Axions & Superfluidity in Weyl Semimetals

From Anomalies in Fundamental QFT to Low Energy EFT, Axion Excitations & Macroscopic Superfluidity in WSMs

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Outline

- **Axial Anomaly in QED_4**
  - Triangle Amplitude & Massless Effective Scalar
  - Relativistic Ideal Hydrodynamics
- **Bosonization in 3 +1 Dimensions**
  - Quantization & Anomalous Commutators
  - Dimensional Reduction to 1+1 in Magnetic Fields
- **Superfluidity from the Anomaly**
  - New Extension of Nambu-Goldstone-like Theorem
  - Electromagnetic Interactions & Higgs-like Mass
- **A general low-energy EFT for WSMs**
  - Describes Fermion Self-Interactions & usual SSB
  - Same Goldstone CDW Mode predicted by Anomaly
- **Prospects for Experimental Detection**

Axions in QCD

Axions—a pseudoscalar **0⁻** particle was postulated by Weinberg and Wilczek to solve the strong CP problem of QCD– to promote the fixed phase angle θ of QCD to a dynamical field by SSB of a new global (Peccei-Quinn) symmetry in order to explain naturally why $\theta \ll 1$ and EDM of neutron is ~0

$$S_{\rm axion} = -\frac{1}{2}\int d^4x \left(\partial_\mu a\, \partial^\mu a + m_a^2\, a^2\right) + \int d^4x \,\frac{a}{f_a} \boldsymbol{E}\cdot\boldsymbol{B}$$

Axions have also attracted attention as a possible dark matter candidate for 25% of the energy content of the Universe

But despite going on 5 decades of extensive searches over many orders of magnitude in coupling strength and mass, no QCD axion has been found

Axial Anomaly in QED_4

Dirac Fermion Lagrangian

$$\mathscr{L}_f = \bar{\psi}\gamma^\mu\big(i\overleftrightarrow{\partial}_\mu + eV_\mu + A^5_\mu\gamma^5\big)\psi - m\bar{\psi}\psi$$

U(1) Vector Gauge Invariance

$$\psi \to e^{i\alpha}\psi\,, \quad \bar{\psi} \to \bar{\psi}e^{-i\alpha}\,, \quad eV_\mu \to eV_\mu + \partial_\mu\alpha$$

Conserved Noether Current $\partial_\mu J^\mu = 0\,, \quad J^\mu = e\bar{\psi}\gamma^\mu\psi$

Apparent U(1)^{ch} Chiral Invariance (if m=0)

$$\psi \to e^{i\beta\gamma^5}\psi\,, \quad \bar{\psi} \to \bar{\psi}e^{i\beta\gamma^5}\,, \quad A^5_\mu \to A^5_\mu + \partial_\mu\beta$$

But Corresponding Current is not conserved (Anomalous)

$$J^\mu_5 = e\bar{\psi}\gamma^\mu\gamma^5\psi\,, \quad \partial_\mu J^\mu_5\big|_{m=0} = \frac{e^2}{16\pi^2}\,\epsilon^{\alpha\beta\mu\nu}F_{\alpha\beta}F_{\mu\nu} = \frac{2\alpha}{\pi}\,\boldsymbol{E}\cdot\boldsymbol{B} \neq 0$$

Axial Anomaly Essentials

$$\Gamma_5^{\mu\alpha\beta}(p,q) = ie^2 \int d^4x \int d^4y\, e^{ip\cdot x + iq\cdot y} \left\langle \mathcal{T} J_5^{\mu}(0) J^{\alpha}(x) J^{\beta}(y) \right\rangle \Big|_{A=0}$$

