CFT in Momentum Space for Parity-odd Interactions, Axions and Dilatons

Claudio Coriano'

Dipartimento di Matematica e Fisica, Universita' del Salento

INFN – Lecce

J_5 JJ, J_5 TT, T JJ

Work with ***S. Lionetti***
M. Creti', S. Lionetti, R. Tommasi
S. Lionetti, D. Melle, L. Torcellini

Braga, Portugal, 26-6- 2025

Are axions always asymptotic ?

Are all axion/gauge field interactions described by a local action?

In the UV, conformal field theory tells us that the anomaly interactions do not necessarily Correspond to asymptotic pseudoscalar states.
One cannot use naively Goldstone's theorem to derive a local action

CFT in momentum space clarifies in a rigorous way the origin of such interactions
And how they are strictly limited to null surfaces

The proof that these interactions live an propagate on the light cone comes from
The analysis of the Conformal Ward identities

CFT constraints on parity-odd interactions with axions and dilatons
arXiv: 2408.02580 *with S. Lionetti*

CFT and anomalies

Gravitational chiral anomaly at finite temperature and density
arXiv: 2404.06272

Axionlike quasiparticles and topological states of matter: Finite density corrections of the chiral anomaly vertex
arXiv: 2402.03151
with M. Creti', Stefano Lionetti, R. Tommasi

Quantum anomalies and parity-odd CFT correlators for chiral states of matter
arXiv: 2409.10480

Parity-violating CFT and the gravitational chiral anomaly
arXiv: 2309.05374 *with S. Lionetti and M.M. Maglio*

CFT correlators and CP-violating trace anomalies
arXiv: 2307.03038 *with S. Lionetti, M. M. Maglio*

The gravitational form factors of hadrons from CFT in momentum space and the dilaton in perturbative QCD

e-Print: 2409.05609

For applications at the EIC

Perturbative analysis of AVV, TJJ

TJJ appears inn the conformal anomaly

M. Giannotti, E. Mottola

Armillis, Delle Rose, CC

Solving the Conformal Constraints from scalar correlators in CFT in momentum space

Delle Rose, Mottola, Serino, CC 2013

General CFT for tensor correlators

Bzowski, McFadden, Skenderis, 2013

Formulation of general methods

(conformal anomaly

Parity even sector reviewed in Maglio, CC, Phys. Rep

Method generalized more recently to

Parity odd sectors + anomalies from CFT

AVV, AAA, ATT, CS TT

Lionetti, Maglio, CC, 2023, 2024

Demonstration of off-shell sum rules in QED (M. Giannotti E. Mottola)

Nonabelian extensions to QCD, **Lionetti, Melle, Torcellini**, 2025

Applications to Gravit. Form Factors

$$\Delta^{\lambda\mu\nu}_{\mathbf{AVV}} = \Delta^{\lambda\mu\nu} = i^3 \int \frac{d^4q}{(2\pi)^4} \frac{Tr\left[\gamma^\mu(\not{q}+m)\gamma^\lambda\gamma^5(\not{q}-\not{k}+m)\gamma^\nu(\not{q}-\not{k}_1+m)\right]}{(q^2-m^2)[(q-k_1)^2-m^2][(q-k)^2-m^2]} +$$

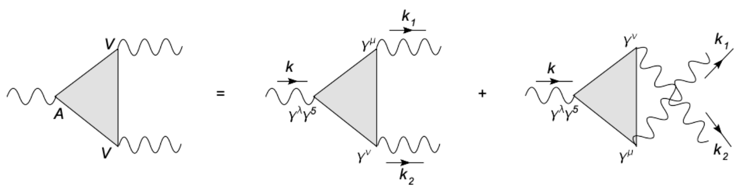
$$k_{1\mu}\Delta^{\lambda\mu\nu}(k_1,k_2) = a_1\epsilon^{\lambda\nu\alpha\beta}k_1^\alpha k_2^\beta$$
$$k_{2\nu}\Delta^{\lambda\mu\nu}(k_1,k_2) = a_2\epsilon^{\lambda\mu\alpha\beta}k_2^\alpha k_1^\beta$$
$$k_\lambda\Delta^{\lambda\mu\nu}(k_1,k_2) = a_3\epsilon^{\mu\nu\alpha\beta}k_1^\alpha k_2^\beta,$$

$$\Delta^{\lambda\mu\nu}(\beta,k_1,k_2) = \Delta^{\lambda\mu\nu}(k_1,k_2) - \frac{i}{4\pi^2}\beta\epsilon^{\lambda\mu\nu\sigma}\left(k_{1\sigma}-k_{2\sigma}\right).$$

$$k_{1\mu}\Delta^{\lambda\mu\nu}(\beta',k_1,k_2) = (a_1 - \frac{i\beta'}{4\pi^2})\varepsilon^{\lambda\nu\alpha\beta}k_1^\alpha k_2^\beta,$$
$$k_{2\nu}\Delta^{\lambda\mu\nu}(\beta',k_1,k_2) = (a_2 - \frac{i\beta'}{4\pi^2})\varepsilon^{\lambda\mu\alpha\beta}k_2^\alpha k_1^\beta,$$
$$k_\lambda\Delta^{\lambda\mu\nu}(\beta',k_1,k_2) = (a_3 + \frac{i\beta'}{2\pi^2})\varepsilon^{\mu\nu\alpha\beta}k_1^\alpha k_2^\beta,$$

$$k_{1\mu}\Delta^{\lambda\mu\nu}(a,k_1,k_2) = 0,$$
$$k_{2\nu}\Delta^{\lambda\mu\nu}(a,k_1,k_2) = 0,$$
$$k_\lambda\Delta^{\lambda\mu\nu}(a,k_1,k_2) = -\frac{i}{2\pi^2}\varepsilon^{\mu\nu\alpha\beta}k_1^\alpha k_2^\beta$$

$$a_1(\beta) = a_2(\beta) = -\frac{i}{8\pi^2} - \frac{i}{4\pi^2}\beta$$
$$a_3(\beta) = -\frac{i}{4\pi^2} + \frac{i}{2\pi^2}\beta,$$



$$a_1(\beta) + a_2(\beta) + a_3(\beta) = a_n = -\frac{i}{2\pi^2}.$$

Rosenberg (1963)

$$\overline{\Delta}_{\lambda\mu\nu} = \hat{a}_1\epsilon[k_1,\mu,\nu,\lambda] + \hat{a}_2\epsilon[k_2,\mu,\nu,\lambda] + \hat{a}_3\epsilon[k_1,k_2,\mu,\lambda]{k_1}^{\nu}$$
$$+\hat{a}_4\epsilon[k_1,k_2,\mu,\lambda]k_2^{\nu} + \hat{a}_5\epsilon[k_1,k_2,\nu,\lambda]k_1^{\mu} + \hat{a}_6\epsilon[k_1,k_2,\nu,\lambda]k_2^{\mu},$$

Ward identities are local

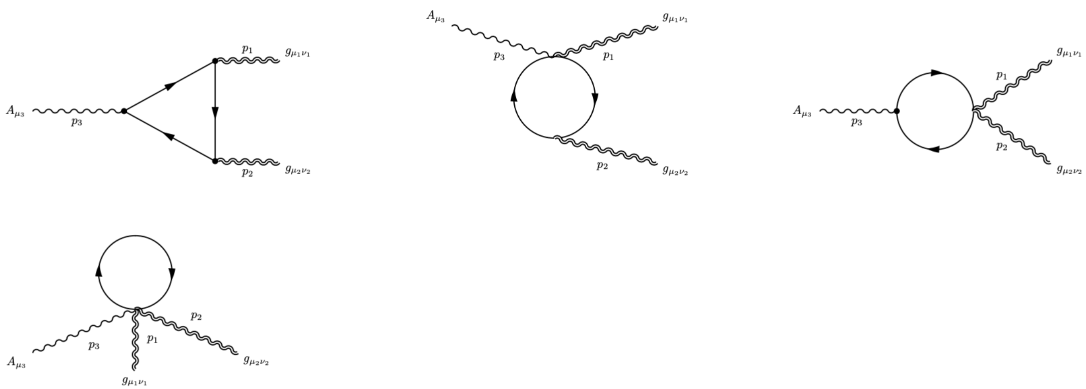
$$\frac{\partial}{\partial x^{\mu}} T^{\lambda\mu\nu}_{\mathbf{AVV}}(x,y,z) = i a_1(\beta)\epsilon^{\lambda\nu\alpha\beta}\frac{\partial}{\partial x^{\alpha}}\frac{\partial}{\partial y^{\beta}}\left(\delta^4(x-z)\delta^4(y-z)\right),$$
$$\frac{\partial}{\partial y^{\nu}} T^{\lambda\mu\nu}_{\mathbf{AVV}}(x,y,z) = i a_2(\beta)\epsilon^{\lambda\mu\alpha\beta}\frac{\partial}{\partial y^{\alpha}}\frac{\partial}{\partial x^{\beta}}\left(\delta^4(x-z)\delta^4(y-z)\right),$$
$$\frac{\partial}{\partial z^{\lambda}} T^{\lambda\mu\nu}_{\mathbf{AVV}}(x,y,z) = i a_3(\beta)\epsilon^{\mu\nu\alpha\beta}\frac{\partial}{\partial x^{\alpha}}\frac{\partial}{\partial y^{\beta}}\left(\delta^4(x-z)\delta^4(y-z)\right),$$

These interactions are finite as far as we impose Ward identities.
Question: can we costruct these interactions directly from the Ward identites ?

