1-form symmetry, Conductivity vs Resistivity & Operator lifetime in chiral MHD

a majestically cute pygmy hippo called "Moo Deng" from Thailand (has nothing to do with this talk)

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Based on 2212.09787 with Arpit Das (Durham → Edinburgh)& Nabil Iqbal (Durham)

And 2309.14438 with Arpit Das, Adrien Florio (Brookhaven → Bielefeld U.) & Nabil Iqbal

Things I'm interested in & some sociology

See e.g. Gaiotto, Kapustin, Seiberg & Willet '14;

* A decade ago, people start pointing out that there are more than ordinary symmetry in QFT

$$U_\theta[vol] = \exp\left(i\theta \int d(vol)\, j^0\right) \qquad U_\theta[vol]\mathcal{O}_q U_{-\theta}[vol] = e^{i\theta q}\mathcal{O}_q$$

  - ✦ There can be U[surface] can act on line or surfaces operators, or U that break some of the group axioms (higher-group & non-invertible)

See e.g. Sharpe '15; Cordova, Dumitrescu & Intriligator '18; Tachikawa '17|
See also TASI lecture by Shu-Sheng Shan '23 & ICTP school lecture by Sakura Schaefer-Nameki '23

  - ✦ A lot of these global non-group (or 'categorical') symmetry came from gauging > connection a lot of math's activity with physics (funded by Simons foundation)

* Questions that appeal to me: Can one formulate a new kind of (physically relevant) fluids with these categorical symmetry?

But why would a physicist care about this?
Can this teach us something we haven't already know?

3 OBVIOUS THINGS THAT SHOULD FIT TOGETHER IN THEORIES WITH DYNAMICAL GAUGE FIELD

* Hydrodynamic limit: If $|\psi\rangle_{IR}$ transformed under $U_\theta \sim \exp\left(i \int d(vol) n\right)$, then $\partial_t n + \nabla \cdot \mathbf{j}(n) = 0$, or anomalous contribution
* Ampere's law $\partial_t \mathbf{E} + \nabla \times \mathbf{B} = \mathbf{j}$ with $\mathbf{j} = \mathbf{j}(n)$ in hydrodynamic description
* Coefficient in constitutive relation $\mathbf{j}(n) = -\sigma\left(T\nabla(\mu/T) - \mathbf{E}\right)$ obtained via Kubo formula

$$\sigma = \lim_{\omega \to 0} \frac{1}{\omega} \mathrm{Im}\langle j^x(\omega) j^x(-\omega)\rangle_{\mathrm{retard}}$$

3 obvious things that should fit together in theories with dynamical gauge field with ABJ anomaly

* Hydrodynamic limit: If $|\psi\rangle_{IR}$ transformed under $U_\theta \sim \exp\left(i\int d(vol)n\right)$,

  then $\partial_t n + \nabla \cdot \mathbf{j}(n) = \boxed{k\mathbf{E}\cdot\mathbf{B}}$ with $(n, \mathbf{j}(n), \mathbf{E}, \mathbf{B})$ dynamical

* Ampere's law $\partial_t \mathbf{E} + \nabla \times \mathbf{B} = \mathbf{j}$ with $\mathbf{j} = \mathbf{j}(n)$ in hydrodynamic description

* Coefficient in constitutive relation $\mathbf{j}(n) = -\sigma\left(T\nabla(\mu/T) - \mathbf{E}\right) - \boxed{2k\mu_5\mathbf{B}}$

  obtained via Kubo formula

$$\sigma = \lim_{\omega\to 0}\frac{1}{\omega}\mathrm{Im}\langle j^x(\omega)j^x(-\omega)\rangle_{\mathrm{retard}}$$

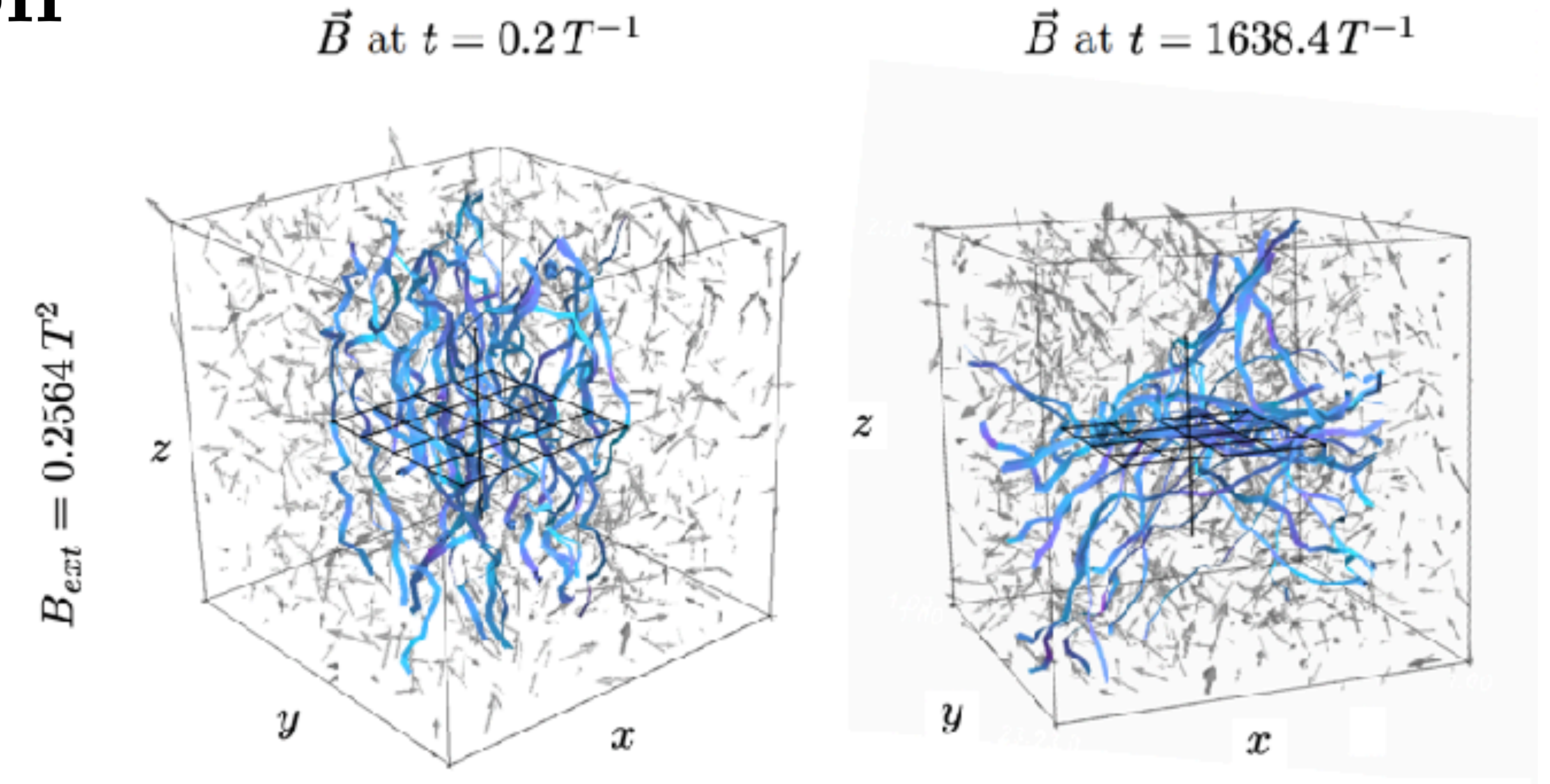
Confusing results from microscopic simulation

Effective theory should emerges from microscopic one.

