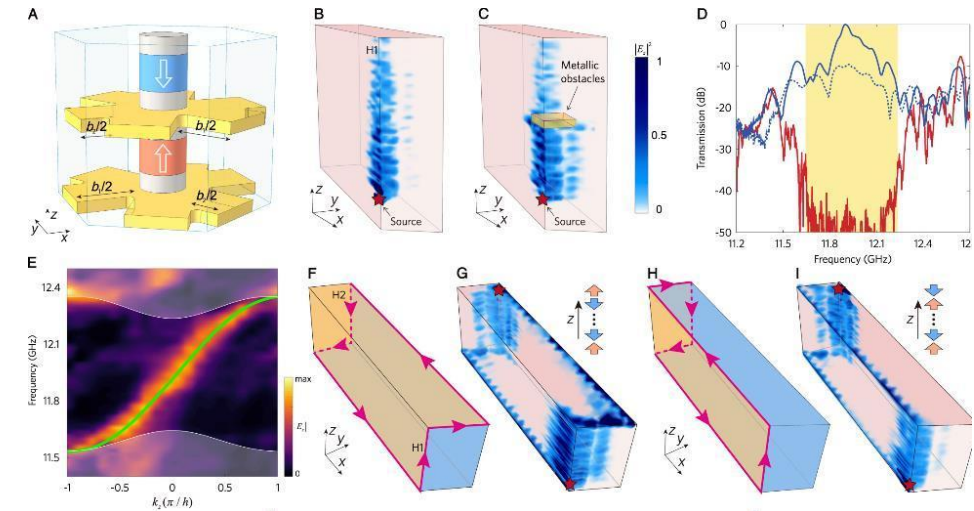
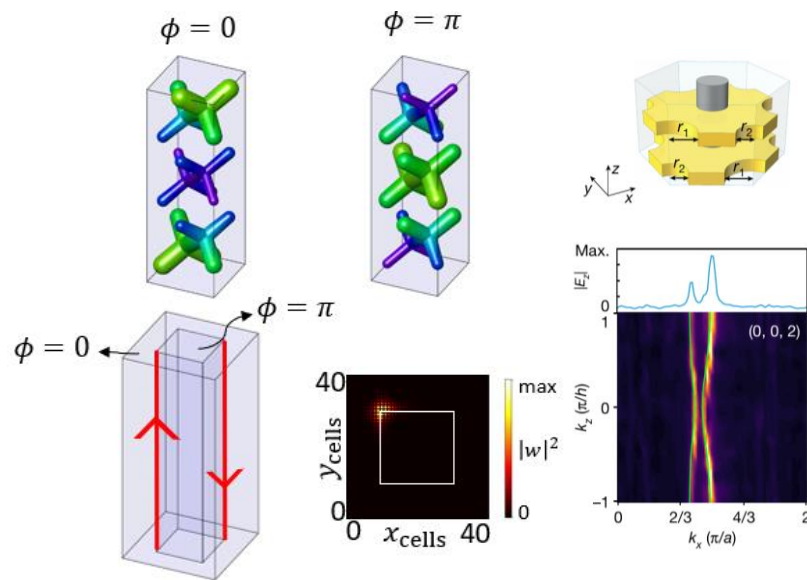


Photonic axion domain walls and insulators

Chiara Devescovi (ETH Zurich)



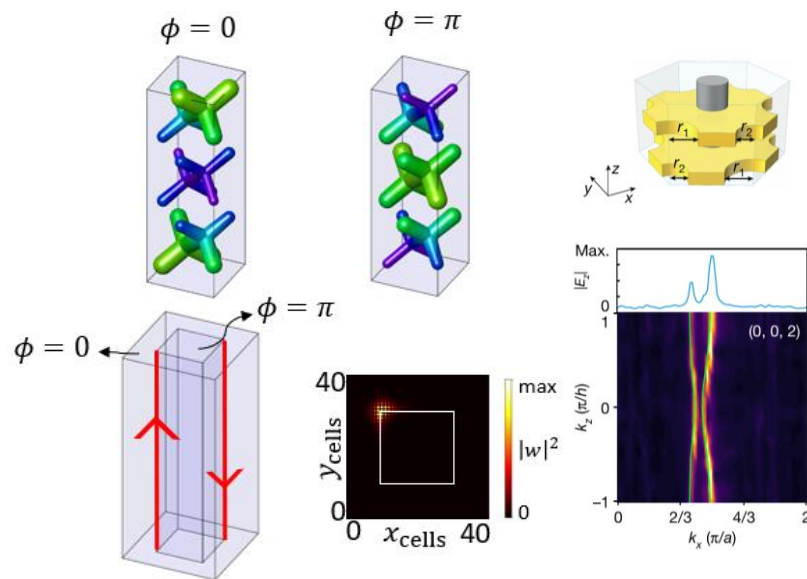
Aitzol García Etxarri
Maia García Vergniory
Antonio Morales Perez