The WIs are sufficient to remove the divergences,
but we need something extra if we want to construct these interactions completely

This “something extra” is Conformal Symmetry

A similar type of analysis can be performed for chiral gravitational anomalies



Lionetti, Maglio, CC 2024

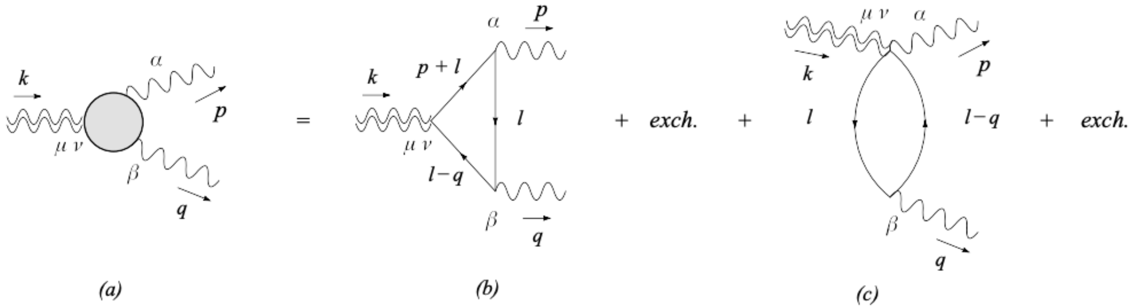
ATT

Conformal Anomalies

TJJ

M. Gannotti, E. Mottola

R. Armillis, L. Delle Rose, CC



AVV (J_5 JJ)
A T T (J_5,TT)

Possible role in the conformal phase of the early universe
Topological materials

Thermal transport, geometry, and anomalies
M. Chernodub, Y. Ferreiros, A. Grushin, K. Landsteiner, M. Vozmediano

- *Phys.Rept.* 977 (2022)

CFT in momentum space

Parity-even Tensor Correlator. (Conformal anomaly correlators)

Bzowski, McFadden Skenderis (2013 and sequel) **general methods for the analsysis of 3 point functions of scalars. Developed the renormalization of the solution, in a completely independent fascion (no reference to field theory realization)**

For correlators involving TJJ, TTT (conformal anomaly), the method can be equivalently formulated in generic free field theories, (Maglio, CC)

Exact Correlators from Conformal Ward Identities in Momentum Space and the Perturbative TJJ Vertex

2018

The general 3-graviton vertex (TTT) of conformal field theories in momentum space in $d = 4$

Scalar Correlators from CWIs. 2013
(Delle Rose, Mottola, Serino, CC

$$\begin{cases} \left[x(1-x)\frac{\partial^2}{\partial x^2} - y^2\frac{\partial^2}{\partial y^2} - 2\,x\,y\frac{\partial^2}{\partial x\partial y} + [\gamma - (\alpha+\beta+1)x]\,\frac{\partial}{\partial x} \right. \\ \qquad\qquad \left. -(\alpha+\beta+1)y\frac{\partial}{\partial y} - \alpha\,\beta\right]\Phi(x,y) = 0\,, \\ \left[y(1-y)\frac{\partial^2}{\partial y^2} - x^2\frac{\partial^2}{\partial x^2} - 2\,x\,y\frac{\partial^2}{\partial x\partial y} + [\gamma' - (\alpha+\beta+1)y]\,\frac{\partial}{\partial y} \right. \\ \qquad\qquad \left. -(\alpha+\beta+1)x\frac{\partial}{\partial x} - \alpha\,\beta\right]\Phi(x,y) = 0\,, \end{cases}$$

[Solving the Conformal Constraints for Scalar Operators in Momentum Space and the Evaluation of Feynman's Master Integrals]()

$$G_{123}(p_1,p_2) = \langle \mathcal{O}_1(p_1)\mathcal{O}_2(p_2)\mathcal{O}_3(-p_1-p_2)\rangle\,.$$

$$\begin{aligned} G_{123}(p_1^2,p_2^2,p_3^2) \;=\; & \frac{c_{123}\;\pi^d\,4^{d-\frac{1}{2}(\eta_1+\eta_2+\eta_3)}\,(p_3^2)^{-d+\frac{1}{2}(\eta_1+\eta_2+\eta_3)}}{\Gamma\left(\frac{\eta_1}{2}+\frac{\eta_2}{2}-\frac{\eta_3}{2}\right)\Gamma\left(\frac{\eta_1}{2}-\frac{\eta_2}{2}+\frac{\eta_3}{2}\right)\Gamma\left(-\frac{\eta_1}{2}+\frac{\eta_2}{2}+\frac{\eta_3}{2}\right)\Gamma\left(-\frac{d}{2}+\frac{\eta_1}{2}+\frac{\eta_2}{2}+\frac{\eta_3}{2}\right)}\Bigg\{ \\ & \Gamma\left(\eta_1-\frac{d}{2}\right)\Gamma\left(\eta_2-\frac{d}{2}\right)\Gamma\left(d-\frac{\eta_1}{2}-\frac{\eta_2}{2}-\frac{\eta_3}{2}\right)\Gamma\left(\frac{d}{2}-\frac{\eta_1}{2}-\frac{\eta_2}{2}+\frac{\eta_3}{2}\right) \\ & \times F_4\left(\frac{d}{2}-\frac{\eta_1+\eta_2-\eta_3}{2}, d-\frac{\eta_1+\eta_2+\eta_3}{2}; \frac{d}{2}-\eta_1+1, \frac{d}{2}-\eta_2+1; x, y\right) \\ & +\Gamma\left(\frac{d}{2}-\eta_1\right)\Gamma\left(\eta_2-\frac{d}{2}\right)\Gamma\left(\frac{\eta_1}{2}-\frac{\eta_2}{2}+\frac{\eta_3}{2}\right)\Gamma\left(\frac{d}{2}+\frac{\eta_1}{2}-\frac{\eta_2}{2}-\frac{\eta_3}{2}\right) \\ & \times x^{\eta_1-\frac{d}{2}}\,F_4\left(\frac{d}{2}-\frac{\eta_2+\eta_3-\eta_1}{2}, \frac{\eta_1+\eta_3-\eta_2}{2}; -\frac{d}{2}+\eta_1+1, \frac{d}{2}-\eta_2+1; x, y\right) \\ & +\Gamma\left(\eta_1-\frac{d}{2}\right)\Gamma\left(\frac{d}{2}-\eta_2\right)\Gamma\left(-\frac{\eta_1}{2}+\frac{\eta_2}{2}+\frac{\eta_3}{2}\right)\Gamma\left(\frac{d}{2}-\frac{\eta_1}{2}+\frac{\eta_2}{2}-\frac{\eta_3}{2}\right) \\ & \times y^{\eta_2-\frac{d}{2}}\,F_4\left(\frac{d}{2}-\frac{\eta_1+\eta_3-\eta_2}{2}, \frac{\eta_2+\eta_3-\eta_1}{2}; \frac{d}{2}-\eta_1+1, -\frac{d}{2}+\eta_2+1; x, y\right) \\ & +\Gamma\left(\frac{d}{2}-\eta_1\right)\Gamma\left(\frac{d}{2}-\eta_2\right)\Gamma\left(\frac{\eta_1}{2}+\frac{\eta_2}{2}-\frac{\eta_3}{2}\right)\Gamma\left(-\frac{d}{2}+\frac{\eta_1}{2}+\frac{\eta_2}{2}+\frac{\eta_3}{2}\right) \\ & \times x^{\eta_1-\frac{d}{2}}y^{\eta_2-\frac{d}{2}}\,F_4\left(-\frac{d}{2}+\frac{\eta_1+\eta_2+\eta_3}{2}, \frac{\eta_1+\eta_2-\eta_3}{2}; -\frac{d}{2}+\eta_1+1, -\frac{d}{2}+\eta_2+1; x, y\right)\Bigg\}. \end{aligned}$$

$$G_{123}(p_1^2,p_2^2,p_3^2) = (p_3^2)^{-d+\frac{1}{2}(\eta_1+\eta_2+\eta_3)}\,\Phi(x,y) \qquad \text{with} \qquad x = \frac{p_1^2}{p_3^2}\,,\quad y = \frac{p_2^2}{p_3^2}\,,$$

extensions to 4-point functions

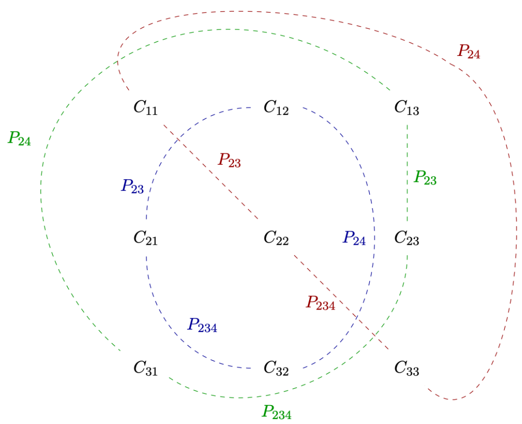
Four-Point Functions in Momentum Space: Conformal Ward Identities in the Scalar/Tensor case

Maglio, Theofilopoulos, CC

• *Eur.Phys.J.C* 80 (2020)

$$\bar{K}(p_2,p_3,p_4,s,t,u) \equiv K_2 + \frac{p_3^2-p_4^2}{s\,t}\frac{\partial}{\partial s\partial t} - \frac{p_3^2-p_4^2}{s\,u}\frac{\partial}{\partial s\partial u} + \frac{1}{t}\frac{\partial}{\partial t}\left(p_2\frac{\partial}{\partial p_2} + p_3\frac{\partial}{\partial p_3} - p_4\frac{\partial}{\partial p_4}\right)$$
$$+ (d-\Delta)\left(\frac{1}{t}\frac{\partial}{\partial t} + \frac{1}{u}\frac{\partial}{\partial u}\right) + \frac{1}{u}\frac{\partial}{\partial u}\left(p_2\frac{\partial}{\partial p_2} - p_3\frac{\partial}{\partial p_3} + p_4\frac{\partial}{\partial p_4}\right) + \frac{2p_2^2+p_3^2+p_4^2-s^2-t^2-u^2}{t\,u}\frac{\partial}{\partial t\partial u}$$