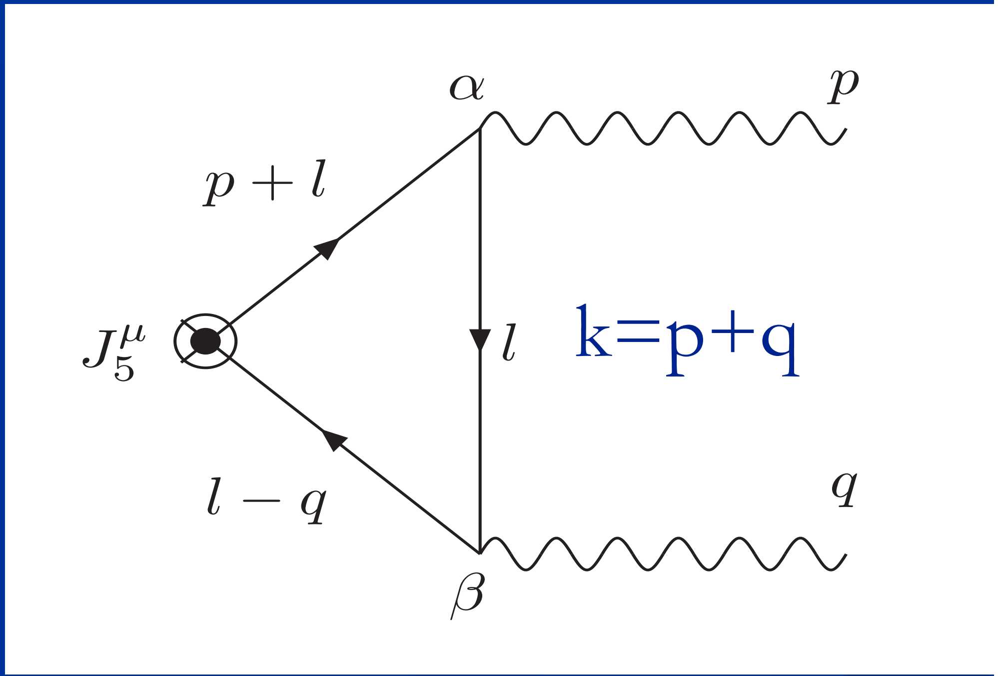
Triangle Diagram linearly UV divergent **naively**

Lorentz invariance of Vacuum **forbids** any linear divergence

+ Vector Current Conservation & **P/T** fixes **finite** Γ_5 completely

Impossible to also require Axial Current Conservation

A well-defined UV **finite** result



$$k_\mu\, \Gamma_5^{\mu\alpha\beta}(p,q) = \mathcal{A}\, \epsilon^{\alpha\beta\rho\sigma} p_\rho q_\sigma\,, \qquad \mathcal{A}\Big|_{m=0} = \frac{e^2}{2\pi^2} \neq 0$$

Massless/Gapless Anomaly Pole

Longitudinal part has a massless pole

$$\Gamma_5^{\mu\alpha\beta}(p,q) = \frac{k^\mu}{k^2}\,\mathcal{A}\,\epsilon^{\alpha\beta\rho\sigma}p_\rho q_\sigma + \Gamma_{5\,\perp}^{\mu\alpha\beta}(p,q)$$

Kramers-Kronig Relation to Imaginary Parts

$$\mathcal{A}(k^2;p^2,q^2;m^2) = \int_0^\infty ds\,\rho_1(s) - m^2\int_0^\infty \frac{ds}{k^2+s}\;\rho_m(s)$$

Spectral fn. obeys a UV finite Sum Rule

$$\int_0^\infty ds\,\rho_1(s;p^2,q^2;m^2) = \frac{e^2}{2\pi^2} = \mathcal{A}(k^2;p^2,q^2)\big|_{m=0}$$

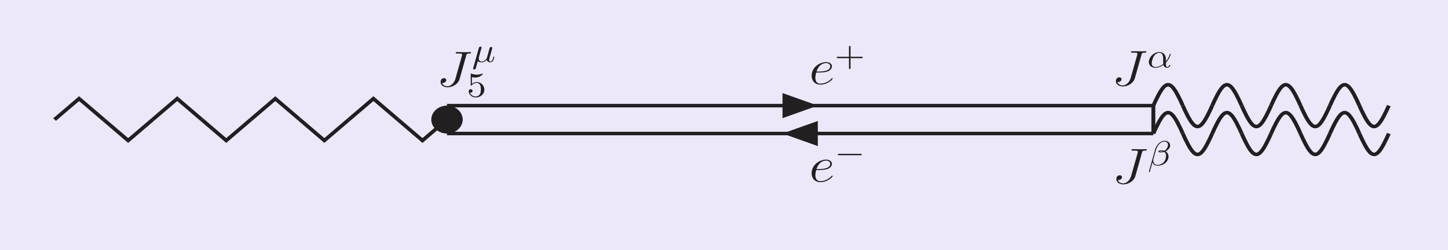
Sum Rule Residue of Pole independent of k^2, p^2, q^2, m^2

