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EXCELENCIA
SEVERO
OCHOA

Exploring Axionic Physics with Weyl Semimetals

Alberto Cortijo

Weyl and Dirac semimetals as a Laboratory for High Energy Physics - Minho 2025

Person power

Joan Bernabeu, current PhD student at UAM (about to finish)

PHYSICAL REVIEW B **110**, L081101 (2024)

Letter

Hysteresis of axionic charge density waves

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PHYSICAL REVIEW B **111**, 205124 (2025)

Axionic acoustic phonons from Weyl semimetals

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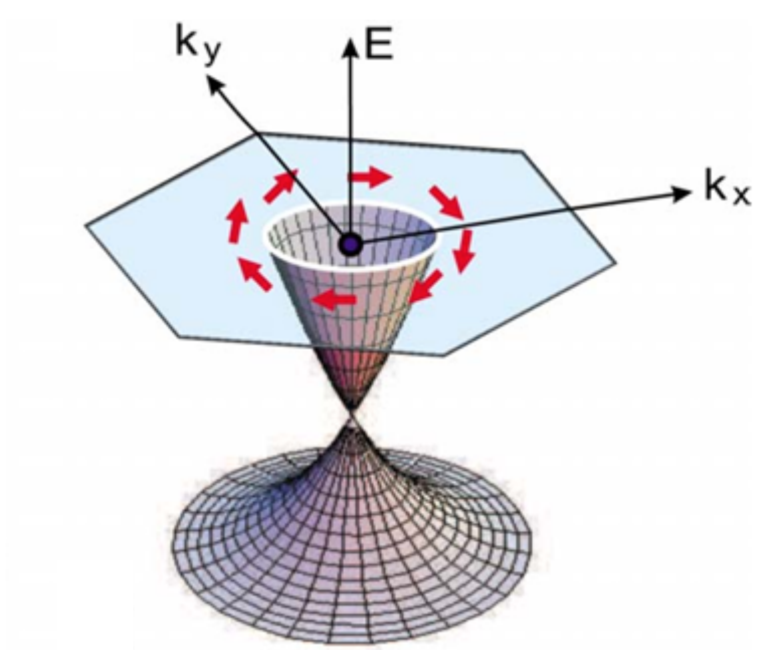
Axions in Condensed Matter Physics

$$\mathcal{S} = \frac{e^2}{4\pi^2\hbar}\int d^3x dt \theta \boldsymbol{E}\cdot\boldsymbol{B}$$

θ is odd under both T and I **from bulk but appears at the surface where I is broken**

$$\theta = (0, \pi)$$

there is 1D another Berry phase at the Fermi surface



$$\sigma_H = \frac{e^2}{2\pi h}(\theta - \phi)$$

Bulk **Surface**

$$\sigma_H = 0$$

As requested by T

N Varnava, D Vanderbilt, PRB 98, 245117 (2018)

Axions in Condensed Matter Physics

$$\mathcal{S} = \frac{e^2}{4\pi^2\hbar}\int d^3x dt \theta \boldsymbol{E}\cdot\boldsymbol{B}$$

$$\sigma_H = \frac{e^2}{2\pi h}(\theta - \phi)$$

Bulk **Surface**

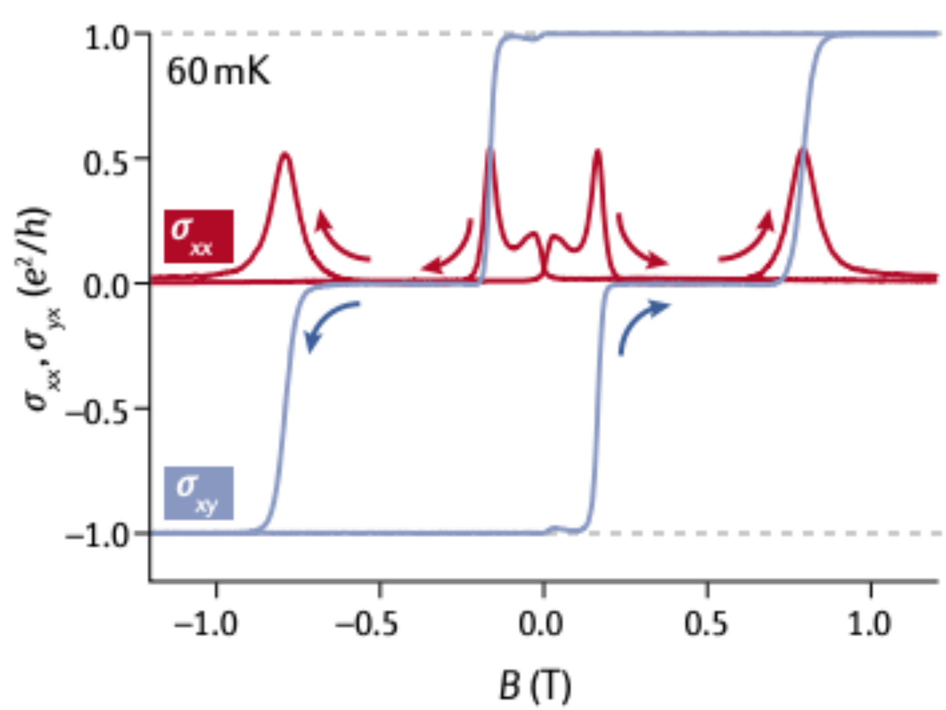
T Is broken

$$\theta \cancel{=} (0, \pi)$$

Axion insulators (static)

N Varnava, D Vanderbilt, PRB 98, 245117 (2018)

V-doped $(\mathrm{Bi}, \mathrm{Sb})_2\mathrm{Te}_3/\mathrm{TI}/\mathrm{Cr}$-doped $(\mathrm{Bi}, \mathrm{Sb})_2\mathrm{Te}_3$



D Xiao et al, PRL 120, 056801 (2018)

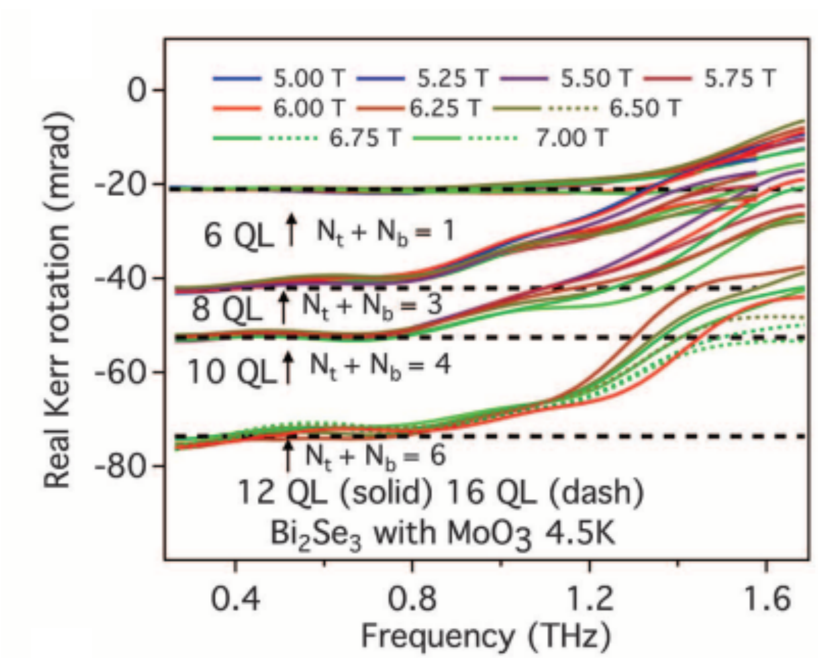
Axions in Condensed Matter Physics

θ can be still π allowed by symmetries of the magnetic point group (but it is not guaranteed to be quantized)

Dynamical axion insulators

I and T are broken

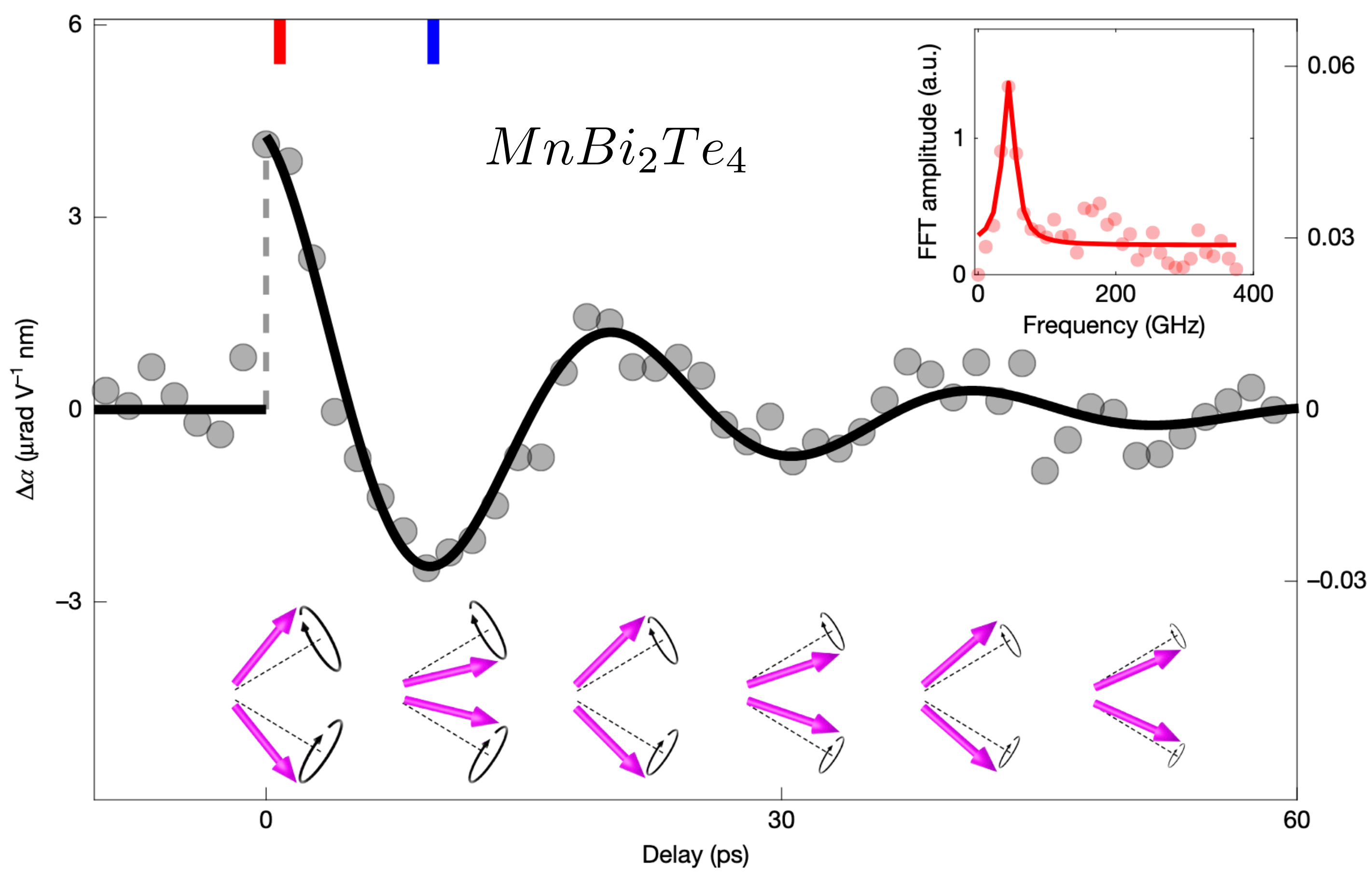
No symmetry protecting edge states



TOPOLOGICAL MATTER

Quantized Faraday and Kerr rotation and axion electrodynamics of a 3D topological insulator

Liang Wu,[1*†] M. Salehi,[2] N. Koirala,[3] J. Moon,[3] S. Oh,[3] N. P. Armitage[1*]



JX Qiu et al. Nature 641, 62 (2025)

Axion insulators (static)

Axion: (Any) light pseudo scalar particle

$$\mathcal{L} = \theta \frac{g^2}{32\pi^2} \overbrace{\varepsilon_{\mu\nu\rho\kappa} F_{\mu\nu} F_{\rho\kappa}}^{\boldsymbol{E}\cdot\boldsymbol{B}} + \bar{\psi}\left(i\gamma^\mu(\partial_\mu - gA_\mu) - me^{i\bar{\theta}\gamma_5}\right)\psi$$

These terms break P and T

$$\theta, \bar{\theta} \neq (0, \pi)$$

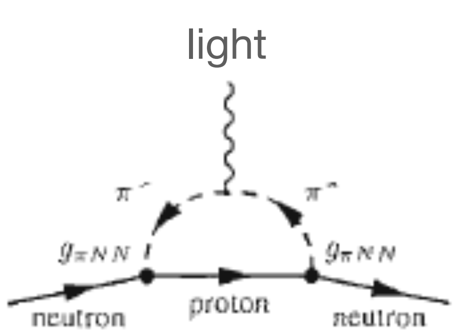
$$\psi \to e^{i\alpha\gamma_5/2}\psi$$

$$\underbrace{\theta = -\bar{\theta}}_{?}$$

$$\theta \to \theta + \alpha$$

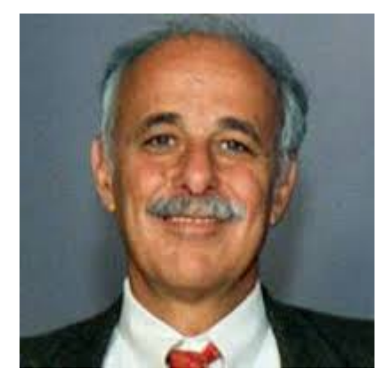
$$\bar{\theta} \to \bar{\theta} - \alpha$$

$$\theta + \bar{\theta} \to \theta + \alpha + \bar{\theta} - \alpha = \theta + \bar{\theta}$$



$$\delta\mathcal{L} = \bar{\psi}(\sigma + i\gamma_5\varphi)\psi$$

propose a yet another U(1) symmetric coupling



R. Peccei

$$\sigma = v\cos\alpha$$

H. Quinn

$$\varphi = v\sin\alpha$$

$$\alpha \sim \alpha_0 + a(x)$$

dynamical axion

$$d_N < 10^{-26} \qquad \theta < 10^{-10}$$

Axion: (Any) light pseudo scalar particle

$$\mathcal{L} = \theta \frac{g^2}{32\pi}$$ ... $$+\bar{\psi}\left(i\gamma^{\mu}\right.$$ ... $$\left.\right)\psi$$

$\neq (0, \pi)$

...ns break P

$$\psi \to e^{i\alpha\gamma_5/2}\psi$$

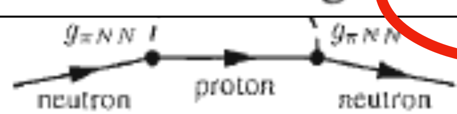
$$\theta \to \theta + \alpha$$

$$\bar{\theta} \to \bar{\theta} - \alpha$$

$$\theta + \bar{\theta} \to \theta + \alpha + \bar{\theta} - \alpha = \theta +$$

Peccei and Quinn[7] have made the ingenious observation that instead of a quark of bare mass zero, we might also consider a chiral symmetry. This leads to a special kind of Higgs boson (which we are calling the *axion*) with zero bare mass.[10]

$)\psi$



propose a yet another U(1) symmetric coupling

H. Quinn

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$$\alpha \sim \alpha_0 + a(x)$$

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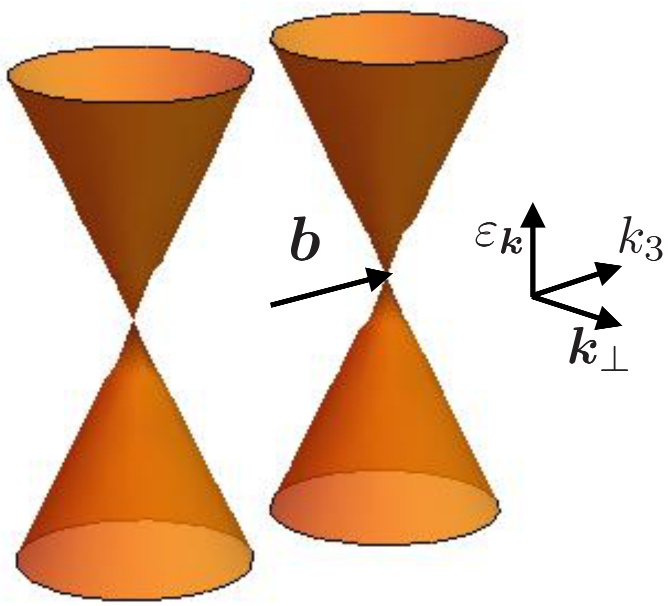
dynamical axion

Weyl semimetals: Emergent chirality

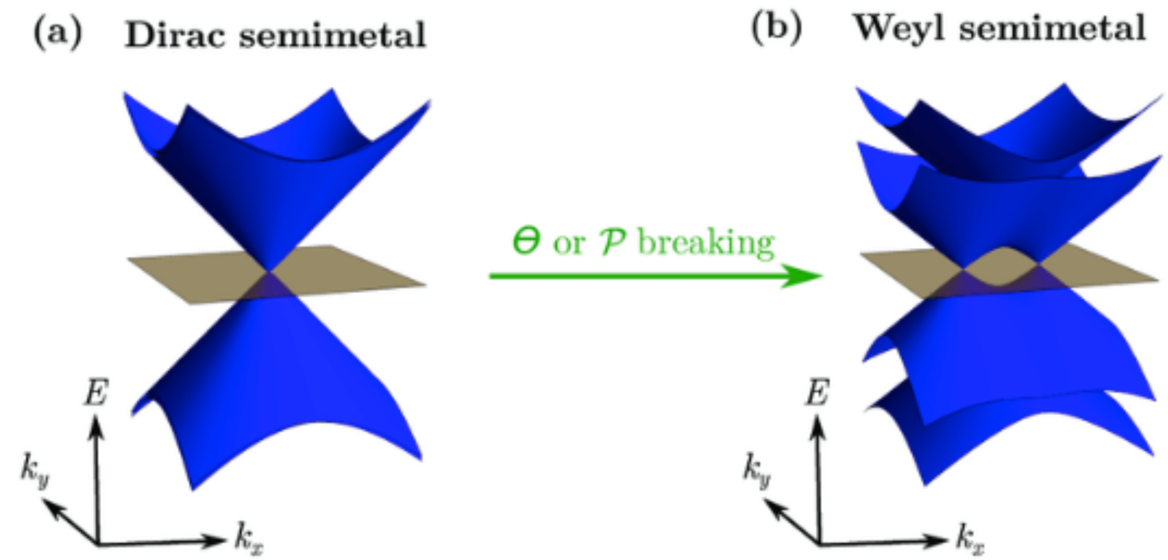


$$H(\boldsymbol{k}) = s\boldsymbol{\sigma}\cdot(\boldsymbol{k} - s\boldsymbol{b})$$

