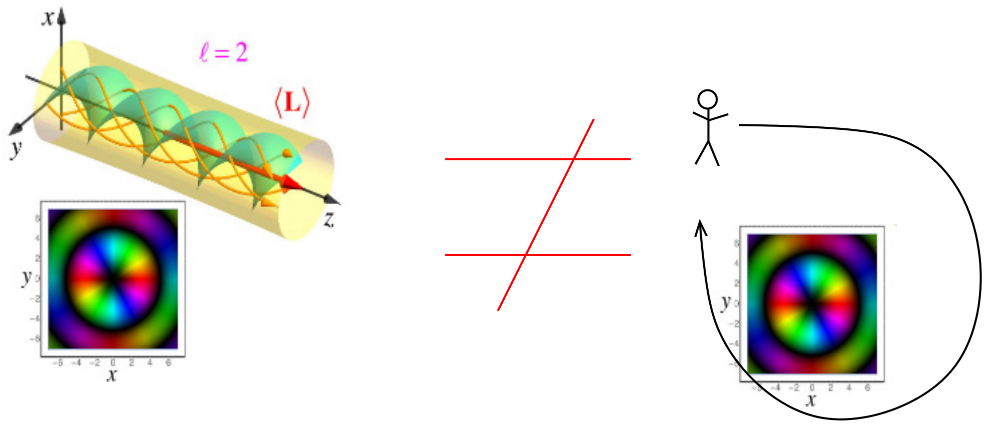
Angular momentum conserved $\left[H, \vec{J}\right]$

"mean field" rotation $\hat{H} \rightarrow \hat{H} - \hat{w}.\hat{J}$

then since $U_{rot} = \exp\left[i\hat{\sigma}.\hat{n}\right]$

$$\hat{H} \rightarrow \hat{H} - \hat{w}.\hat{J} \quad , \quad e^{i\hat{H}t}\left|\psi\right\rangle \equiv \hat{U}_{rot}\left|\psi\right\rangle$$

Putting such a system in a mean field potential ("active" rotation) always the same as rotating the detector.



Angular momentum conserved $\left[H,\vec{J}\right]$

“mean field“ rotation $\hat{H} \rightarrow \hat{H} - \hat{w}.\hat{J}$

But breaking any of these conditions introduces difference. In particular (unlike in classical mechanics) breaking <u>one</u> generator enough to spoil equivalence! This is a <u>quantum effect</u> . Wavepacket “stretches“ along broken generator in a way different from passive rotation!

A quantum recap: Are passive and active rotations the same

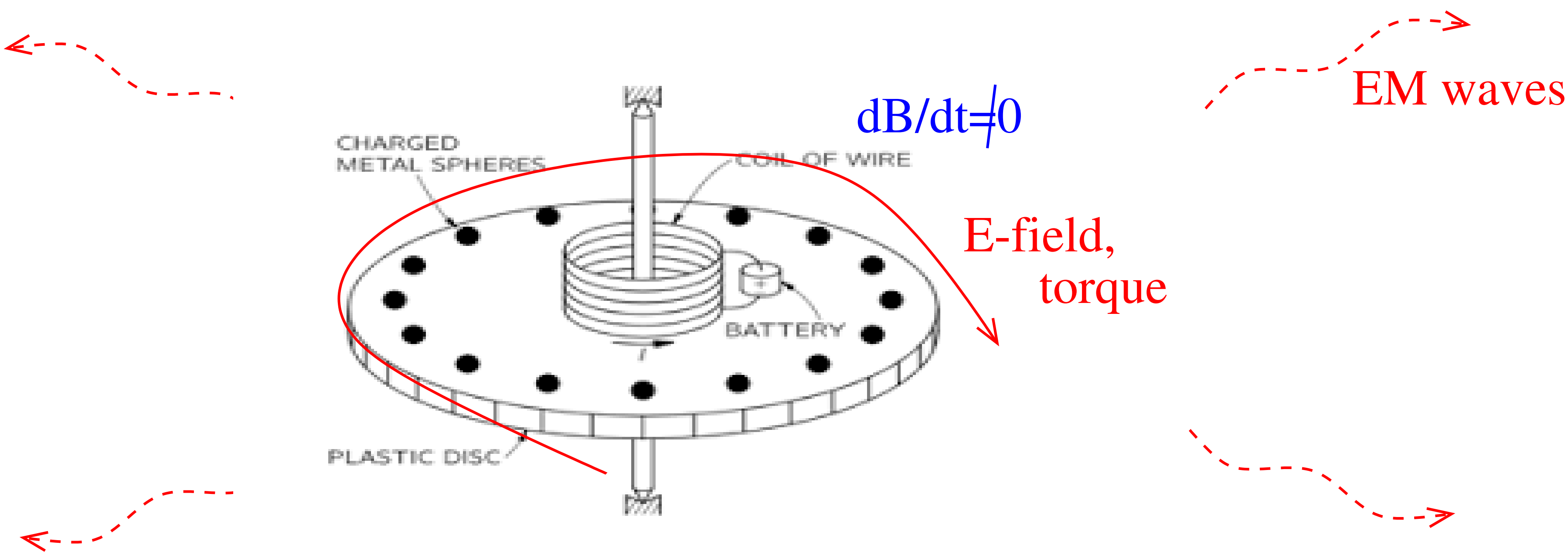
Scalar interactions yes! full rotational symmetry