A Gapless 'Goldstone' Mode from Anomaly

ρ_1 and Chirality Violating Transition of 2-particle virtual $e^+ e^-$ pair state

$$\sum_{e^+,e^-} \langle 0|J_5^\mu|e^+ e^-\rangle\langle e^+ e^-|J^\alpha J^\beta|0\rangle \neq 0$$

do not vanish because of degeneracy with soft infrared photons
Dirac fermions do not become a decoupled pair of Weyl fermions
in the presence of electromagnetism even at m=0



An effective massless scalar (0^-) degree of freedom from a correlated massless $e^+ e^-$ pair with opposite helicities moving co-linearly at v= c

A relativistic Cooper pair & Gapless Collective Mode from the Axial Anomaly

Anomaly Bosonic Effective Action

The anomaly/longitudinal part of $\Gamma_5^{\mu\alpha\beta}$ is protected against higher order corrections by Adler-Bardeen Thm. & remains at all scales

Non-local 1PI Effective Action (note massless propagator)

$$\mathcal{S}_{\rm 1PI} = \int d^4x \int d^4y \; A_\mu^5(x)\; \partial_x^\mu \,\square_{xy}^{-1}\, \mathscr{A}_4(y)$$

Variation w.r.t. A_μ^5 produces the axial anomaly

$$\partial_\mu J_5^\mu = \mathscr{A}_4 \equiv \frac{\alpha}{2\pi} F_{\mu\nu}\widetilde{F}^{\mu\nu}$$

Local Effective Action

$$\mathcal{S}_{\rm anom} = \int d^4x \left[\left(\partial_\mu \eta + A_\mu^5\right) J_5^\mu + \eta \mathscr{A}_4 \right]$$

Variation w.r.t. η reproduces the axial anomaly

Fluid variation w.r.t. J_5^μ produces the constraint $\partial_\mu \eta + A_\mu^5 = 0$

Solved by $\eta = -\square^{-1}\, \partial^\mu A_\mu^5$, substituted into $\mathcal{S}_{\rm anom}$ reproduces $\mathcal{S}_{\rm 1PI}$

$\mathcal{S}_{\rm anom}$ is a hydrodynamic action & η is a Clebsch potential

Weyl Nodes at Finite Chiral Density

$$\mathcal{S}_{\rm Fluid} = \int d^4x \Big[\big(\partial_\lambda \eta + A^5_\lambda\big) J_5^\lambda + \eta \mathscr{A}_4 - \varepsilon(n_5)\Big] = \int d^4x \, P$$

Chiral Density $\quad n_5^2 = -J_5^\lambda J_{5\,\lambda} > 0$

Chiral Chemical Potential $\quad \dfrac{d\,\varepsilon}{dn_5} = \mu_5 = \bar{A}^5_0$ in equilibrium rest frame

Variation w.r.t. J_5^μ now gives

$$\partial_\lambda \eta + A^5_\lambda - \left(\frac{d\varepsilon}{dn_5}\right)\left(\frac{dn_5}{dJ_5^\lambda}\right) = 0$$