This can be done in **Real-time classical lattice simulation**

Figueroa & Shaposhnikv 1707.09967

Figueroa, Florio & Shaposhnikov 1904.11892



We use scalar QED with additional scalar field

$$S = S_{QED} + \int d^4x \left((\partial_t \theta)^2 + k\theta\epsilon^{\mu\nu\rho\sigma} F_{\mu\nu} F_{\rho\sigma} \right)$$

which produce the ABJ Ward identity

$$\chi_a \dot{\mu} = k\epsilon^{\mu\nu\rho\sigma} F_{\mu\nu} F_{\rho\sigma}$$

Using chiral MHD equation, to replace $\mathbf{E}, \mathbf{B}$, we expect to find

$$\boxed{\frac{d}{dt}\mu = -\Gamma_5 \mu\,, \quad \Gamma_5 \sim \frac{B^2}{\sigma}}$$

$$\boxed{\begin{aligned} \partial_t \mathbf{E} &= -\sigma \mathbf{E} - \nabla \times \mathbf{B} - k\mu \mathbf{B} \\ \partial_t \mathbf{B} &= \nabla \times \mathbf{E} = 0 \end{aligned}}$$

Using the microscopic definition of σ

$$\sigma = \sigma_{el} = \lim_{\omega \to 0} \frac{1}{\omega} \mathrm{Im} \langle j^x_{el} j^x_{el} \rangle_{\text{retard}}$$

which was computed in QED

For fermion QED: Arnold, Moore & Yaffe '00 & '03

For scalar QED: Sobol 1905.08190

BUT $\Gamma_5 \sim B^2/\sigma$ is off from $-\frac{d}{dt}\log\mu$ by a factor of 10

Resolution:

1) Let's take hydrodynamic principle seriously

2) Let's carefully look at the symmetry of QED plasma

Symmetry and hydrodynamics

Symmetry

Transformations of states/operators

$$\hat{U}_\theta[M]\ :\ \hat{X} \to \exp(i\theta\hat{Q})\,\hat{X}\,\exp(-i\theta\hat{Q}) = e^{i\theta q_X}\hat{X}$$

Some operators are "conserved"

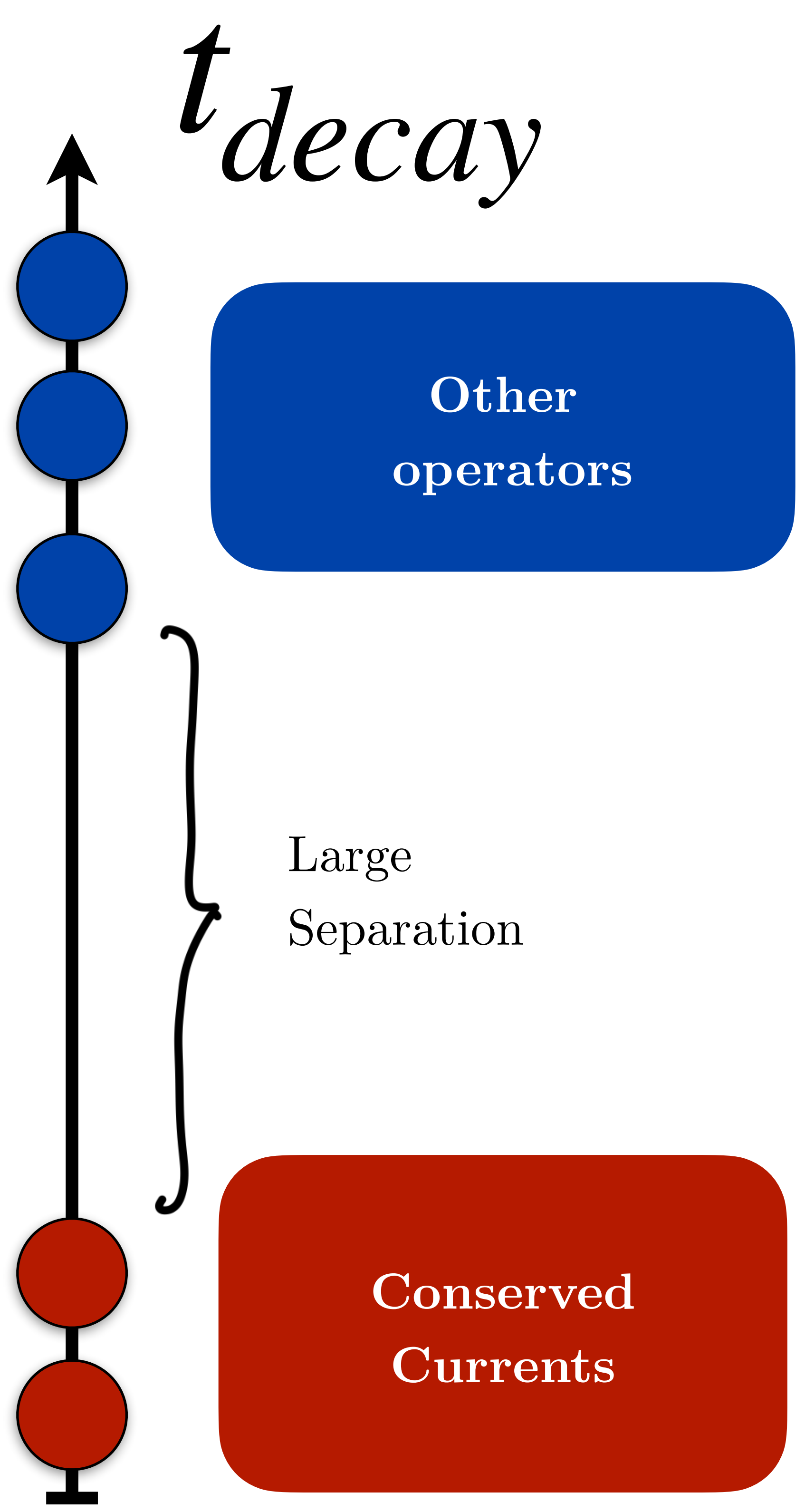
$$\frac{d}{dt}\hat{Q} = 0\,, \quad Q[M] = \int_M d(vol)\,n$$

$$\partial_t n + \partial_i j^i = 0$$

$$j^i = D\partial_i n + \mathcal{O}(\partial^2)$$

Transport coefficients D determined by microscopic

But "flow" and correlation $\langle j^i(t,x) j^j(t',x')\rangle$ fixed by symmetry



