Quantum materials, lattice field theory, chiral anomaly

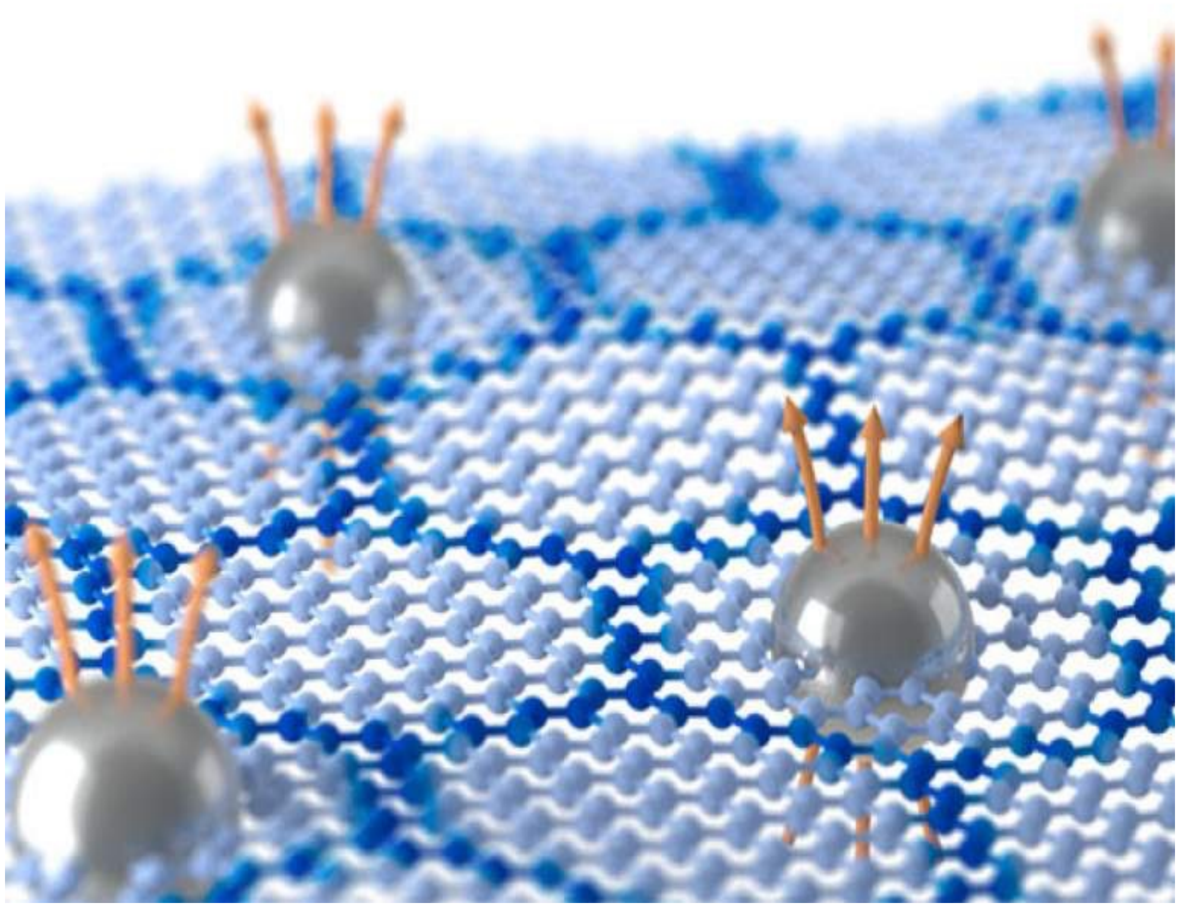
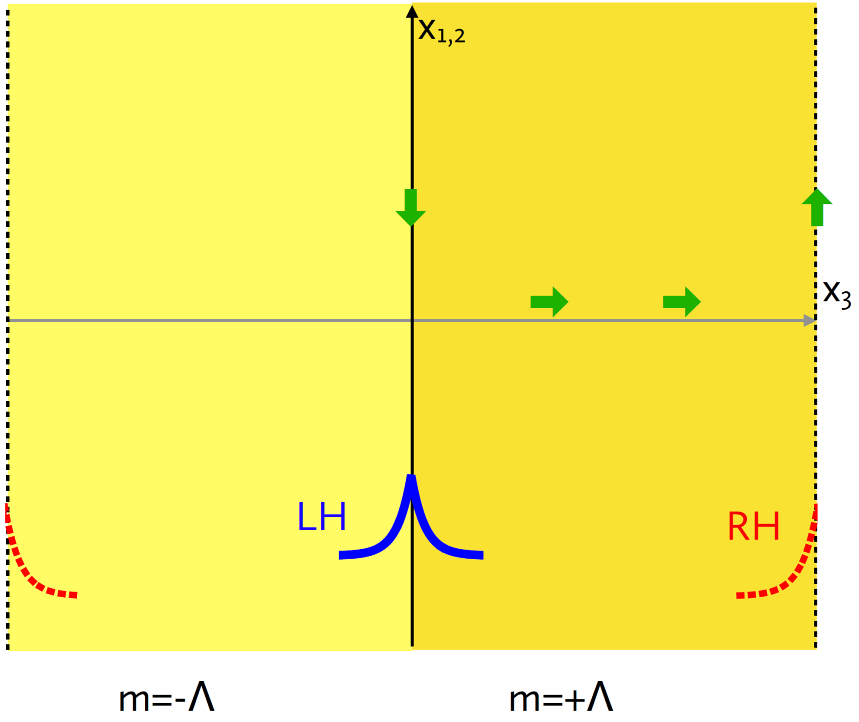


Figure courtesy: Nationalmaglab, Phys.org, David Kaplan



Srimoyee Sen,
Iowa State University

work with David Kaplan, University of Washington

Phys.Rev.Lett. 132 (2024) 14, 141603,

Phys.Rev.Lett. 132 (2024) 14, 141604

arxiv: 2412.02024

“Weyl and Dirac Semimetals as a Laboratory for High-Energy Physics” 2025, University of Minho, Braga,

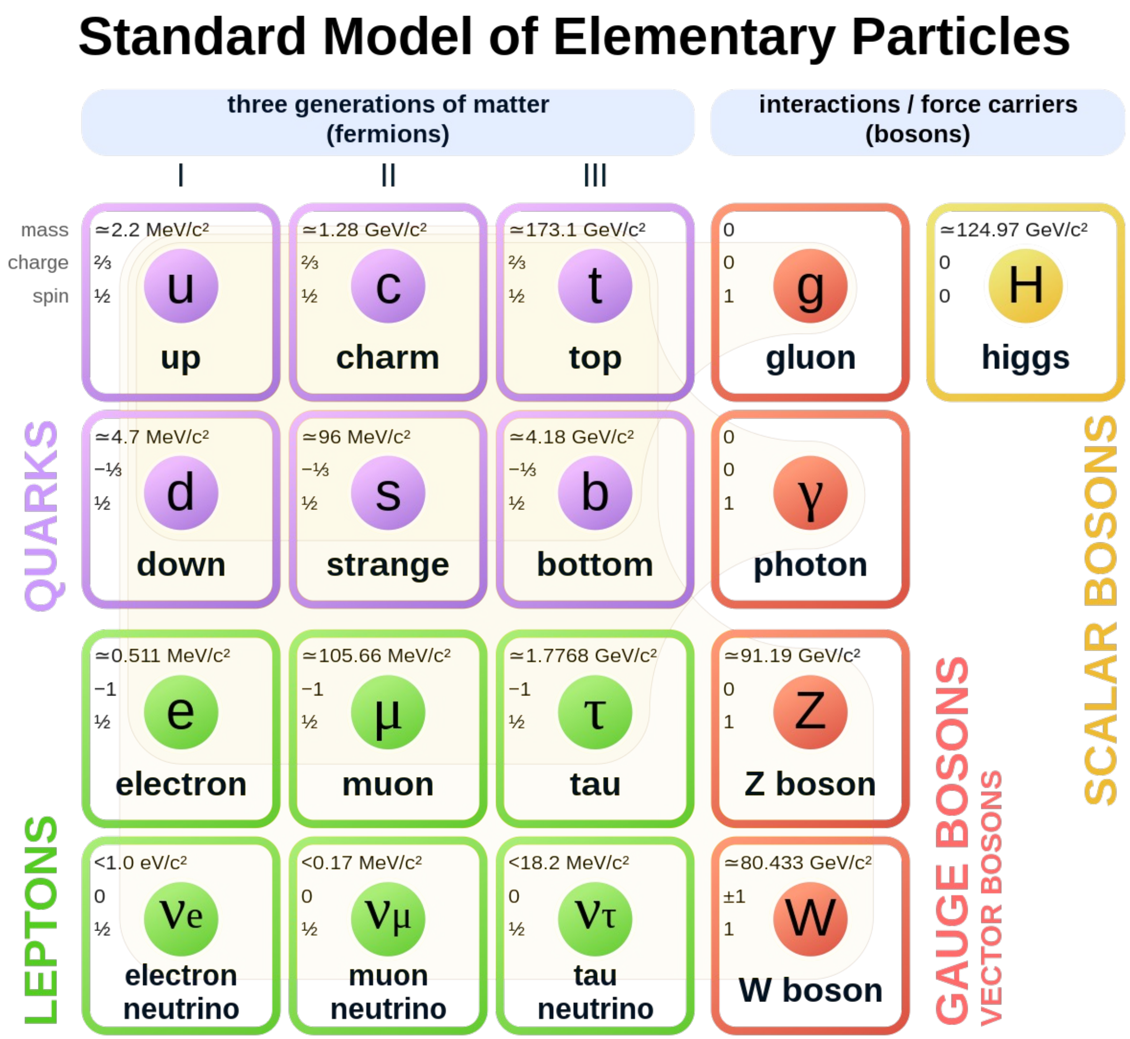
Plan of the talk

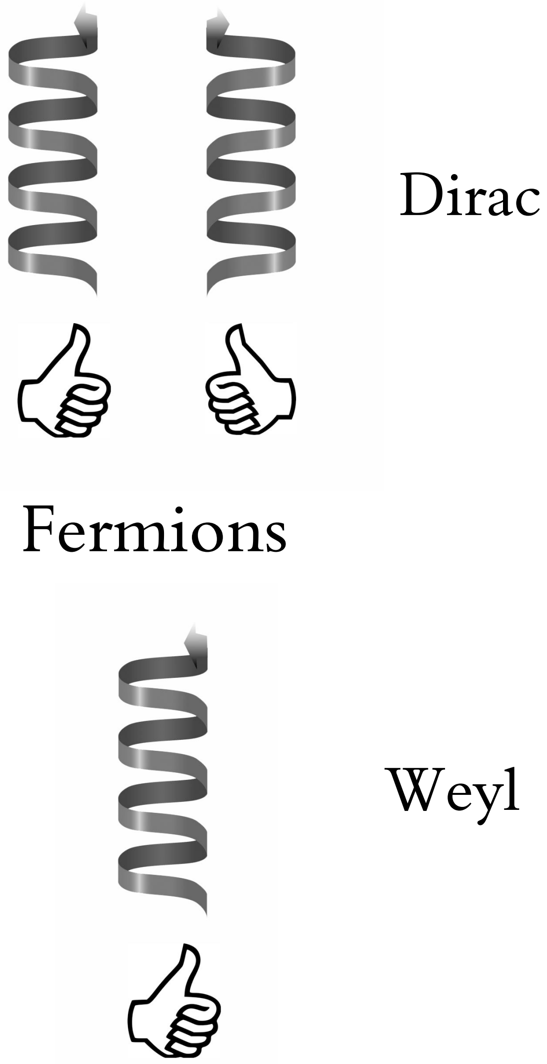
- Thinking about ideas at the intersection of condensed matter physics (topological phases, weyl and Dirac semimetals), nuclear and particle physics profitable for all the fields involved.