Vector interactions no! In QM magnetism “source of angular momentum“ breaking spherical rotation symmetry

Tensor interactions If $h_{\mu\nu}T^{\mu\nu}$ probably yes to linear order

Photon, graviton angular momentum: But the world (and phase transitions) are QFT, not QM!

A paradox by Feynman When magnetic field collapses, it generates a strong circular electric field tangent to the disk's perimeter due to the presence of the charges. Angular momentum apparently not conserved ,but one must consider the EM waves which go to infinity . no such waves in passive rotation. Lesson: Must consider asymptotic states



QM→QFT: What changes? Physics from

$$\frac{\delta}{\delta J}\ln \mathcal{Z}[J] \quad , \quad Z[J] = \int \mathcal{D}\phi \exp\left[\int d^4x\,(-L(\phi) + J(x,t)\phi)\right]$$

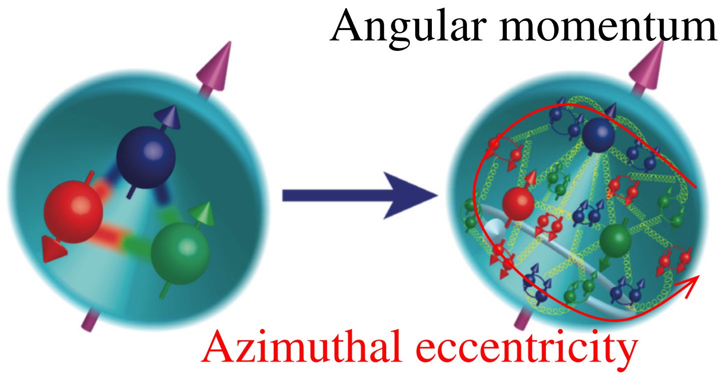
Active rotation in terms of $\langle A_\mu \rangle$ the mean field causing rotation/accelleration

$$\hat{p}_\mu \to \hat{p}_\mu - e\,\langle A_\mu \rangle \quad , \quad \langle A_0 \rangle = V\vec{x} \quad , \quad \langle A_i \rangle = \epsilon_{ijk} B_j x_k$$

Passive rotation $J(x,t)$ source/sink of rotating detector

$$S = \int d^4x L(A,\psi) \to \int d^4x\,[L(A,\psi) + J_\mu(x,t)A^\mu]$$

Unitary inequivalence boundary conditions change the system generic QFT? Ji decomposition $\hat{J}_{nucleon} = \hat{S}_{quark} + \hat{L}_{quark} + \hat{J}_{gluon}$



Terms in red ncessarily non-local by gauge invariance L_{quark} related to Wilson loop, J_{gluon} combines spin and angular momentum

Coefficients fixed by sum rules Kim et al, 2503.20225

Expect non-local terms different between active and passive case, at finite temperature $\mathrm{Im}\left[\hat{O}(x), \hat{O}(x`)\right]$ (Passive rotations cannot break a state)

What about hydrodynamics and thermodynamics? Let's analyze the bucket

$$H = \frac{I\dot{\phi}_B}{2} + \sum_{i=1}^{N} \frac{p_i^2}{2m} + V_{edge} + V_{water} + V_g,$$

$$V_{edge} \propto \sum_{i=1}^{N} \delta\left(r_i - R\right)\left(\dot{\phi}_i - \dot{\phi}_b\right)^2, V_{water} \sim \sum_{IJ=1}^{N} V(\vec{x}_i - \vec{x}_j), V_g \sim \sum_{i=1}^{N} mg(\vec{x}_i)_z$$

- V_g breaks spherical symmetry to cylindrical
- $\lim_{N\to\infty} V_{water} \to$ volume preserving diffeomorphisms,break rigidity

Both do not commute with some angular momentum generators. Lesson: mean field interacting with fluid important

Global equilibrium: Killing it!