Microscopic Theory

Reach local thermal equilibrium →

Large scale, late time

Symmetry of "Q"ED (without anomaly)

Symmetry

Transformations of states created by 't Hooft line

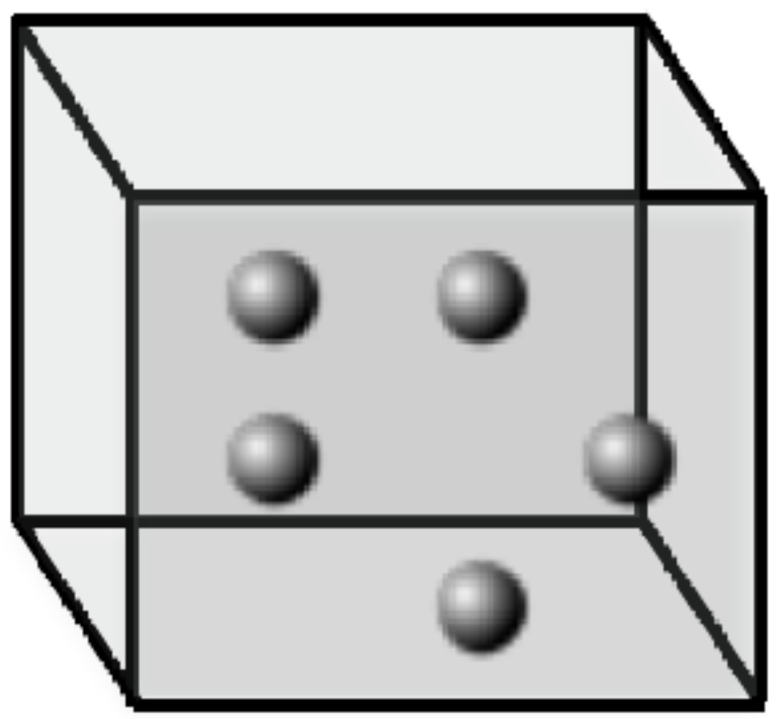
$$\hat{U}_\theta[M] \;:\; \hat{X} \to \exp(i\theta\hat{Q})\,\hat{X}\,\exp(-i\theta\hat{Q}) = e^{i\theta q_X}\hat{X}$$

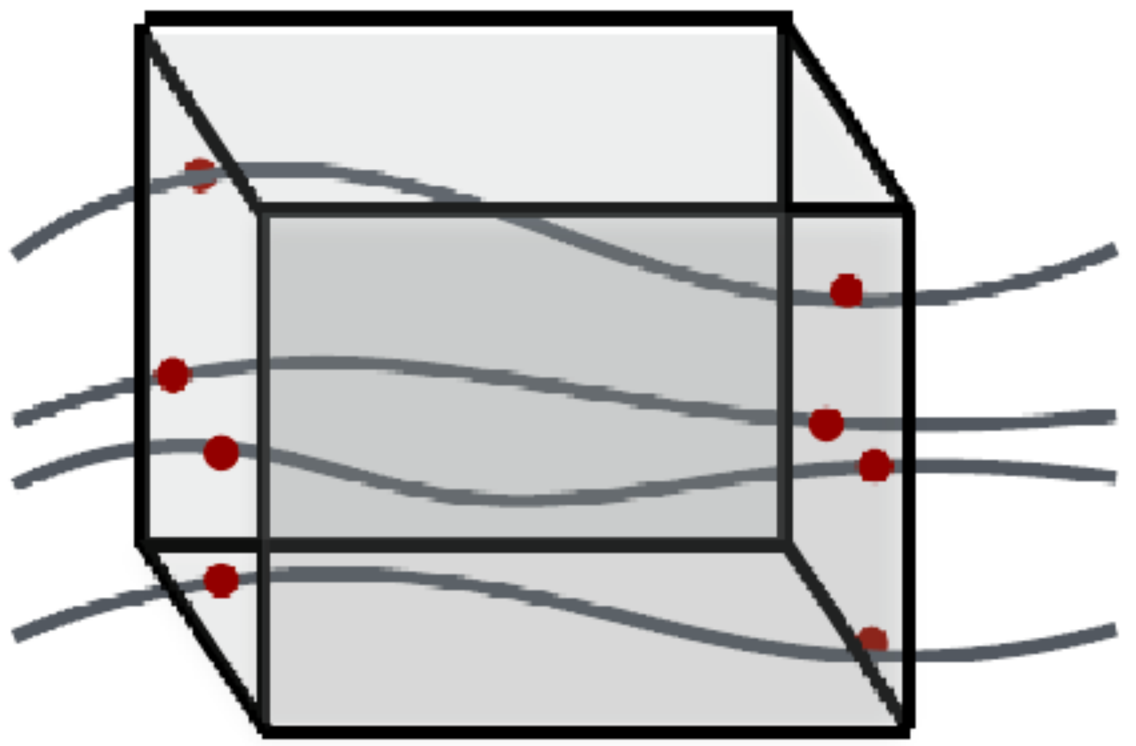
Some operators are "conserved"

$$\frac{d}{dt}\hat{Q} = 0\,, \quad Q[M] = \int_M d(surface)\cdot\mathbf{B}$$

M is now 2d closed surface

$$\boxed{\partial_t\mathbf{B} + \nabla\times\mathbf{E} = 0}$$





Electric flux ↔ number of Wilson line is not conserved and can be written as

$$\boxed{\mathbf{E} = \rho(\nabla\times\mathbf{B}) + \mathcal{O}(\partial^2)}$$

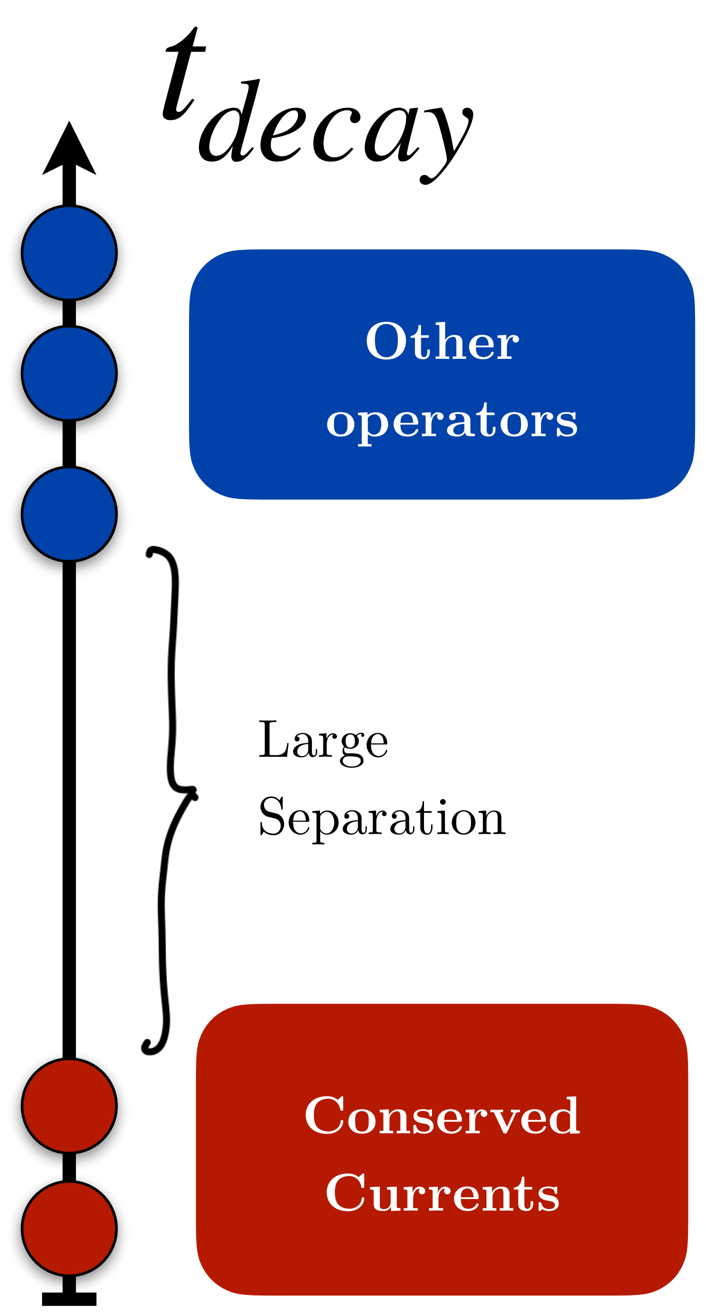
Transport coefficients ρ determine response of electric field