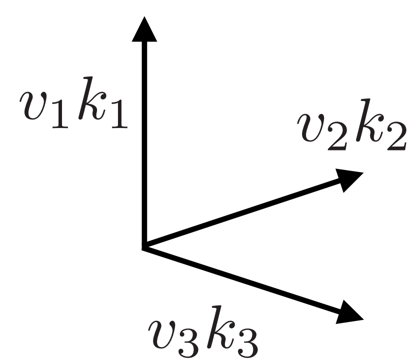
Herrig PR 52, 365 (1937)

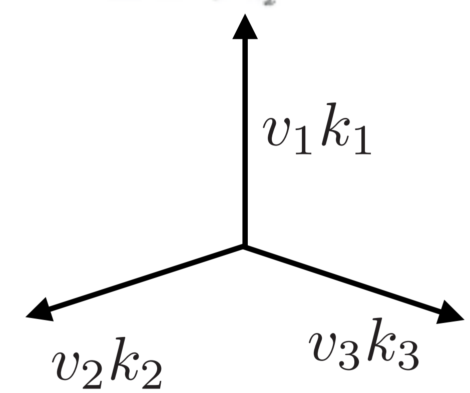
S is the chirality

$$\mathcal{L}_{\mathrm{W}} = i\bar{\psi}\left(\gamma^\mu\partial_\mu - \gamma^\mu\gamma_5 b_\mu\right)\psi$$



chirality means handedness





T is broken

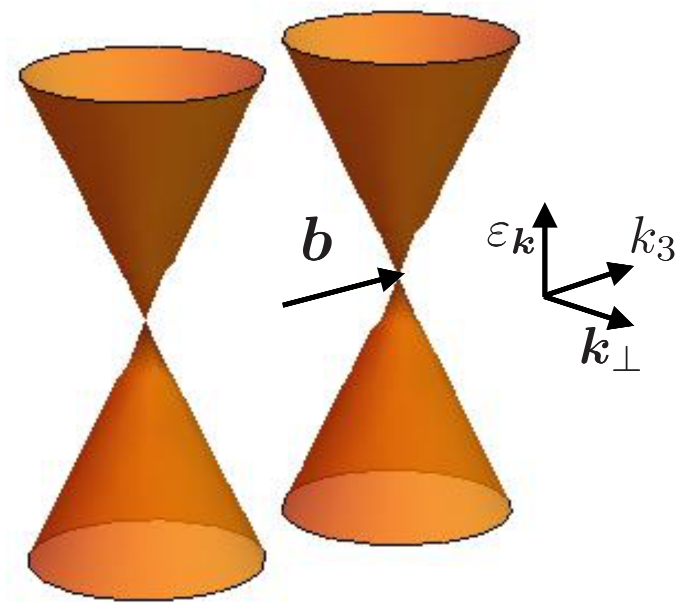
$$\theta(\boldsymbol{x}) = \boldsymbol{b}\cdot\boldsymbol{x}$$

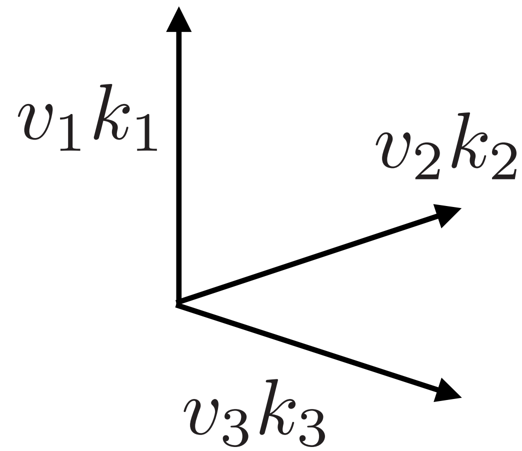
$$\boldsymbol{J}_H = \boldsymbol{b}\times\boldsymbol{E}$$

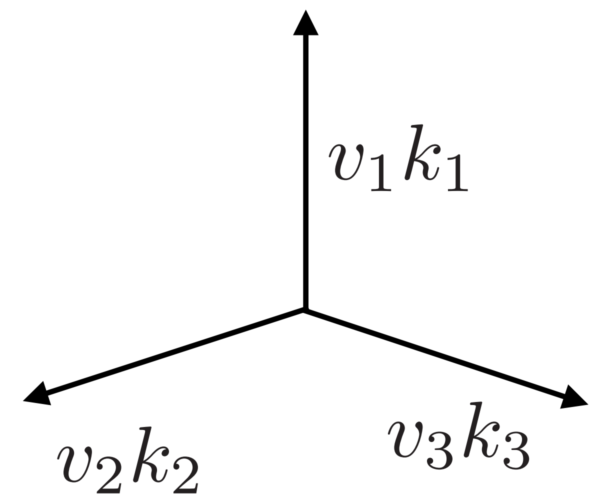
Topological Hall current

“Axion metal”

Weyl semimetals: Emergent chirality







chirality means handedness

Nielsen, Ninomiya, NucPhysB 185, 20 (1981)

$$H(\boldsymbol{k}) = s\boldsymbol{\sigma}\cdot(\boldsymbol{k} - s\boldsymbol{b})$$

Herrig PR 52, 365 (1937)

$$\psi_s \to e^{i\theta_s}\psi_s$$

S **is the chirality**

$$\gamma_5 \psi_s = s\psi_s$$

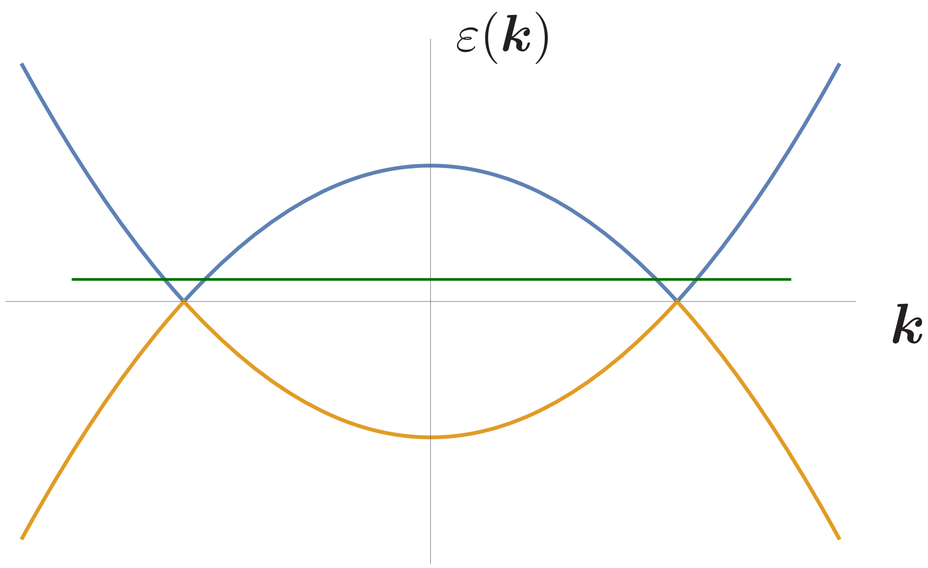
$$\theta_+ = \theta + \varphi$$
$$\theta_- = \theta - \varphi$$

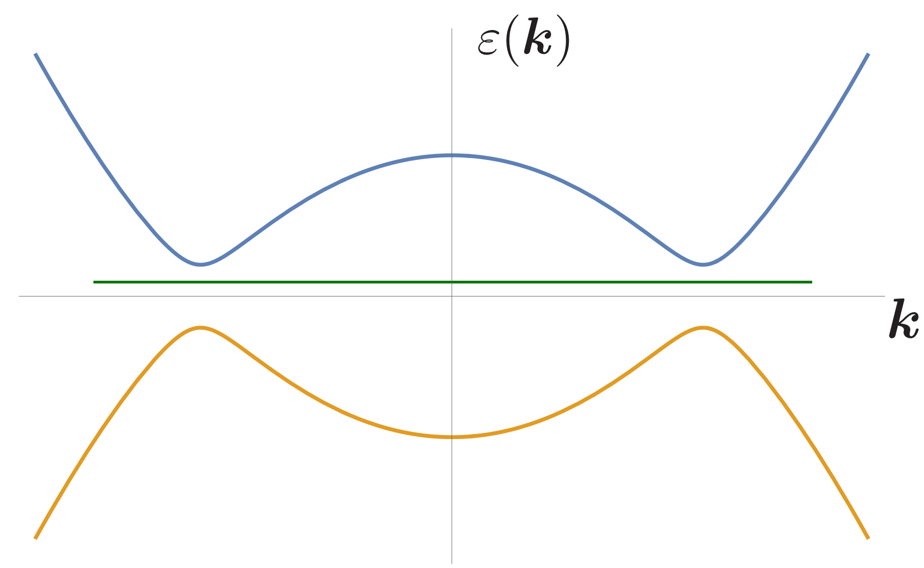
θ U(1) electromagnetic symmetry

φ U(1) emergent symmetry: chiral symmetry

$$U_L(1)\times U_R(1)$$

Axionic CDW instability in Weyls: Breaking chirality





$$H(\boldsymbol{k}) = \boldsymbol{\alpha}\cdot\boldsymbol{k}$$

Weyl (semi)metal

$$H(\boldsymbol{k}) = \boldsymbol{\alpha}\cdot\boldsymbol{k} + \beta m$$

insulator

$$\psi \to e^{i\theta}\psi$$

$$\psi \to e^{i\varphi\gamma_5}\psi$$

Mass in the Dirac equation breaks chiral symmetry explicitly

$$\psi \to e^{i\theta}\psi$$

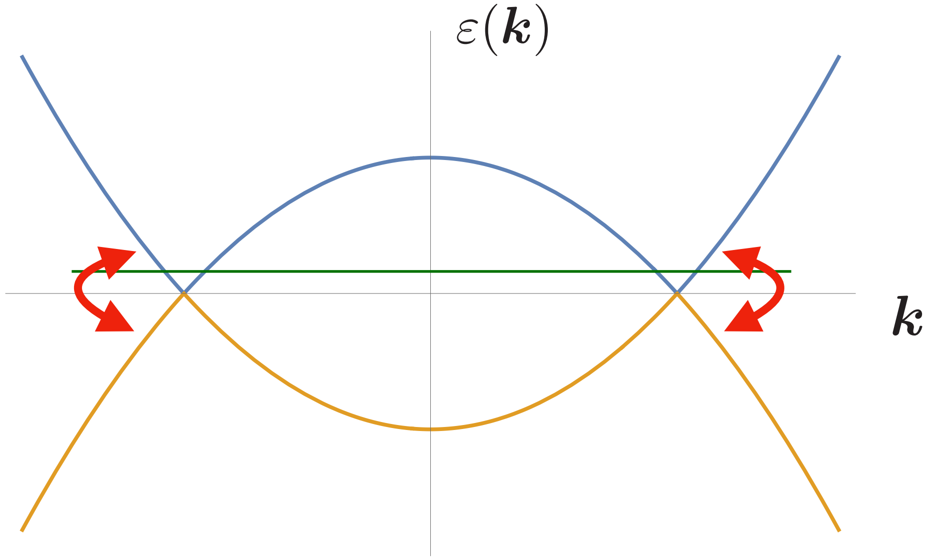
$$\cancel{\psi \to e^{i\varphi\gamma_5}\psi}$$

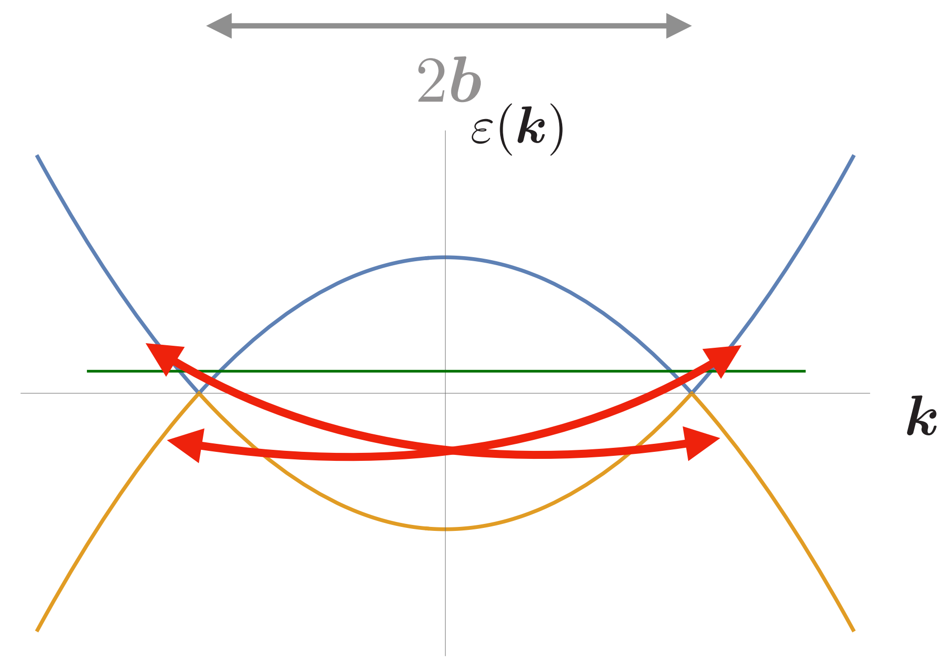
$$\Delta \equiv \langle\bar{\psi}\psi\rangle = m \to m e^{i\theta\gamma_5}$$

$$U_L(1)\times U_R(1)$$

$$U_L(1)\times U_R(1) \to U_{\mathrm{em}}(1)$$

Axionic CDW instability in Weyls: Breaking chirality





$$H(\boldsymbol{k}) = v_0 \begin{pmatrix} k_3 & k_1 - ik_2 \\ k_1 + ik_2 & -k_3 \end{pmatrix}$$

$$H(\boldsymbol{k}) \simeq \begin{pmatrix} v_0 \boldsymbol{\sigma} \cdot \boldsymbol{k} & m(\boldsymbol{b}) \\ m(\boldsymbol{b}) & -v_0 \boldsymbol{\sigma} \cdot \boldsymbol{k} \end{pmatrix}$$

$$\psi(x) \sim e^{i\boldsymbol{b}\cdot\boldsymbol{x}} \psi_+(x) + e^{-i\boldsymbol{b}\cdot\boldsymbol{x}} \psi_-(x)$$

$$< \bar{\psi}\gamma^0\psi > \sim < \bar{\psi}_+\gamma^0\psi_+ > + < \bar{\psi}_-\gamma^0\psi_- > + m\cos(2\boldsymbol{b}\cdot\boldsymbol{x})$$

spatial modulation of the order parameter
Charge Density Wave

Axionic CDW instability in Weyls: Breaking chirality

How to get this axion phase mode from microscopics

We have a symmetry breaking pattern here, Chiral U(1): Mean field theory

$$\mathcal{H} = v_0 \overbrace{\psi^+ \gamma^0}^{\bar{\psi}} \gamma^i k_i \psi - \overset{?}{g \psi_+^+ \psi_- \psi_-^+ \psi_+}$$

$$\psi = \begin{pmatrix} \psi_+ \\ \psi_- \end{pmatrix} \equiv \begin{pmatrix} \psi(\boldsymbol{b} + \boldsymbol{k}) \\ \psi(-\boldsymbol{b} + \boldsymbol{k}) \end{pmatrix}$$

$$\gamma^0 = \tau_1 \sigma_0$$

$$\gamma_i = -i\tau_2 \sigma_i$$

$$\gamma^0 \gamma_i = \tau_3 \sigma_i$$

$$\gamma^5 = -\tau_3 \sigma_0$$

$$\gamma^0 \gamma_5 = -i\tau_2 \sigma_0$$

$$\mathcal{H}_{\mathrm{MF}} = \psi^+ \begin{pmatrix} v_0 \boldsymbol{\sigma} \cdot \boldsymbol{k} & (m_0 - i m_1)\sigma_0 \\ (m_0 + i m_1)\sigma_0 & -v_0 \boldsymbol{\sigma} \cdot \boldsymbol{k} \end{pmatrix} \psi$$

$$\varepsilon(\boldsymbol{k}) = \pm \sqrt{v_0^2 |\boldsymbol{k}|^2 + \underbrace{m_0^2 + m_1^2}_{|\Delta|^2}}$$

$$\delta \mathcal{H}_{\mathrm{MF}} = m_0 \psi^+ \gamma_0 \psi + i m_1 \psi^+ \gamma_0 \gamma_5 \psi$$

most general U(1) breaking mass operator

Axionic CDW instability in Weyls: Breaking chirality

How to get this axion phase mode from microscopics

We have a symmetry breaking pattern here, Chiral U(1): Mean field theory

$$\mathcal{H} = v_0 \, \overbrace{\psi^+\gamma^0}^{\bar\psi} \gamma^i k_i \psi - g\psi_+^+\psi_-\psi_-^+\psi_+$$

$$\psi = \begin{pmatrix} \psi_+ \\ \psi_- \end{pmatrix} \equiv \begin{pmatrix} \psi(\boldsymbol{b}+\boldsymbol{k}) \\ \psi(-\boldsymbol{b}+\boldsymbol{k}) \end{pmatrix}$$

$$\gamma^0 = \tau_1\sigma_0$$

$$\gamma_i = -i\tau_2\sigma_i$$

$$\gamma^0\gamma_i = \tau_3\sigma_i$$

$$\gamma^5 = -\tau_3\sigma_0$$

$$\gamma^0\gamma_5 = -i\tau_2\sigma_0$$

$$\mathcal{H}_{\mathrm{MF}} = \psi^+(\tau_3\boldsymbol{\sigma}\cdot\boldsymbol{k} + \Delta\gamma_0 e^{-i\gamma_5\theta})\psi$$

$$\delta\mathcal{H}_{\mathrm{int}} = -\frac{g}{4}[(\psi^+\gamma_0\psi)^2 - (\psi^+\gamma_0\gamma_5\psi)^2] = -g\psi_+^+\psi_-\psi_-^+\psi_+$$

most general U(1) breaking mass operator

Axionic CDW instability in Weyls: Breaking chirality

How to get this axion phase mode from microscopics

We have a symmetry breaking pattern here, Chiral U(1): Mean field theory

$$\delta\mathcal{H}_{\text{int}} = -\frac{g}{4}[(\psi^+\gamma_0\psi)^2 - (\psi^+\gamma_0\gamma_5\psi)^2] = -g\psi_+^+\psi_-\psi_-^+\psi_+$$

$$\delta\mathcal{H}_{\text{int}} \approx \frac{\lambda^2}{\omega^2 - \Omega_0^2(2\boldsymbol{b})}\psi_+^+\psi_-\psi_-^+\psi_+$$

$$\omega^2 \ll \Omega_0^2$$

Virtual e-ph coupling in the static limit

$$\delta\mathcal{H}_{\text{int}} \approx -\frac{\lambda^2}{\Omega_0^2}\psi_+^+\psi_-\psi_-^+\psi_+$$

the effective coupling constant has dimensions of L^2

$$\delta\mathcal{H}_{\text{U}} \approx \frac{U_0}{1-\chi(0,2\boldsymbol{b})}\psi_+^+\psi_-\psi_-^+\psi_+$$

e-e interaction in the static limit

$$\delta\mathcal{H}_{\text{e-ph}} \approx \lambda(2\boldsymbol{b})(\phi_{2\boldsymbol{b}} + \phi_{-2\boldsymbol{b}}^+)\psi_+^+\psi_- + \omega_{\boldsymbol{k}}\phi_{\boldsymbol{k}}^+\phi_{\boldsymbol{k}}$$