- Will discuss a striking example from the past connecting lattice field theory, chiral anomalies and quantum Hall effect

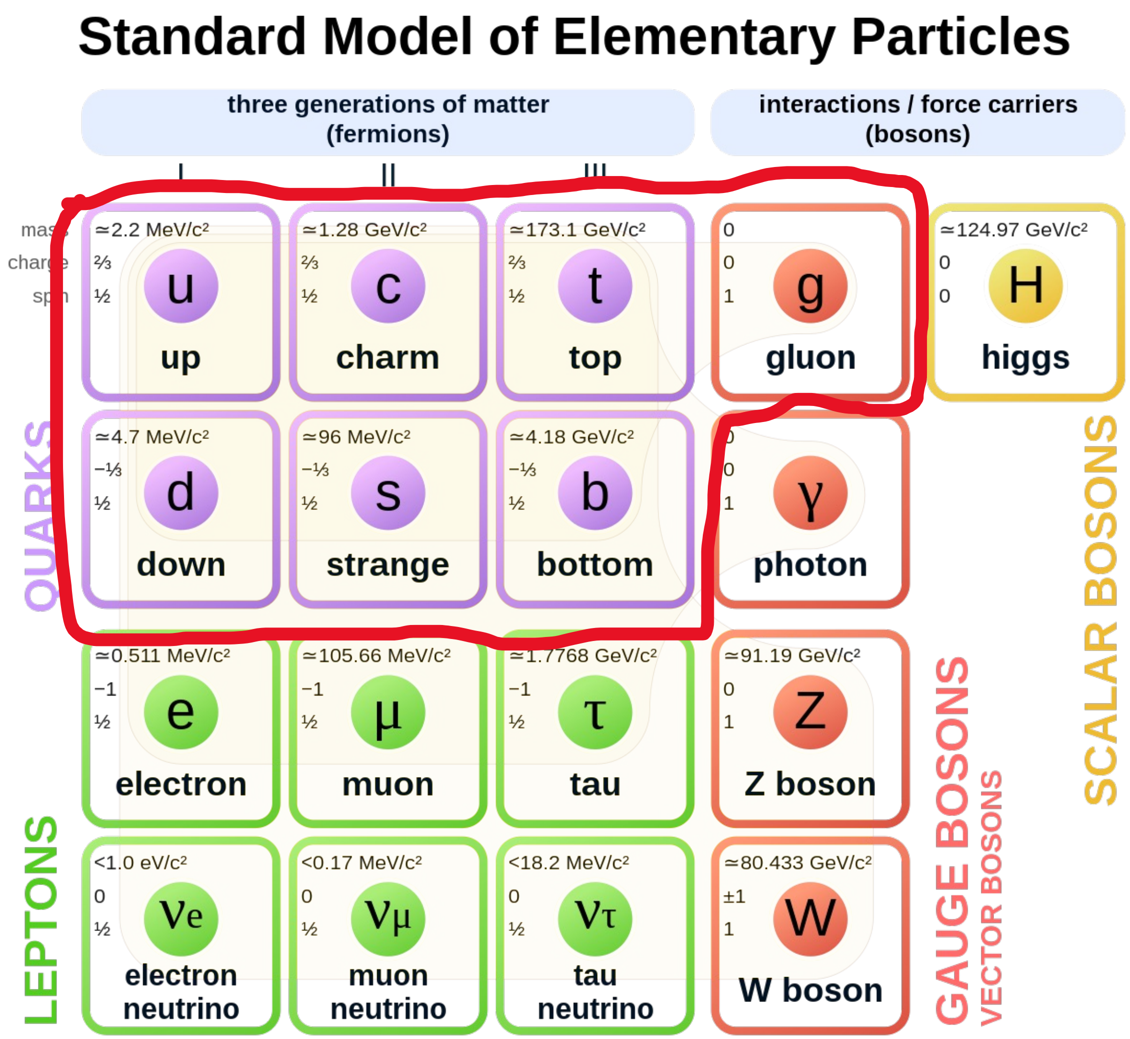
- Discuss problems where a similar line of thinking may help make progress.

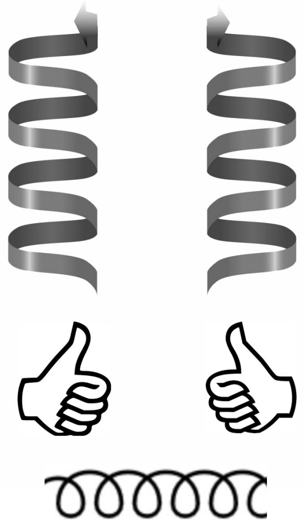
Field Theory ingredients: fundamental forces and fermions (massless)





Field Theory ingredients: fundamental forces and fermions (massless)





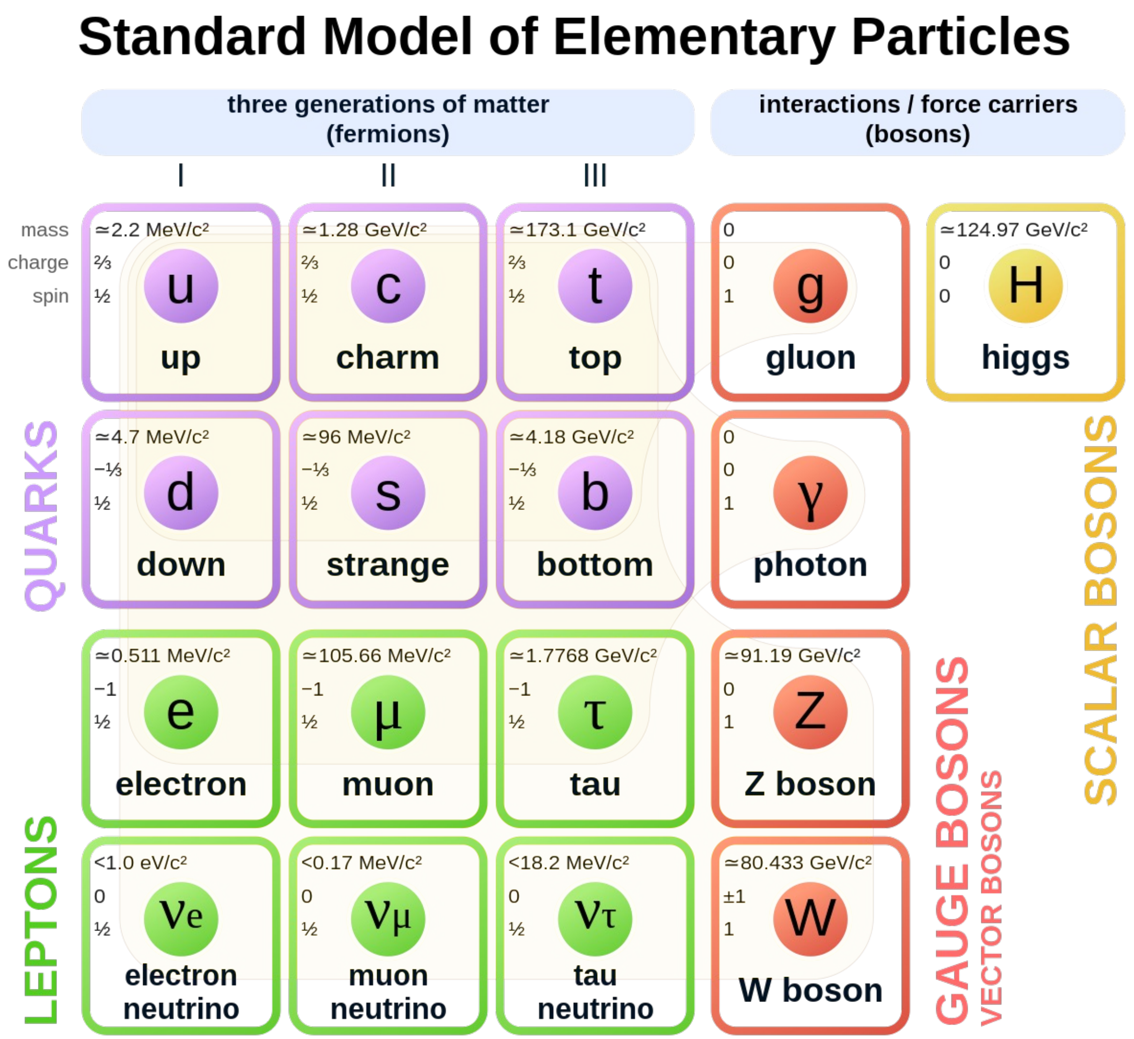
Dirac

Quantum chromodynamics:

Gluons talk to fermions of both chirality equally

Vector gauge theory

Field Theory ingredients: fundamental forces and fermions (massless/gapless)



Weyl

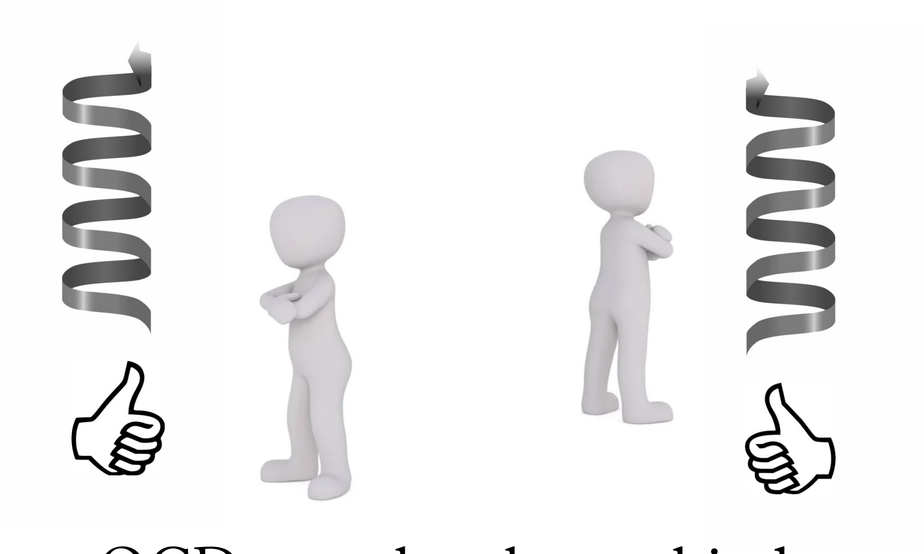
Standard model, weak interactions:

Talks only to one chirality

Chiral gauge theory

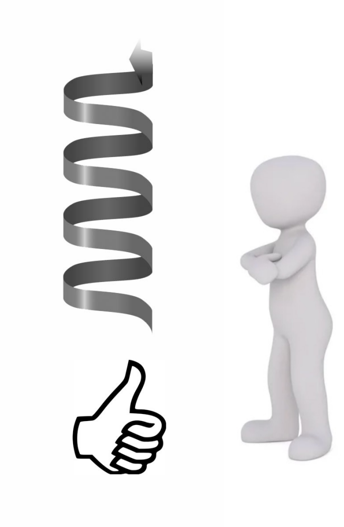
QCD vs weak interactions

$m = 0 \Rightarrow$ chiral symmetry



QCD: good to have chiral symmetry, i.e. exact masslessness, can't allow the chiralities to talk.

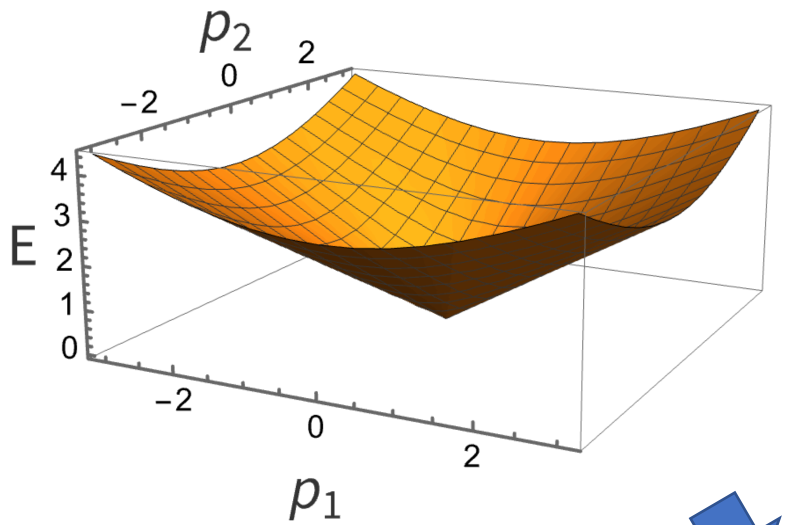
Solved on the lattice.