$$\rho \sim \lim_{\omega\to 0}\frac{1}{\omega}\mathrm{Im}\;\langle E^x(\omega)E^x(-\omega)\rangle_{\mathrm{retard}}$$

But "flow" and correlation $\langle\mathbf{E}(t,x)\mathbf{E}(t',x')\rangle$ fixed by symmetry

See e.g. Gaiotto, Kapustin, Seiberg & Willet '14
Grozdanov, Hofman & Iqbal '16

Hydrodynamics of plasma



Conserved quantities

$J^0 = \{\text{Energy } E, \text{ momentum } \mathbf{P}, \text{ magnetic flux } \mathbf{B}\}$

(Badly)Non-conserved quantity

Electric flux $\mathbf{E}$ due to screening

Conserved but trivial

j^μ_{el} in $\partial_\mu j^\mu_{el} = 0$ since Hilbert/phase space cannot transform under gauged symmetry

Resulting hydrodynamics

$$\mathbf{E} = -\rho(\nabla \times \mathbf{B}) \qquad \partial_t \mathbf{B} - \nabla \times \mathbf{E} = 0$$

& all other observables are expressed in terms of J^0

Grozdanov, Hofman & Iqbal '16

$\Rightarrow$

Direct consequence

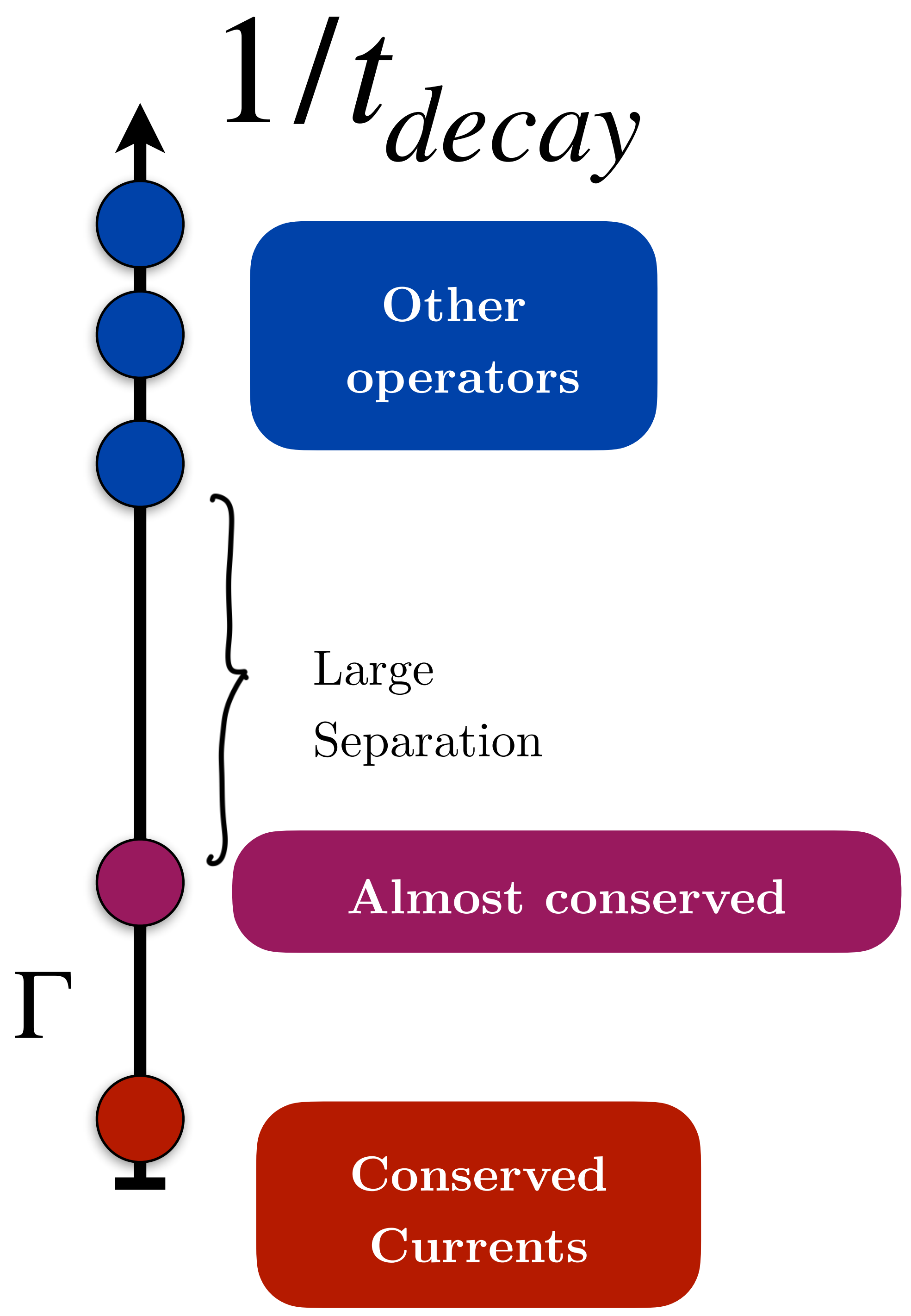
Express 'gauged' charge and current via Gauss-law

$$n_{el} = \nabla \cdot \mathbf{E} \qquad \partial_t n_{el} + \partial_i j^i_{el} = 0$$

$$\boxed{\sigma_{el} = \lim_{\omega \to 0} \frac{1}{\omega} \mathrm{Im} \langle j^x_{el} j^x_{el} \rangle_{\text{retard}} \Big|_{k=0} \propto \lim_{\omega \to 0} \rho \omega^2}$$

Not only that $\sigma_{el} \neq 1/\rho$, it is actually zero…
what is going on here?