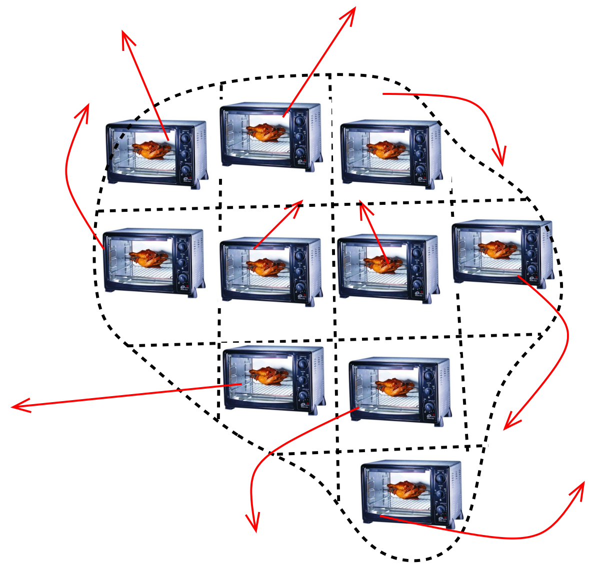
If equilibrium is global, β_μ killing field. maximally mixed state will become another maximally mixed state when translated by a Killing vector. $\left[\hat{H},\hat{J}\right]$ irrelevant . But equilibrium has to be perfect , which for arbitrary β_μ is not necessarily stable! rigidity is Killing field of Galileo group!

...Or Killing Global equilibrium? Equilibrium under rotation/accelleration

The thermodynamic limit presupposes $V \to \infty, \Delta t \to \infty$, not covariant

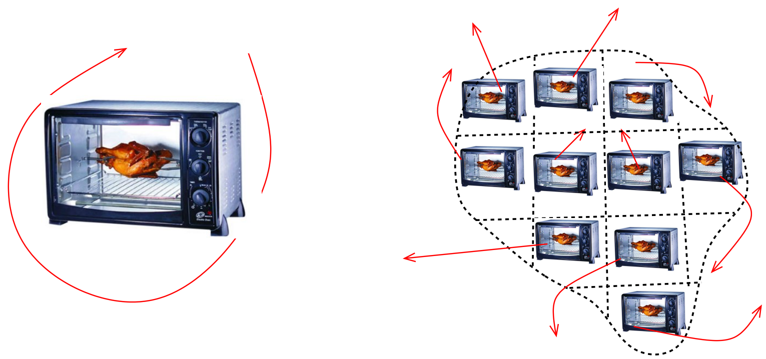
Fluctuation scale vs $\partial \beta_\mu$ gives covariant limit. but if two are comparable, this is not equilibrium! not an extremum of partition function fluctuations in β_μ (i.e. momentum!) break “global equilibrium“ need local equilibrium

Local equilibrium (No mean fields) Killing argument should still hold...