SM: Chiral symmetry absolutely essential, unpair the chirality

"Chiral gauge theories", yet to be formulated on the lattice.

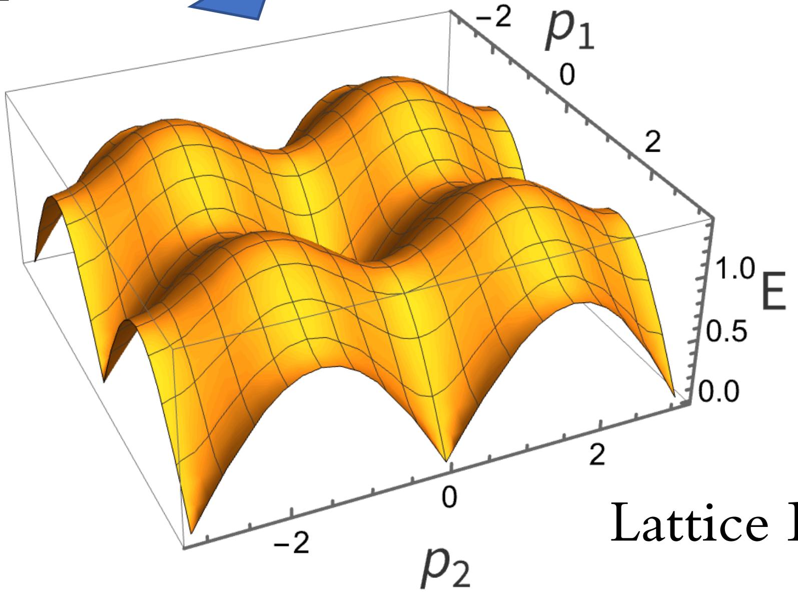
On the lattice (massless Dirac fermion problem for QCD)



Continuum dispersion

We want to formulate fermion theories on the lattice.

We want to keep the fermions practically gapless (to simulate light quarks).

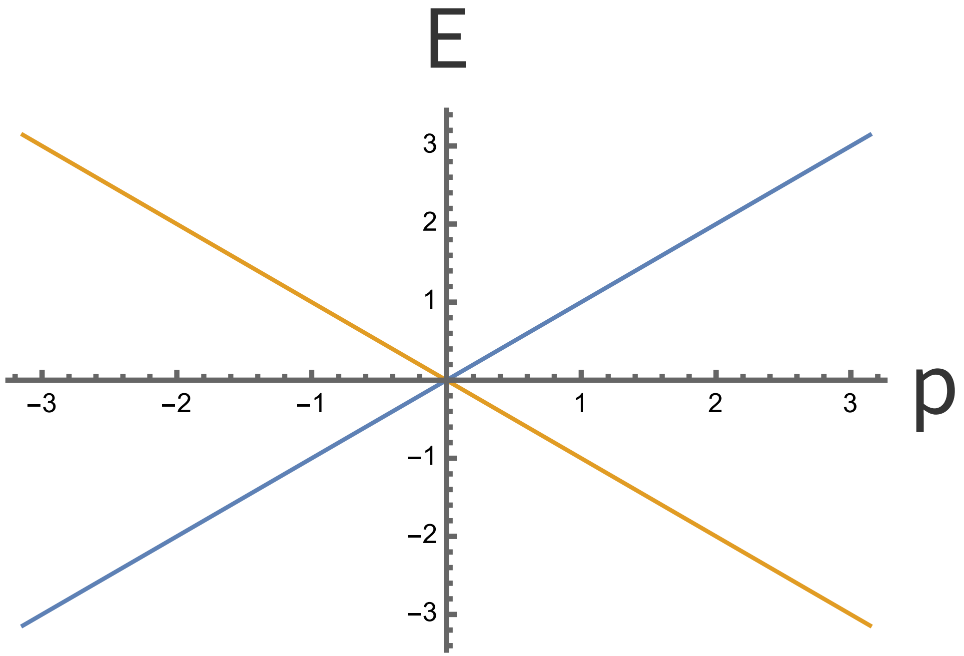


Lattice Brillouin zone

This combination i.e. “discrete space-time + gapless fermion”: extremely difficult to engineer.

Nielsen-Ninomiya No-Go

Why chiral symmetry is hard: Dispersion (1 spatial dimension)



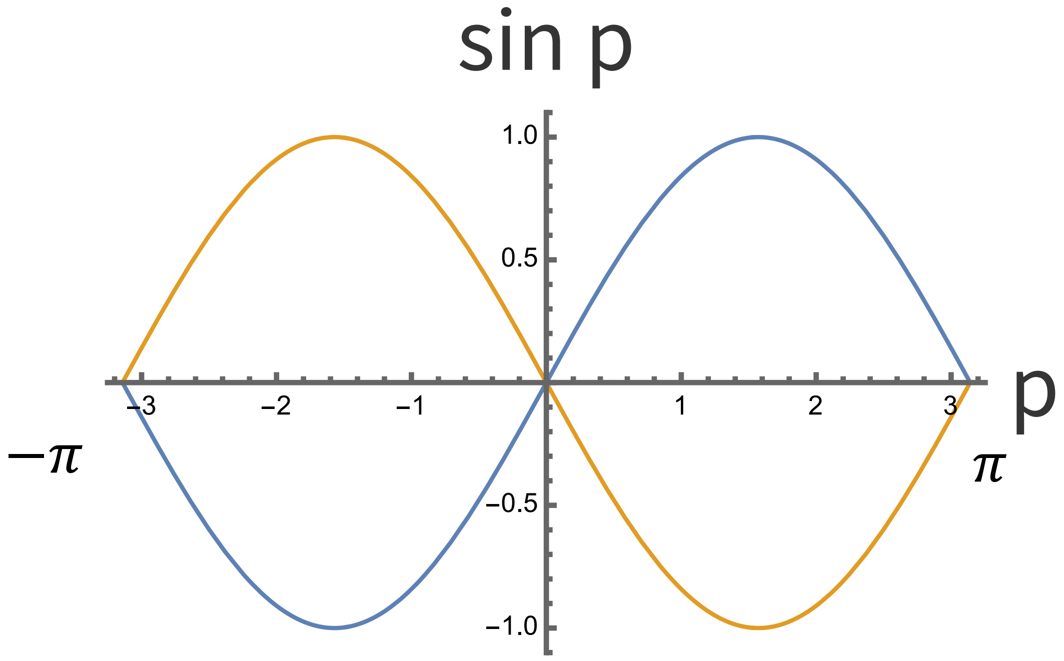
Continuum dispersion for a Dirac Hamiltonian

The no-go is better visualized using dispersion relation in Minkowski space-time (time continuous).

Hamiltonian formulation.

$$E = \pm p$$

Brilliuoin zones (Dirac)



Two Dirac fermions

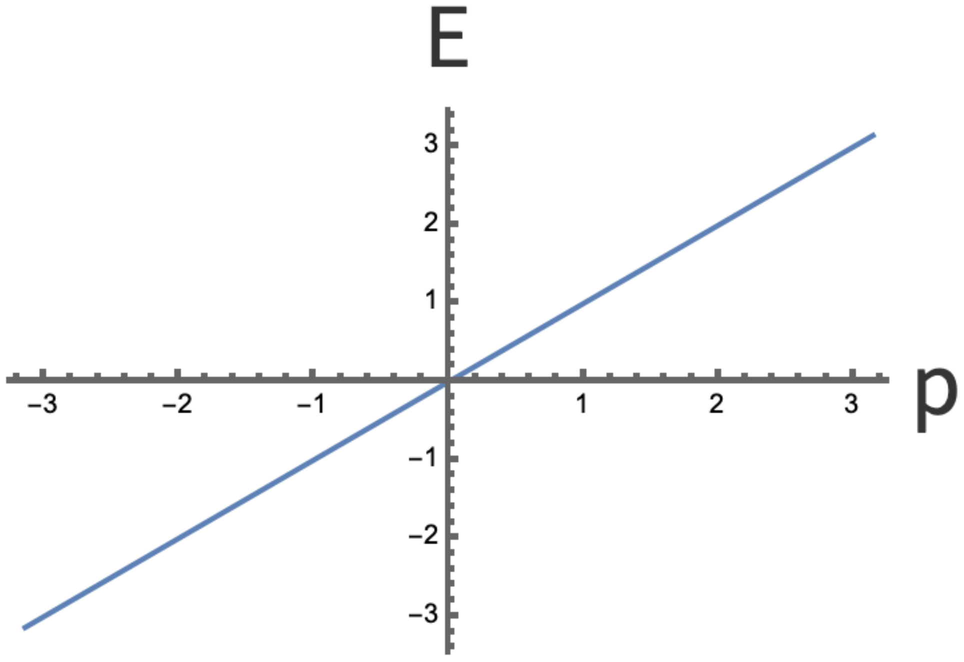
Lattice in space.

Time not discretized.

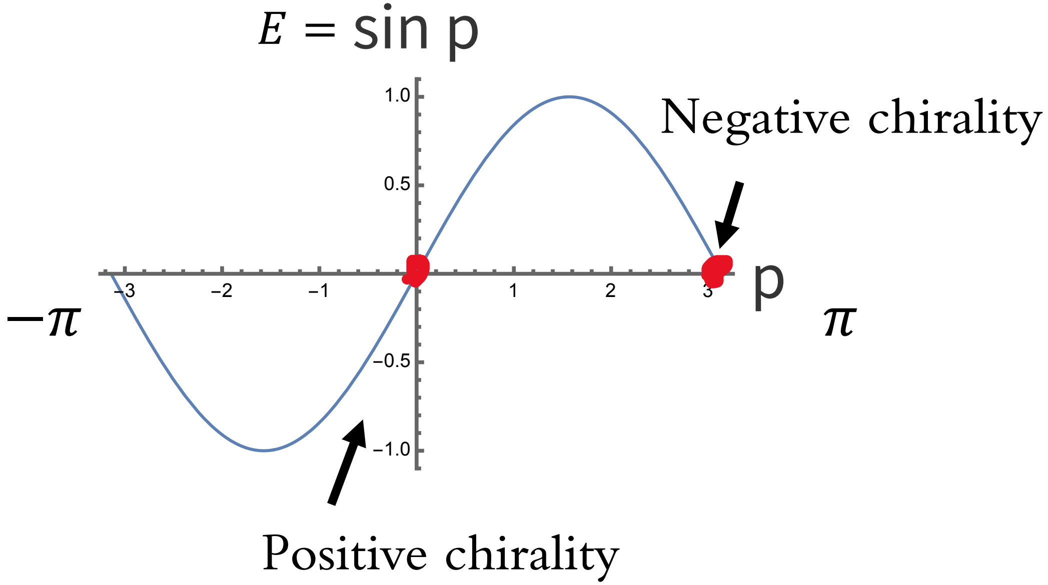
Solving the naively discretized Dirac Hamiltonian with eigenvalues $\pm\sin p$

$$E = \pm\sin p$$

Weyl fermion



Continuum



Lattice

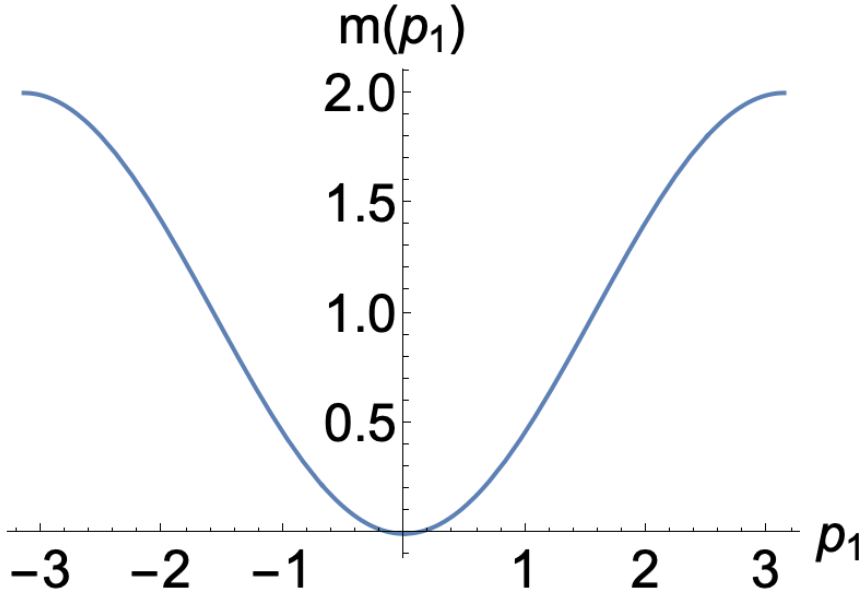
Even number of zero crossing of periodic functions

Quick fix for Dirac (leads to other problems)

Add a momentum dependent mass:

$$(p_0)^2 - (\sin p_1)^2 - m(p_1)^2 = 0$$

$$(p_0 \sim \pm p_1 \sim 0) \ , \ (p_0 \sim \pi \pm p_1 \sim 0)$$



Extra species gone. But this causes other problems.