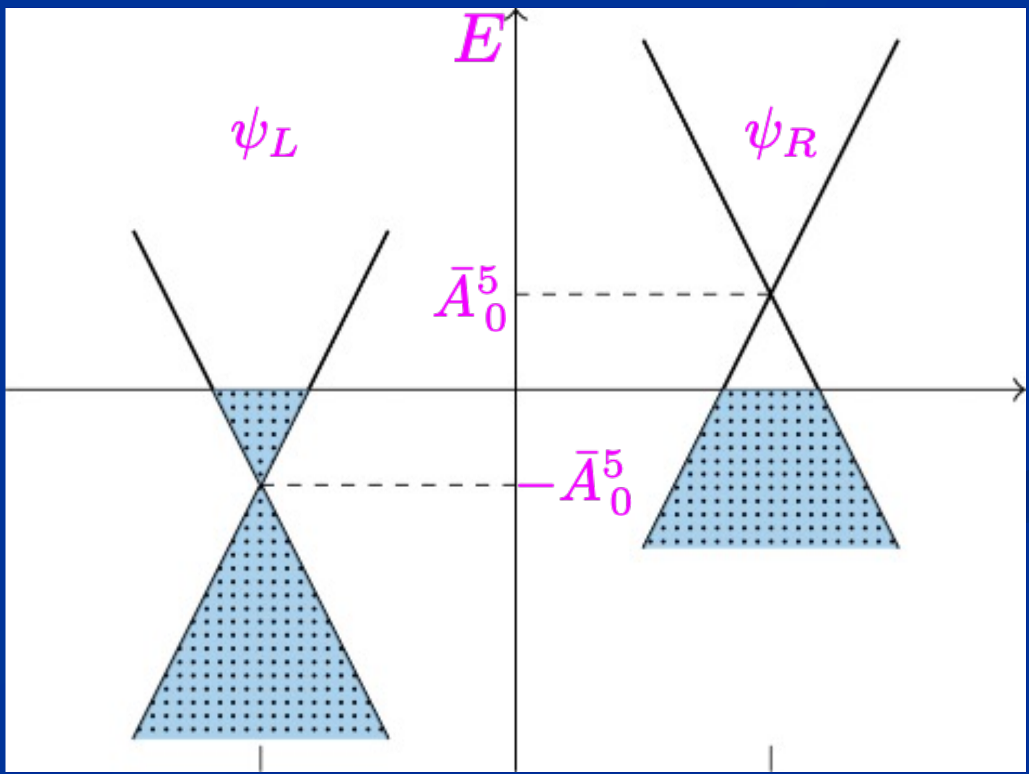
$$J_5^\lambda = -\frac{n_5}{\mu_5}\left(\partial^\lambda \eta + \bar{A}^{5\,\lambda}\right)$$

Perturbations from Equilibrium

$$\delta\mu_5 = \delta\dot{\eta}$$

$$\delta J_5^0 = \delta n_5 = \frac{dn_5}{d\mu_5}\,\delta\dot{\eta}$$

$$\delta J_5^i = -\frac{n_5}{\mu_5}\nabla^i\,\delta\eta$$



Chiral Density Waves (Axions) on the Fermi-Dirac Sea

$$\partial_\lambda(\delta J_5^\lambda) = \frac{dn_5}{d\mu_5}\left(\frac{\partial^2}{\partial t^2} - v_s^2\,\nabla^2\right)\delta\eta = \mathscr{A}_4$$

Propagating Axion from Anomaly

Sound Speed $\quad v_s^2 = \frac{dP}{d\varepsilon} = \frac{n_5\,d\mu_5}{\mu_5\,dn_5} = \frac{n_5}{\mu_5}\left(\frac{dn_5}{d\mu_5}\right)^{-1}$

Irrotatational & Dissipationless (at T=0, no impurities)

Pressure Wave = Superfluid First Sound

Simple Example: Free Massless Dirac Fermions in 4D

$$n_5 = 2\int \frac{d^3\boldsymbol{p}}{(2\pi)^3}\bigg|_{|\boldsymbol{p}|\leqslant\mu_5} = \frac{\mu_5^3}{3\pi^2} \qquad P(\mu_5) = \mu_5 n_5 - \varepsilon = \frac{\mu_5^4}{12\pi^2} = \frac{\varepsilon}{3}$$

$$\varepsilon(n_5) = 2\int \frac{d^3\boldsymbol{p}}{(2\pi)^3}\,|\boldsymbol{p}|\bigg|_{|\boldsymbol{p}|\leqslant\mu_5} = \frac{\mu_5^4}{4\pi^2} = \frac{3}{4}\,(3\pi^2)^{\frac{1}{3}}\,n_5^{\frac{4}{3}} \qquad v_s^2\big|_{\text{free}} = \frac{1}{3}c^2 \to \frac{1}{3}v_F^2$$

Dimensional Reduction in B Field

Uniform Constant ***B*** field $\quad \boldsymbol{B} = F_{23}\big|_{q=0}\,\hat{\mathbf{x}}$

Induced E/M current of fermions in bosonic form

$$J^\lambda = \frac{\delta S_{\rm anom}}{\delta A_\lambda} = \frac{\alpha}{2\pi}\frac{\delta}{\delta A_\lambda}\int d^4x\,\eta\,F^{\mu\nu}\,\widetilde{F}_{\mu\nu} = \frac{2\alpha}{\pi}\,\widetilde{F}^{\lambda\nu}\,\partial_\nu\eta$$

$$J^0 = \rho = -\frac{2\alpha}{\pi}\boldsymbol{B}\cdot\nabla\eta \qquad \boldsymbol{J} = \frac{2\alpha}{\pi}\boldsymbol{B}\,\dot{\eta}$$