Peierls Instability

$$\langle\phi_{2\boldsymbol{b}} + \phi_{-2\boldsymbol{b}}^+\rangle = \Delta$$

Axionic CDW instability in Weyls: Breaking chirality

How to get this axion phase mode from microscopics

$$\mathcal{L} = \bar{\psi}(\gamma^0\omega - \gamma^i k_i)\psi + g[(\bar{\psi}\psi)^2 + (i\bar{\psi}\gamma_5\psi)^2]$$

$$\downarrow$$

$$\mathcal{L} = \bar{\psi}(\gamma^0\omega - \gamma^i k_i)\psi + \bar{\psi}(\sigma + i\gamma_5\pi)\psi - \frac{1}{2g}(\sigma^2 + \pi^2)$$

$\sigma = \Delta\cos\theta \qquad \pi = \Delta\sin\theta$

θ **Is the Goldstone mode**

$$\mathcal{L} = \bar{\psi}(\gamma^0\omega - \gamma^i k_i)\psi + \bar{\psi}(\Delta e^{i\gamma_5\theta})\psi - \frac{1}{2g}\Delta^2$$

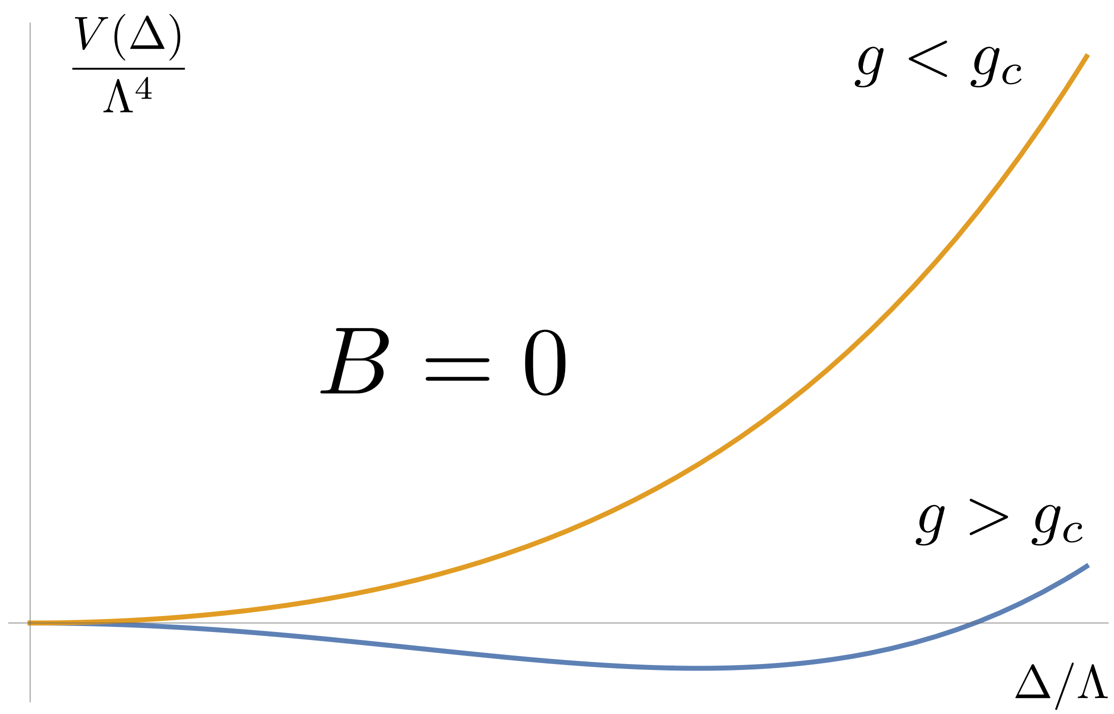
$$\psi \to \chi = e^{i\frac{\theta}{2}}\psi$$

$$\bar{\psi} \to \bar{\chi} = \bar{\psi}e^{i\frac{\theta}{2}}$$

$$\mathcal{L} = \bar{\chi}(\underbrace{\gamma^0\omega - \gamma^i k_i}_{G_0^{-1}(\omega,\boldsymbol{k})})\chi + \Delta\bar{\chi}\chi - \frac{1}{2g}\Delta^2$$

(at the classical level)

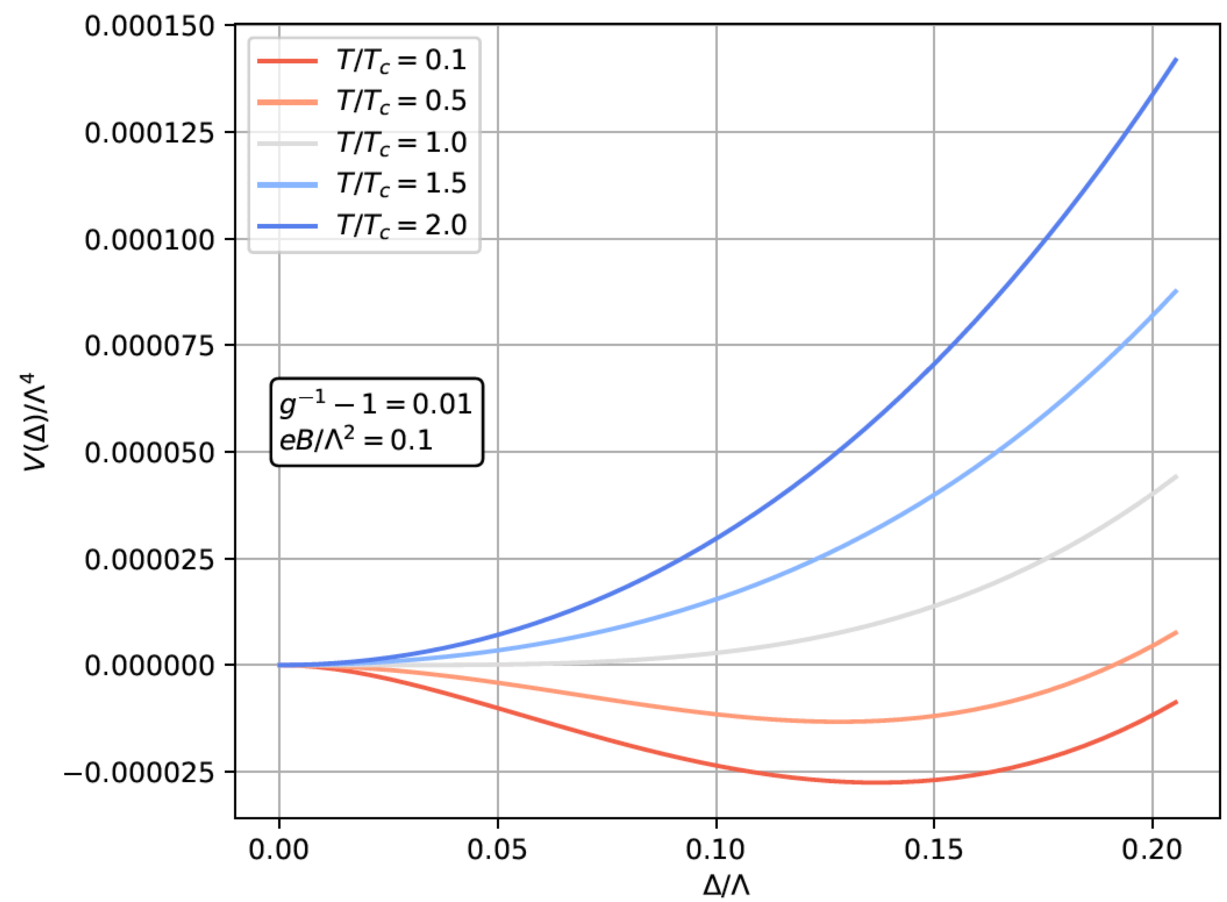
Axionic CDW instability in Weyls: Magnetic catalysis



$$\Delta \sim (g - g_c)^{1/2}$$

$$\Delta_0^2 = \begin{cases} \frac{eB}{\pi} \exp\left[-\frac{\Lambda^2}{eB}(g^{-1}-1) - \gamma_E\right] & \text{for } g \ll 1, \\ \Delta^2_{\mathrm{NJL},0} & \text{for } g > 1, \end{cases}$$

$$B \neq 0$$



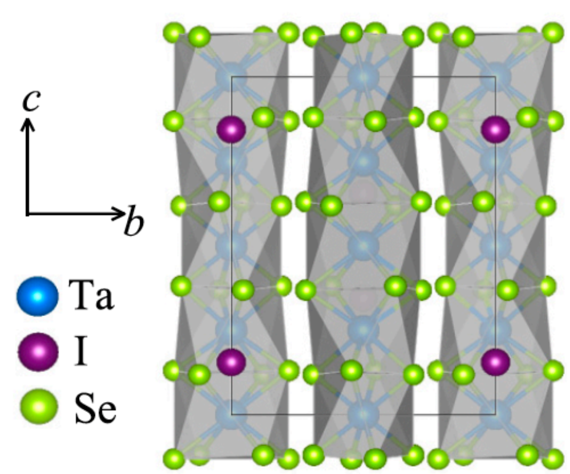
At nonzero magnetic field and T=0, there is always phase transition: Magnetic catalysis

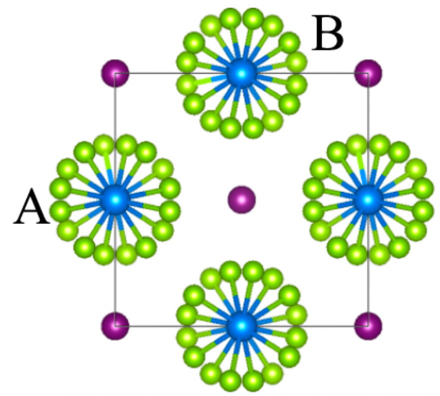
At nonzero T, phase transition is 2nd order

VA Miransky, IA Shovkovy PRD 66, 045006 (2002)

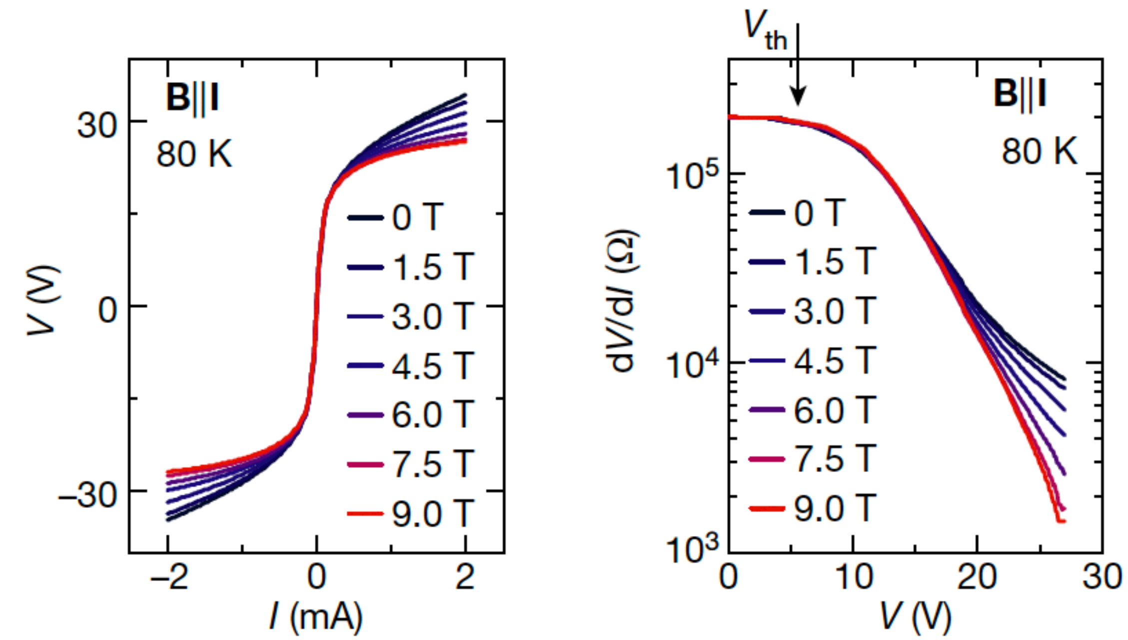
Dynamical axion insulators: Experimental status

$(TaS_4)_2I$



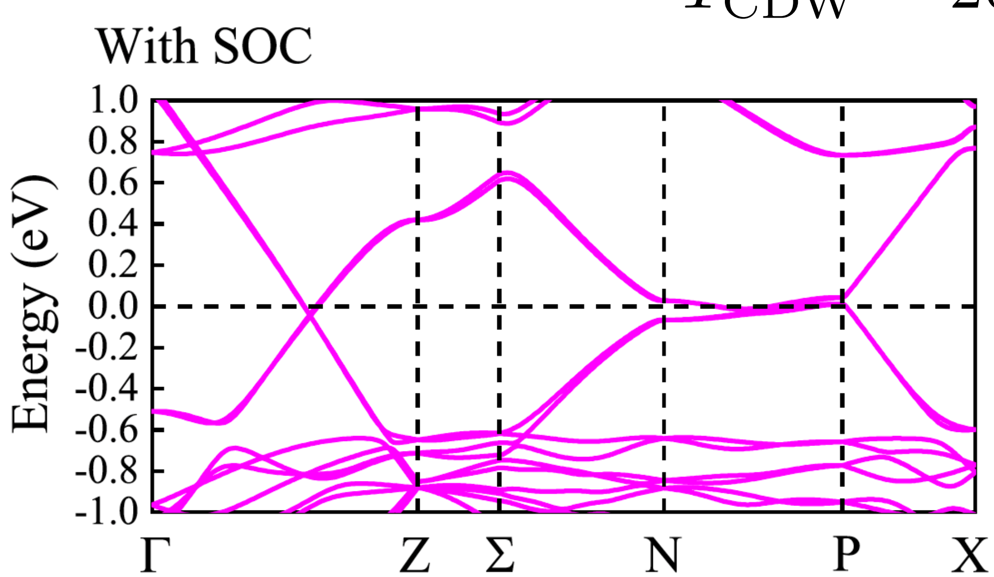


J Gooth et al Nature 575, 315 (2019)

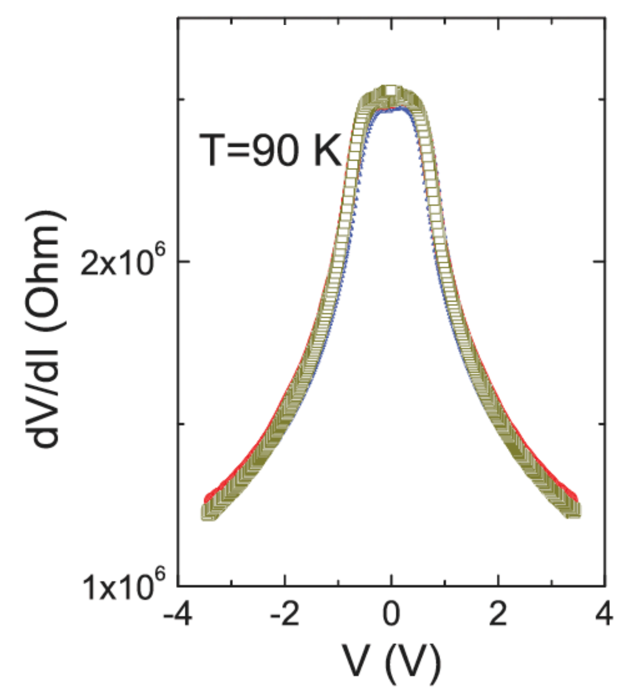


$$J = -\frac{e}{\pi}\frac{d\delta\theta}{dt}$$

$$T_{\mathrm{CDW}} = 263K$$



Y Zhang et al. PRB 101, 174106 (2020)



$$\mathcal{L} = \frac{e^2}{2\pi h}\delta\theta(t, \boldsymbol{r})\boldsymbol{E}\cdot\boldsymbol{B}$$

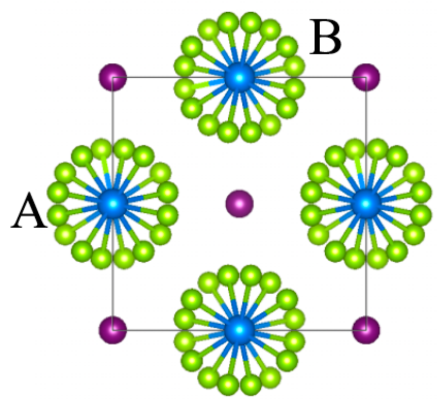
Repeated experiment:
No trace of dependence with B

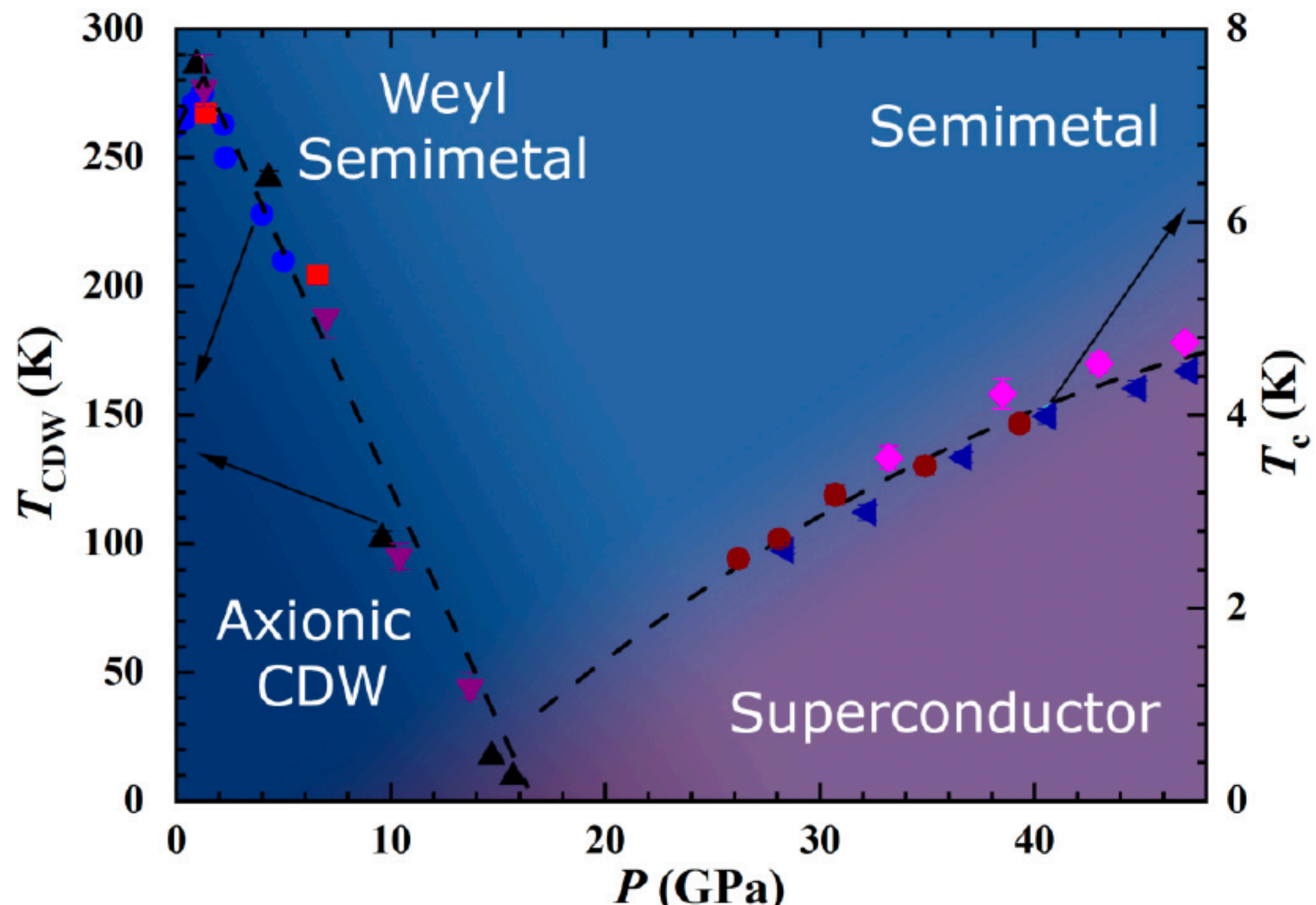
AA Shinchenko et al. App. Phys. Lett. 120, 063102 (2022)