Orbits of form factors



The decomposition of the $TOOO$

$$\langle T^{\mu_1\nu_1}(\mathbf{p_1})O(\mathbf{p_2})O(\mathbf{p_3})O(\bar{\mathbf{p}}_4)\rangle = \langle t^{\mu_1\nu_1}(\mathbf{p_1})O(\mathbf{p_2})O(\mathbf{p_3})O(\bar{\mathbf{p}}_4)\rangle + \langle t_{\text{loc}}^{\mu_1\nu_1}(\mathbf{p_1})O(\mathbf{p_2})O(\mathbf{p_3})O(\bar{\mathbf{p}}_4)\rangle,$$

4K INTEGRALS

$$I_{\alpha\{\beta_1,\beta_2,\beta_3,\beta_4\}}(p_1,p_2,p_3,p_4) = \int_0^\infty dx\, x^\alpha \prod_{i=1}^4 (p_i)^{\beta_i} K_{\beta_i}(p_i\, x),$$

Scalar reduction of the conformal constraints

Maglio, Theofilopoulos, CC

$$\frac{\partial}{\partial p_{1\mu}} = \frac{p_1^\mu}{p_1}\frac{\partial}{\partial p_1} - \frac{\bar{p_4}^\mu}{p_4}\frac{\partial}{\partial p_4} + \frac{p_1^\mu + p_2^\mu}{s}\frac{\partial}{\partial s},$$
$$\frac{\partial}{\partial p_{2\mu}} = \frac{p_2^\mu}{p_2}\frac{\partial}{\partial p_1} - \frac{\bar{p_4}^\mu}{p_4}\frac{\partial}{\partial p_4} + \frac{p_1^\mu + p_2^\mu}{s}\frac{\partial}{\partial s} + \frac{p_2^\mu + p_3^\mu}{t}\frac{\partial}{\partial t},$$
$$\frac{\partial}{\partial p_{3\mu}} = \frac{p_3^\mu}{p_3}\frac{\partial}{\partial p_3} - \frac{\bar{p_4}^\mu}{p_4}\frac{\partial}{\partial p_4} + \frac{p_2^\mu + p_3^\mu}{t}\frac{\partial}{\partial t}.$$

For Yangian symmetry scalars

$$\begin{aligned}
\langle O(p_1)O(p_2)O(p_3)O(p_4)\rangle = 2^{\frac{d}{2}-4}\; C \sum_{\lambda,\mu=0,\Delta-\frac{d}{2}} \xi(\lambda,\mu)\Bigg[&\left(s^2\,t^2\right)^{\Delta-\frac{3}{4}d}\left(\frac{p_1^2p_3^2}{s^2t^2}\right)^\lambda\left(\frac{p_2^2p_4^2}{s^2t^2}\right)^\mu \\
\times\, F_4\left(\frac{3}{4}d-\Delta+\lambda+\mu, \frac{3}{4}d-\Delta+\lambda+\mu, 1-\Delta+\frac{d}{2}+\lambda, 1-\Delta+\frac{d}{2}+\mu, \frac{p_1^2p_3^2}{s^2t^2}, \frac{p_2^2p_4^2}{s^2t^2}\right)& \\
+\left(s^2\,u^2\right)^{\Delta-\frac{3}{4}d}\left(\frac{p_2^2p_3^2}{s^2u^2}\right)^\lambda\left(\frac{p_1^2p_4^2}{s^2u^2}\right)^\mu& \\
\times\, F_4\left(\frac{3}{4}d-\Delta+\lambda+\mu, \frac{3}{4}d-\Delta+\lambda+\mu, 1-\Delta+\frac{d}{2}+\lambda, 1-\Delta+\frac{d}{2}+\mu, \frac{p_2^2p_3^2}{s^2u^2}, \frac{p_1^2p_4^2}{s^2u^2}\right)& \\
+\left(t^2\,u^2\right)^{\Delta-\frac{3}{4}d}\left(\frac{p_1^2p_2^2}{t^2u^2}\right)^\lambda\left(\frac{p_3^2p_4^2}{t^2u^2}\right)^\mu& \\
\times\, F_4\left(\frac{3}{4}d-\Delta+\lambda+\mu, \frac{3}{4}d-\Delta+\lambda+\mu, 1-\Delta+\frac{d}{2}+\lambda, 1-\Delta+\frac{d}{2}+\mu, \frac{p_1^2p_2^2}{t^2u^2}, \frac{p_3^2p_4^2}{t^2u^2}\right)\Bigg],&
\end{aligned}$$

Parity-Odd 3-Point Functions from CFT in Momentum Space and the Chiral Anomaly

Lionetti, Maglio, CC, 2023

AVV

$$\nabla_\mu \langle J^\mu \rangle = 0, \qquad \nabla_\mu \langle J_5^\mu \rangle = a\, \varepsilon^{\mu\nu\rho\sigma} F_{\mu\nu} F_{\rho\sigma}$$

Defining equations

$$p_{i\mu_i}\ \langle J^{\mu_1}(p_1) J^{\mu_2}(p_2) J_5^{\mu_3}(p_3) \rangle = 0, \qquad i = 1, 2$$

$$p_{3\mu_3}\ \langle J^{\mu_1}(p_1) J^{\mu_2}(p_2) J_5^{\mu_3}(p_3) \rangle = -8\, a\, i\, \varepsilon^{p_1 p_2 \mu_1 \mu_2}$$

By functional differentiations wrt backgrounds

$$J^\mu(p) = j^\mu(p) + j^\mu_{loc}(p),$$
$$j^\mu = \pi^\mu_\alpha(p)\, J^\alpha(p), \quad \pi^\mu_\alpha(p) \equiv \delta^\mu_\alpha - \frac{p_\alpha p^\mu}{p^2},$$
$$j^\mu_{loc}(p) = \frac{p^\mu}{p^2}\, p \cdot J(p)$$

$$J_5^\mu(p) = j_5^\mu(p) + j^\mu_{5loc}(p),$$
$$j_5^\mu = \pi^\mu_\alpha(p)\, J_5^\alpha(p),$$
$$j^\mu_{5loc}(p) = \frac{p^\mu}{p^2}\, p \cdot J_5(p)$$

L/T decomposition

$$\langle J^{\mu_1}(p_1) J^{\mu_2}(p_2) J_5^{\mu_3}(p_3) \rangle = \langle j^{\mu_1}(p_1) j^{\mu_2}(p_2) j_5^{\mu_3}(p_3) \rangle + \langle J^{\mu_1}(p_1) J^{\mu_2}(p_2)\, j_{5\,\mathrm{loc}}^{\mu_3}(p_3) \rangle$$

transverse — longitudinal

Longitudinal sector determined by the anomaly: the inclusion of an anomaly pole

$$\langle J^{\mu_1}(p_1)J^{\mu_2}(p_2)\, j_{5\ \mathrm{loc}}^{\mu_3}\,(p_3)\rangle = \frac{p_3^{\mu_3}}{p_3^2}\, p_{3\,\alpha_3}\ \langle J^{\mu_1}(p_1)J^{\mu_2}(p_2)J_5^{\alpha_3}(p_3)\rangle = -\frac{8\,a\,i}{p_3^2}\varepsilon^{p_1p_2\mu_1\mu_2}\, p_3^{\mu_3}$$

TRANSVERSE SECTOR

On the other hand, the transverse part can be formally written as

$$\langle j^{\mu_1}(p_1)j^{\mu_2}(p_2)j_5^{\mu_3}(p_3)\rangle = \pi^{\mu_1}_{\alpha_1}(p_1)\pi^{\mu_2}_{\alpha_2}(p_2)\pi^{\mu_3}_{\alpha_3}(p_3)\Big[A_1(p_1,p_2,p_3)\,\varepsilon^{p_1p_2\alpha_1\alpha_2}p_1^{\alpha_3} + A_2(p_1,p_2,p_3)\,\varepsilon^{p_1p_2\alpha_1\alpha_3}p_3^{\alpha_2}$$
$$+ A_3(p_1,p_2,p_3)\,\varepsilon^{p_1p_2\alpha_2\alpha_3}p_2^{\alpha_1} + A_4(p_1,p_2,p_3)\,\varepsilon^{p_1\alpha_1\alpha_2\alpha_3} + A_5(p_1,p_2,p_3)\,\varepsilon^{p_2\alpha_1\alpha_2\alpha_3}\Big]$$

SCHOUTEN RELATIONS

$$\delta^{\beta_3[\alpha_1}\varepsilon^{\alpha_2\alpha_3\beta_1\beta_2]} = 0.$$

Simplification

$$\langle j^{\mu_1}(p_1)j^{\mu_2}(p_2)j_5^{\mu_3}(p_3)\rangle = \pi^{\mu_1}_{\alpha_1}(p_1)\pi^{\mu_2}_{\alpha_2}(p_2)\pi^{\mu_3}_{\alpha_3}(p_3)\Big[A_1(p_1,p_2,p_3)\,\varepsilon^{p_1p_2\alpha_1\alpha_2}p_1^{\alpha_3}$$
$$+ A_2(p_1,p_2,p_3)\,\varepsilon^{p_1\alpha_1\alpha_2\alpha_3} - A_2(p_2,p_1,p_3)\,\varepsilon^{p_2\alpha_1\alpha_2\alpha_3}\Big]$$