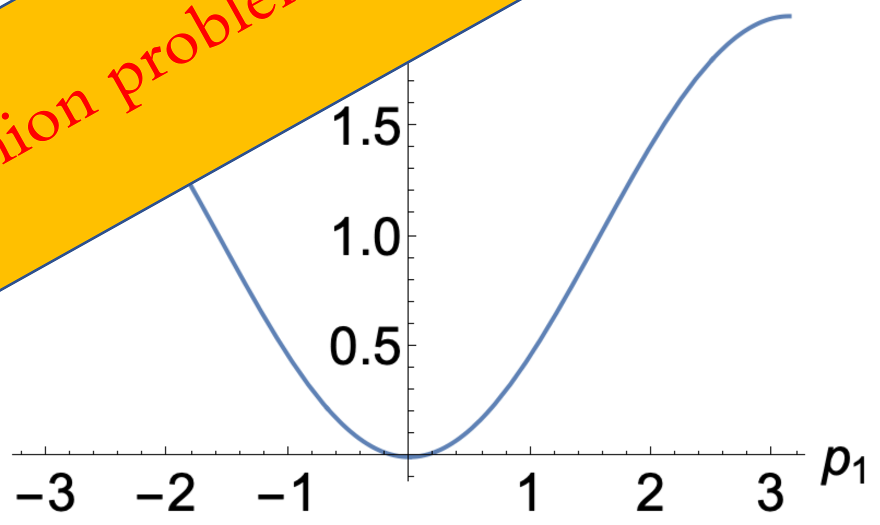
Chiral symmetry is killed.

Quick fix for Dirac (leads to other problems)

Add a momentum dependent mass:

$$(p_0)^2 - (\sin p_1)^2 - m(p_1)^2 = 0$$

$(p_0 \sim \pm p_1 \sim 0$

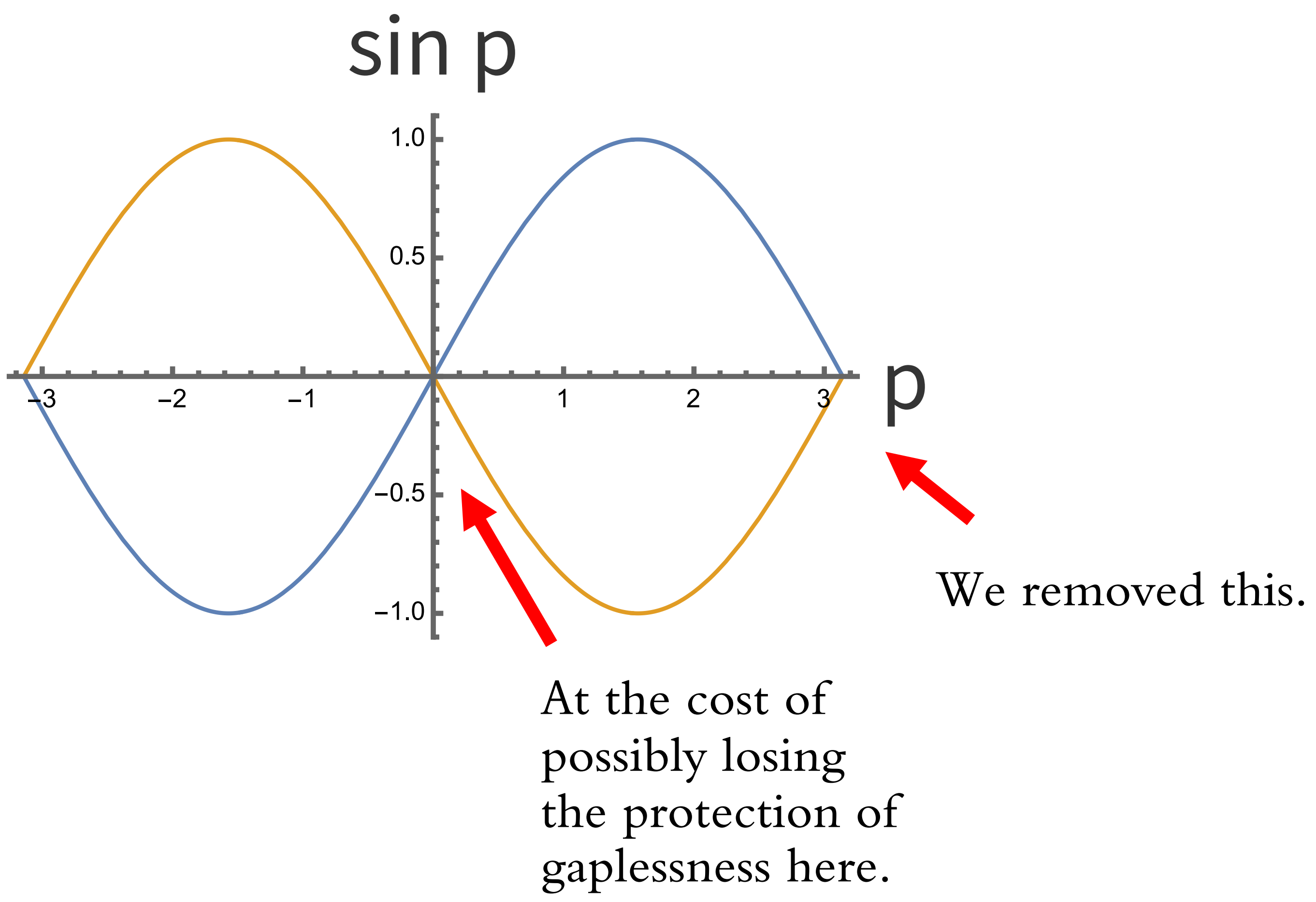


This is the chiral fermion problem

Extra species gone. But this causes other problems.

Chiral symmetry is killed.

Let's take another look at momentum dependent mass



What if you could physically separate the two chiralities?

By going to one higher dimension?

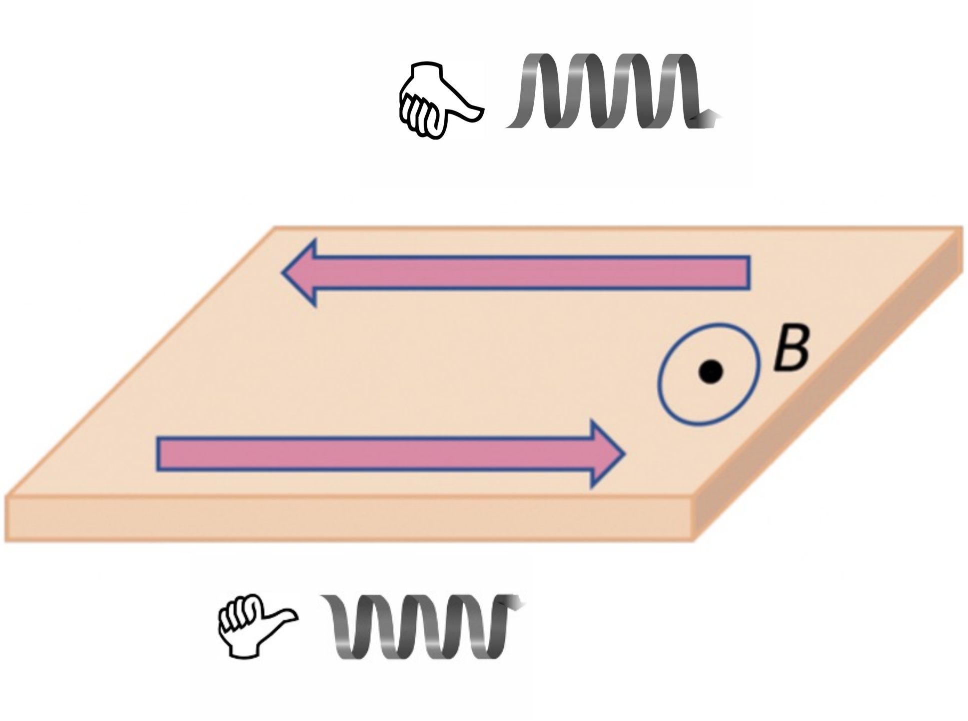
Where does this happen?

What if you could physically separate the two chiralities?

Quantum Hall Effect

Relativistic Analog: Dirac fermion in $2+1$ dimensions

Anomalous Hall effect.

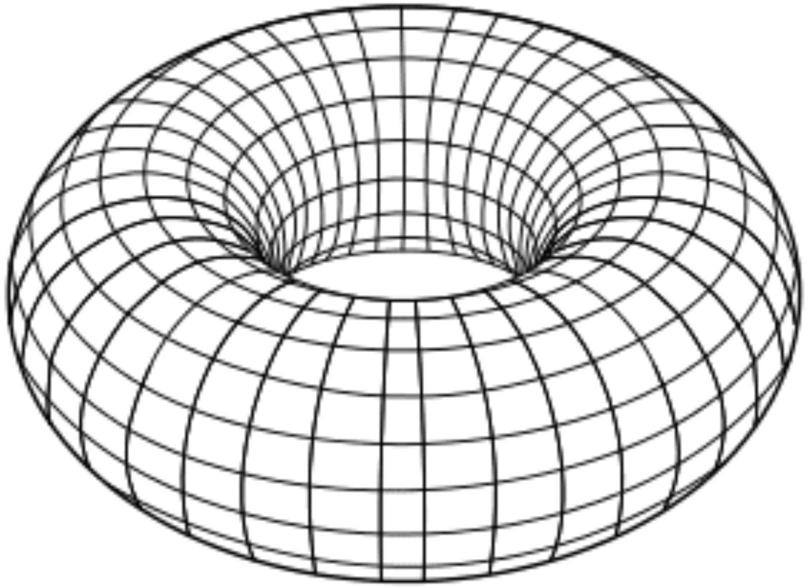


How do you achieve this in a relativistic theory?

Go to higher dimension. Consider 2+1 D Dirac fermion. Let's go to finite volume.