(QUASI)HYDRODYNAMICS OF PLASMA



Conserved quantities

$J^0 = \{\text{Energy } E, \text{ momentum } \mathbf{P}, \text{ magnetic flux } \mathbf{B}\}$

Almost conserved quantity

Electric flux $\mathbf{E}$ due to screening

Resulting (quasi) hydrodynamics

$$\partial_t \mathbf{B} - \nabla \times \mathbf{E} = 0 \qquad \partial_t \mathbf{E} + d_1 \nabla \times \mathbf{B} = -\Gamma \mathbf{E}$$

Compare to Ampere's law $\Gamma = \sigma/\epsilon$
play a role of decay rate (1/lifetime)
of the *independent* electric field operator

D. Forster's book
Stephanov & Yin '17
Grozdanov, Lucas & NP '18

$\Rightarrow$

Express 'gauged' charge and current via Gauss-law

$$n_{el} = \nabla \cdot \mathbf{E} \qquad \partial_t n_{el} + \partial_i j^i_{el} = 0$$

$$\sigma_{el} = \lim_{\omega \to 0} \frac{1}{\omega} \mathrm{Im} \langle j^x_{el} j^x_{el} \rangle_{\text{retard}} \Big|_{k=0} \propto \frac{\rho \omega^2}{1 + (\omega/\Gamma)^2}$$

Only in regime $\omega \gg \Gamma$ and $\rho \sim 1/\Gamma$ that we find $\sigma_{el} = 1/\rho$.
This turns out to be true when $e^2 \ll 1$

I want to emphasise

* In late time limit, low energy EFT governed by conserved charge $J^0 = \{E, \mathbf{P}, \mathbf{B}\}$ (and almost conserved charge $\tilde{J}^0 = \{\mathbf{E}\}$).
  => All other observable expressed in term of $J^0, \tilde{J}^0$

* Conductivity in (ungauged) theory play a role of **E**-lifetime and, a priori, has nothing to do with resistivity

$$\sigma = \lim_{\omega \to 0} \frac{1}{\omega} \mathrm{Im} \langle j^x(\omega) j^x(-\omega) \rangle_{\text{retard}} \qquad J^0 = \{E, \mathbf{P}, n\}$$

$$\rho \sim \lim_{\omega \to 0} \frac{1}{\omega} \mathrm{Im} \ \langle E^x(\omega) E^x(-\omega) \rangle_{\text{retard}} \qquad J^0 = \{E, \mathbf{P}, \mathbf{B}\}$$

=> Only in special limit where **E** is long-lived, that we have $\sigma = 1/\rho$

Our resolution to FFS's paradox:
E operator is short-lived

As consequences

A. 'Naive conductivity vanishes

$$\sigma_{el} = \lim_{\omega \to 0} \frac{1}{\omega} \mathrm{Im} \langle j^x_{el} j^x_{el} \rangle_{\text{retard}} \Big|_{k=0} \propto \lim_{\omega \to 0} \rho \omega^2$$

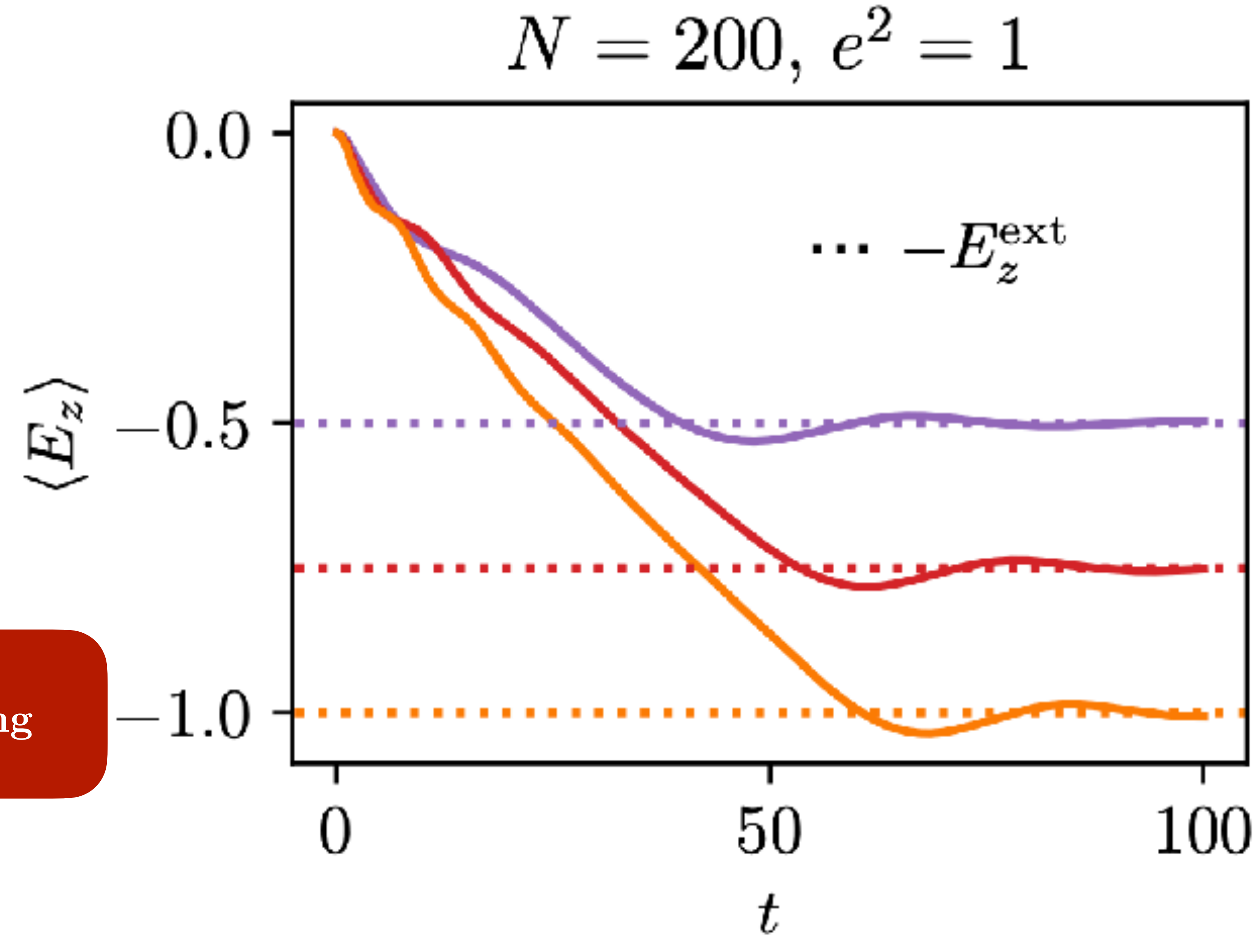
B. Dynamic of **B** is purely diffusive (no 'Israel-Stewart' crossover to ballistic)