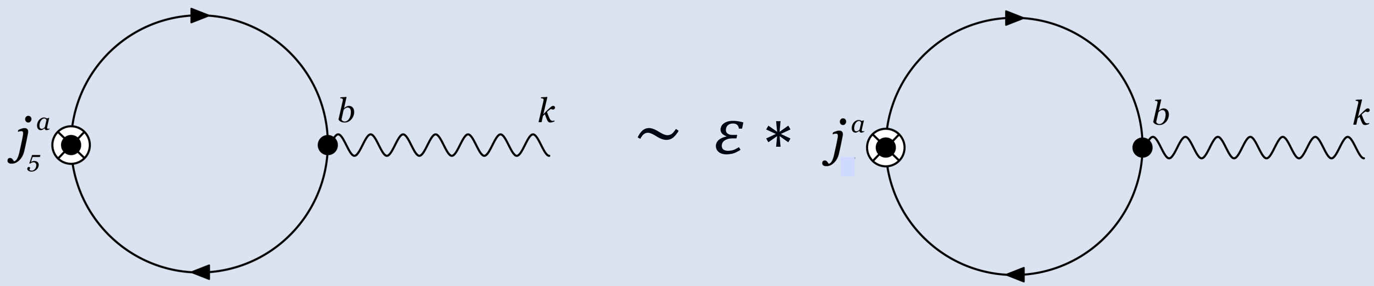
Transverse part of $\Gamma_5{}^{\lambda\alpha\beta}$ vanishes & anomalous part becomes exact

$$\lim_{q\to 0}\Gamma_5^{\lambda\alpha\beta}(k-q,q)A_\beta(q)\Big|_{\boldsymbol{k}_\perp=0} = \frac{2i\alpha}{\pi k^2}\left(k^2\delta^\alpha_{\ \nu} - k^\alpha k_\nu\right)\tilde{F}^{\lambda\nu}$$

$$= \begin{cases} -2i\alpha B\,\Pi_2^{ac}(k)\,\epsilon_c{}^\ell & \text{if } \lambda = \ell = 0{,}1 \text{ and } \alpha = a = 0{,}1 \\ 0 & \text{otherwise} \end{cases}$$

4D Triangle Reduces to $1/k^2$ pole of 2D Schwinger Model

2D Anomaly in Schwinger Model



Simply related to 2D Vacuum Polarization

$$\Pi_2^{ab}(k) = \frac{1}{\pi k^2}\left(k^a k^b - g^{ab} k^2\right)$$

$$j_5^a = \frac{1}{\pi}\frac{\partial^a}{\Box}E$$

Dimensional Reduction in B Field

4D Anomaly in Constant ***B*** field becomes 2D anomaly

$$\mathscr{A}_4\big|_B = \frac{e^2}{2\pi^2}\boldsymbol{E}\cdot\boldsymbol{B} = \frac{eB}{2\pi}\,\mathscr{A}_2\,, \qquad \mathscr{A}_2 = \frac{eE}{\pi}$$ of electrons in LLL

2D Fluid Effective Action

$$S_{\mathrm{anom},B} \propto \frac{eB}{2\pi}\int d^2x\,\left[(\partial^a\eta + \tilde{A}^a)\tilde{j}_a + \eta\,\mathscr{A}_2 - \varepsilon_2(\tilde{n})\right] = \frac{eB}{2\pi}\,S_{\mathbf{2D\ Fluid}}$$

$$\varepsilon(n_5) = \frac{eB}{2\pi}\,\varepsilon_2(\tilde{n})\,, \qquad \varepsilon_2(\tilde{n}) = \frac{\pi}{2}\tilde{n}^2 = p_2 = \frac{\tilde{\mu}^2}{2\pi}$$

Variation w.r.t. $\tilde{j}^a$ gives

$$\tilde{j}^a = -\frac{\tilde{n}}{\tilde{\mu}}\left(\partial^a\eta + \tilde{A}^a\right) = -\frac{1}{\pi}\,\partial^a\eta$$ Bosonization