$$A_1(p_1,p_2,p_3) = -A_1(p_2,p_1,p_3).$$

The anomaly enters in the CWIs

$$\mathcal{A} \equiv -\frac{16\, a\, i\, (\Delta_3 - 1)}{p_3^2}$$

$$0 = \sum_{j=1}^{2} \left[-2 \frac{\partial}{\partial p_{j\kappa}} - 2 p_j^\alpha \frac{\partial^2}{\partial p_j^\alpha \, \partial p_{j\kappa}} + p_j^\kappa \frac{\partial^2}{\partial p_j^\alpha \, \partial p_{j\alpha}} \right] \langle J^{\mu_1}(p_1) J^{\mu_2}(p_2) J_5^{\mu_3}(p_3) \rangle$$
$$+ 2 \left(\delta^{\mu_1 \kappa} \frac{\partial}{\partial p_1^{\alpha_1}} - \delta^\kappa_{\alpha_1} \frac{\partial}{\partial p_{1\mu_1}} \right) \langle J^{\alpha_1}(p_1) J^{\mu_2}(p_2) J_5^{\mu_3}(p_3) \rangle$$
$$+ 2 \left(\delta^{\mu_2 \kappa} \frac{\partial}{\partial p_2^{\alpha_2}} - \delta^\kappa_{\alpha_2} \frac{\partial}{\partial p_{2\mu_2}} \right) \langle J^{\mu_1}(p_1) J^{\alpha_2}(p_2) J_5^{\mu_3}(p_3) \rangle \equiv \mathcal{K}^\kappa \langle J^{\mu_1}(p_1) J^{\mu_2}(p_2) J_5^{\mu_3}(p_3) \rangle \, .$$

$$\pi^{\lambda_1}_{\mu_1}(p_1) \pi^{\lambda_2}_{\mu_2}(p_2) \pi^{\lambda_3}_{\mu_3}(p_3) \left(\mathcal{K}^\kappa \langle J^{\mu_1}(p_1) J^{\mu_2}(p_2) J_5^{\mu_3}(p_3) \rangle \right) = 0$$

equations

$$C_{ij} = 0, \quad i = 1,2, \quad j = 1,2,3$$
$$C_{31} = 0,$$
$$C_{3j} + \mathcal{A} = 0, \quad j = 2,3,5$$
$$C_{34} - \mathcal{A} = 0,$$
$$C_{36} = 0$$

$$\pi^{\lambda_1}_{\mu_1}(p_1) \pi^{\lambda_2}_{\mu_2}(p_2) \pi^{\lambda_3}_{\mu_3}(p_3) \left(\mathcal{K}^\kappa \langle j^{\mu_1}(p_1) j^{\mu_2}(p_2) j_5^{\mu_3}(p_3) \rangle \right) =$$
$$= \pi^{\lambda_1}_{\mu_1}(p_1) \pi^{\lambda_2}_{\mu_2}(p_2) \pi^{\lambda_3}_{\mu_3}(p_3) \Big[p_1^\kappa \Big(C_{11}\, \varepsilon^{\mu_1\mu_2\mu_3 p_1} + C_{12}\, \varepsilon^{\mu_1\mu_2\mu_3 p_2} + C_{13}\, \varepsilon^{\mu_1\mu_2 p_1 p_2} p_1^{\mu_3} \Big)$$
$$+ p_2^\kappa \Big(C_{21}\, \varepsilon^{\mu_1\mu_2\mu_3 p_1} + C_{22}\, \varepsilon^{\mu_1\mu_2\mu_3 p_2} + C_{23}\, \varepsilon^{\mu_1\mu_2 p_1 p_2} p_1^{\mu_3} \Big) + C_{31} \varepsilon^{\kappa\mu_1\mu_2\mu_3} + C_{32} \varepsilon^{\kappa\mu_1\mu_2 p_1} p_1^{\mu_3}$$
$$+ C_{33} \varepsilon^{\kappa\mu_1\mu_2 p_2} p_1^{\mu_3} + C_{34} \varepsilon^{\kappa\mu_1 p_1 p_2} \delta^{\mu_2\mu_3} + C_{35} \varepsilon^{\kappa\mu_2 p_1 p_2} \delta^{\mu_1\mu_3} + C_{36} \varepsilon^{\kappa\mu_3 p_1 p_2} \delta^{\mu_1\mu_2} \Big],$$

$$
\begin{aligned}
&K_{31}\,A_1 = 0, && K_{32}\,A_1 = 0,\\
&K_{31}\,A_2 = 0, && K_{32}\,A_2 = \left(\frac{4}{p_1^2} - \frac{2}{p_1}\frac{\partial}{\partial p_1}\right) A_2(p_1 \leftrightarrow p_2) + 2A_1,\\
&K_{31}\,A_2(p_1 \leftrightarrow p_2) = \left(\frac{4}{p_2^2} - \frac{2}{p_2}\frac{\partial}{\partial p_2}\right) A_2 - 2A_1, && K_{32}\,A_2(p_1 \leftrightarrow p_2) = 0,
\end{aligned}
$$

where we have defined

$$
K_i = \frac{\partial^2}{\partial p_i^2} + \frac{(d+1-2\Delta_i)}{p_i}\frac{\partial}{\partial p_i}, \qquad K_{ij} = K_i - K_j.
$$

These equations can also be reduced to a set of homogenous equations by repeatedly applying the operator K_{ij} and we have

$$
\begin{aligned}
&K_{31}\,A_1 = 0, \qquad && K_{32}\,A_1 = 0,\\
&K_{31}\,A_2 = 0, \qquad && K_{32}K_{32}\,A_2 = 0.
\end{aligned}
$$

3K Integrals

$$
I_{\alpha\{\beta_1\beta_2\beta_3\}}\left(p_1, p_2, p_3\right) = \int dx x^{\alpha} \prod_{j=1}^{3} p_j^{\beta_j} K_{\beta_j}\left(p_j x\right)
$$

$$K_\nu(x) = \frac{\pi}{2}\frac{I_{-\nu}(x) - I_\nu(x)}{\sin(\nu\pi)}, \qquad \nu \notin \mathbb{Z} \qquad\qquad I_\nu(x) = \left(\frac{x}{2}\right)^\nu \sum_{k=0}^{\infty} \frac{1}{\Gamma(k+1)\Gamma(\nu+1+k)} \left(\frac{x}{2}\right)^{2k}$$

Reducing the 3K integral in the solution

$$I_{1\{0,0,0\}} = (2\pi)^2 K_{4,\{1,1,1\}} = (2\pi)^2 \int \frac{d^4k}{(2\pi)^4} \frac{1}{k^2\,(k-p_1)^2\,(k+p_2)^2} = \frac{1}{4} C_0(p_1^2, p_2^2, p_3^2)$$

$$A_2^{(CFT)}(p_1, p_2, p_3) = 8ia\, p_2^2\, I_{3\{1,0,1\}}(p_1^2, p_2^2, p_3^2).$$

A_1=0

$$\begin{aligned} I_{3\{1,0,1\}}(p_1^2, p_2^2, p_3^2) = & \frac{1}{\lambda^2}\Bigg\{ -2p_1^2 p_3^2 \Big[p_1^2\left(p_2^2 - 2p_3^2\right) + p_1^4 + p_2^2 p_3^2 - 2p_2^4 + p_3^4 \Big] C_0\left(p_1^2, p_2^2, p_3^2\right) \\ & + p_1^2 \left(\left(p_1^2 - p_2^2\right)^2 + 4p_2^2 p_3^2 - p_3^4 \right) \log\left(\frac{p_1^2}{p_2^2}\right) + 4p_1^2 p_3^2 \left(p_1^2 - p_3^2\right) \log\left(\frac{p_1^2}{p_3^2}\right) \\ & - p_3^2 \Big((p_2^2 - p_3^2)^2 + 4p_1^2 p_2^2 - p_1^4 \Big) \log\left(\frac{p_2^2}{p_3^2}\right) - \lambda(p_1^2 - p_2^2 + p_3^2) \Bigg\} \end{aligned}$$

$$\begin{aligned} \langle j^{\mu_1}(p_1) j^{\mu_2}(p_2) j_5^{\mu_3}(p_3) \rangle = \pi^{\mu_1}_{\alpha_1}(p_1)\pi^{\mu_2}_{\alpha_2}(p_2)\pi^{\mu_3}_{\alpha_3}(p_3) \Big[& A_1(p_1, p_2, p_3)\, \varepsilon^{p_1 p_2 \alpha_1 \alpha_2} p_1^{\alpha_3} \\ & + A_2(p_1, p_2, p_3)\, \varepsilon^{p_1 \alpha_1 \alpha_2 \alpha_3} - A_2(p_2, p_1, p_3)\, \varepsilon^{p_2 \alpha_1 \alpha_2 \alpha_3} \Big] \end{aligned}$$

$$A_1(p_1, p_2, p_3) = -A_1(p_2, p_1, p_3).$$

SUMMARY AVV

Notice that CWIs have "forced" the anomaly coefficient into the transverse sector as well.
This is in agreement with Furry's theorem (vanishing of VVV conserved currents), without using C-invariance, but just conformal symmetry

$$\langle J^{\mu_1}(p_1) J^{\mu_2}(p_2) J_A^{\mu_3}(p_3)\rangle = -\frac{8i\,a_1}{p_3^2}\varepsilon^{p_1 p_2 \mu_1 \mu_2}\, p_3^{\mu_3}$$

$$8ia_1\,\pi^{\mu_1}_{\alpha_1}(p_1)\pi^{\mu_2}_{\alpha_2}(p_2)\pi^{\mu_3}_{\alpha_3}(p_3)\left[p_2^2\, I_{3\{1,0,1\}}\,\varepsilon^{p_1\alpha_1\alpha_2\alpha_3} - p_1^2\, I_{3\{0,1,1\}}\,\varepsilon^{p_2\alpha_1\alpha_2\alpha_3}\right].$$

$$\langle J^{\mu_1}(p_1) J^{\mu_2}(p_2) J_A^{\mu_3}(p_3)\rangle = \langle J^{\mu_1}(p_1) J^{\mu_2}(p_2)\, j_{A\,\mathrm{loc}}^{\mu_3}(p_3)\rangle + \pi^{\mu_1}_{\alpha_1}(p_1)\pi^{\mu_2}_{\alpha_2}(p_2)\pi^{\mu_3}_{\alpha_3}(p_3)\,\Delta_T^{\alpha_1\alpha_2\alpha_3}(p_1^2,p_2^2,p_3^2,m^2)$$