Dynamical axion insulators: Experimental status

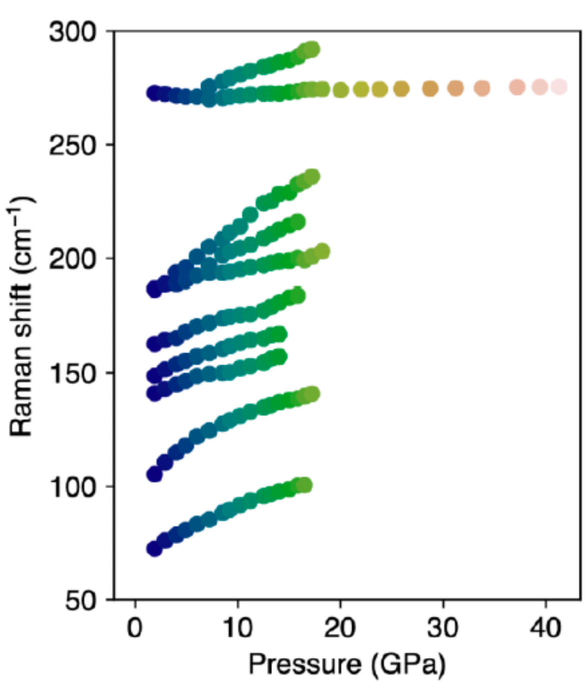
$$(TaS_4)_2I$$

$$T_{\mathrm{CDW}} = 263K$$

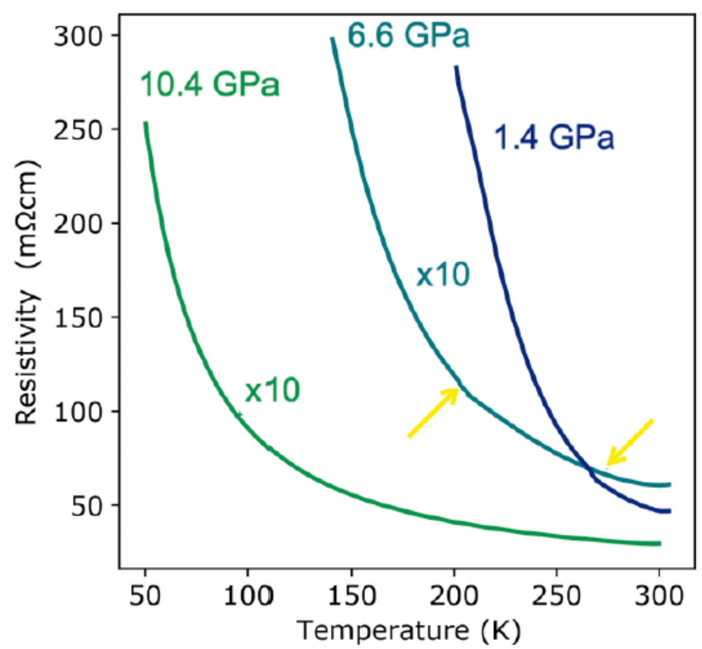




Raman spectrum under P



Kink in resistivity (CDW)



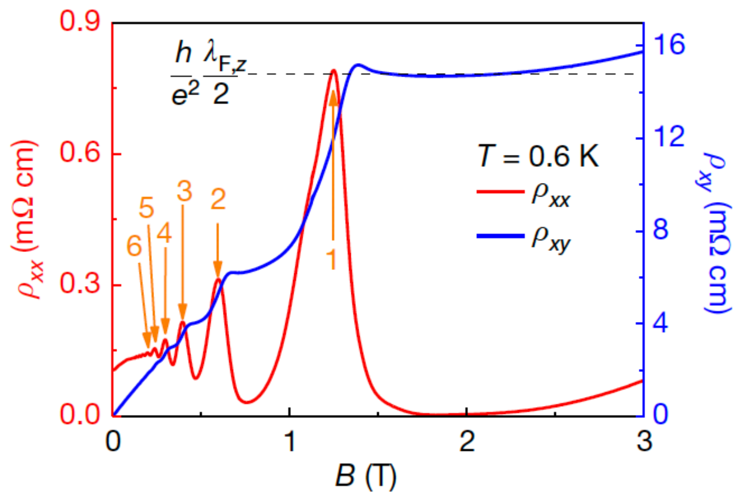
Pressure favors connectivity among chains that is bad for the quasi 1D character

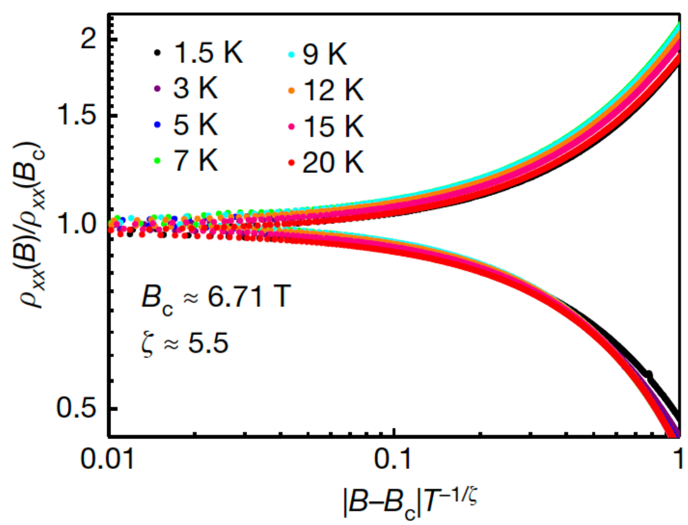
QG Mu et al. Phys. Rev. Research 5, 084201 (2021)

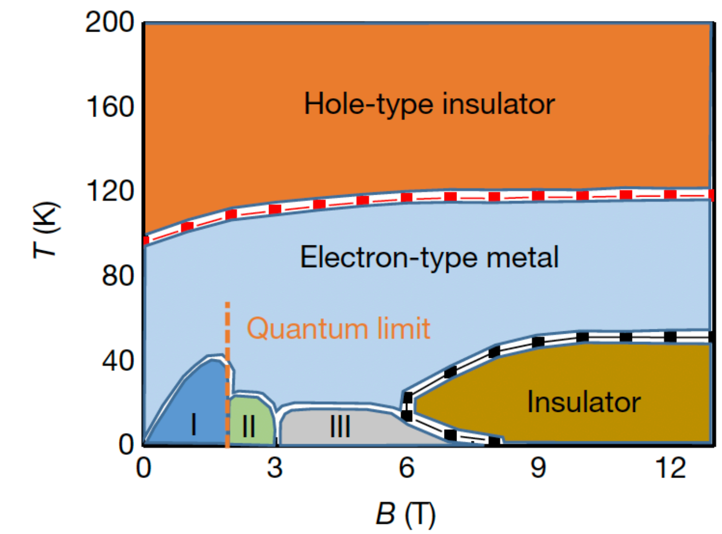
Dynamical axion insulators: Experimental status

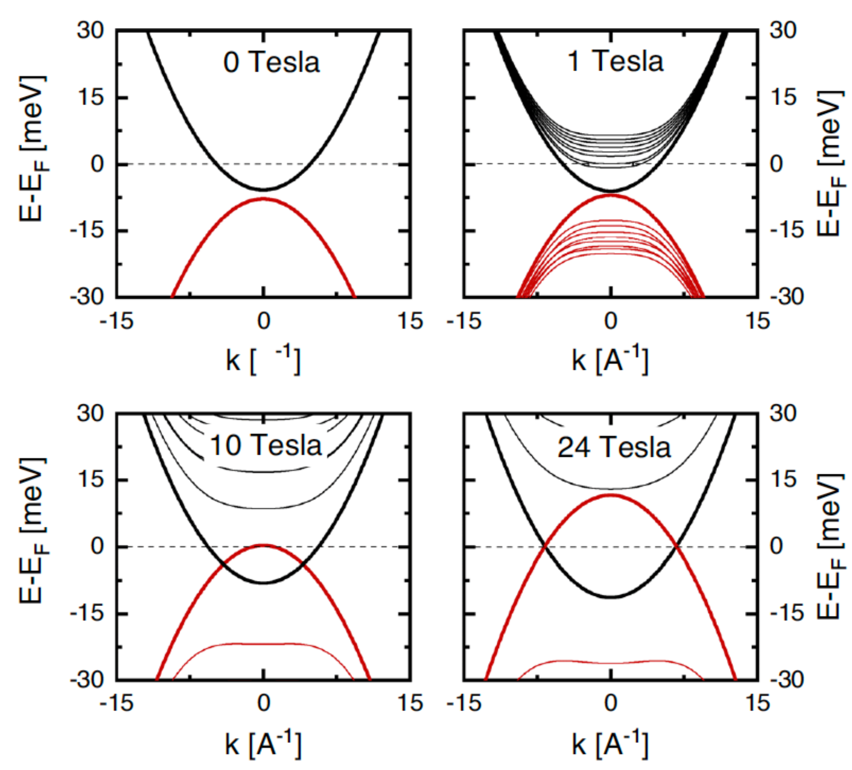
F Tang et al. Nature 569, 537 (2019)

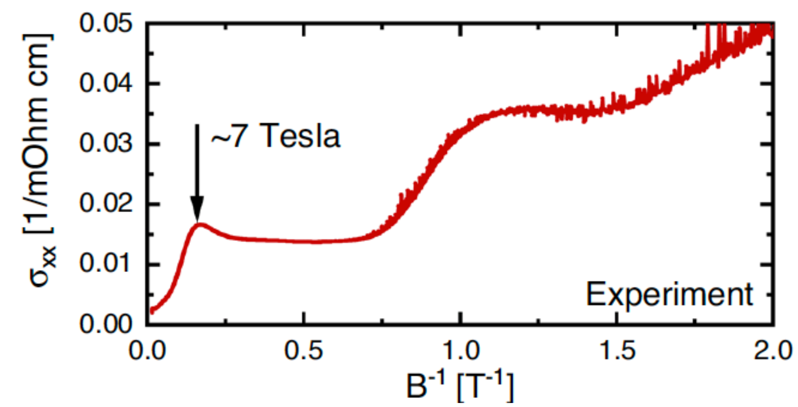
$ZrTe_5$

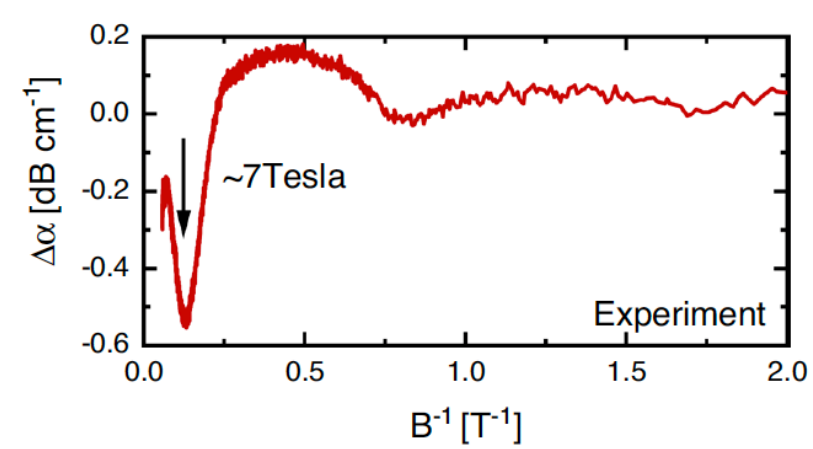








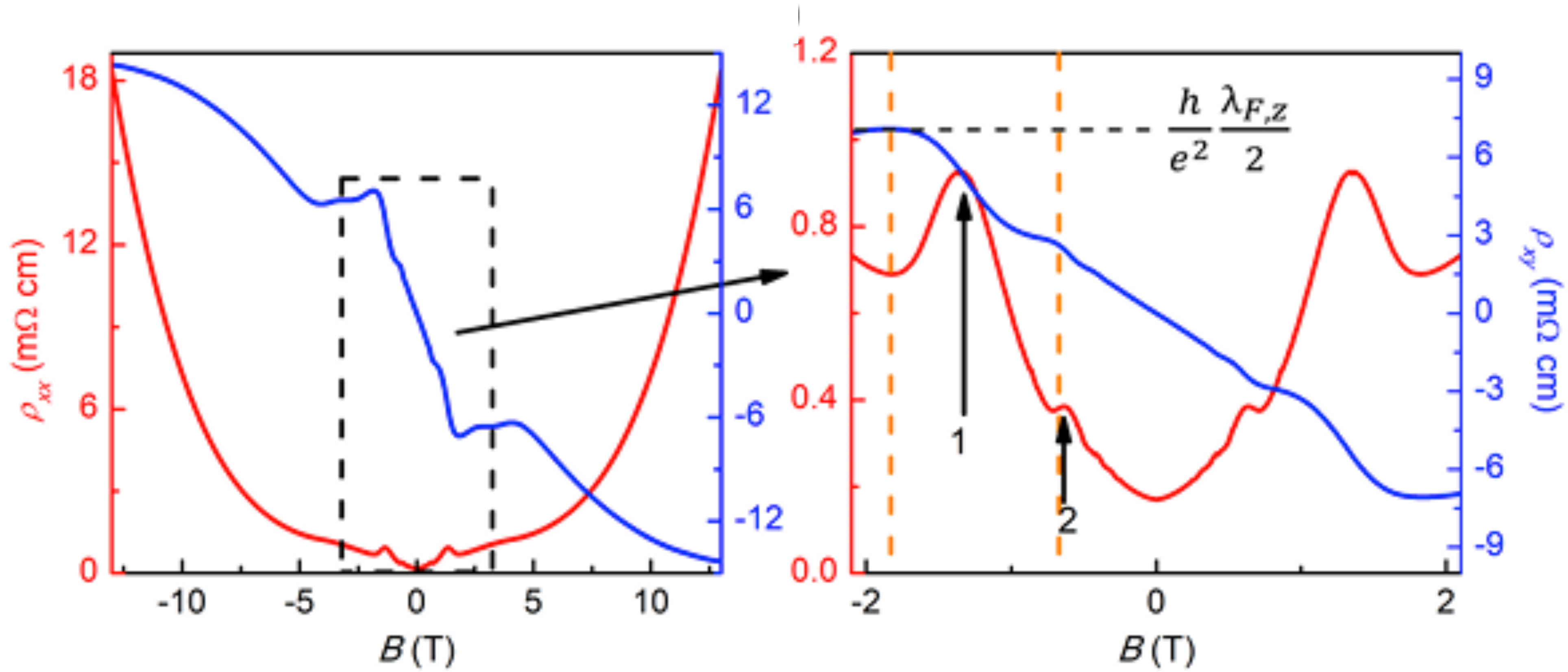




S Galeski et al Nat. Comm. 13, 7418 (2022)

Dynamical axion insulators: Experimental status

$HfTe_5$

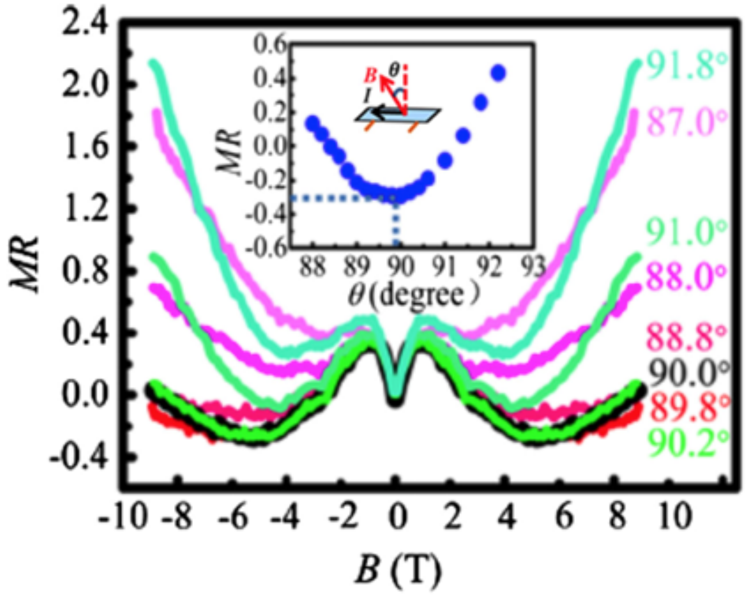


The Hall plateau is proportional to the Fermi wavelength instead of a lattice constant: Signature of electron instability?