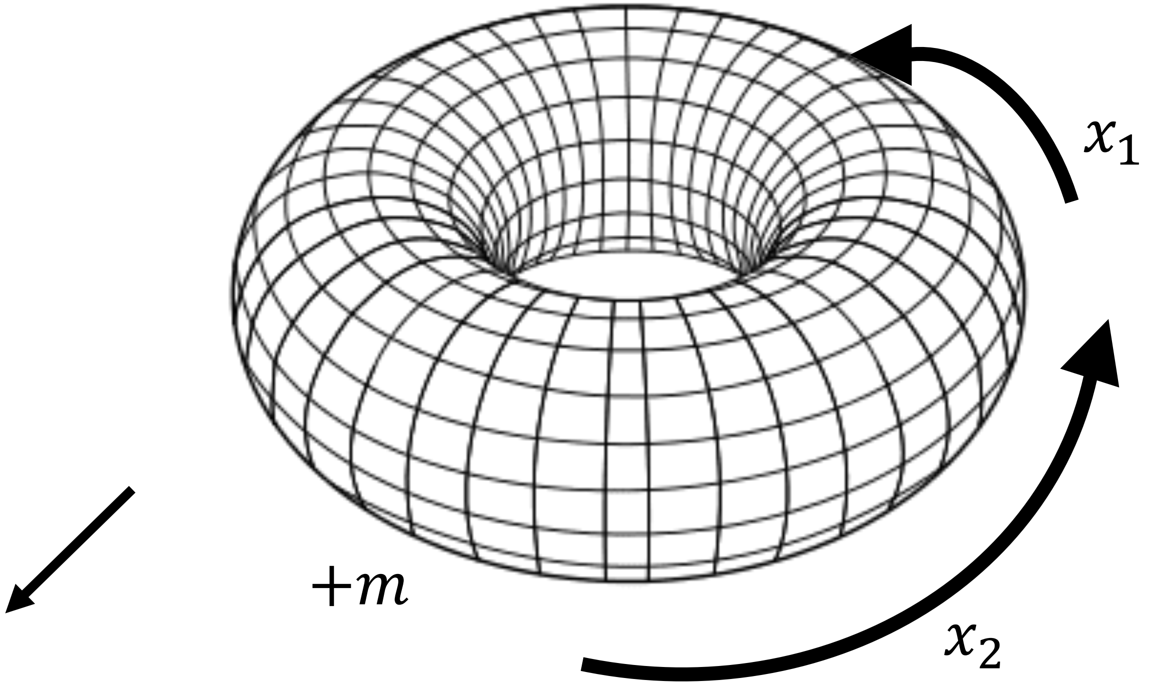
Periodic boundary condition in all directions with uniform Dirac mass.

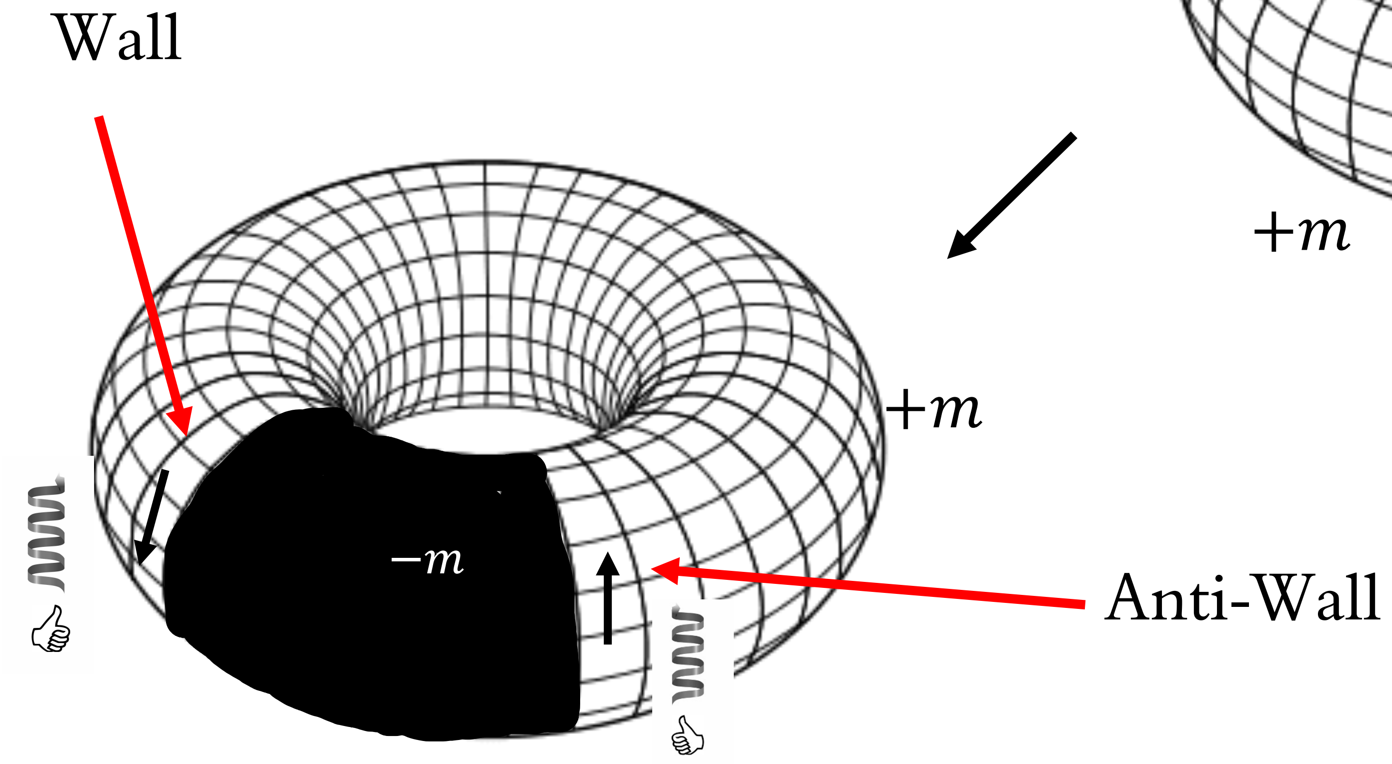
$+m$



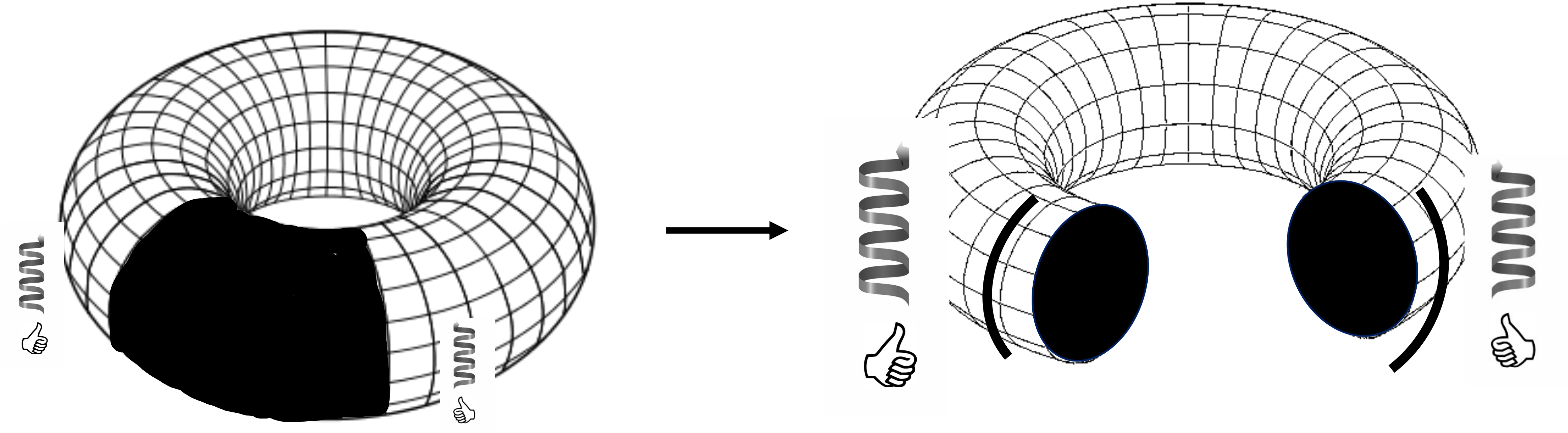
How do you do separate the two chirality?

Introduce domain walls in
fermion mass





Mass defect to cylinder (open boundary condition)



So, how do you simulate QCD?

- Physically separate the right and the left: right lives on the wall and left on the anti-wall (Kaplan 1992). Quantum corrections suppressed in the separation.
- Allow the gluons to talk to both walls.
- And the physics of the two walls together will give you QCD.

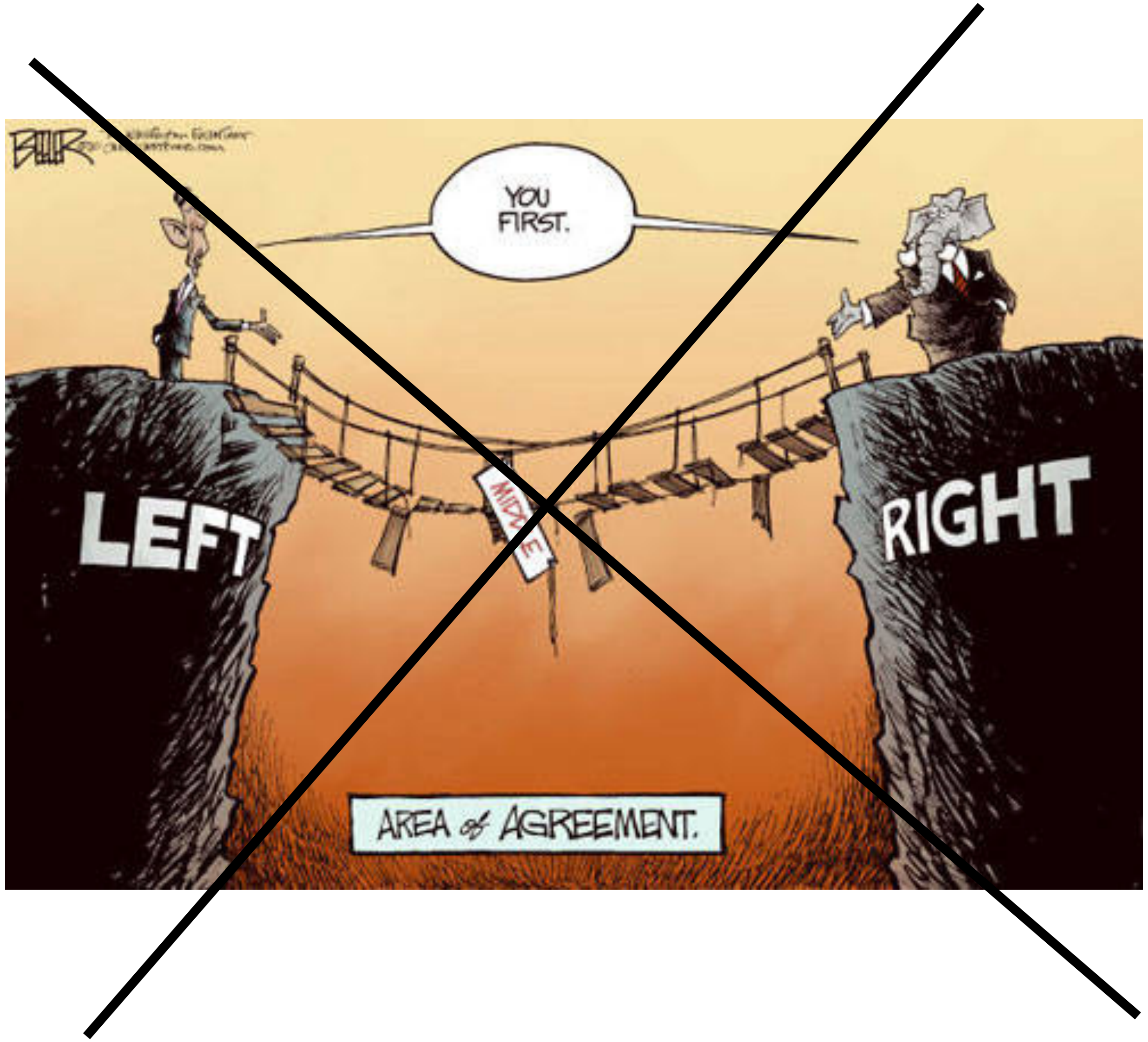
Quantum Hall effect

Doesn't work for chiral gauge theories

The idea does not work for chiral gauge theories though.