C. The correct decay rate is of chiral MHD characterise by 'hydro' parameter ρ

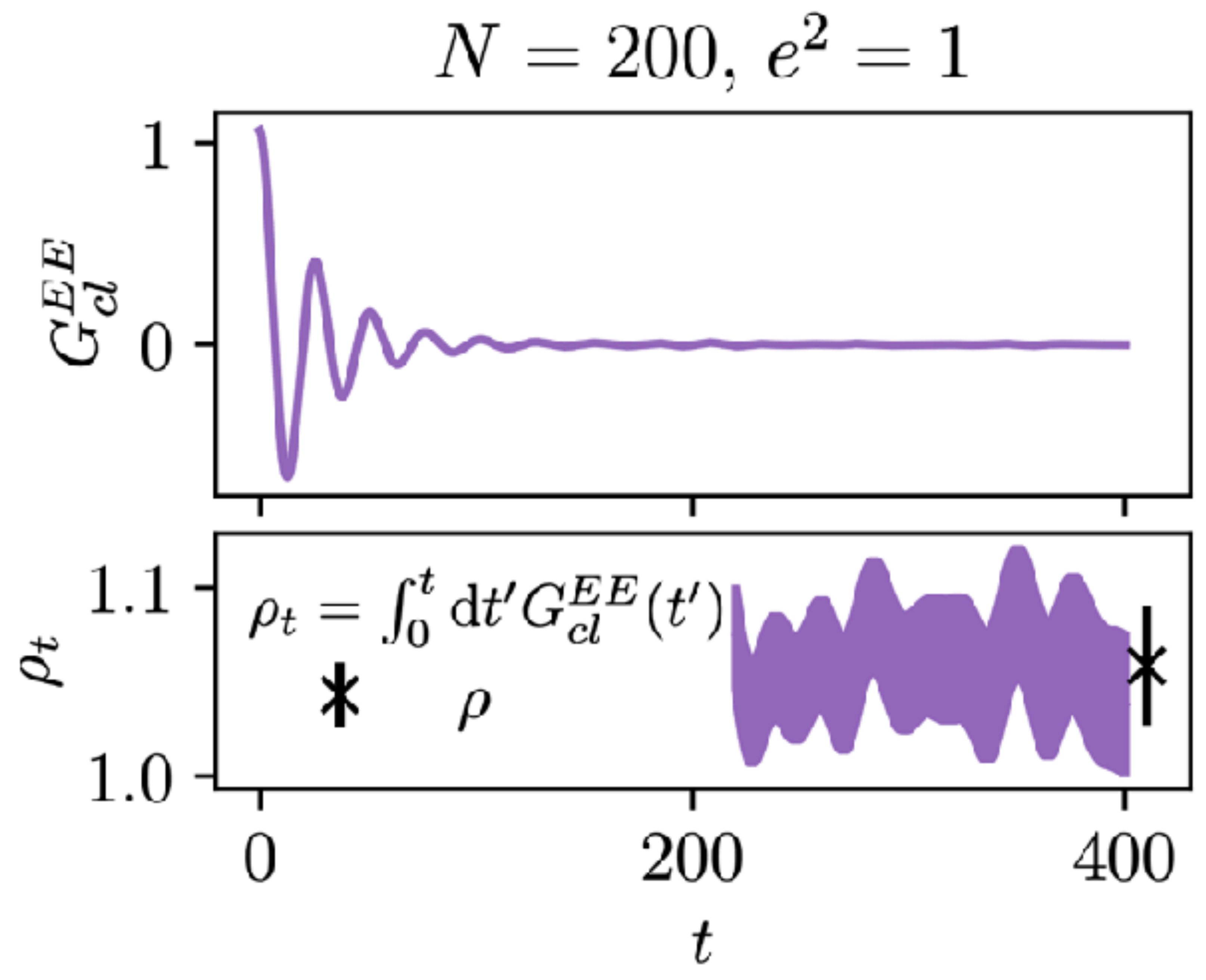
Numerical evidence

Is the electric field really short-lived?