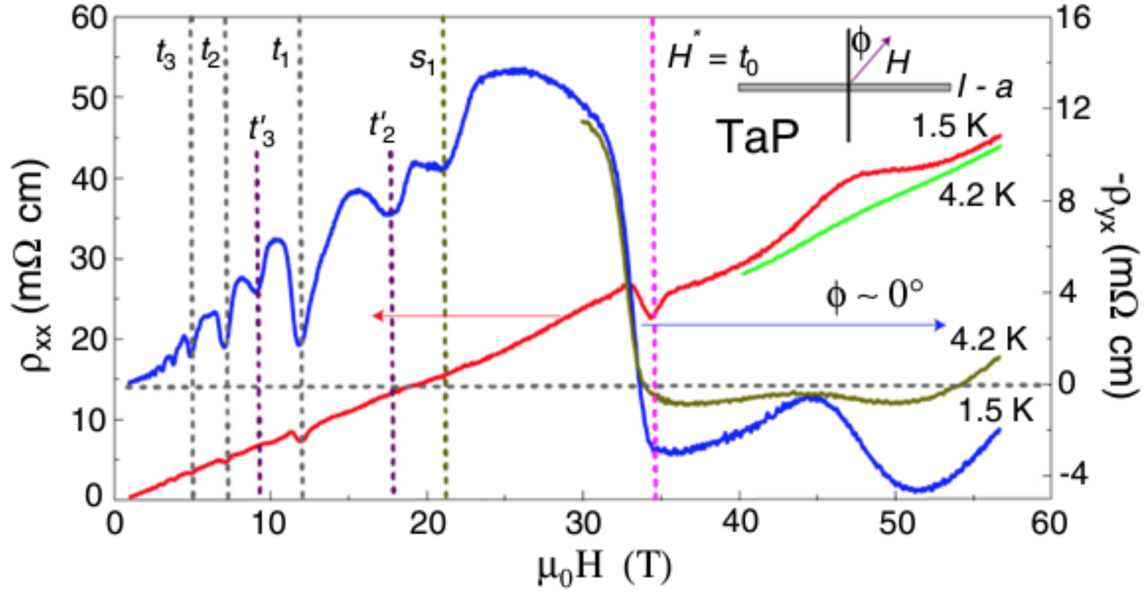
P Wang et al. PRB 101. 161201R (2020)

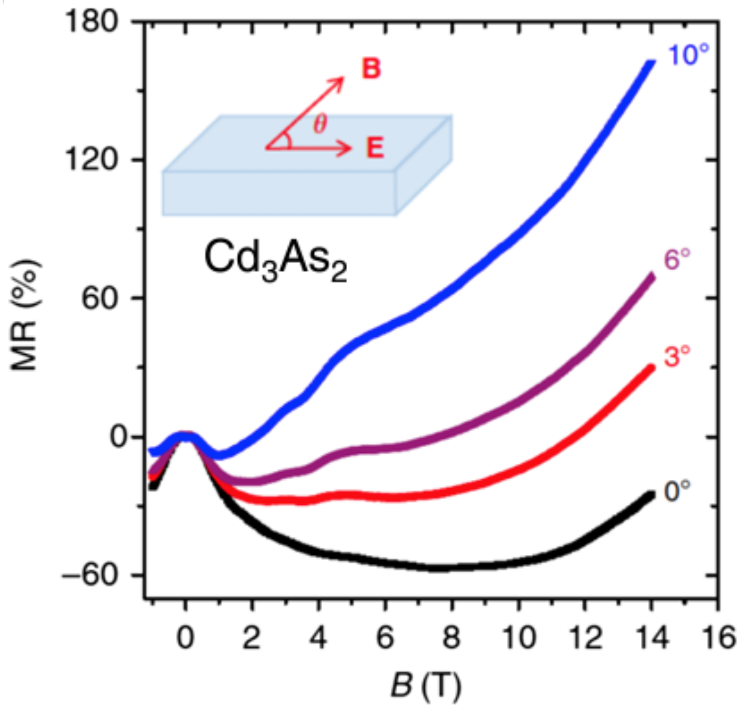
Dynamical axion insulators: Experimental status

$TaAs$

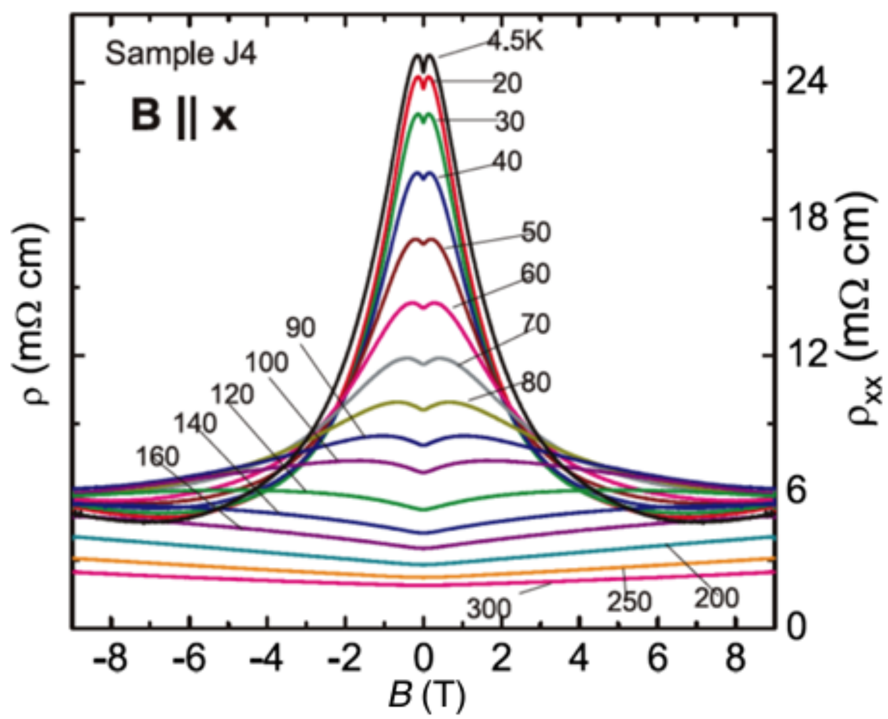


TaP





Na_3Bi



BQ Lv et al. Rev. Mod. Phys. 93, 025002

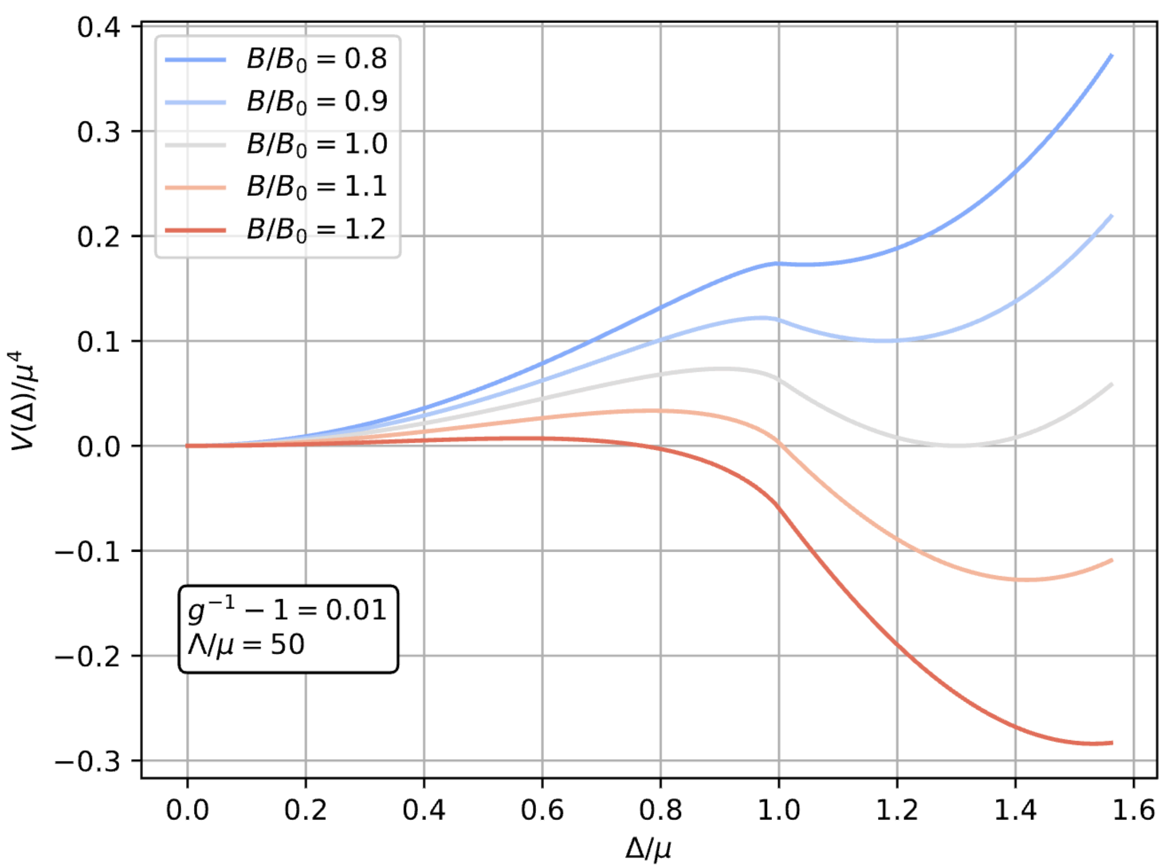
Dynamical axion insulators: Chemical potential

Weyls under B fields have been extensively studied since 2015 and MC appears with any finite B field. Why is not observed? Finite chemical potential

$$V_0(\Delta) = \frac{\Delta^2}{2G} + \frac{eB}{8\pi^2}\int_{\Lambda^{-2}}^{\infty} \frac{ds}{s^2} e^{-s\Delta^2} \coth(eBs)$$

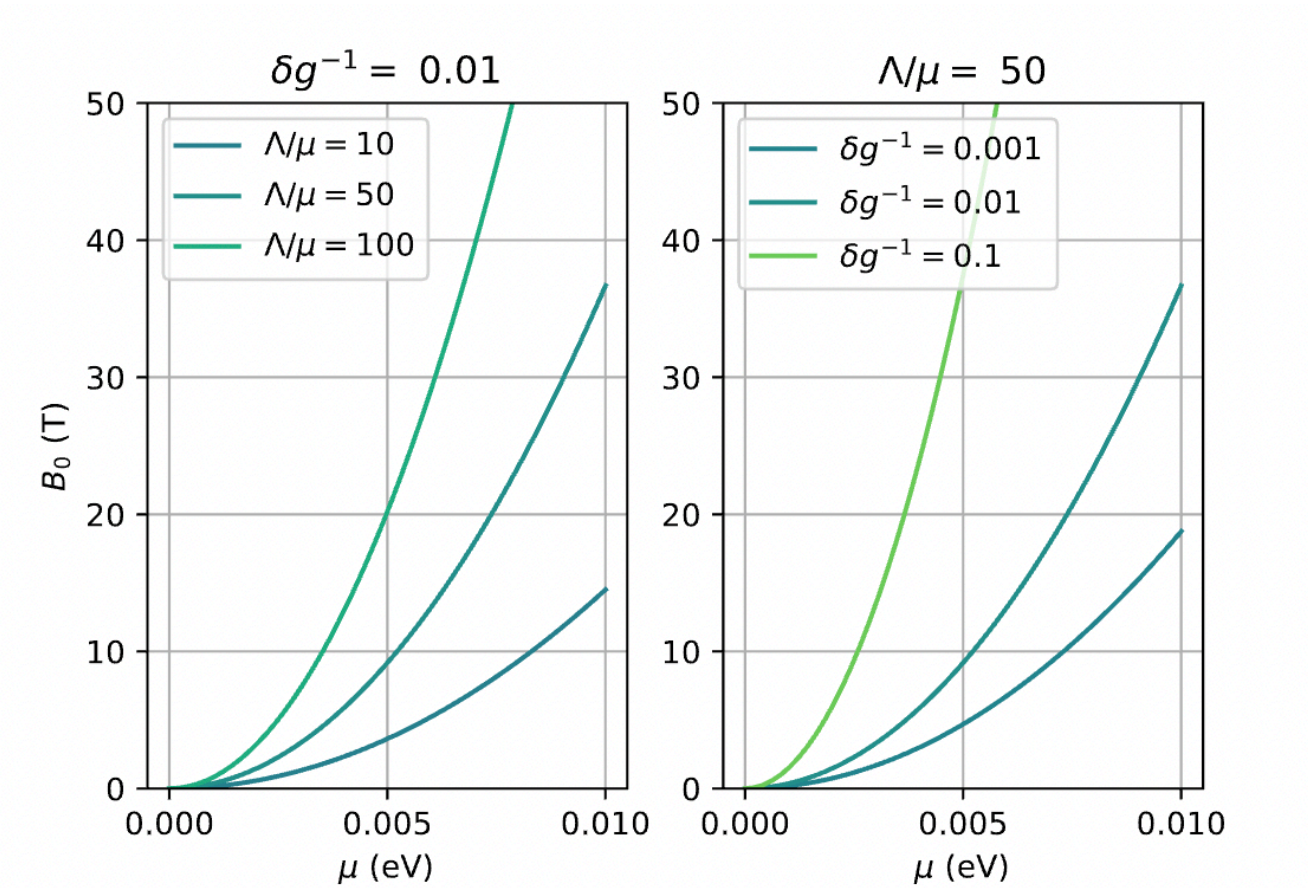
$$V_{\mu,0}(\Delta) = -\frac{eB}{4\pi^2}\sum_{n=0}^{\infty} \alpha_n \int_{-\infty}^{\infty} dp\, \Theta\left[\mu - \varepsilon_n(p)\right] \cdot \left[\mu - \varepsilon_n(p)\right]$$

K Klimenko, arXiv:hep-th/9809218 (1998)



Chemical potential competes against magnetic field

First order phase transition: Two minima



J Bernabeu, A Cortijo PRB 110, L081101 (2024)

Dynamical axion insulators: First order phase transition

Minima coexist

$$B_a < B_1 < B_0 < B_2 < B_b$$

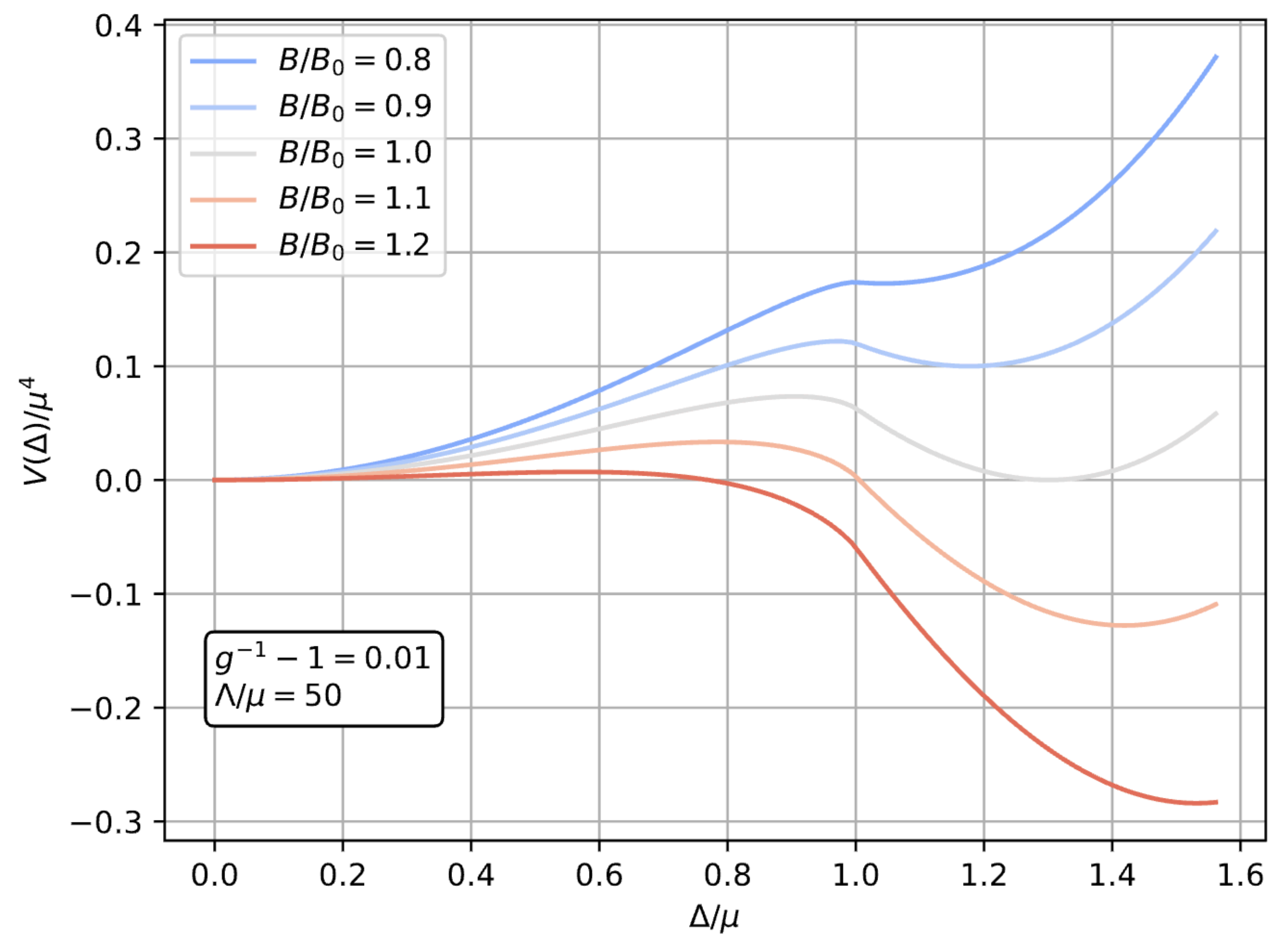
gap no longer a (local) minimum

0 no longer a (local) minimum

Phase transition takes place by nucleation of bubbles

B_0 is the magnetic field where the two minima are degenerate

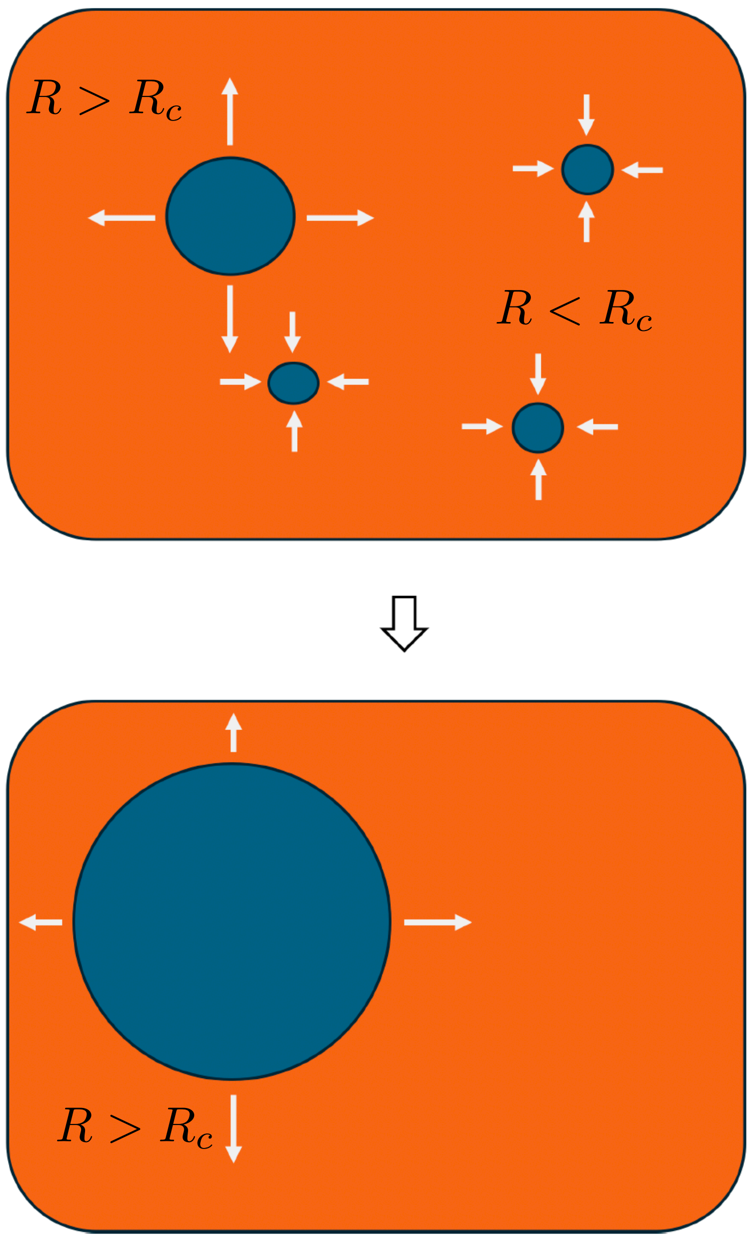
How close are these values to B_0 indicate the possibility of hysteresis