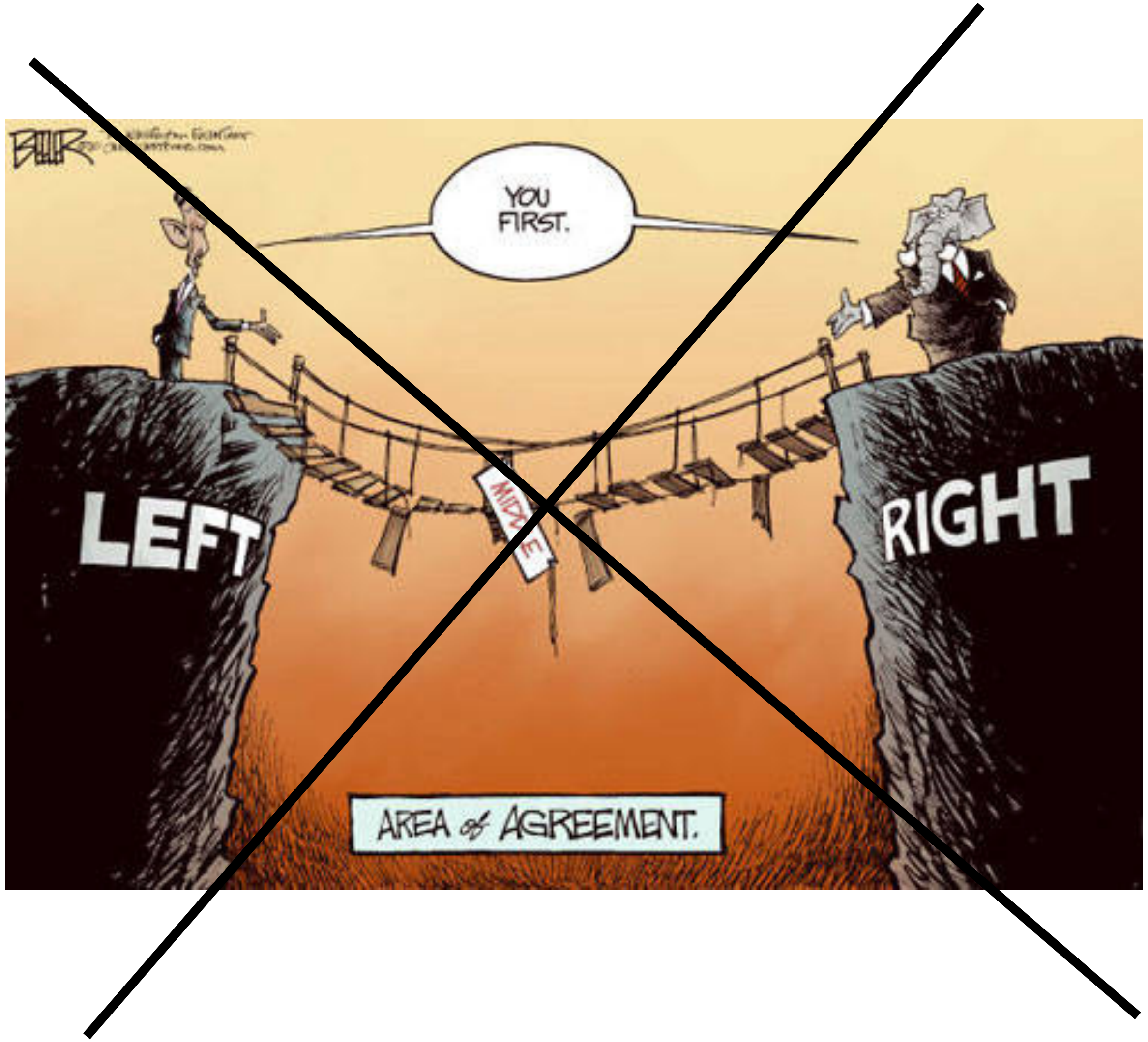
The construction in finite volume necessarily has two defects.

Two defects lead to opposite chiralities producing non-chiral theory.

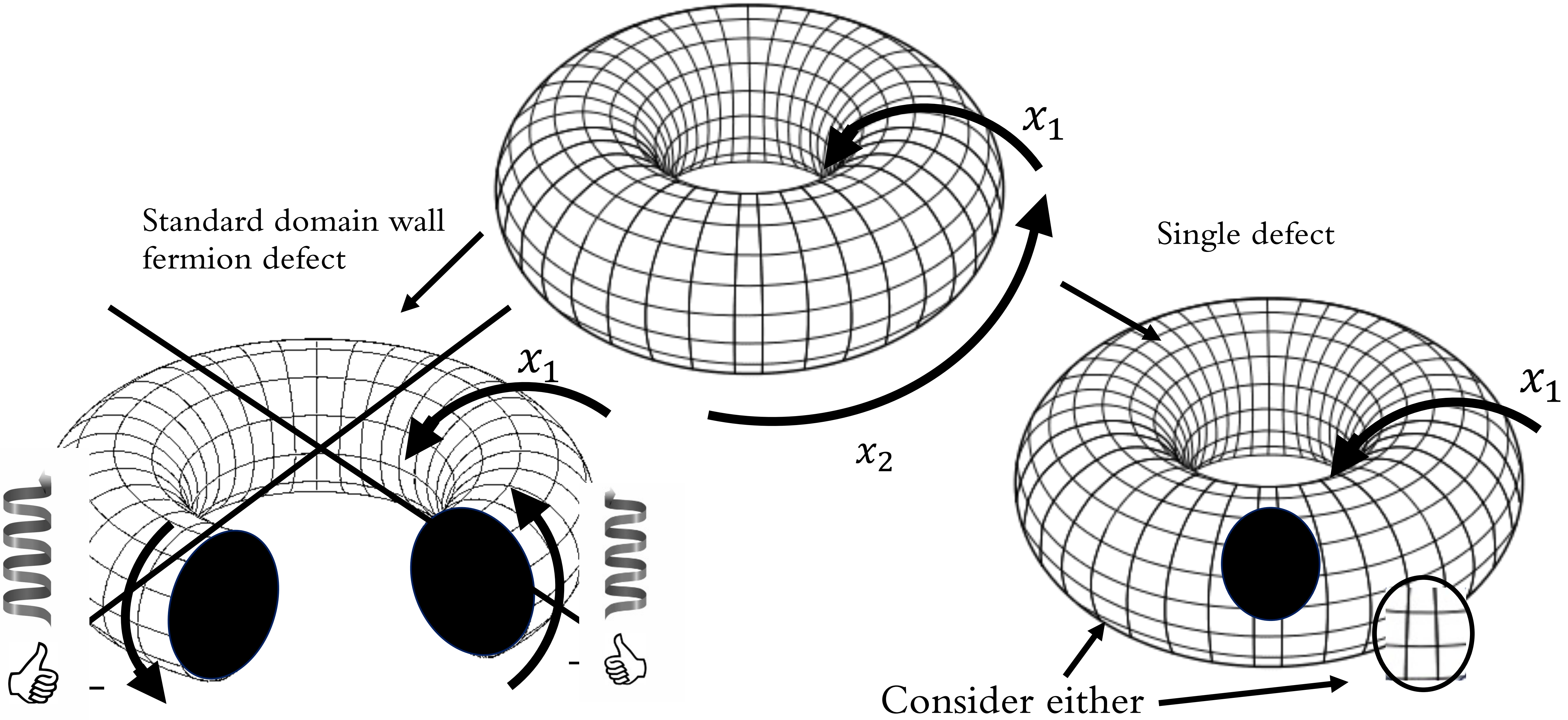


Doesn't work for chiral gauge theories

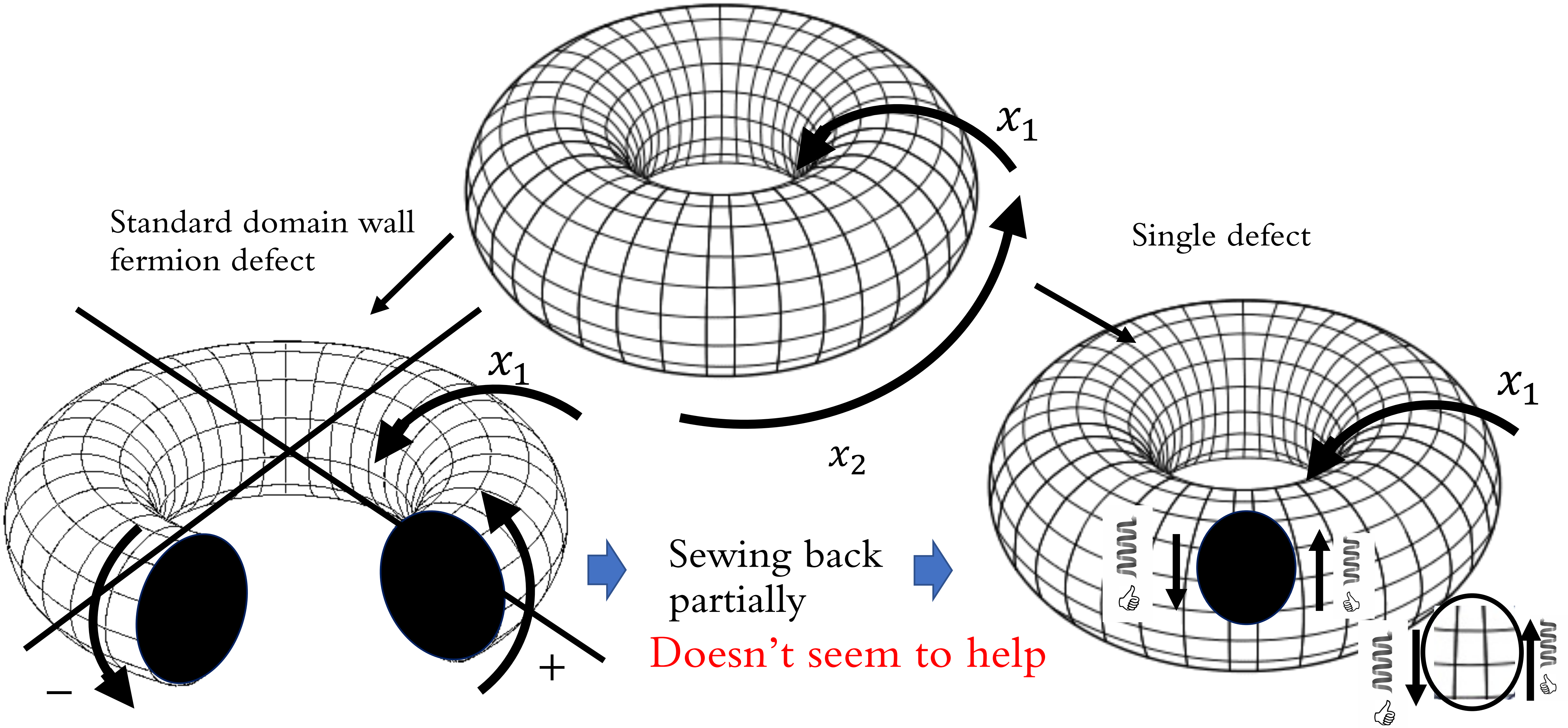
We need to isolate a single Weyl fermion of a particular chirality --- impossible with the standard domain wall setup.



What if we had a single defect?