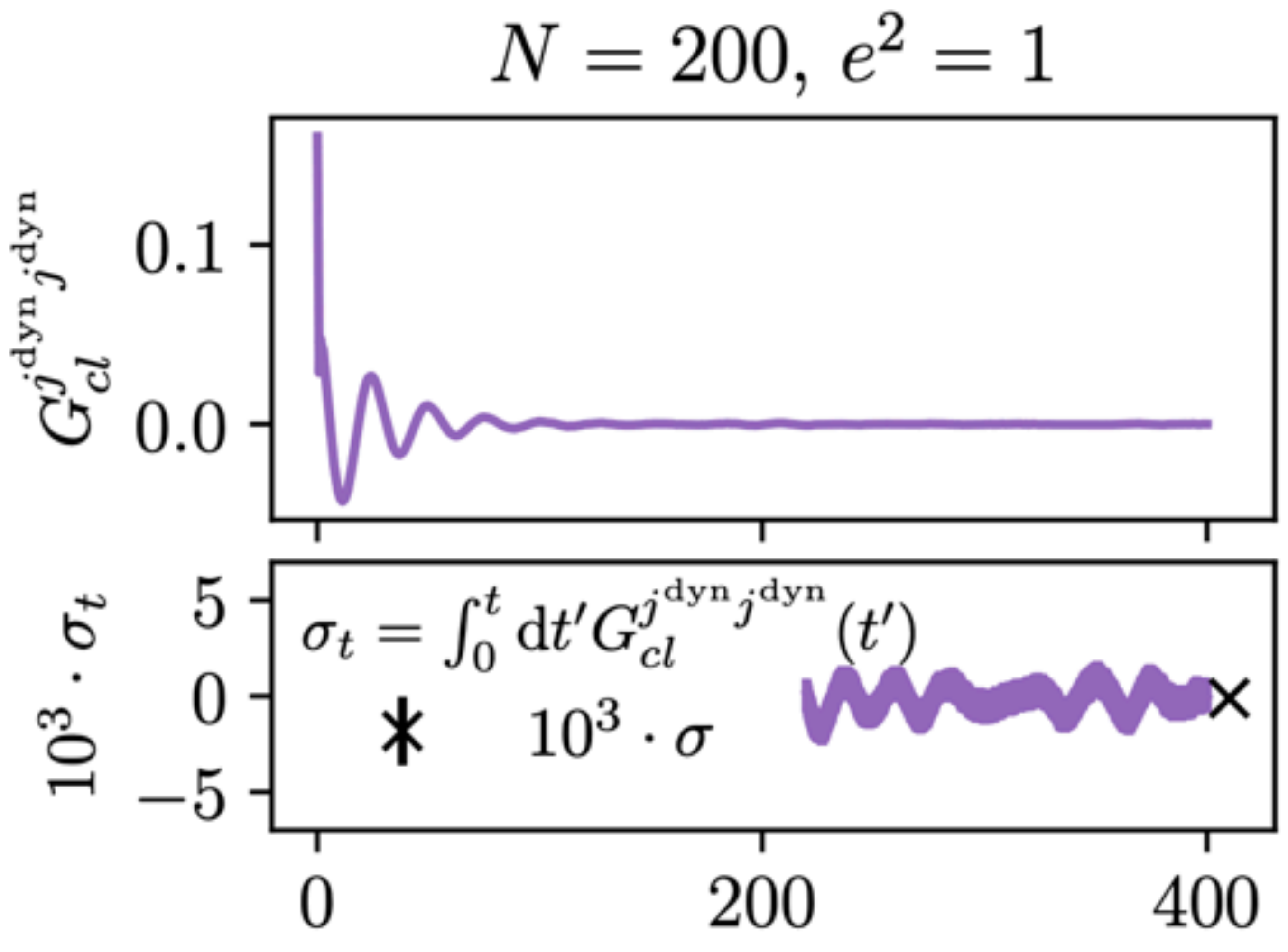
Quenching

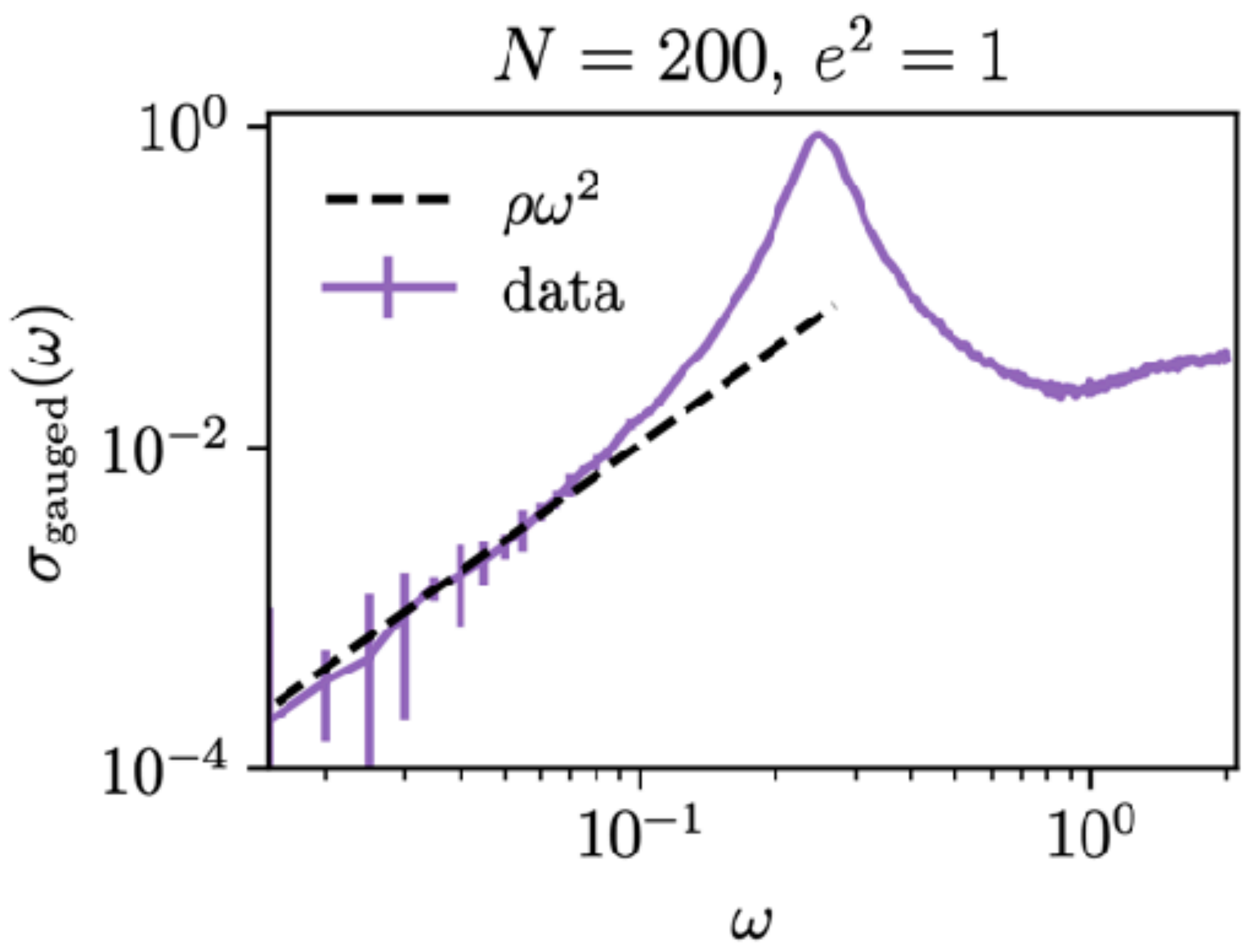


Correlator



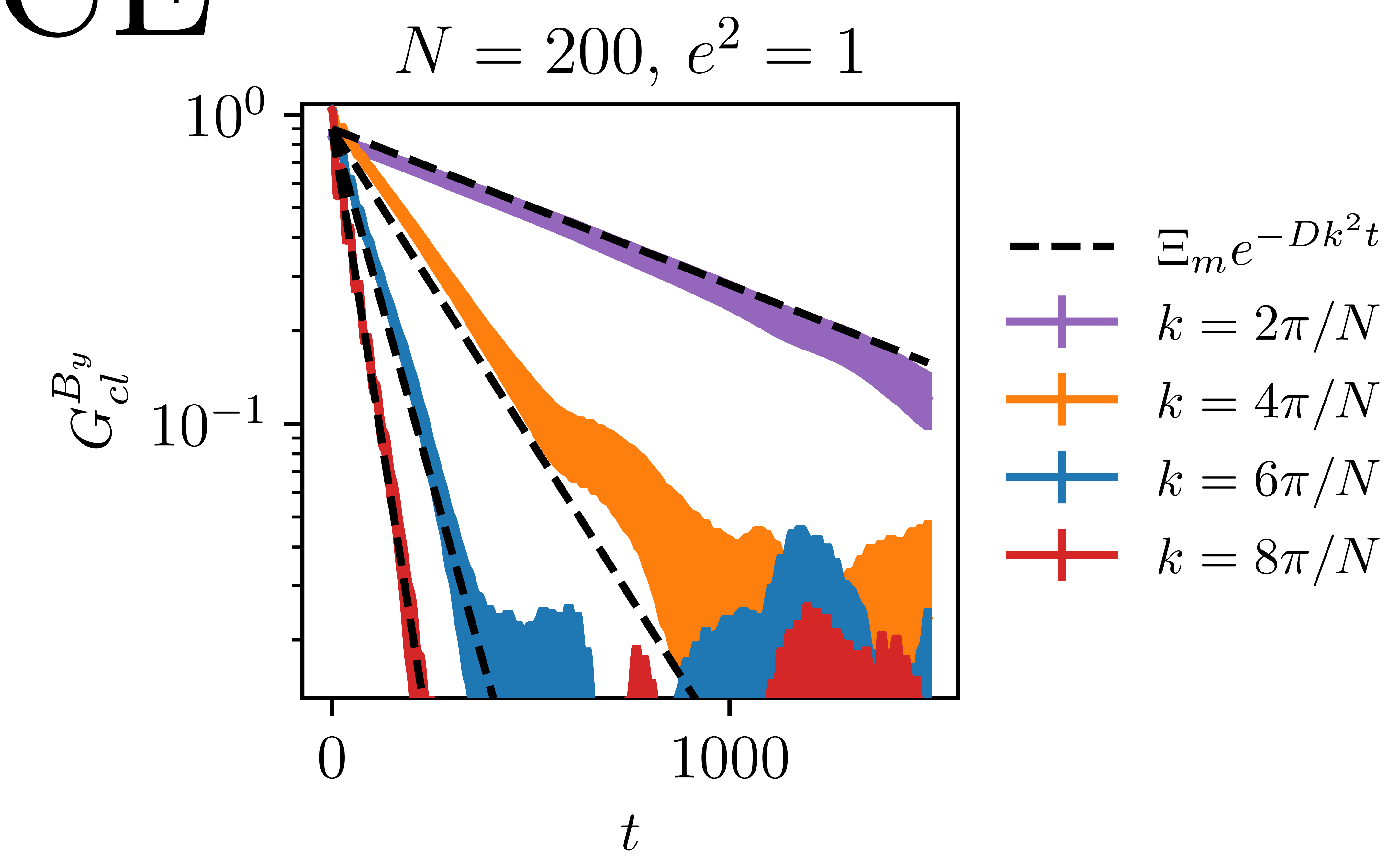
What if we try to extract conductivity?