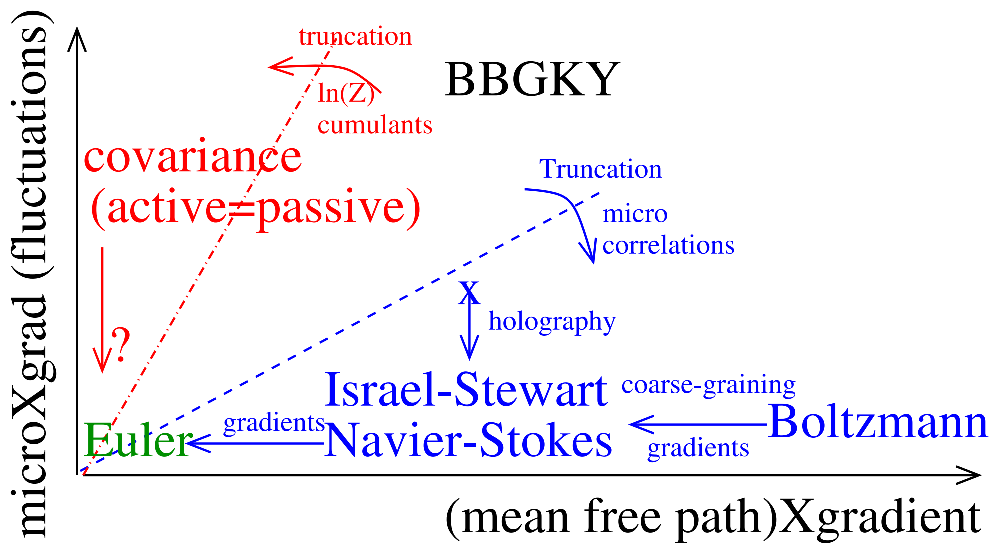
Number of localized microstates in cell scalar under general coordinate transformations , so local dynamics generally covariant. Not true for most hydro EFTs , because defined on intrisically non-local scale l_{mfp} .
My speculation: Ambiguity of hydrodynamics Eckart,Israel-Stewart,BDNK gauges . EFTs defined with them generally non-covariant

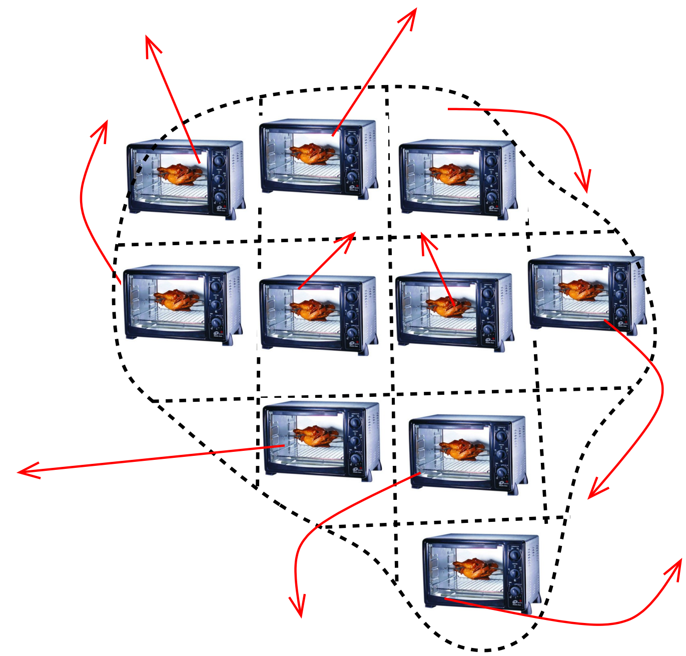
GT,Soares,Sampaio,2504.17152 If $l_{micro} \sim l_{mfp}$ dynamics locally indistinguishable from fluctuations in generally covariant way...



- $\langle T_{\mu\nu} \rangle$, $G_{\mu\nu\alpha\beta} = \langle T\left[T_{\mu\nu}(x), T_{\alpha\beta}(x')\right]\rangle$ via Gravitational ward identity
$$\nabla_\mu \mathcal{W}^{\mu\nu\alpha\beta} = 0 \quad , \quad \mathcal{W}^{\mu\nu\alpha\beta} = G^{\mu\nu,\alpha\beta}\left(\Sigma_\mu, \Sigma'_\nu\right) - \frac{1}{\sqrt{g}}\delta\left(\Sigma' - \Sigma\right) \times$$
$$\times \left(g^{\beta\mu} \left\langle \hat{T}^{\alpha\nu}\left(x'\right)\right\rangle_\Sigma + g^{\beta\nu}\left\langle \hat{T}^{\alpha\mu}\left(x'\right)\right\rangle_\Sigma - g^{\beta\alpha}\left\langle \hat{T}^{\mu\nu}\left(x'\right)\right\rangle_\Sigma \right)$$

- Linear response $\langle T_{\mu\nu}\rangle\left(\Sigma\right) = \int e^{\epsilon \Sigma_0} \mathcal{G}^{\mu\nu,\alpha\beta}(\Sigma_0' - \Sigma_0) T_{\alpha\beta}(\Sigma_0') d\Sigma_0$ generally covariant if volume preserving diffeos ($\sim$ ideal hydro symmetry!)