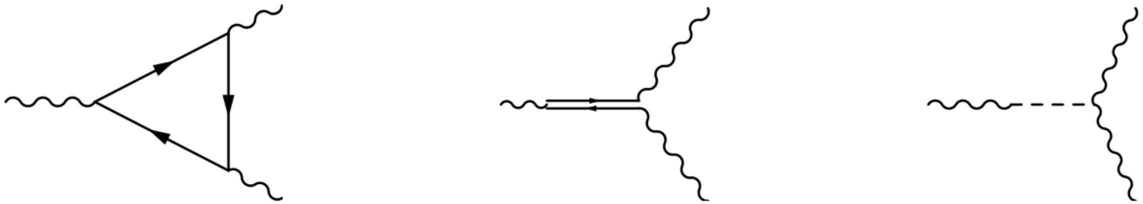
If we move away from the conformal limit, the decomposition is still valid, but we need to identify the Anomaly form factor (AFF)

$$\Phi_0(p_1^2,p_2^2,p_3^2,m^2)$$

$$\frac{m^2}{\pi^2}\,\frac{1}{q^2}C_0\left(q^2,p_1^2,p_2^2,m^2\right) + \frac{1}{2\pi^2}\,\frac{1}{q^2},$$

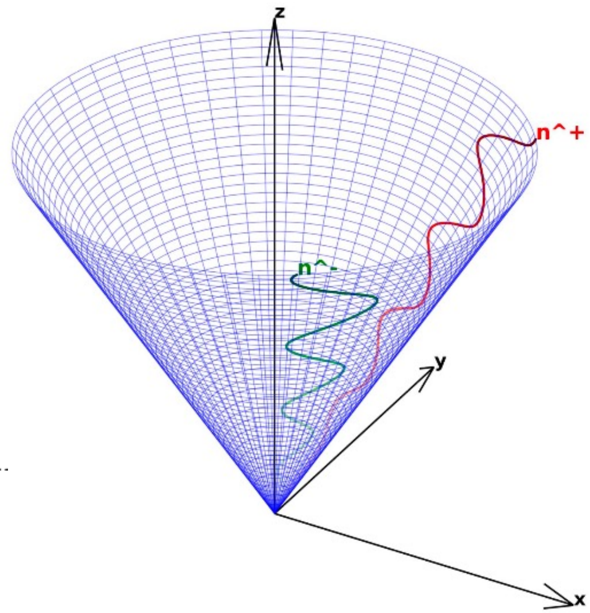
$$C_0\left(p_1^2,p_2^2,q^2,m^2,m^2,m^2\right) = \int_0^1 dx\int_0^{1-x} dy\,\frac{1}{\Delta(x,y)}$$

$$\Delta(x,y) = m^2 + q^2 y(x+y-1) - p_1^2 xy + p_2^2 x(x+y-1)$$



(a) (b) (c)

$$\frac{\varphi}{f_a}\,\partial\cdot\langle J_5\rangle$$



$$\mathcal{S}_{eff}\sim\frac{e^2}{2\pi}\int d^4x\ d^4y\ \partial\cdot B\ \Box^{-1}(x,y)\ F\tilde{F}(y).$$

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J_5 T T

$$T^{\mu_i\nu_i}(p_i) = t^{\mu_i\nu_i}(p_i) + t_{loc}^{\mu_i\nu_i}(p_i),$$
$$J_5^{\mu_i}(p_i) = j_5^{\mu_i}(p_i) + j_{5\,loc}^{\mu_i}(p_i),$$

$$t^{\mu_i\nu_i}(p_i) = \Pi^{\mu_i\nu_i}_{\alpha_i\beta_i}(p_i)\,T^{\alpha_i\beta_i}(p_i), \qquad t_{loc}^{\mu_i\nu_i}(p_i) = \Sigma^{\mu_i\nu_i}_{\alpha_i\beta_i}(p)\,T^{\alpha_i\beta_i}(p_i),$$

$$j_5^{\mu_i}(p_i) = \pi^{\mu_i}_{\alpha_i}(p_i)\,J_5^{\alpha_i}(p_i), \qquad j_{5\,loc}^{\mu_i}(p_i) = \frac{p_i^{\mu_i}\,p_{i\,\alpha_i}}{p_i^2}\,J_5^{\alpha_i}(p_i),$$

$$\Pi^{\mu\nu}_{\alpha\beta} = \frac{1}{2}\left(\pi^{\mu}_{\alpha}\pi^{\nu}_{\beta} + \pi^{\mu}_{\beta}\pi^{\nu}_{\alpha}\right) - \frac{1}{d-1}\pi^{\mu\nu}\pi_{\alpha\beta},$$
$$\Sigma^{\mu_i\nu_i}_{\alpha_i\beta_i} = \frac{p_{i\,\beta_i}}{p_i^2}\left[2\delta^{(\nu_i}_{\alpha_i}p_i^{\mu_i)} - \frac{p_{i\alpha_i}}{(d-1)}\left(\delta^{\mu_i\nu_i} + (d-2)\frac{p_i^{\mu_i}p_i^{\nu_i}}{p_i^2}\right)\right] + \frac{\pi^{\mu_i\nu_i}(p_i)}{(d-1)}\delta_{\alpha_i\beta_i}.$$

$$\begin{aligned}\langle T^{\mu_1\nu_1}T^{\mu_2\nu_2}J_5^{\mu_3}\rangle =& \langle t^{\mu_1\nu_1}t^{\mu_2\nu_2}j_5^{\mu_3}\rangle + \left\langle T^{\mu_1\nu_1}T^{\mu_2\nu_2}j_{5\,loc}^{\mu_3}\right\rangle + \left\langle T^{\mu_1\nu_1}t_{loc}^{\mu_2\nu_2}J_5^{\mu_3}\right\rangle + \left\langle t_{loc}^{\mu_1\nu_1}T^{\mu_2\nu_2}J_5^{\mu_3}\right\rangle \\ &- \left\langle T^{\mu_1\nu_1}t_{loc}^{\mu_2\nu_2}j_{5\,loc}^{\mu_3}\right\rangle - \left\langle t_{loc}^{\mu_1\nu_1}t_{loc}^{\mu_2\nu_2}J_5^{\mu_3}\right\rangle - \left\langle t_{loc}^{\mu_1\nu_1}T^{\mu_2\nu_2}j_{5\,loc}^{\mu_3}\right\rangle + \left\langle t_{loc}^{\mu_1\nu_1}t_{loc}^{\mu_2\nu_2}j_{5\,loc}^{\mu_3}\right\rangle\end{aligned}$$

$$\delta_{\mu_i\nu_i}\,\langle T^{\mu_1\nu_1}(p_1)T^{\mu_2\nu_2}(p_2)J_5^{\mu_3}(p_3)\rangle = 0,$$
$$p_{i\mu_i}\,\langle T^{\mu_1\nu_1}(p_1)T^{\mu_2\nu_2}(p_2)J_5^{\mu_3}(p_3)\rangle = 0,$$

Simplifications

$$\langle T^{\mu_1\nu_1}T^{\mu_2\nu_2}J_5^{\mu_3}\rangle = \langle t^{\mu_1\nu_1}t^{\mu_2\nu_2}j_5^{\mu_3}\rangle + \left\langle T^{\mu_1\nu_1}T^{\mu_2\nu_2}j_{5\,loc}^{\mu_3}\right\rangle = \langle t^{\mu_1\nu_1}t^{\mu_2\nu_2}j_5^{\mu_3}\rangle + \left\langle t^{\mu_1\nu_1}t^{\mu_2\nu_2}j_{5\,loc}^{\mu_3}\right\rangle.$$

$$p_{3\mu_3}\,\langle T^{\mu_1\nu_1}(p_1)T^{\mu_2\nu_2}(p_2)J_5^{\mu_3}(p_3)\rangle = 4\,i\,a_2\,(p_1\cdot p_2)\left\{\left[\varepsilon^{\nu_1\nu_2p_1p_2}\left(g^{\mu_1\mu_2}-\frac{p_1^{\mu_2}p_2^{\mu_1}}{p_1\cdot p_2}\right)+(\mu_1\leftrightarrow\nu_1)\right]+(\mu_2\leftrightarrow\nu_2)\right\},$$

Anomaly pole in the longitudinal sector. (a2)

$$\left\langle t^{\mu_1\nu_1}t^{\mu_2\nu_2}j_{5\,loc}^{\mu_3}\right\rangle = 4ia_2\frac{p_3^{\mu_3}}{p_3^2}\,(p_1\cdot p_2)\left\{\left[\varepsilon^{\nu_1\nu_2p_1p_2}\left(g^{\mu_1\mu_2}-\frac{p_1^{\mu_2}p_2^{\mu_1}}{p_1\cdot p_2}\right)+(\mu_1\leftrightarrow\nu_1)\right]+(\mu_2\leftrightarrow\nu_2)\right\}.$$

$$
\begin{aligned}
0 =& \sum_{i=1}^{3}\left[2x_i^\kappa\left(\Delta_i + x_i^\alpha\frac{\partial}{\partial x_i^\alpha}\right) - x_i^2\delta^{\kappa\alpha}\frac{\partial}{\partial x_i^\alpha}\right]\langle T^{\mu_1\nu_1}(x_1)T^{\mu_2\nu_2}(x_2)J_5^{\mu_3}(x_3)\rangle \\
&+ 2\left[\delta^{\kappa\mu_1}x_{1\alpha} - \delta^\kappa_\alpha x_1^{\mu_1}\right]\langle T^{\alpha\nu_1}(x_1)T^{\mu_2\nu_2}(x_2)J_5^{\mu_3}(x_3)\rangle + 2\left[\delta^{\kappa\nu_1}x_{1\alpha} - \delta^\kappa_\alpha x_1^{\nu_1}\right]\langle T^{\mu_1\alpha}(x_1)T^{\mu_2\nu_2}(x_2)J_5^{\mu_3}(x_3)\rangle \\
&+ 2\left[\delta^{\kappa\mu_2}x_{2\alpha} - \delta^\kappa_\alpha x_2^{\mu_2}\right]\langle T^{\mu_1\nu_1}(x_1)T^{\alpha\nu_2}(x_2)J_5^{\mu_3}(x_3)\rangle + 2\left[\delta^{\kappa\nu_2}x_{2\alpha} - \delta^\kappa_\alpha x_2^{\nu_2}\right]\langle T^{\mu_1\nu_1}(x_1)T^{\mu_2\alpha}(x_2)J_5^{\mu_3}(x_3)\rangle \\
&+ 2\left[\delta^{\kappa\mu_3}x_{3\alpha} - \delta^\kappa_\alpha x_3^{\mu_3}\right]\langle T^{\mu_1\nu_1}(x_1)T^{\mu_2\nu_2}(x_2)J_5^{\alpha}(x_3)\rangle\,,
\end{aligned}
$$