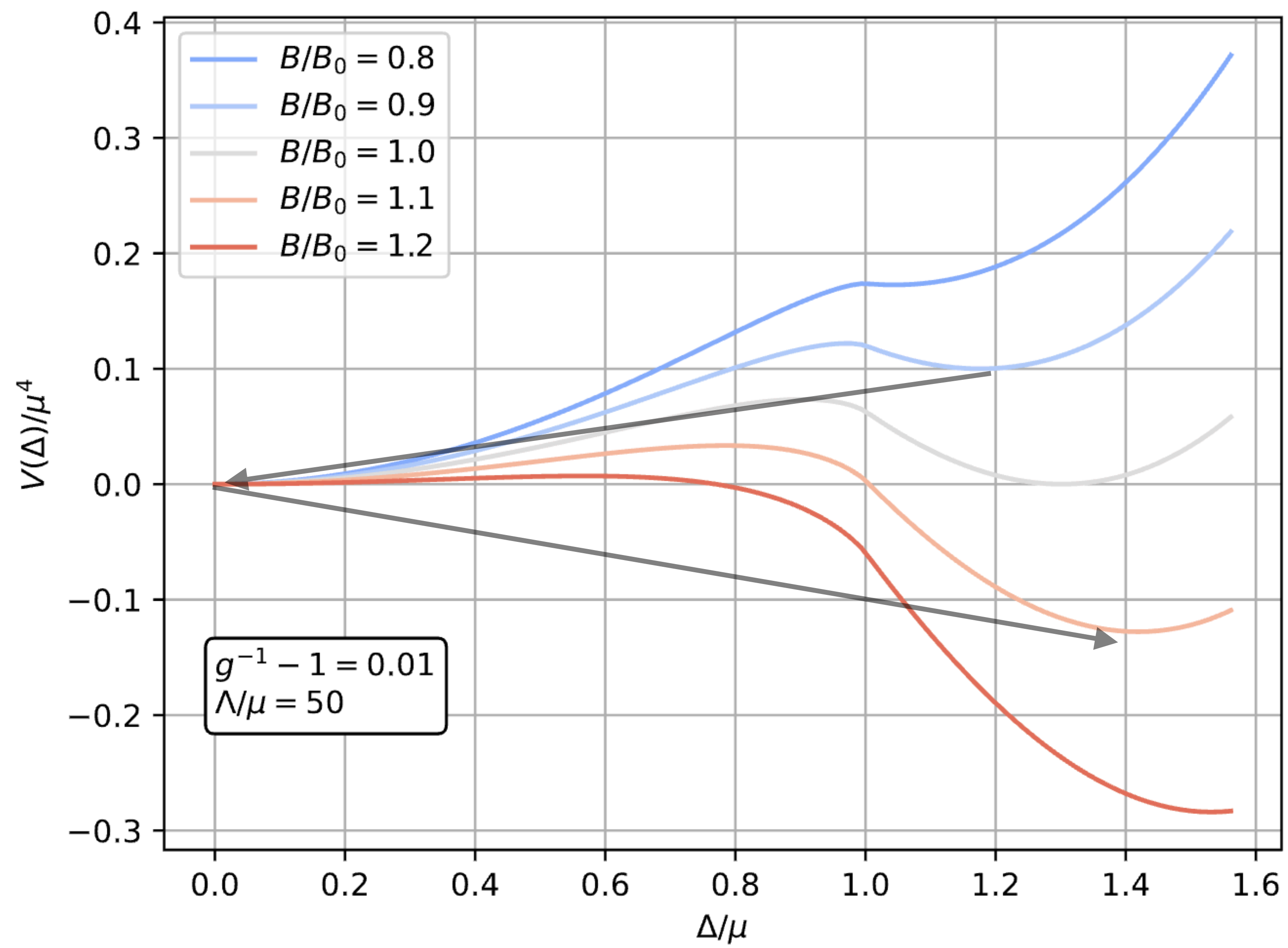


J Bernabeu, A Cortijo PRB 110, L081101 (2024)

Dynamical axion insulators: 1st order (Q)PT and nucleation

The idea behind transition by bubble nucleation is universal





Phys. Rev. A 23, 2719 (1981)

$$\frac{1}{t_n} \sim R_b^3 \Gamma$$

Time bubble takes to be unstable

$$t_n(B_{1,2}) < t_r$$

Some reference time, larger than the experiment takes to be performed

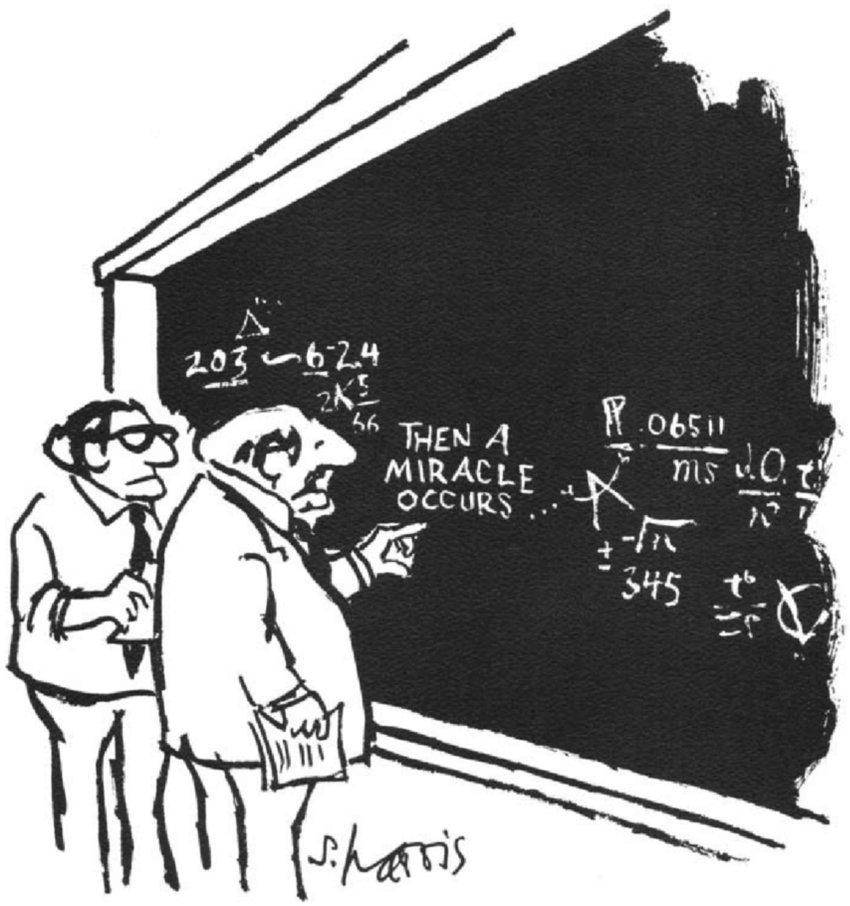
Dynamical axion insulators: 1st order (Q)PT and nucleation

$$t_n^{-1} \simeq R_b^3 \Gamma = \frac{S_b^2}{4\pi R_b} e^{-S_b}$$

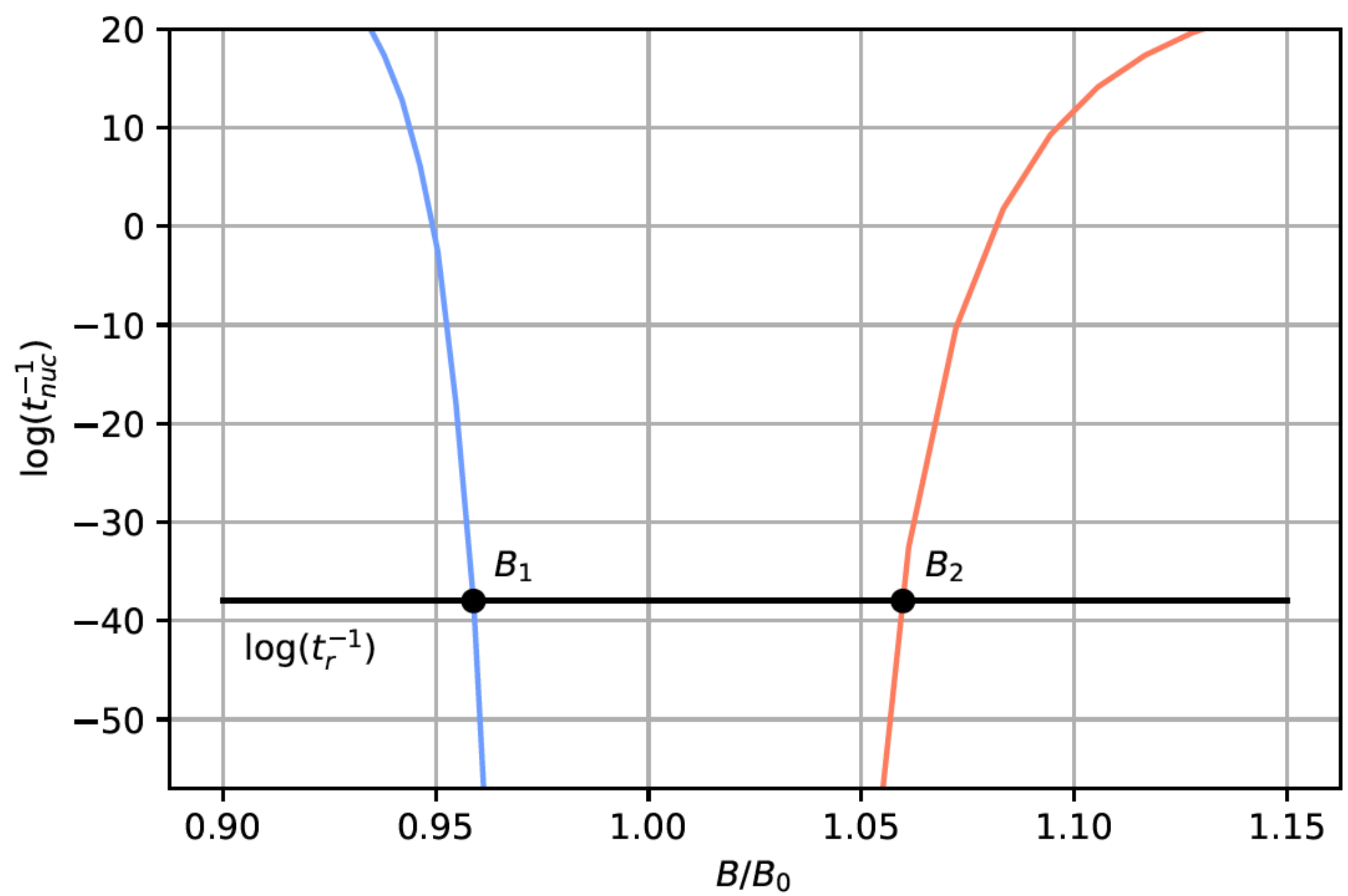
$$\mathcal{L}_\Delta = \frac{Z^{-1}(\Delta)}{2}\left[(\partial_0\Delta)^2 - \sum_i v_i^2(\partial_i\Delta)^2\right] - V_{\text{eff}}(\Delta)$$

Kinematic effects of the order parameter needed: radiative corrections as it is a composite field

$$Z^{-1}(\Delta) = \frac{eB}{24\pi^2\Delta^2}\left[1 - \left(1 - \frac{\Delta^2}{\mu^2}\right)^{\frac{3}{2}}\Theta(\mu - \Delta)\right]$$



"I THINK YOU SHOULD BE MORE EXPLICIT HERE IN STEP TWO."



J Bernabeu, A Cortijo PRB 110, L081101 (2024)

Dynamical axion insulators: Hysteresis

$$t_n^{-1} \simeq R_b^3 \Gamma = \frac{S_b^2}{4\pi R_b} e^{-S_b}$$

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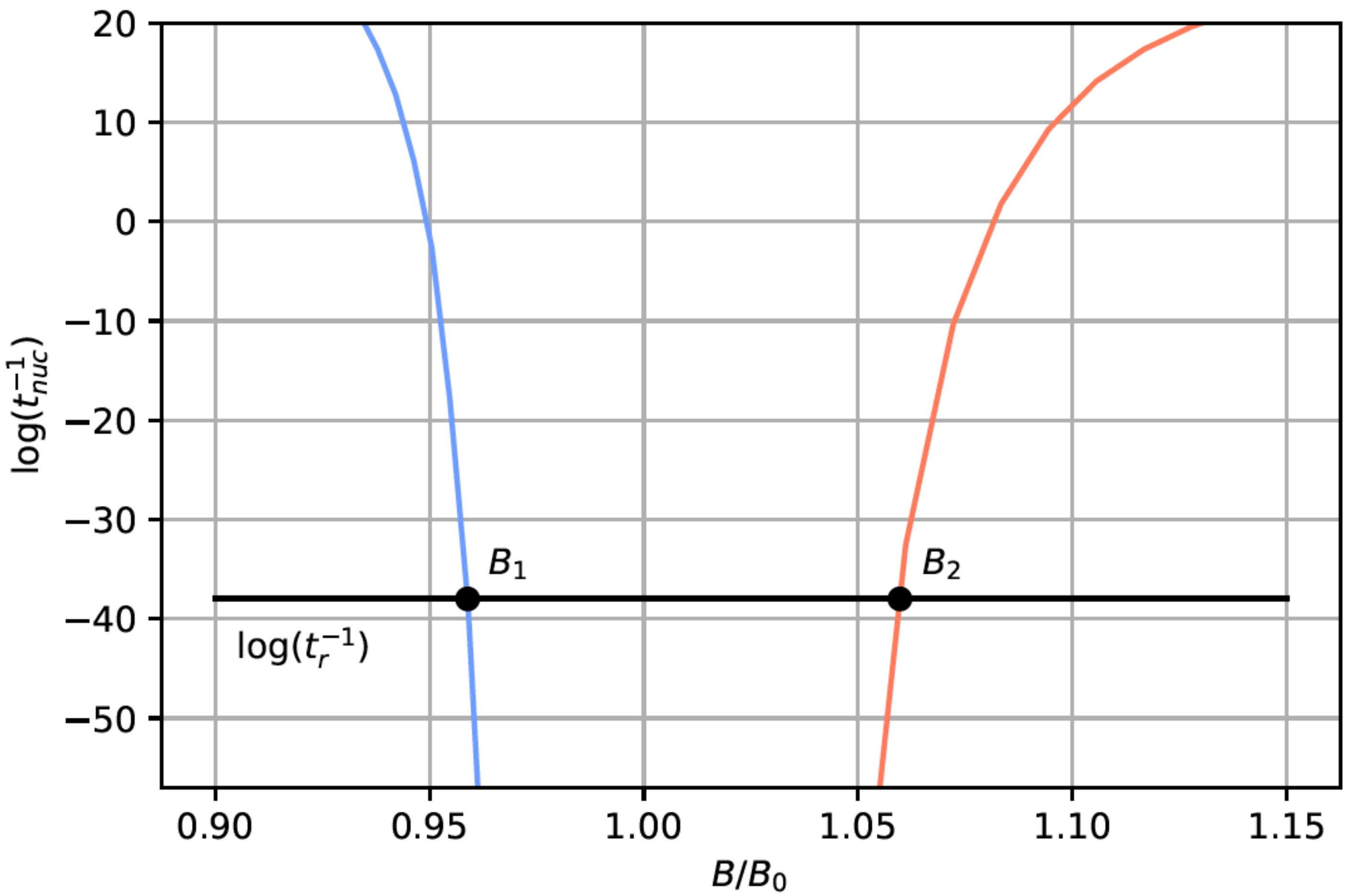
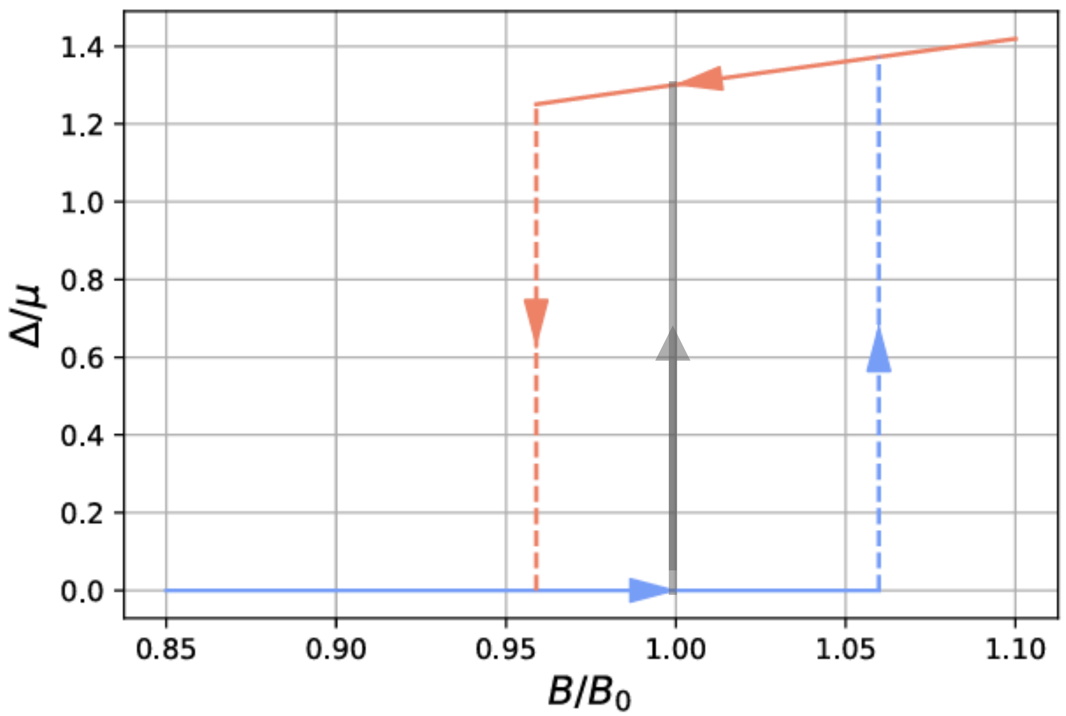
$B_{1,2}$ **Are not sensitive to the reference time**