Numerical evidence

Resolving the factor 10 paradox?



Do we see only diffusion?

Dynamic governed by pure magnetic diffusion as in genuine hydrodynamic descriptions

If using result in 'genuine' hydro regime with $\Gamma \to \infty$

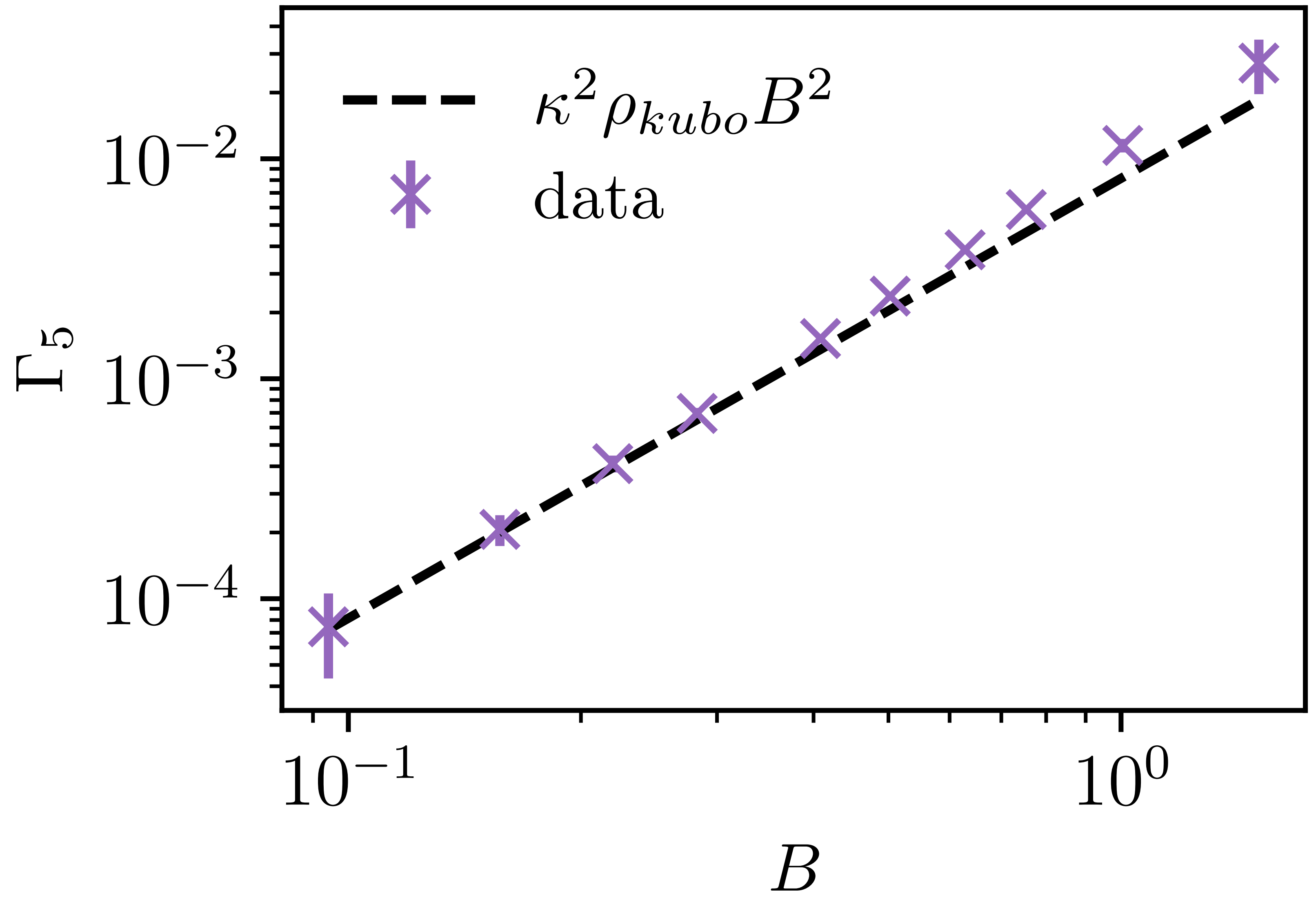
$$\mathbf{E} = \frac{\rho}{\chi}\nabla \times \mathbf{B} - 2k\rho\mu\mathbf{B}$$

No weakly coupled is assumed !

The chiral decay rate should be

$$\frac{d}{dt}\mu = -\Gamma_5\mu\,, \quad \Gamma_5 \sim \rho B^2$$

$$\rho = \lim_{\omega\to 0} \mathrm{Im}\ \langle E^i E^i\rangle_{\mathrm{retard}}\Big|_{k=0}$$



Almost perfectly fit the data $\sim \frac{d}{dt}\log\mu$ at small $\mathbf{B}$

Things that I omitted

- If you should throw away non-conserved quantity, why not throw away $U(1)_5$

$$\partial_t n_5 \propto \mathbf{E} \cdot \mathbf{B} \qquad \text{broken by ABJ anomaly}$$

  - It is a manifestation of a global symmetry without inverse!

Choi, Lam & Shao '22

Karasik '22 ; Garcia-Etxebarria & Iqbal '22

  - Using hydrodynamic EFT framework, we can show, without gauging, that

$$\mathbf{E} = \frac{\rho}{\chi} \nabla \times \mathbf{B} - 2k\rho\mu\mathbf{B}$$

No weakly coupled is assumed !

Das, Iqbal & NP '23

- I only focus on the normal phase. But there what about spontaneously broken? What about anomaly of these non-group symmetry? Phase diagram? Contraints on other experimental setup?

  There are really A LOT more that we can learn!