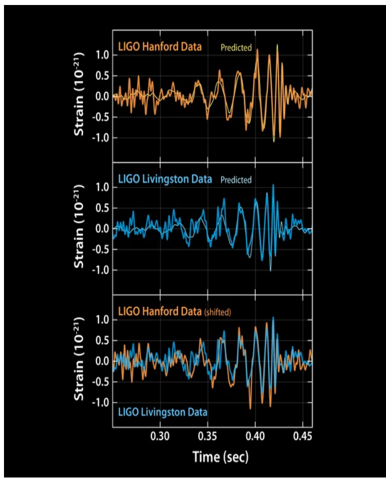
So with no mean field at ensemble level for local extensive and intensive quantities active and passive rotations are the same if $l_{fluct} \simeq l_{mfp} \ll R$. (general covariance could explain hydro in small systems . Non-local observables (moment of inertia) probably not covariant!

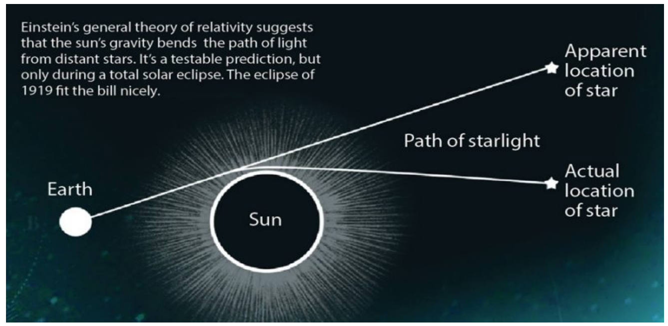
Summarizing: Active and passive rotations equivalent when

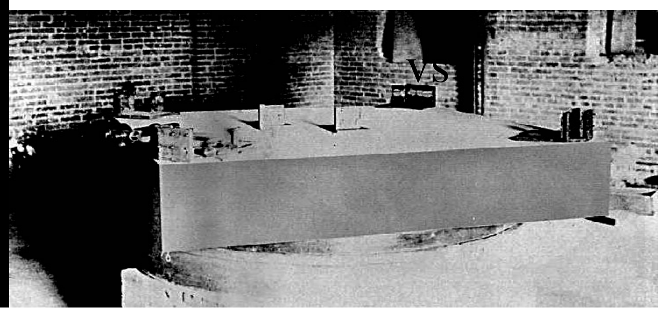
- Angular momentum conserved and
  - no mean field breaking spherical symmetry
  - no asymptotic states
- Perfect equilibrium with no fluctuations

Doubtful this is physical! difference needs to be included at EFT level before phase diagrams in non-inertial frames examined