$$
\begin{aligned}
0 =& \,\mathcal{K}^\kappa\left\langle T^{\mu_1\nu_1}(p_1)\,T^{\mu_2\nu_2}(p_2)\,J_5^{\mu_3}(p_3)\right\rangle \\
=& \sum_{j=1}^{2}\left(2\left(\Delta_j - d\right)\frac{\partial}{\partial p_{j\kappa}} - 2p_j^\alpha\frac{\partial}{\partial p_j^\alpha}\frac{\partial}{\partial p_{j\kappa}} + (p_j)^\kappa\frac{\partial}{\partial p_j^\alpha}\frac{\partial}{\partial p_{j\alpha}}\right)\left\langle T^{\mu_1\nu_1}(p_1)\,T^{\mu_2\nu_2}(p_2)\,J_5^{\mu_3}(p_3)\right\rangle \\
&+ 4\left(\delta^{\kappa(\mu_1}\frac{\partial}{\partial p_1^{\alpha_1}} - \delta^\kappa_{\alpha_1}\delta^{(\mu_1}_\lambda\frac{\partial}{\partial p_{1\lambda}}\right)\left\langle T^{\nu_1)\alpha_1}(p_1)\,T^{\mu_2\nu_2}(p_2)\,J_5^{\mu_3}(p_3)\right\rangle \\
&+ 4\left(\delta^{\kappa(\mu_2}\frac{\partial}{\partial p_2^{\alpha_2}} - \delta^\kappa_{\alpha_2}\delta^{(\mu_2}_\lambda\frac{\partial}{\partial p_{2\lambda}}\right)\left\langle T^{\nu_2)\alpha_2}(p_2)\,T^{\mu_1\nu_1}(p_1)\,J_5^{\mu_3}(p_3)\right\rangle\,.
\end{aligned}
$$

(4

$$\langle t^{\mu_1\nu_1}(p_1) t^{\mu_2\nu_2}(p_2)\, j_5^{\mu_3}(p_3)\rangle = \Pi^{\mu_1\nu_1}_{\alpha_1\beta_1}(p_1)\,\Pi^{\mu_2\nu_2}_{\alpha_2\beta_2}(p_2)\,\pi^{\mu_3}_{\alpha_3}(p_3)\Big[A_1 \varepsilon^{p_1\alpha_1\alpha_2\alpha_3} p_2^{\beta_1} p_3^{\beta_2} - A_1(p_1\leftrightarrow p_2)\varepsilon^{p_2\alpha_1\alpha_2\alpha_3} p_2^{\beta_1} p_3^{\beta_2} + A_2 \varepsilon^{p_1\alpha_1\alpha_2\alpha_3}\delta^{\beta_1\beta_2} - A_2(p_1\leftrightarrow p_2)\varepsilon^{p_2\alpha_1\alpha_2\alpha_3}\delta^{\beta_1\beta_2} + A_3 \varepsilon^{p_1 p_2\alpha_1\alpha_2} p_2^{\beta_1} p_3^{\beta_2} p_1^{\alpha_3} + A_4 \varepsilon^{p_1 p_2 \alpha_1\alpha_2}\delta^{\beta_1\beta_2} p_1^{\alpha_3}\Big]$$

$$0 = \Pi^{\rho_1\sigma_1}_{\mu_1\nu_1}(p_1)\,\Pi^{\rho_2\sigma_2}_{\mu_2\nu_2}(p_2)\,\pi^{\rho_3}_{\mu_3}(p_3)\left(\mathcal{K}^\kappa \langle T^{\mu_1\nu_1}(p_1)\, T^{\mu_2\nu_2}(p_2)\, J_5^{\mu_3}(p_3)\rangle\right)$$

SCT act endomorphically
On the tt sector

Equations can be solved by a method discussed in detail in the paper

$$0 = K_{31} A_1, \qquad 0 = K_{32} A_1 + \frac{2}{p_1^2}\left(p_1 \frac{\partial}{\partial p_1} - 4\right) A_1(p_1 \leftrightarrow p_2)$$

$$0 = K_{31} A_2 + 4A_1, \qquad 0 = K_{32} A_2 + \frac{2}{p_1^2}\left(p_1 \frac{\partial}{\partial p_1} - 4\right) A_2(p_1 \leftrightarrow p_2) + 4A_1$$

$$0 = K_{31} A_1, \qquad 0 = K_{32} K_{32} A_1$$

$$0 = K_{31} K_{31} A_2, \qquad 0 = K_{32} K_{32} K_{32} A_2.$$

$$
\begin{aligned}
0 = \Pi^{\rho_1\sigma_1}_{\mu_1\nu_1}(p_1)\,\Pi^{\rho_2\sigma_2}_{\mu_2\nu_2}(p_2)\,\pi^{\rho_3}_{\mu_3}(p_3)\left(\mathcal{K}^{\kappa}\left\langle T^{\mu_1\nu_1}(p_1)\,T^{\mu_2\nu_2}(p_2)\,J_5^{\mu_3}(p_3)\right\rangle\right) = \Pi^{\rho_1\sigma_1}_{\mu_1\nu_1}(p_1)\,\Pi^{\rho_2\sigma_2}_{\mu_2\nu_2}(p_2)\,\pi^{\rho_3}_{\mu_3}(p_3)\Bigg[\\
p_1^{\kappa}\Big(C_{11}\varepsilon^{p_1\mu_1\mu_2\mu_3}p_2^{\nu_1}p_3^{\nu_2} + C_{12}\varepsilon^{p_2\mu_1\mu_2\mu_3}p_2^{\nu_1}p_3^{\nu_2} + C_{13}\varepsilon^{p_1\mu_1\mu_2\mu_3}\delta^{\nu_1\nu_2} + C_{14}\varepsilon^{p_2\mu_1\mu_2\mu_3}\delta^{\nu_1\nu_2} \\
+ C_{15}\varepsilon^{p_1p_2\mu_1\mu_2}p_2^{\nu_1}p_3^{\nu_2}p_1^{\mu_3} + C_{16}\varepsilon^{p_1p_2\mu_1\mu_2}\delta^{\nu_1\nu_2}p_1^{\mu_3}\Big) \\
+ p_2^{\kappa}\Big(C_{21}\varepsilon^{p_1\mu_1\mu_2\mu_3}p_2^{\nu_1}p_3^{\nu_2} + C_{22}\varepsilon^{p_2\mu_1\mu_2\mu_3}p_2^{\nu_1}p_3^{\nu_2} + C_{23}\varepsilon^{p_1\mu_1\mu_2\mu_3}\delta^{\nu_1\nu_2} + C_{24}\varepsilon^{p_2\mu_1\mu_2\mu_3}\delta^{\nu_1\nu_2} \\
+ C_{25}\varepsilon^{p_1p_2\mu_1\mu_2}p_2^{\nu_1}p_3^{\nu_2}p_1^{\mu_3} + C_{26}\varepsilon^{p_1p_2\mu_1\mu_2}\delta^{\nu_1\nu_2}p_1^{\mu_3}\Big) \\
+ \delta^{\kappa\mu_1}\Big(C_{31}\varepsilon^{p_1\mu_2\mu_3\nu_1}p_3^{\nu_2} + C_{32}\varepsilon^{p_2\mu_2\mu_3\nu_1}p_3^{\nu_2} + C_{33}\varepsilon^{p_1p_2\mu_2\nu_1}p_1^{\mu_3}p_3^{\nu_2} + C_{34}\varepsilon^{p_1p_2\mu_2\mu_3}\delta^{\nu_1\nu_2}\Big) \\
+ \delta^{\kappa\mu_2}\Big(C_{41}\varepsilon^{p_1\mu_1\mu_3\nu_2}p_2^{\nu_1} + C_{42}\varepsilon^{p_2\mu_1\mu_3\nu_2}p_2^{\nu_1} + C_{43}\varepsilon^{p_1p_2\mu_1\nu_2}p_1^{\mu_3}p_2^{\nu_1} + C_{44}\varepsilon^{p_1p_2\mu_1\mu_3}\delta^{\nu_1\nu_2}\Big) \\
+ C_{51}\varepsilon^{\kappa\mu_1\mu_2\mu_3}\delta^{\nu_1\nu_2} + C_{52}\varepsilon^{\kappa\mu_1\mu_2\mu_3}p_2^{\nu_1}p_3^{\nu_2} + C_{53}\varepsilon^{p_1\kappa\mu_1\mu_2}p_1^{\mu_3}\delta^{\nu_1\nu_2} + C_{54}\varepsilon^{p_2\kappa\mu_1\mu_2}p_1^{\mu_3}\delta^{\nu_1\nu_2}\Bigg]
\end{aligned}
\tag{78}
$$