As $B_1 \neq B_0 \neq B_2$, there is hysteresis



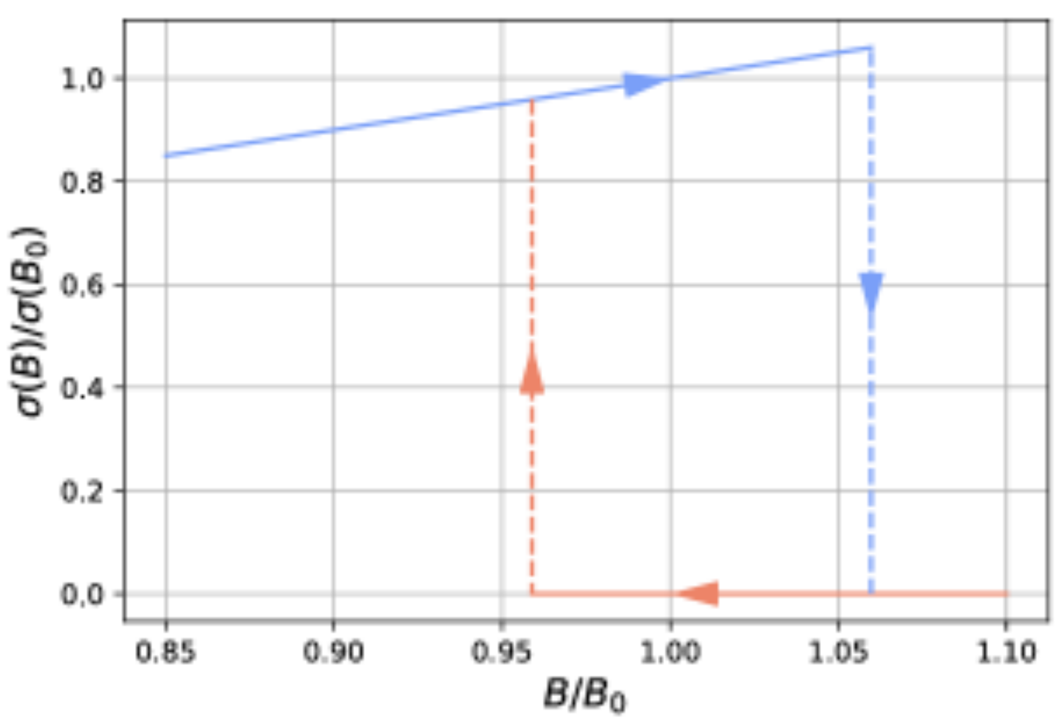
J Bernabeu, A Cortijo PRB 110, L081101 (2024)

Dynamical axion insulators: Hysteresis

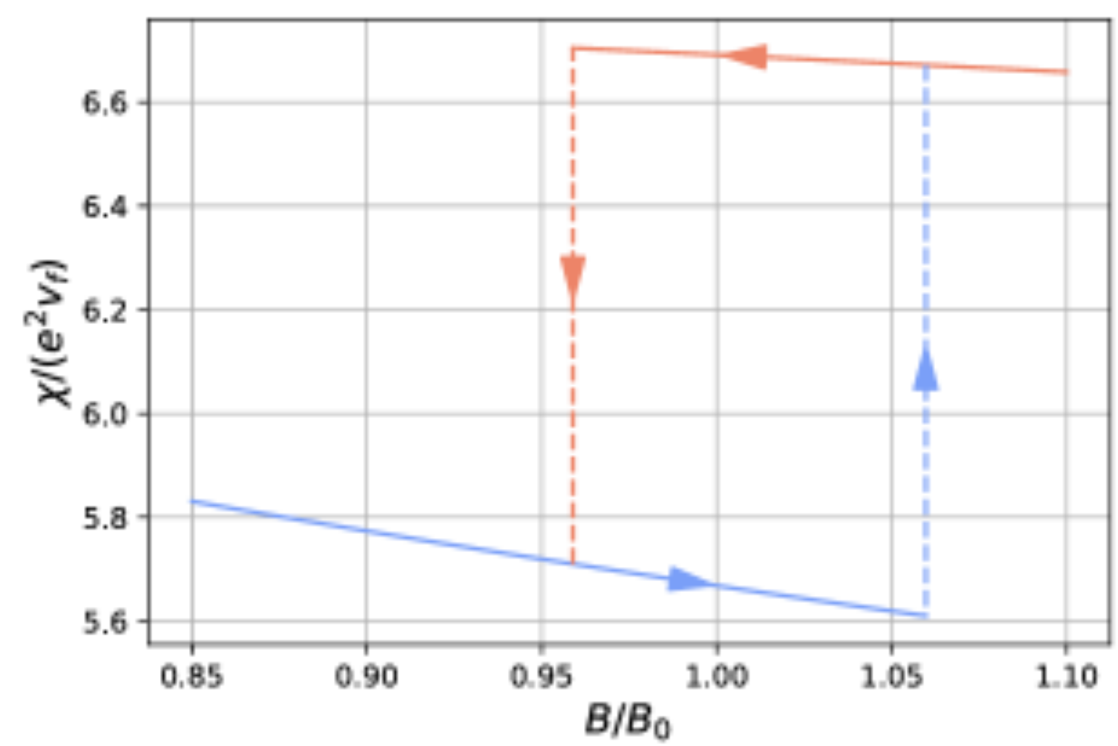


As $B_1 \neq B_0 \neq B_2$ **, there is hysteresis**

Magnetic susceptibility



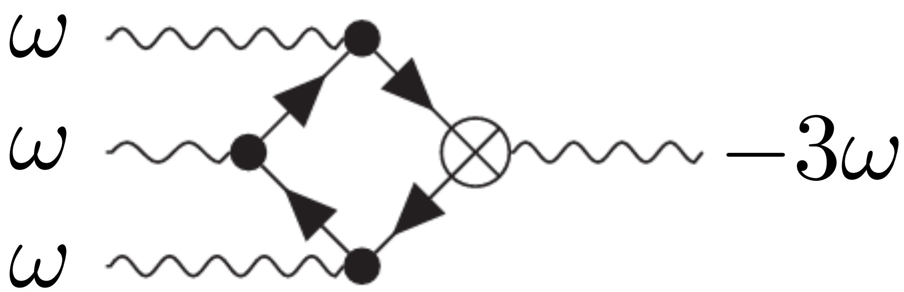
Metallic magnetoconductivity



J Bernabeu, A Cortijo PRB 110, L081101 (2024)

Dynamical axion insulators: Hysteresis

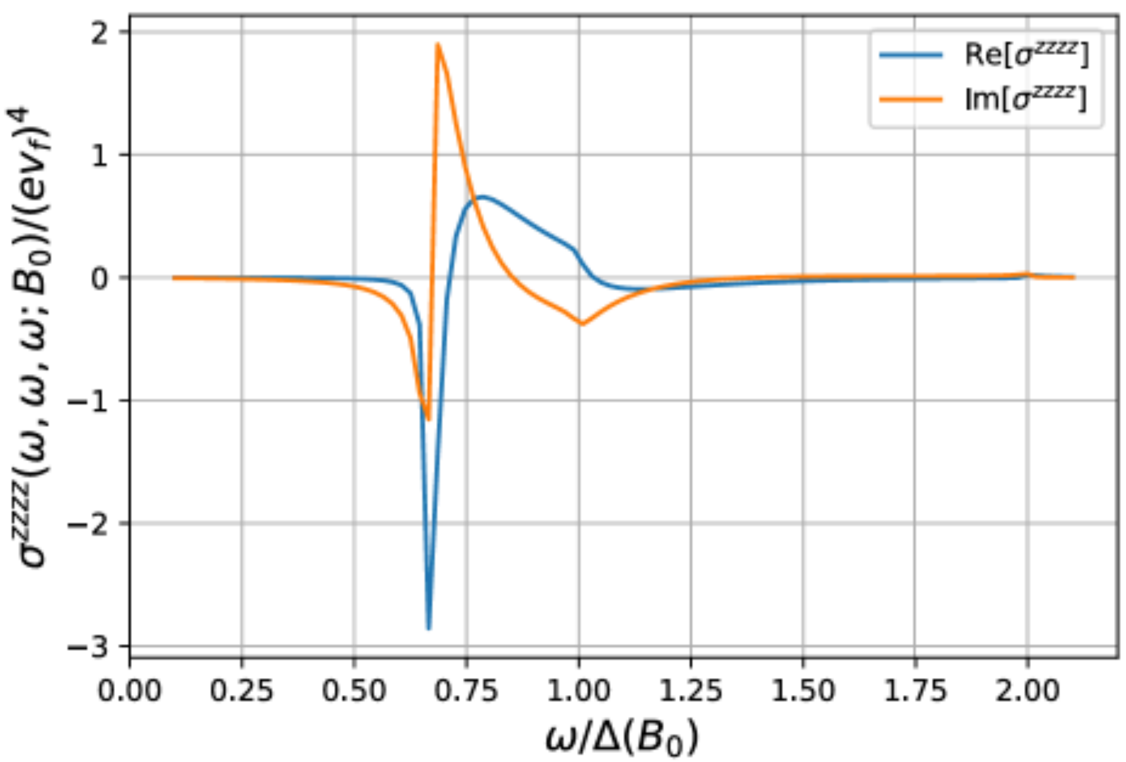
Nonlinear optics (3rd Harmonic generation)

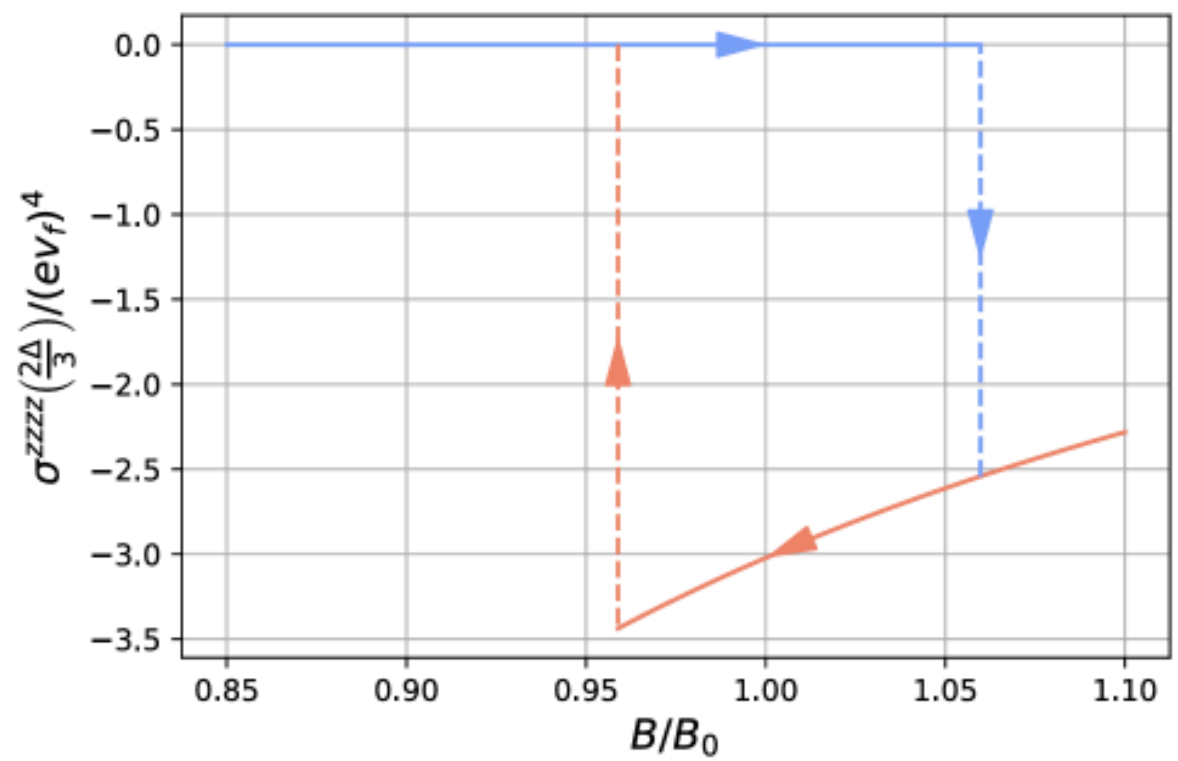


$$J(-3\omega) = \sigma^{(3)}(\omega) E^3(\omega)$$

Resonance at $3\omega \sim 2\Delta$

In the ultra quantum limit, this is zero in the metallic phase





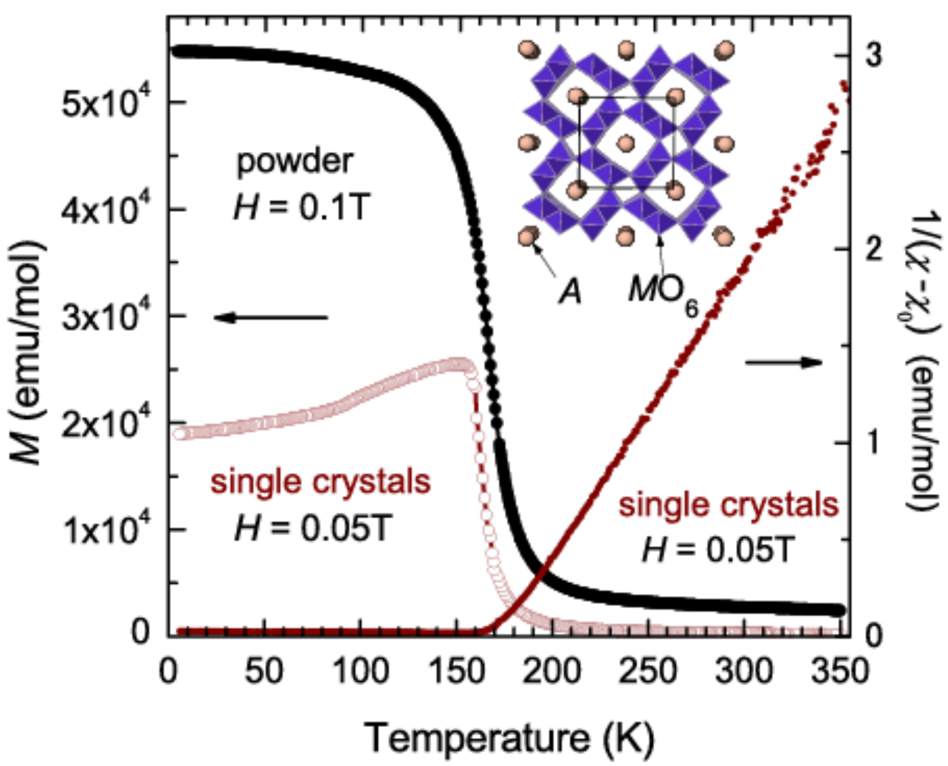
J Bernabeu, A Cortijo PRB 110, L081101 (2024)

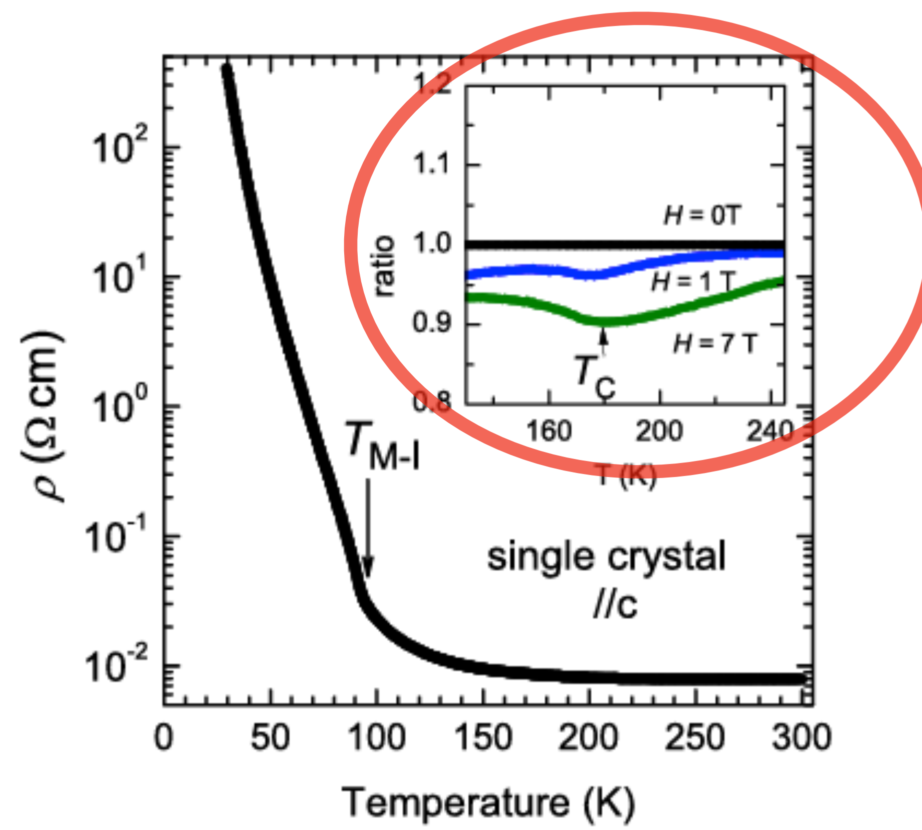
Dynamical axion insulators: More experimental status

$K_2Cr_8O_{16}$

Ferromagnetic metal - Ferromagnetic insulator phase transition!