Finally, onto the topic of this conference







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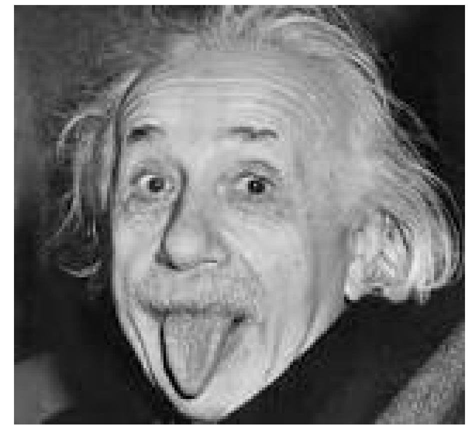
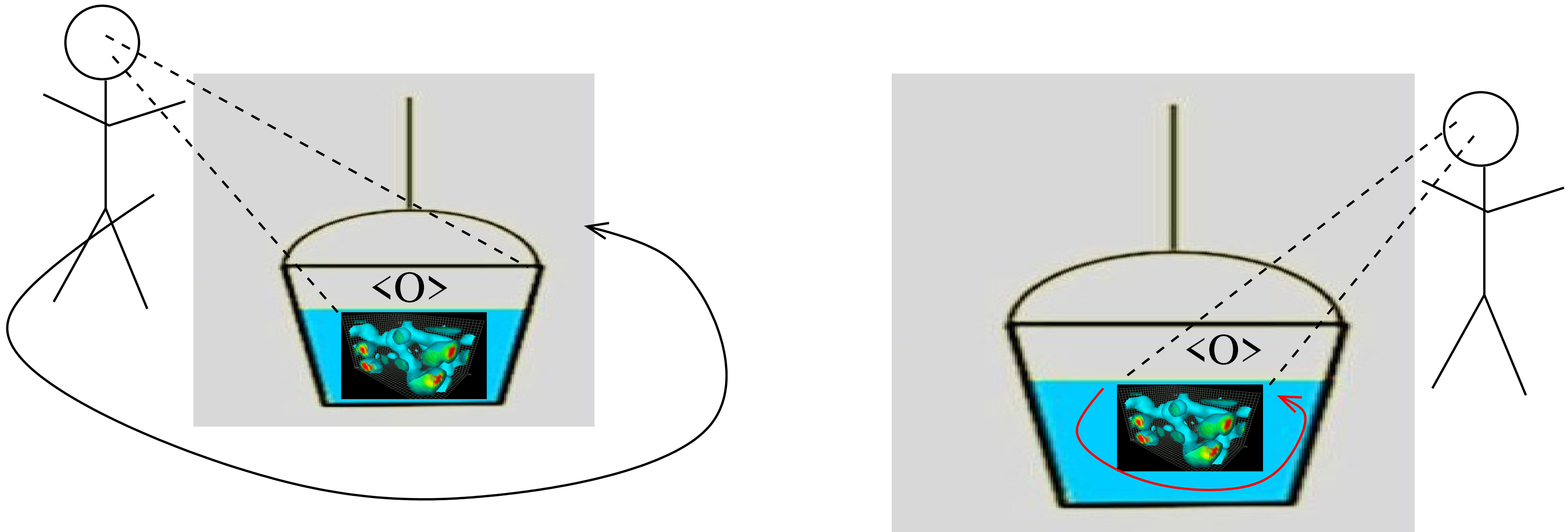


Physics is ultimately an <u>experimental</u> science. A confusing theoretical situation should be checked <u>experimentally</u>

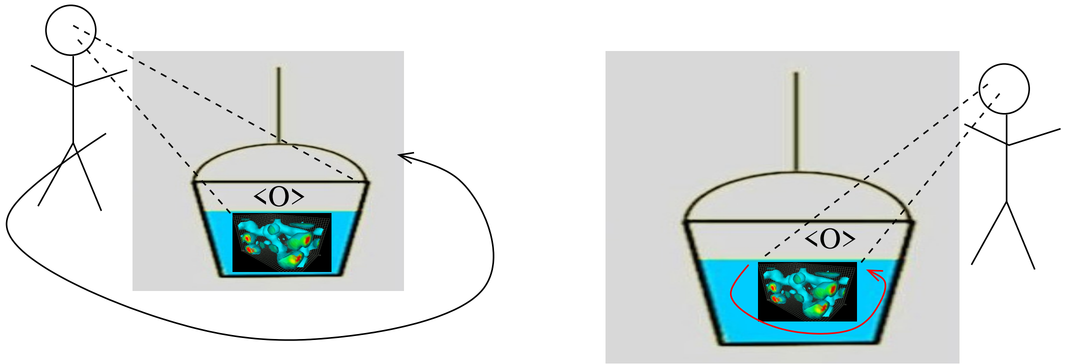
Finally, onto the topic of this conference



Construct analogue system with phase transition, measure order parameter in the two situations!

"relativisticlike,topological, but can both spin it and spin while measuring it

Water: Weyl semimetal Bucket: "measuring device"($\vec{J}$? EM field with $\hat{L}$?)



What we need can we get this from Weyl semimetals?

- Phase transition Chiral order parameter (NJL-like)
- Relativistic-like dispersion relation
- Order parameter changes in scales where both <u>perhaps</u> active,passive rotations possible (”c“ still “fast“,but “non-relativistic“, currents and fields?)

If so, probe of fundamental physics! general covariance of QM,QFT

Conclusions

Active and passive rotations generally not the same

Calculations in a rotating frame tend to be passive, experiments (obviously) active

QFT has some subtleties w.r.t. QM

Local equilibrium signature

Enough theory confusion to warrant an experimental investigation. Weyl semimetals possible candidate