Primary second order PDE’s

$$0 = C_{ij} \qquad\qquad i = \{1, 2\}, \quad j = \{1, \ldots 6\}$$

They correspond to second order differential equations. The secondary equations are instead given by

$$0 = C_{ij} \qquad\qquad i = \{3, 4, 5\}, \quad j = \{1, \ldots 4\}$$

Solution

$$A_1 = \eta_1\, J_{3\{0,0,0\}} + \eta_2\, J_{4\{0,1,0\}},$$
$$A_2 = \theta_1\, J_{4\{1,2,0\}} + \theta_2\, J_{3\{0,2,0\}} + \theta_3\, J_{3\{1,1,0\}} + \theta_4\, J_{2\{0,1,0\}} + \theta_5\, J_{2\{1,0,0\}} + \theta_6\, J_{1\{0,0,0\}} + \theta_7\, J_{3\{0,1,1\}} + \theta_8\, J_{2\{0,0,1\}},$$

Intermediate renormalization

As in BMS, modified to account for the chiral nature of The anomaly

$$\theta_1 = \theta_1^{(0)} + \theta_1^{(1)}\epsilon + \theta_1^{(2)}\epsilon^2, \qquad \theta_2 = \theta_2^{(0)} + \theta_2^{(1)}\epsilon + \theta_2^{(2)}\epsilon^2, \qquad \theta_3 = \theta_3^{(0)} + \theta_3^{(1)}\epsilon + \theta_3^{(2)}\epsilon^2,$$
$$\theta_4 = \theta_4^{(0)} + \theta_4^{(1)}\epsilon + \theta_4^{(2)}\epsilon^2, \qquad \theta_5 = \theta_5^{(0)} + \theta_5^{(1)}\epsilon + \theta_5^{(2)}\epsilon^2, \qquad \theta_6 = \theta_6^{(0)} + \theta_6^{(1)}\epsilon + \theta_6^{(2)}\epsilon^2,$$
$$\theta_7 = \theta_7^{(0)} + \theta_7^{(1)}\epsilon, \qquad \theta_8 = \theta_8^{(0)} + \theta_8^{(1)}\epsilon.$$

$$\alpha + 1 \pm \beta_1 \pm \beta_2 \pm \beta_3 = -2k \quad , \quad k = 0, 1, 2, \dots$$

$$I_{\alpha\{\beta_1,\beta_2,\beta_3\}} \longmapsto I_{\alpha+u\epsilon\{\beta_1+v_1\epsilon,\, \beta_2+v_2\epsilon,\, \beta_3+v_3\epsilon\}}$$
$$J_{N\{k_1,k_2,k_3\}} \longmapsto J_{N+u\epsilon\{k_1+v_1\epsilon,\, k_2+v_2\epsilon,\, k_3+v_3\epsilon\}}.$$

The solution

$$A_1 = -4\, i\, a_2\, p_2^2\, I_{5\{2,1,1\}}$$
$$A_2 = -8\, i\, a_2\, p_2^2 \left(p_3^2\, I_{4\{2,1,0\}} - 1 \right)$$
$$A_3 = 0$$
$$A_4 = 0.$$

MESS (secondary equations)

$$0 = -2p_1 \frac{\partial}{\partial p_1} A_1 + 2p_2 \frac{\partial}{\partial p_2} A_1 (p_1 \leftrightarrow p_2)$$

$$0 = -\left(p_1^2 - p_2^2 + p_3^2\right) A_1 + \left(-p_1^2 + p_2^2 + p_3^2\right) A_1(p_1 \leftrightarrow p_2) - 2p_1 \frac{\partial}{\partial p_1} A_2 + 2p_2 \frac{\partial}{\partial p_2} A_2(p_1 \leftrightarrow p_2)$$
$$+ 2A_2 - 2A_2(p_1 \leftrightarrow p_2)$$

$$0 = -\frac{2p_2^3}{p_3^2} \frac{\partial}{\partial p_2} A_1(p_1 \leftrightarrow p_2) - 2\left(\frac{p_2^2 + p_3^2}{p_3^2}\right) p_2 \frac{\partial}{\partial p_2} A_1 + \left(-\frac{2p_2^2}{p_3^2} + \frac{p_3^2 - p_2^2 - p_1^2}{p_1^2}\right) p_1 \frac{\partial}{\partial p_1} A_1$$
$$+ 2p_2^2 \left(\frac{p_3^2 - p_1^2}{p_3^2 p_1}\right) \frac{\partial}{\partial p_1} A_1(p_1 \leftrightarrow p_2) - 4p_2^2 \left(\frac{2}{p_1^2} + \frac{1}{p_3^2}\right) A_1(p_1 \leftrightarrow p_2) + 4\left(\frac{p_1^2 + p_2^2 - p_3^2}{p_1^2} - \frac{p_2^2}{p_3^2}\right) A_1$$
$$- \frac{2}{p_1} \frac{\partial}{\partial p_1} A_2 + \frac{8}{p_1^2} A_2 - \frac{64 i a p_2^2}{p_3^2}$$

$$0 = -\left(\frac{p_1^2 + p_2^2 - p_3^2}{p_3^2}\right) p_1 \frac{\partial}{\partial p_1} A_1 - \frac{\left(p_1^2 - 2p_3^2\right)\left(p_1^2 + p_2^2 - p_3^2\right)}{p_1 p_3^2} \frac{\partial}{\partial p_1} A_1(p_1 \leftrightarrow p_2) - \left(\frac{p_1^2 + p_2^2 - p_3^2}{p_3^2}\right) p_2 \frac{\partial}{\partial p_2} A_1$$
$$- \left(\frac{p_1^2 + p_2^2 - 3p_3^2}{p_3^2}\right) p_2 \frac{\partial}{\partial p_2} A_1(p_1 \leftrightarrow p_2) - 2\left(\frac{p_1^2 + p_2^2 - 2p_3^2}{p_3^2} + 4\frac{p_1^2 + p_2^2 - p_3^2}{p_1^2}\right) A_1(p_1 \leftrightarrow p_2)$$
$$- 2\left(\frac{p_1^2 + p_2^2 - 2p_3^2}{p_3^2}\right) A_1 + \frac{2}{p_1} \frac{\partial}{\partial p_1} A_2(p_1 \leftrightarrow p_2) - \frac{8}{p_1^2} A_2(p_1 \leftrightarrow p_2) - \frac{32 i a \left(p_1^2 + p_2^2 - p_3^2\right)}{p_3^2}$$

$$0 = \frac{2p_1}{p_3^2} \frac{\partial}{\partial p_1} A_1 + 2\left(\frac{p_1^2 - p_3^2}{p_3^2 p_1}\right) \frac{\partial}{\partial p_1} A_1(p_1 \leftrightarrow p_2) + \frac{2p_2}{p_3^2} \frac{\partial}{\partial p_2} A_1 + \frac{2p_2}{p_3^2} \frac{\partial}{\partial p_2} A_1(p_1 \leftrightarrow p_2)$$
$$+ 4\left(\frac{2}{p_1^2} + \frac{1}{p_3^2}\right) A_1(p_1 \leftrightarrow p_2) + \frac{4}{p_3^2} A_1 + \frac{64 i a}{p_3^2}$$

$$0 = -\frac{2p_1}{p_3^2} \frac{\partial}{\partial p_1} A_2 + 2\left(\frac{p_3^2 - p_1^2}{p_3^2 p_1}\right) \frac{\partial}{\partial p_1} A_2(p_1 \leftrightarrow p_2) - \frac{2p_2}{p_3^2} \frac{\partial}{\partial p_2} A_2 - \frac{2p_2}{p_3^2} \frac{\partial}{\partial p_2} A_2(p_1 \leftrightarrow p_2) - \frac{8}{p_1^2} A_2(p_1 \leftrightarrow p_2)$$
$$+ \frac{32 i a \left(p_1^2 + p_2^2 - p_3^2\right)}{p_3^2}$$

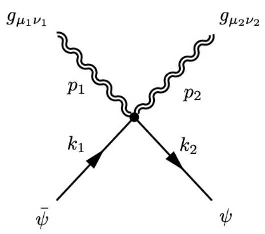
(105)

$$S_0 = \int d^d x \, \frac{e}{2} \, e_a^\mu \left[i\bar{\psi}\gamma^a \left(D_\mu \psi\right) - i \left(D_\mu \bar{\psi}\right) \gamma^a \psi \right]$$

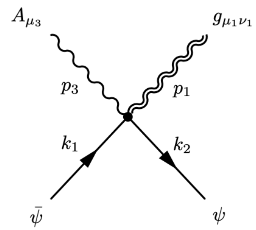
$$\omega_{\mu ab} \equiv e_a^\nu \left(\partial_\mu e_{\nu b} - \Gamma^\lambda_{\mu\nu} e_{\lambda b} \right).$$

$$S_0 = \int d^d x \, e \left[\frac{i}{2} \bar{\psi} e_a^\mu \gamma^a \left(\partial_\mu \psi\right) - \frac{i}{2} \left(\partial_\mu \bar{\psi}\right) e_a^\mu \gamma^a \psi - g A_\mu \bar{\psi} e_a^\mu \gamma^a \gamma_5 \psi + \frac{i}{4} \omega_{\mu ab} e_c^\mu \bar{\psi} \gamma^{abc} \psi \right]$$