Variation w.r.t. η now gives the 2D axial anomaly of electrons in LLL

$$\partial_a\tilde{j}^a = -\frac{1}{\pi}\,\partial_a\partial^a\eta = \mathscr{A}_2 = \frac{eE}{\pi}$$

Equivalent to 2D Schwinger Model

Induced ***E*** field in ***B*** direction by Gauss Law

$$\nabla \cdot \mathbf{E} = \rho = -\frac{2\alpha}{\pi}\,\mathbf{B} \cdot \boldsymbol{\nabla}\eta \Longrightarrow e\mathbf{E} = -\frac{2\alpha}{\pi} e\boldsymbol{B}\,\eta$$

so $\left(\partial_t^2 - \partial_x^2 + M^2\right)\eta = 0\,, \qquad M^2 = 2\alpha\left(\frac{eB}{\pi}\right)$

η is the Schwinger boson, now massive from the E/M interaction

And the 2D Fluid Effective Action is the Action of massless QED_2

$$(\partial^a \eta)\,\tilde{j}_a - \varepsilon(\tilde{n}) = -\frac{1}{2\pi}\,\partial_a\eta\,\partial^a\eta$$

$$-\frac{1}{e_2^2} F_{ab}F^{ab} + \eta\,\mathscr{A}_2 = -\frac{1}{2\pi} M^2\,\eta^2\,, \qquad M^2 = \frac{e_2^2}{\pi}$$

$$S_{\text{2D Fluid}} = -\frac{1}{2\pi}\int d^2x\,\left(\partial_a\eta\,\partial^a\eta + M^2\eta^2\right) = S_{\text{Schw}}$$

Remarkable Identity of Superfluid EFT and Microscopic QFT

Fermion Self-Interactions in 4D EFT

Often done by NJL Model to describe SSB

$$\mathscr{L}_{\rm NJL} = \tfrac{1}{4}G\left[(\bar\psi\psi)^2 - (\bar\psi\gamma^5\psi)^2\right]$$

Replace by Yukawa interaction

$$\mathscr{L}_Y = -g\bar\psi\Phi\psi = -g\bar\psi\left(\phi_1 + i\phi_2\gamma^5\right)\psi$$

Together with a $U(1)\bigotimes U^{\rm ch}(1)$ chiral invariant potential

$$V = \frac{\kappa}{2}\left(\phi_1^2+\phi_2^2\right) + \frac{\lambda}{4}\left(\phi_1^2+\phi_2^2\right)^2$$

Allows usual SSB if $\kappa < 0$, as well as ASB anomaly symmetry breaking

And kinetic term invariant under $\Phi \to e^{-2i\beta\gamma^5}\,\Phi\,, \quad A^5_\mu \to A^5_\mu + \partial_\mu\beta$

$$\mathscr{L}_{\rm kin} = -\frac{1}{2}\left(\partial_\mu\phi_1 - 2A^5_\mu\phi_2\right)^2 - \frac{1}{2}\left(\partial_\mu\phi_2 + 2A^5_\mu\phi_1\right)^2$$

All IR Relevant Terms in a Renormalizable EFT

SSB with Chiral Chemical Potential

Polar Representation $\Phi = \sigma \exp\left(2i\zeta\gamma^5\right)$

Chiral Chemical Potential $\mu_5^2 = -(\partial^\mu\zeta + A^{5\mu})(\partial_\mu\zeta + A^{5\mu})$

Effective Potential $V_{\text{eff}}(\sigma, \mu_5) = -2\mu_5^2\sigma^2 + V(\sigma)$

SSB $\bar{\sigma} = \sqrt{\frac{4\mu_5^2 - \kappa}{\lambda}} \neq 0\,,$ if $4\mu_5^2 > \kappa$

Note: Non-zero μ_5 favors SSB solution

Scalar Contribution to Axial Current

$$J_5^\lambda[\Phi] = 4\,\phi_2\,\overleftrightarrow{\partial}^\lambda\phi_1 - 4\,A^{5\lambda}(\phi_1^2 + \phi_2^2) = -4\sigma^2\left(\partial^\lambda\zeta + A^{5\lambda}\right)$$