K Hasegawa et al. PRL 103, 146403 (2009)





$\rho(B)/\rho(0)$

Inset: magnetoconductivity increasing with B

First principles calculations suggest Weyl ferro phase

JZ Zhao et al New J Phys 22, 073062 (2020)

PRL **107,** 266402 (2011) PHYSICAL REVIEW LETTERS week ending 23 DECEMBER 2011

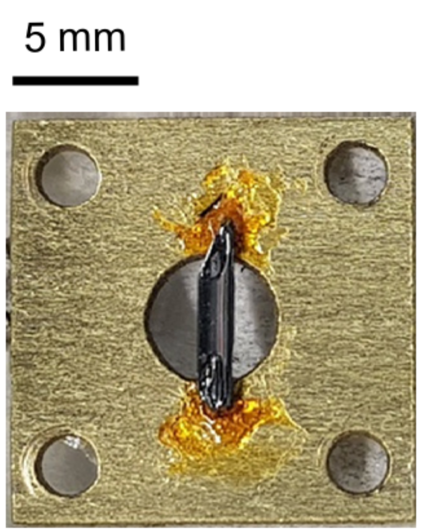
Peierls Mechanism of the Metal-Insulator Transition in Ferromagnetic Hollandite $K_2Cr_8O_{16}$

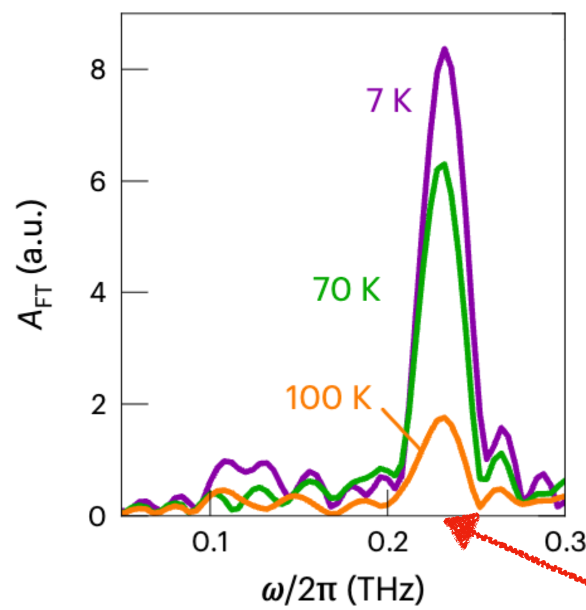
T. Toriyama,[1] A. Nakao,[2] Y. Yamaki,[3] H. Nakao,[2] Y. Murakami,[2] K. Hasegawa,[4] M. Isobe,[4] Y. Ueda,[4]
A. V. Ushakov,[5] D. I. Khomskii,[5] S. V. Streltsov,[6,7] T. Konishi,[8] and Y. Ohta[1]

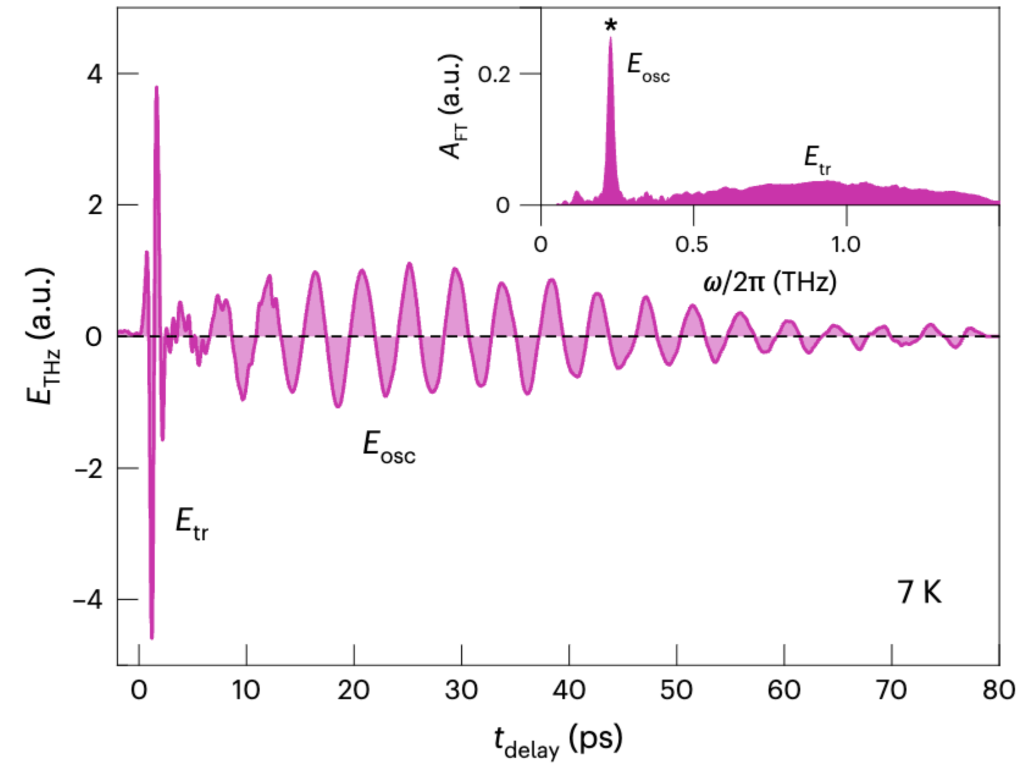
Dynamical axion insulators: More experimental status

S Kim et al Nat. Materials 22, 429 (2023)

$$(TaSe_4)_2I \qquad 2\Delta \sim 250meV$$







Gapped phason (axion?) mode

Damping due to mixing with phonons

$$\ddot{p} + \gamma_P \dot{p} + \omega_0^2 p + \kappa q = 0$$
$$\ddot{q} + \gamma_Q \dot{q} + \omega_0^2 q + \kappa p = 0,$$

PHYSICAL REVIEW B **111**, 205124 (2025)

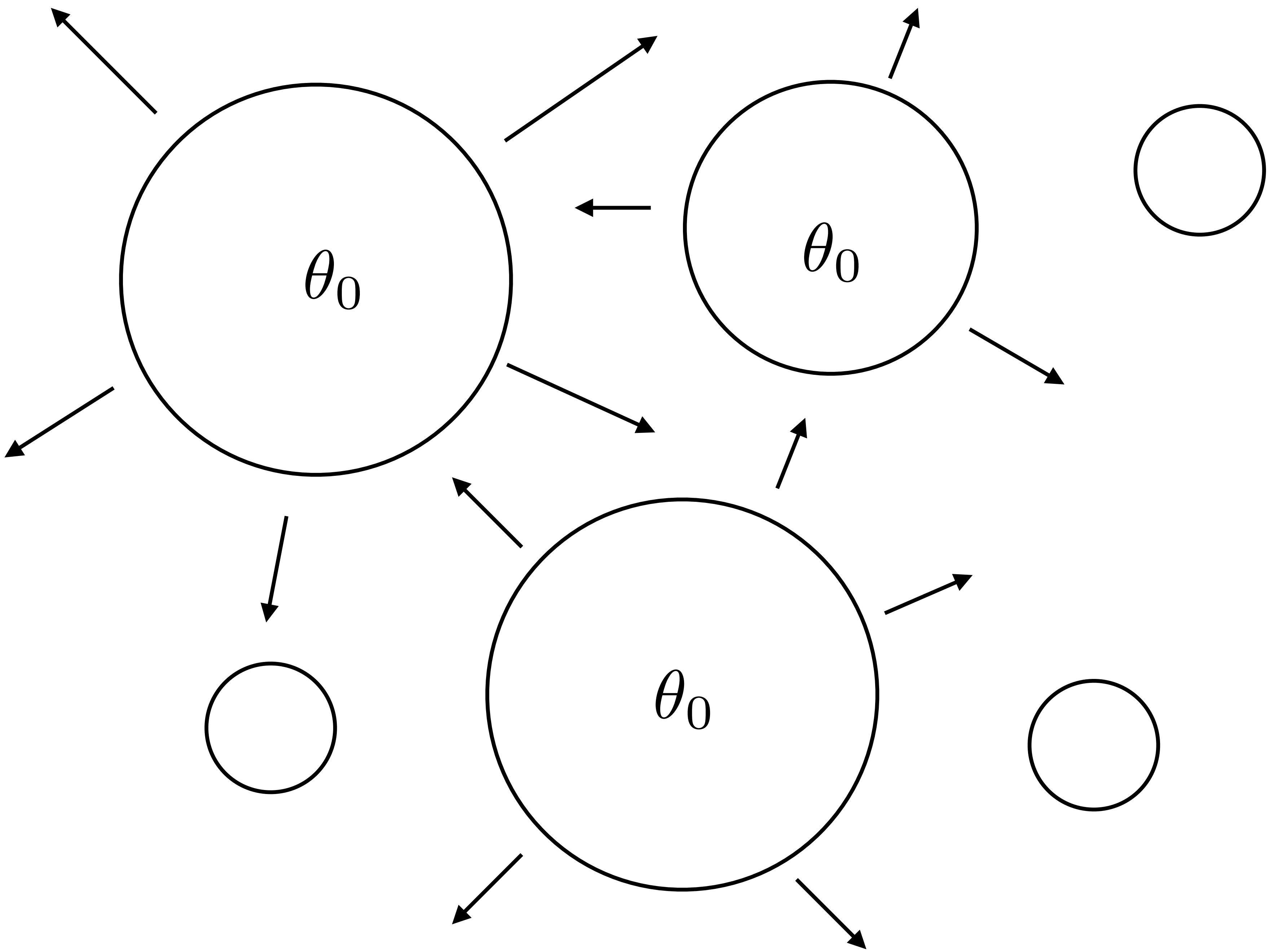
Axionic acoustic phonons from Weyl semimetals

Joan Bernabeu*
Departamento de Física de la Materia Condensada, Universidad Autónoma de Madrid, Cantoblanco, E-28049 Madrid, Spain

Alberto Cortijo†
Instituto de Ciencia de Materiales de Madrid (ICMM), Consejo Superior de Investigaciones Científicas (CSIC), and Sor Juana Inés de la Cruz 3, 28049 Madrid, Spain

Dynamical axion insulators: Hysteresis

In the previous theory, we have made the important assumption that all symmetry broken bubbles choose the save value for θ



Dynamical axion insulators: Hysteresis

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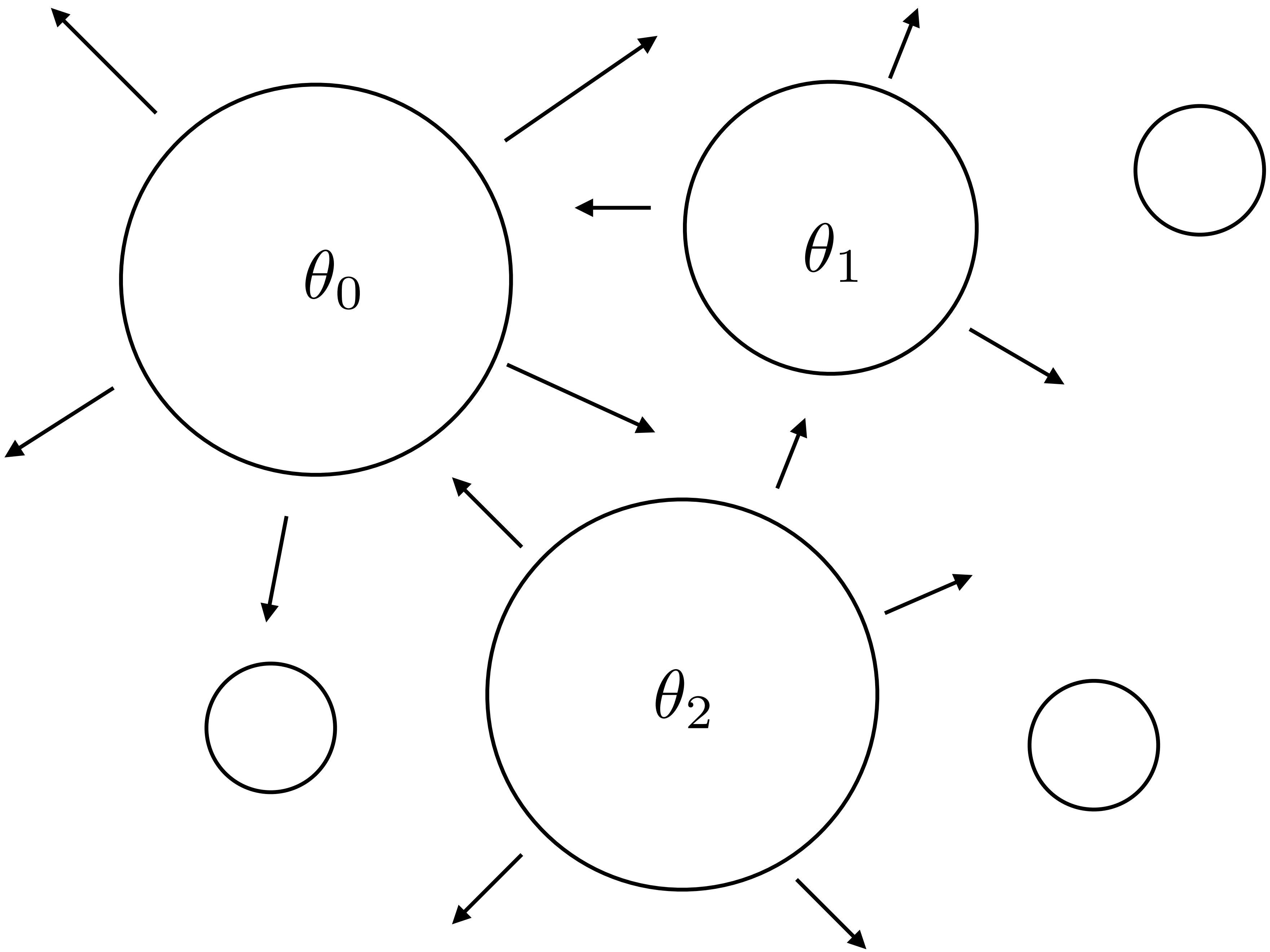
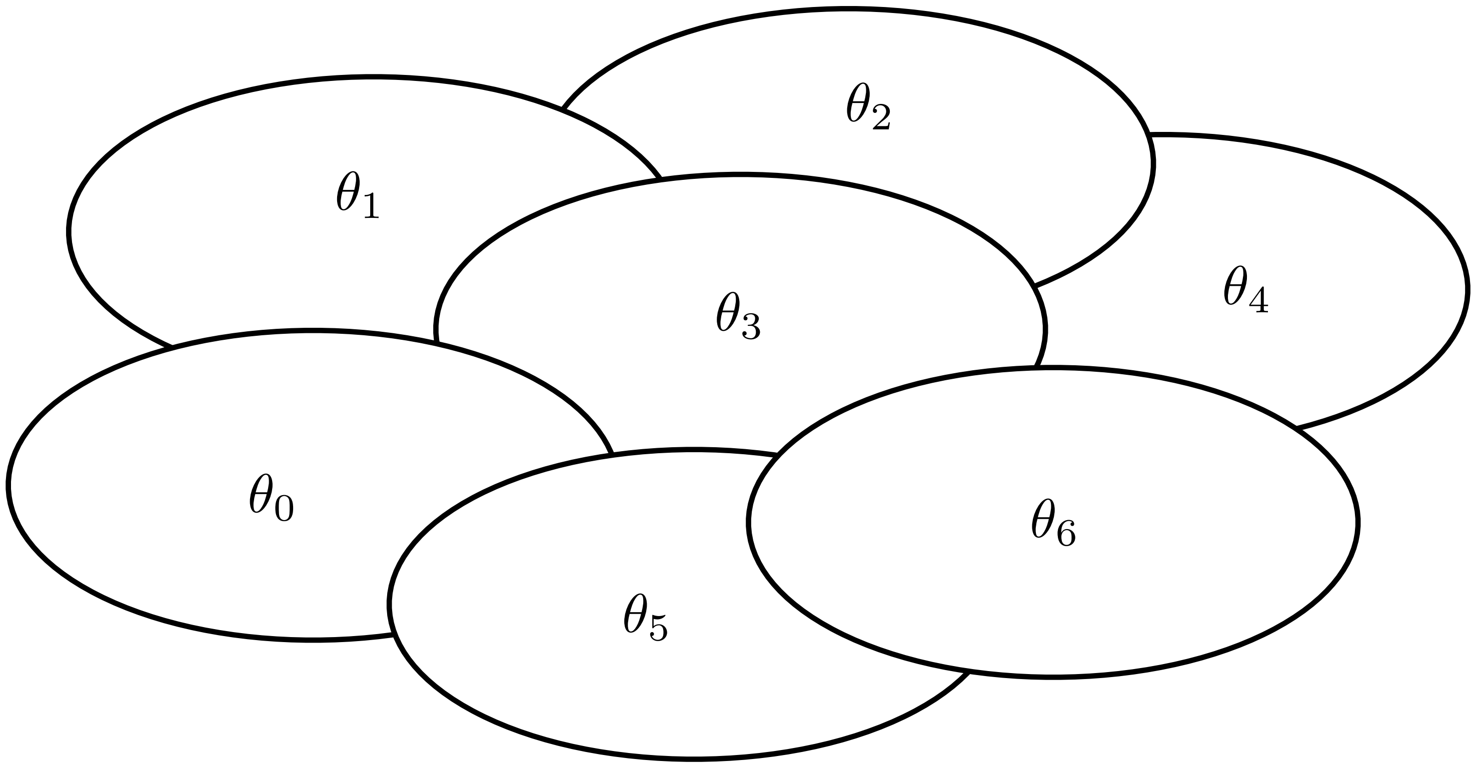
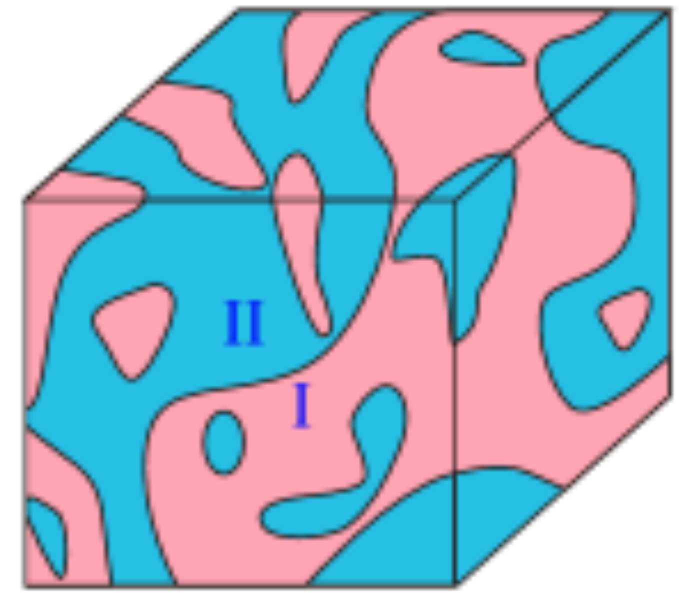
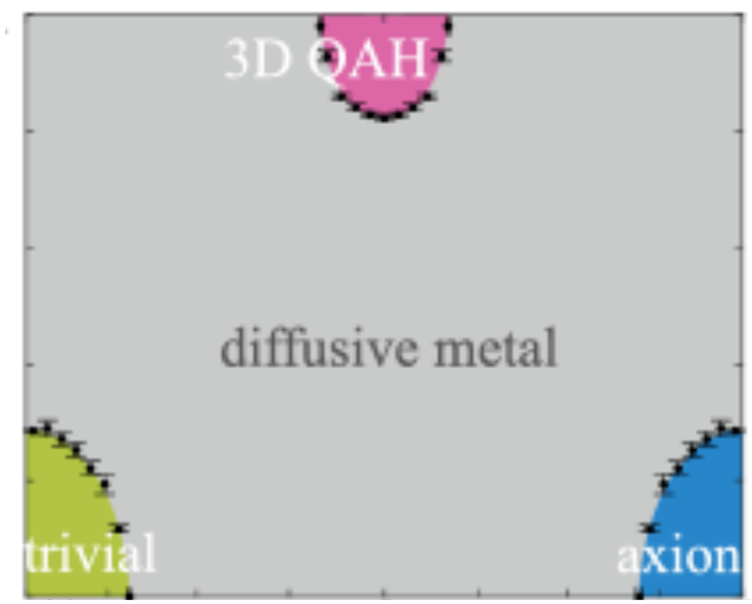


Dynamical axion insulators: Hysteresis







**As each bubble represents a topologically distinct insulator, but as T (and I) are broken, no symmetry enforce the domain walls to host boundary modes,
Bulk might remain insulating**

ZD Song et al. PRL 127, 016602 (2021)

Might gapping the axions pin the value of θ_0 in each bubble?

But this is I invariant, implying percolating boundary modes and diffusive metallic phase

Conclusions

- Dynamical axionic phases are complicated to observe in condensed matter systems as well

- Magnetic catalysis might help out, but chemical potential competes against so 1st order transition is predicted

- Nucleation theory predicts a rather “universal” hysteresis behavior that permeates to several measurable properties

- This theory is based on the growth of a single bubble, but many bubbles can be produced: Randomized theta vacuum

Thank you!