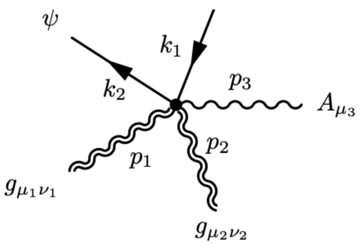
$$T^{\mu\nu} = -\frac{i}{2} \left[\bar{\psi}\gamma^{(\mu}\nabla^{\nu)}\psi - \nabla^{(\mu}\bar{\psi}\gamma^{\nu)}\psi - g^{\mu\nu}\left(\bar{\psi}\gamma^\lambda \nabla_\lambda \psi - \nabla_\lambda \bar{\psi}\gamma^\lambda \psi \right) \right] - g\bar{\psi}\left(g^{\mu\nu}\gamma^\lambda A_\lambda - \gamma^{(\mu} A^{\nu)} \right) \gamma_5 \, \psi.$$



$$V^{\mu_1\nu_1\mu_2\nu_2}_{gg\bar{\psi}\psi} = - \frac{i}{8} \left(B^{\mu_1\nu_1\mu_2\nu_2\alpha\beta} - C^{\mu_1\nu_1\mu_2\nu_2\alpha\beta} + D^{\mu_1\nu_1\mu_2\nu_2\alpha\beta} \right) \gamma_\alpha (k_1 + k_2)_\beta + \frac{i}{128} G^{\alpha\beta\gamma} A^{\mu_1\nu_1\gamma\rho} p_2^\sigma \times \left(g^{\alpha\mu_2} g^{\beta\sigma} g^{\nu_2\rho} + g^{\alpha\nu_2} g^{\beta\sigma} g^{\mu_2\rho} - g^{\alpha\sigma} g^{\beta\nu_2} g^{\mu_2\rho} - g^{\alpha\sigma} g^{\beta\mu_2} g^{\nu_2\rho} \right)$$



$$V^{\mu_1\nu_1\mu_3}_{gA\bar{\psi}\psi} = -\frac{ig}{2} A^{\mu_1\nu_1\mu_3\rho}\gamma_\rho\gamma_5$$



$$V^{\mu_1\nu_1\mu_2\nu_2\mu_3}_{ggA\bar{\psi}\psi} = - \frac{ig}{4} \left(B^{\mu_1\nu_1\mu_2\nu_2\mu_3\lambda} - C^{\mu_1\nu_1\mu_2\nu_2\mu_3\lambda} + D^{\mu_1\nu_1\mu_2\nu_2\mu_3\lambda} \right) \gamma_\lambda \gamma_5$$

Perturbative checks. We show how to map 3K integrals into scalar master integrals

$$C_0(p_1^2, p_2^2, p_3^2) \equiv \frac{1}{i\pi^2} \int d^d l \frac{1}{l^2(l-p_1)^2(l+p_2)^2}$$

$$I_{\alpha\{\beta_1\beta_2\beta_3\}} = (-1)^{\beta_t}\ \mathrm{K}_{j,\beta_j}^{|n_0|-1} \left[p_1^{2\beta_1} p_2^{2\beta_2} p_3^{2\beta_3} \left(\frac{1}{p_1}\frac{\partial}{\partial p_1}\right)^{\beta_1} \left(\frac{1}{p_2}\frac{\partial}{\partial p_2}\right)^{\beta_2} \left(\frac{1}{p_3}\frac{\partial}{\partial p_3}\right)^{\beta_3} I_{1\{000\}} \right]$$

$$K_{d\{\delta_1\delta_2\delta_3\}} \equiv \int \frac{d^d k}{(2\pi)^d} \frac{1}{(k^2)^{\delta_3}\left((k-p_1)^2\right)^{\delta_2}\left((k+p_2)^2\right)^{\delta_1}}.$$

$$I_{5\{2,1,1\}} = \frac{i}{4} p_1 p_2 p_3 \left(p_1 \frac{\partial}{\partial p_1} - 1\right) \frac{\partial^3}{\partial p_1 \partial p_2 \partial p_3} C_0(p_1^2, p_2^2, p_3^2)$$

$$I_{4\{2,1,0\}} = -\frac{i}{4} p_1 p_2 \left(p_1 \frac{\partial}{\partial p_1} - 1\right) \frac{\partial^2}{\partial p_1 \partial p_2} C_0(p_1^2, p_2^2, p_3^2)$$

Result for the conformal solution

Amazingly simple

$$\langle t^{\mu_1\nu_1}(p_1) t^{\mu_2\nu_2}(p_2)\, j_5^{\mu_3}(p_3)\rangle = \Pi^{\mu_1\nu_1}_{\alpha_1\beta_1}(p_1)\,\Pi^{\mu_2\nu_2}_{\alpha_2\beta_2}(p_2)\,\pi^{\mu_3}_{\alpha_3}(p_3)\Big[A_1 \varepsilon^{p_1\alpha_1\alpha_2\alpha_3} p_2^{\beta_1} p_3^{\beta_2} - A_1(p_1 \leftrightarrow p_2)\varepsilon^{p_2\alpha_1\alpha_2\alpha_3} p_2^{\beta_1} p_3^{\beta_2} + A_2 \varepsilon^{p_1\alpha_1\alpha_2\alpha_3}\delta^{\beta_1\beta_2} - A_2(p_1 \leftrightarrow p_2)\varepsilon^{p_2\alpha_1\alpha_2\alpha_3}\delta^{\beta_1\beta_2} + A_3 \varepsilon^{p_1 p_2\alpha_1\alpha_2} p_2^{\beta_1} p_3^{\beta_2} p_1^{\alpha_3} + A_4 \varepsilon^{p_1 p_2 \alpha_1\alpha_2}\delta^{\beta_1\beta_2} p_1^{\alpha_3}\Big]$$

$$A_1 = \frac{g p_2^2}{24\pi^2\lambda^4}\left\{A_{11} + A_{12}\log\left(\frac{p_1^2}{p_2^2}\right) + A_{13}\log\left(\frac{p_1^2}{p_3^2}\right) + A_{14}\, C_0(p_1^2, p_2^2, p_3^2)\right\},$$

$$A_2 = \frac{g p_2^2}{48\pi^2\lambda^3}\left\{A_{21} + A_{22}\log\left(\frac{p_1^2}{p_2^2}\right) + A_{23}\log\left(\frac{p_1^2}{p_3^2}\right) + A_{24}\, C_0(p_1^2, p_2^2, p_3^2)\right\},$$

$$A_3 = 0,$$

$$A_4 = 0$$

$$a_2 = -\frac{ig}{384\pi^2}.$$

$$\langle T^{\mu_1\nu_1} T^{\mu_2\nu_2} J_5^{\mu_3}\rangle = \langle t^{\mu_1\nu_1} t^{\mu_2\nu_2} j_5^{\mu_3}\rangle + \langle t^{\mu_1\nu_1} t^{\mu_2\nu_2} j_{5\,loc}^{\mu_3}\rangle\,.$$

$$\langle t^{\mu_1\nu_1} t^{\mu_2\nu_2} j_{5\,loc}^{\mu_3}\rangle = 4 i a_2 \frac{p_3^{\mu_3}}{p_3^2}(p_1\cdot p_2)\left\{\left[\varepsilon^{\nu_1\nu_2 p_1 p_2}\left(g^{\mu_1\mu_2} - \frac{p_1^{\mu_2} p_2^{\mu_1}}{p_1\cdot p_2}\right) + (\mu_1 \leftrightarrow \nu_1)\right] + (\mu_2 \leftrightarrow \nu_2)\right\}.$$

$$\mathcal{S}_{eff}[\eta,\chi;A,B] = \int d^4x \, \left\{ (\partial^\mu \eta)\,(\partial_\mu \chi) - \chi\, \partial^\mu B_\mu + \frac{e^2}{8\pi^2}\, \eta\, F_{\mu\nu}\tilde{F}^{\mu\nu} \right\}$$

Giannotti and Mottola

$$\Box\, \eta = -\partial^\lambda B_\lambda\,,$$

$$\Box\, \chi = \frac{e^2}{8\pi^2}\, F_{\mu\nu}\tilde{F}^{\mu\nu} = \frac{e^2}{16\pi^2}\, \epsilon^{\mu\nu\rho\sigma} F_{\mu\nu} F_{\rho\sigma}\,.$$

$$\mathcal{S}_{eff}[\eta,\chi;A,B] = \int d^4x \, \left\{ (\partial^\mu \eta_g)\,(\partial_\mu \chi_g) - \chi_g\, \partial^\mu B_\mu + a_g\, \eta_g\, R_{\mu\nu}\tilde{R}^{\mu\nu} \right\}$$

Gravitational anomaly

Lionetti, Melle, CC

$$\Box\, \eta_g = -\partial^\lambda B_\lambda\,,$$

$$\Box\, \chi_g = a_g R_{\mu\nu}\tilde{R}^{\mu\nu}.$$

The residue of the particle pole in the light-cone variables

$$g_p \equiv \lim_{q^2 \to 0} q^2 \langle J^{\mu_1}(p_1) J^{\mu_2}(p_2) J_A^{\mu_3}(q) \rangle \neq 0$$

Condition to be satisfied in order to define a particle pole

with light-cone versors defined by $(n^{\pm})^2 = 0$, $n^+ \cdot n^- = 1$. In the light cone limit q^+ is large as well as p_1^+ and p_2^+, while the components

$$p_3 \equiv q = q^+ n^+ + q^- n^- \qquad p_1 = p_1^+ n^+ + p_1^- n^- + p_\perp \qquad p_2 = p_2^+ n^+ + p_2^- n^- - p_\perp$$

$$\langle J^{\mu_1}(p_1) J^{\mu_2}(p_2) J_A^{\mu_3}(p_3) \rangle = \langle J^{\mu_1}(p_1) J^{\mu_2}(p_2)\, j_{A\,\mathrm{loc}}^{\mu_3}(p_3) \rangle + \pi^{\mu_1}_{\alpha_1}(p_1) \pi^{\mu_2}_{\alpha_2}(p_2) \pi^{\mu_3}_{\alpha_3}(p_3)\, \Delta_T^{\alpha_1 \alpha_2 \alpha_3}(p_1^2, p_2^2, p_3^2, m^2)$$

$$q^- = \frac{q^2}{2q^+} \qquad p_1^- = \frac{p_1^2 + p_\perp^2}{2p_1^+} \qquad p_2^- = \frac{p_2^2 + p_\perp^2}{2p_2^+}$$

No residue

Only if the two vector lines are on shell, the dynamica generates a nonzero residue

CONCLUSIONS

CFT in momentum space. Provides a consistent framework for the analysis of anomaly correlators

Axion-like interactions need a mechanism of vacuum misalignment to be defined as asymptotic states

Naïve extensions of Goldstone’s theorem are not justified