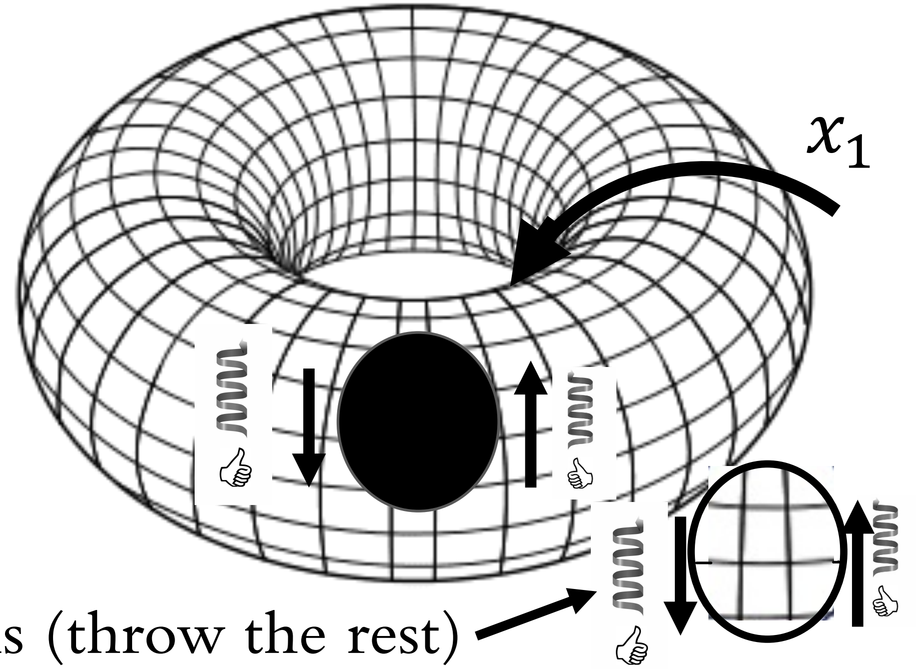


Opposite chirality on the two sides..



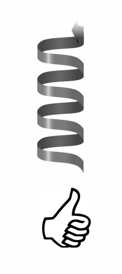
Opposite chirality on the two sides..

Maybe the problem is that we are keeping the definition of chirality position independent.

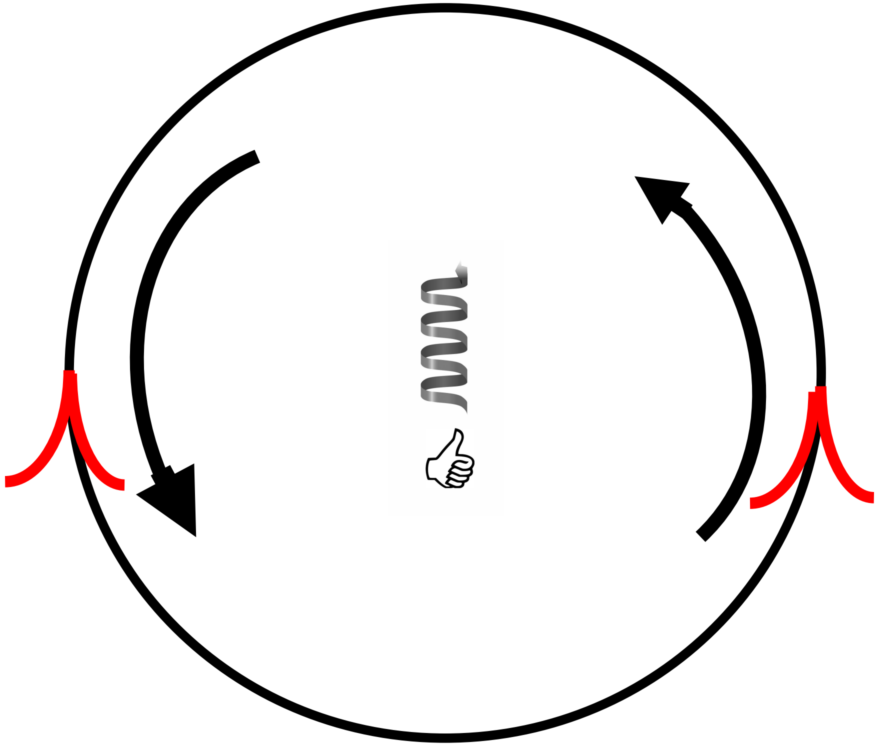


Define chirality in a position dependent manner

Define chirality as clockwise travel vs anticlockwise travel:

counter-clockwise 

clockwise 



Single chirality: Weyl mode

Phys.Rev.Lett. 132 (2024) 14, 141603, *Phys.Rev.Lett.* 132 (2024) 14, 141604

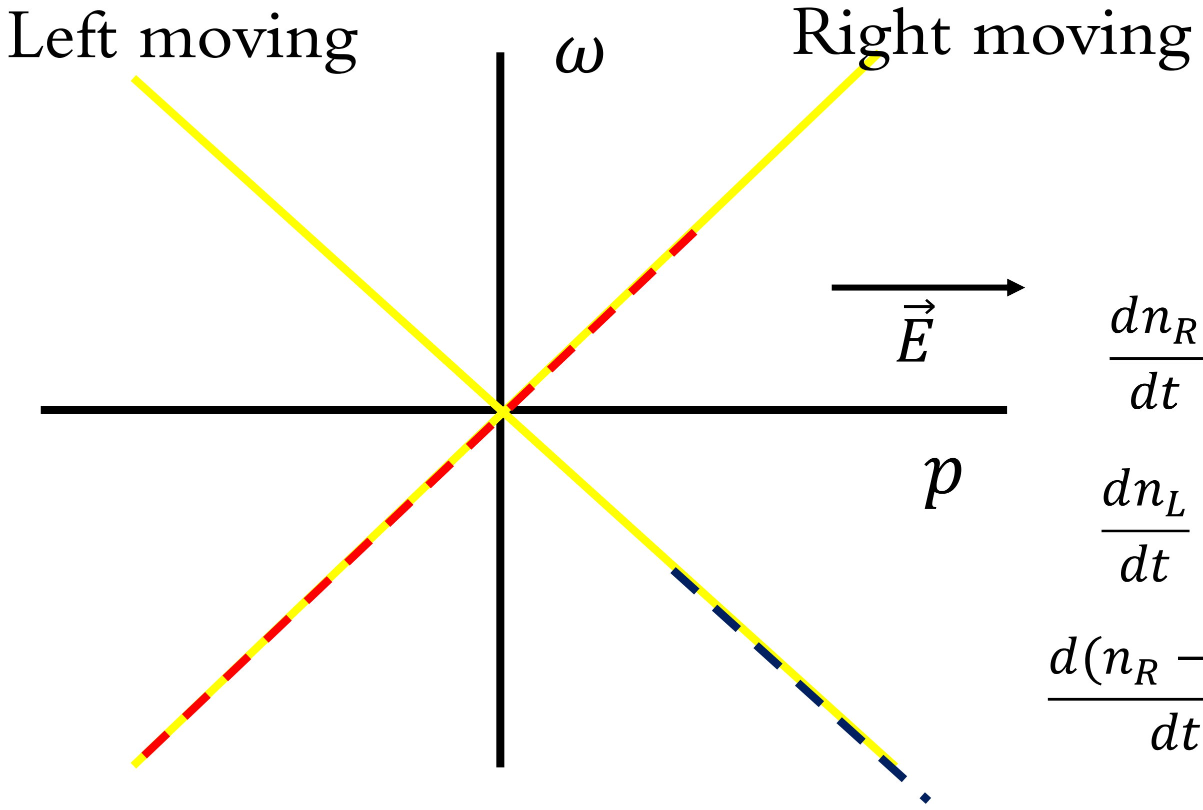
Gauging

Can engineer any number of Weyl fermions on the boundary.

We can gauge any subgroup of the available global symmetry of the free fermion theory. ---- Must make sense only if the theory is anomaly free.

How do we see this?

1+1 D massless Dirac fermion spectrum



$$\frac{dn_R}{dt} = \frac{eE}{2\pi}$$

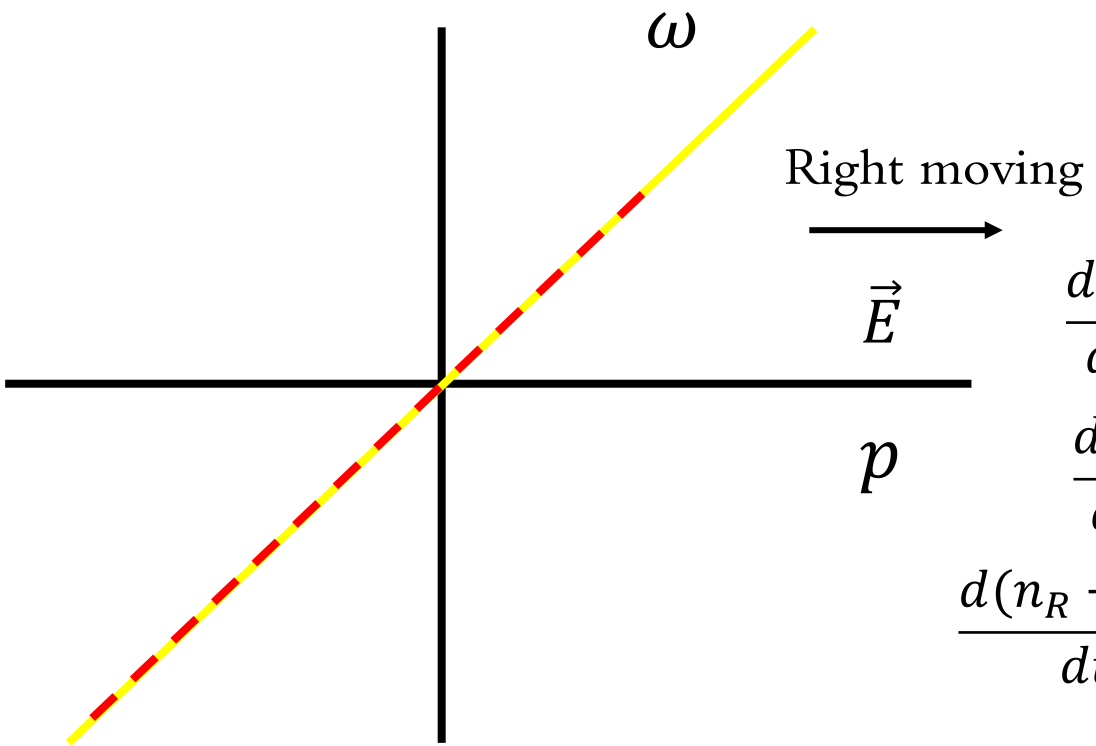
$$\frac{dn_L}{dt} = -\frac{eE}{2\pi}$$

$$\frac{d(n_R - n_L)}{dt} = \frac{eE}{\pi}$$

Current $\propto E$

Vector current or charge $n_R + n_L$ conserved, axial not so.