Chiral Density in SSB equilibrium state in terms of scalar vev

$$\frac{n_5}{\mu_5} = \frac{1}{\mu_5}\sqrt{-J_5^\lambda J_{5\lambda}} = 4\bar{\sigma}^2 \qquad \varepsilon - \mu_5\, n_5 = P = \frac{1}{4}\lambda\bar{\sigma}^4$$

Goldstone Sound Mode

Perturbations from Equilibrium: $\sigma = \bar{\sigma} + \delta\sigma \,, \quad \zeta = \bar{\zeta} + \delta\zeta = \delta\zeta$

Quadratic Action:

$$S_{\Phi}^{(2)} = -\frac{1}{2}\int d^4x \left[\delta\sigma\,(-\Box + M_\sigma^2)\,\delta\sigma - 4\bar{\sigma}^2\,\delta\zeta\,\Box\,\delta\zeta - 16\mu_5\,\bar{\sigma}\,\delta\sigma\,\delta\dot{\zeta}\right]$$

2 x 2 matrix in Fourier Space:

$$\begin{pmatrix} -\omega^2 + \boldsymbol{k}^2 + M_\sigma^2 & 4i\omega\mu_5 \\ -4i\omega\mu_5 & -\omega^2 + \boldsymbol{k}^2 \end{pmatrix}, \qquad M_\sigma^2 = V''_{\rm eff}(\bar{\sigma}) = 2\lambda\bar{\sigma}^2$$

Gapless Sound Mode:

$$\omega^2 = v_s^2\,\boldsymbol{k}^2 + \mathcal{O}\left(\frac{k^4}{M_\sigma^2}\right), \qquad v_s^2 = \frac{\lambda\bar{\sigma}^2}{2\kappa + 3\lambda\bar{\sigma}^2}$$

Chiral Current:

$$\delta J_5^0 = \frac{n_5}{\mu_5}\frac{1}{v_s^2}\,\delta\dot{\zeta} = \frac{dn_5}{d\mu_5}\,\delta\dot{\zeta} \qquad \delta\boldsymbol{J}_5 = -\frac{\bar{n}_5}{\bar{\mu}_5}\nabla(\delta\zeta)$$

Gapless Sound Mode = Chiral Anomaly

$$\partial_\mu(\delta J_5^\mu) = \frac{dn_5}{d\mu_5}\left(\frac{\partial^2}{\partial t^2} - v_s^2\nabla^2\right)\delta\zeta = \frac{2\alpha}{\pi}\delta(\boldsymbol{E}\cdot\boldsymbol{B})$$

Identical (up to prefactor) as Gapless Mode $\delta\zeta \Leftrightarrow \delta\eta$

found previously for free fermions and **no explicit SSB**

Equivalence between standard (classical) SSB

& Quantum Anomaly Symmetry Brealing (ASB)

A new variation of Goldstone theorem

Chiral Anomaly is completely unaffected by fermion self-interactions (even strong ones) & onset of SSB due to Adler-Bardeen Thm. and ‘t Hooft Anomaly matching

Can Axion be Observed in WSMs?

One possibility: For a WSM in a strong $\boldsymbol{B}$ field, a time dependent $\delta\zeta \Leftrightarrow \delta\eta$ generates a time dependent electric current. This current can be excited by a time dependent $\boldsymbol{E}$ parallel to $\boldsymbol{B}$ through the anomaly source.

As the frequency ω of the $\boldsymbol{E}$ source is varied it can sweep through the resonance expected from the axion pole denominator

$$\boldsymbol{J}_\omega \propto \delta\eta_\omega \propto \left(-\omega^2 + v_s^2\boldsymbol{k}^2 + \Gamma^2\right)^{-1} E_\omega\, B$$

Estimates indicate this may be feasible

Axion Electrodynamics in the Laboratory

Summary

- QED Chirall Anomaly contains a Massless 0^- Collective Mode due to Cooper Pairing of Massless Electrons
- Non-Dissipative Irrotational Macroscopic Superfluid derived from First Principles QFT of the Chiral Anomaly
- Partial Bosonization in 3+1 closely related to 1+1 in Magnetic Fields, Lowest Landau Level
- Quantum Chiral Anomaly itself is responsible for a Goldstone Axionic Mode in WSMs without classical SSB

$\Longrightarrow$ Anomaly Symmetry Breaking (ASB)

WSM Laboratory for the QCD Axion