Edge world: Anomaly, Weyl fermion



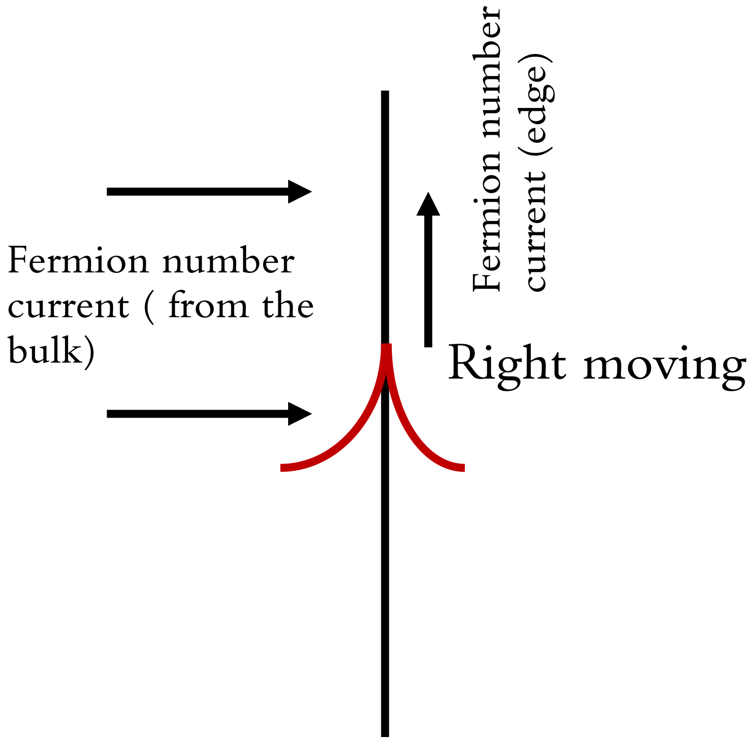
$$\frac{dn_R}{dt} = \frac{eE}{2\pi}$$

$$\frac{dn_L}{dt} = 0$$

$$\frac{d(n_R + n_L)}{dt} = \frac{eE}{\pi}$$

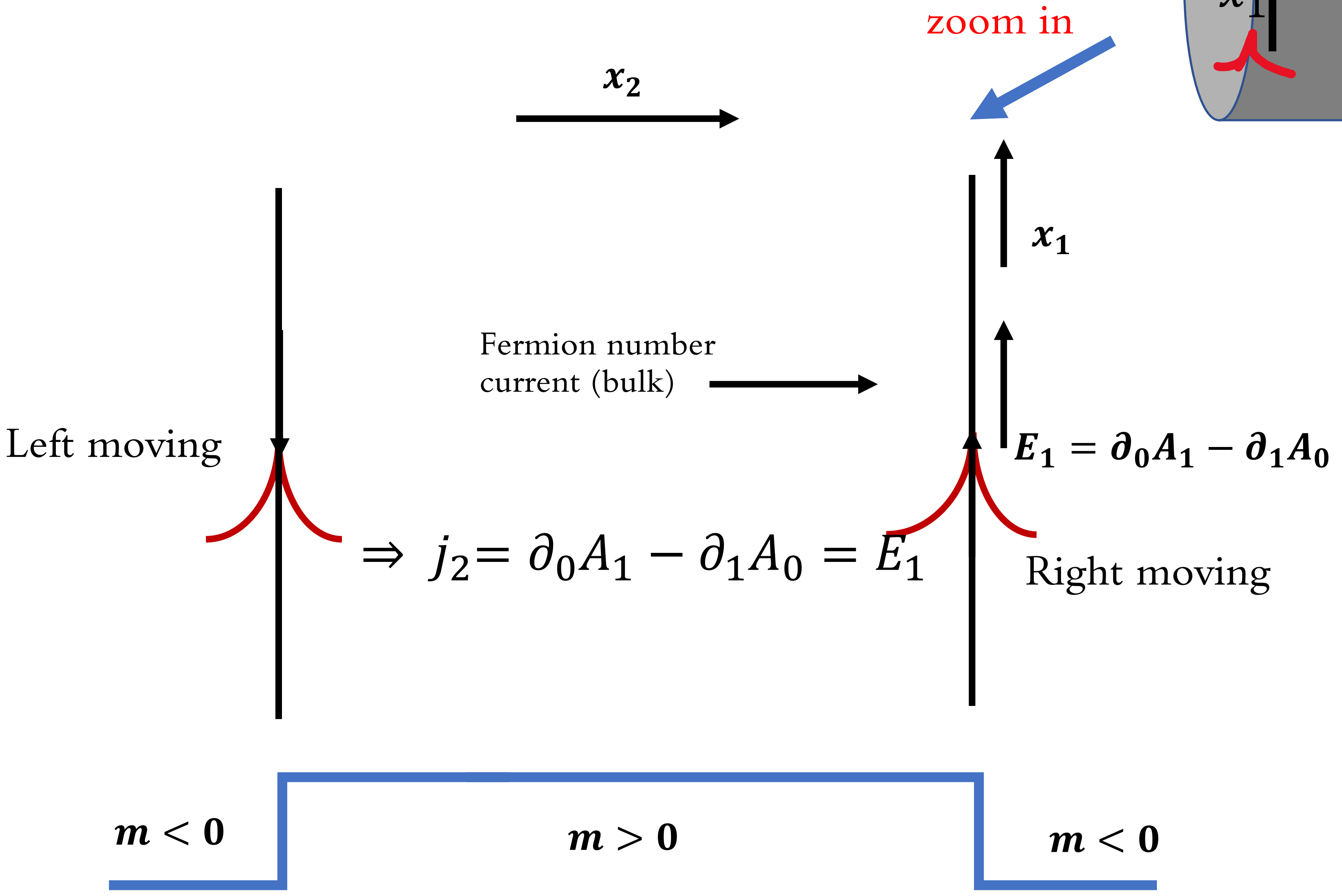
Current $\propto E$

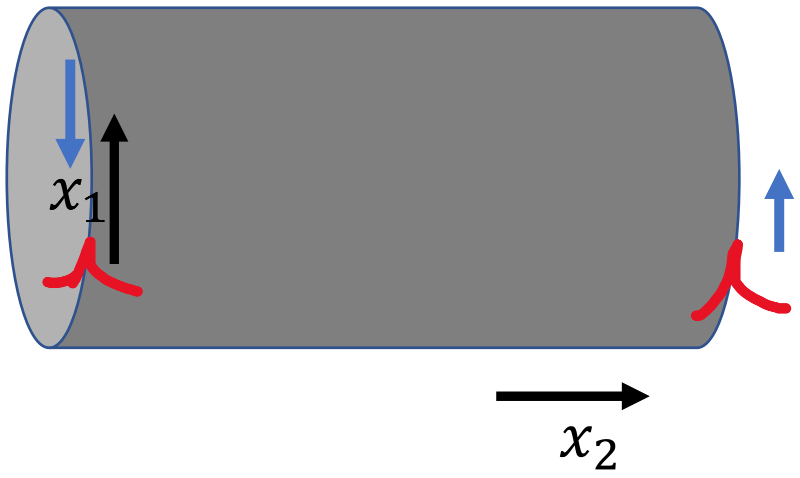
Vector current not conserved, by itself is sick in an electric field.



Can exist on the boundary of a higher dimensional theory

Domain wall + anomaly



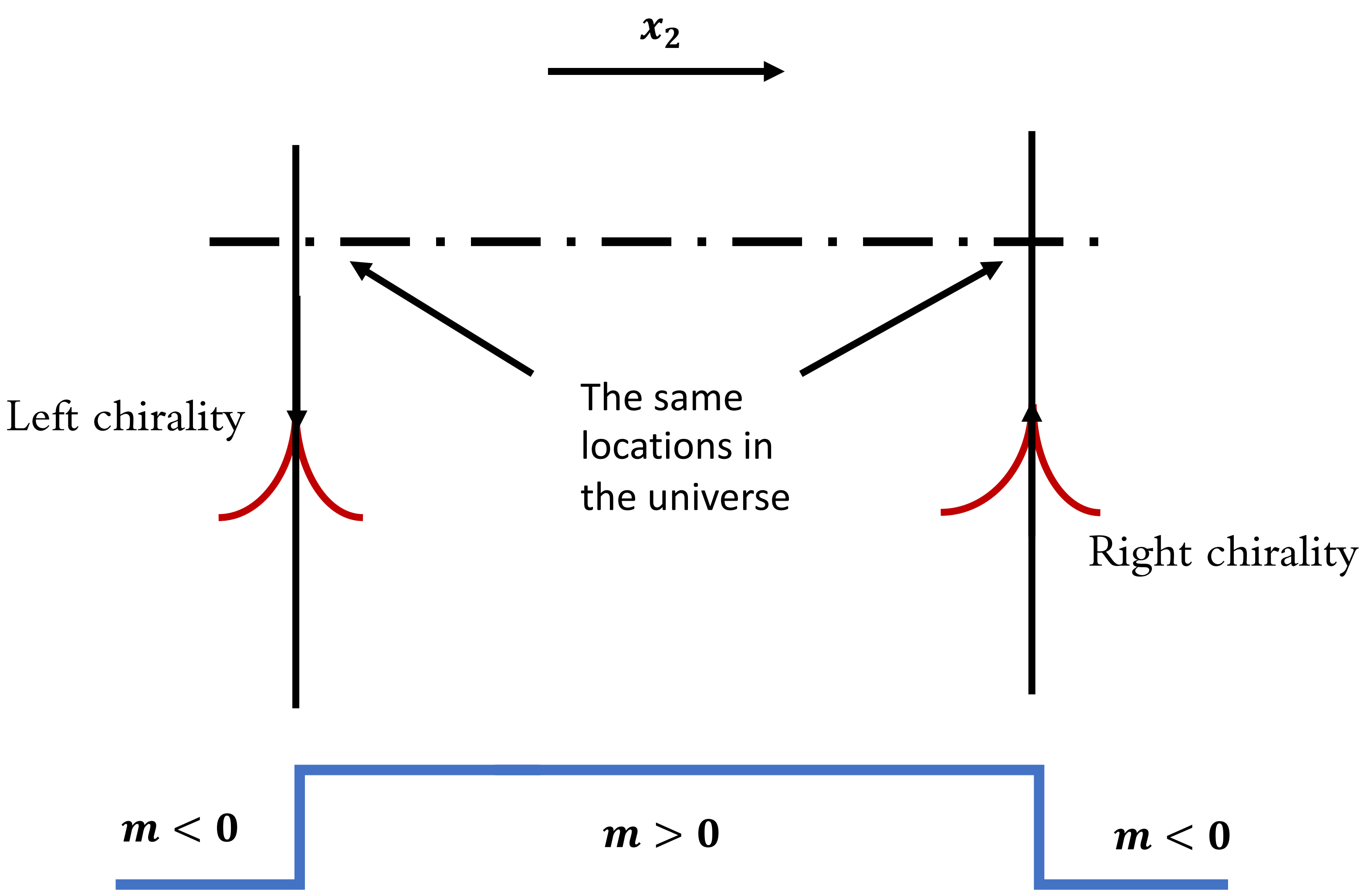


2 + 1 D Dirac fermion setup with domain walls

Gauge the Dirac fermion

Bulk talks to the boundary, great for QCD

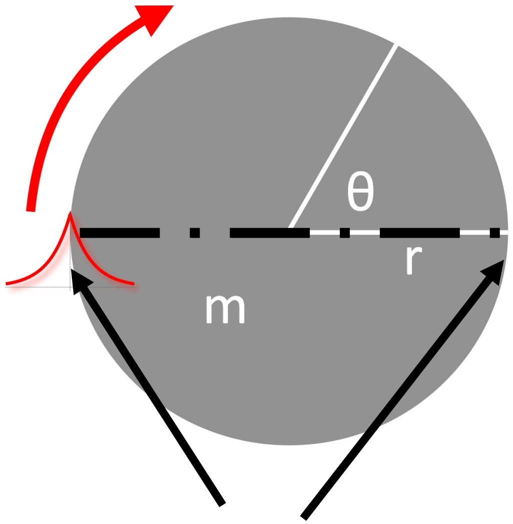
Domain wall + anomaly + QCD



Bulk talks to the boundary, great for QCD

The two walls talk to each other

disk + anomaly



Different locations in the universe, shouldn't communicate across the defect

But they will if the boundary theory has gauge anomaly: nonlocal!!

Thankfully, the standard model is anomaly free.

So, the disk construction makes sense, and the boundary theory is local.

Summary and outlook

The plan is to engineer the fermion content of the CGT (e.g. the standard model) on the boundary and then turn on the desirable gauge interactions.

Several outstanding questions remain relating to the gauging of the theory.

The ideas discussed here rely on separating the chiralities in extra dimension. Demonstrate ties between quantum Hall physics and lattice field theory.

In Dirac and Weyl semimetals, one may be able to separate different Weyl nodes in momentum space.

Could they be used in lattice formulation of QCD or CGT in real time?