Gui-Geng Liu
Subhaskar Mandal
Yidong Chong
Baile Zhang

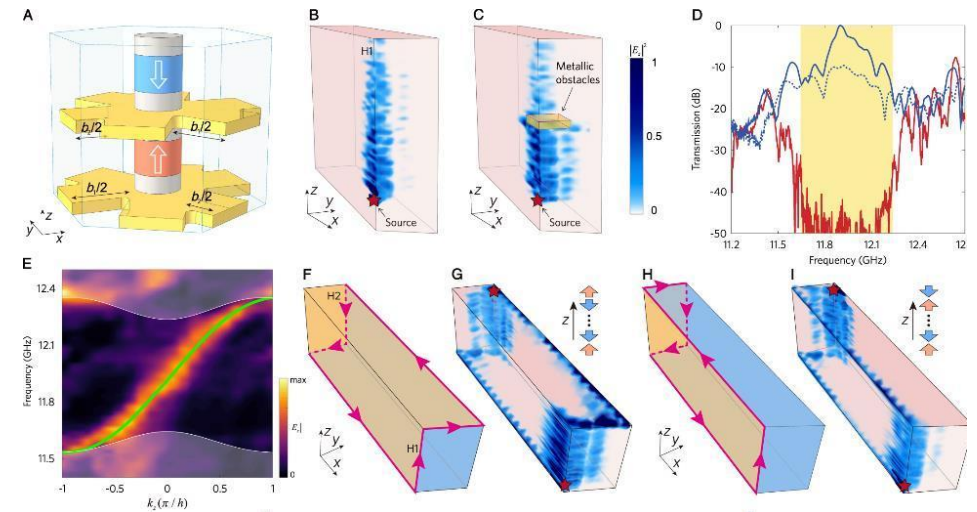
In this presentation:

Realizing an AXI for light



Axion topology in photonic crystals
Nat. Comm. 15, 6814 (2024)

From **Relative** to **Intrinsic** photonic AXI



Photonic Axion Insulator
Science 387, 162–166 (2025)

Outline

Outline

- Context:

Outline

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 - Axion Insulator (AXI)

Outline

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1- From 3D Chern insulators to Relative AXI

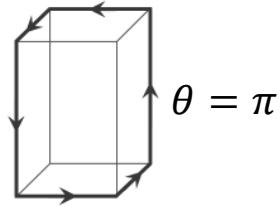
2- From Relative to Intrinsic AXI

- Outlook and conclusions

What is an Axion Insulator (AXI)?

What is an Axion Insulator (AXI)?

3D magnetic
higher-order topological
insulators (HOTI)

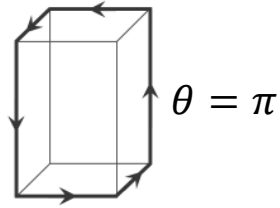


$$\theta = \frac{1}{12\pi} \int_{BZ} \text{Tr} \varepsilon^{\mu\nu\rho} \left[A_\mu \partial_\nu A_\rho - \frac{2i}{3} A_\mu A_\nu A_\rho \right] d^3k$$

X.L. Qi, T. L. Hughes, and S.C. Zhang, "Topological field theory of time-reversal invariant insulators" PRB 78, 195424 (2008)

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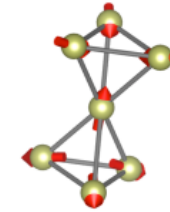
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Invariant quantization:

$$\theta \rightarrow -\theta$$

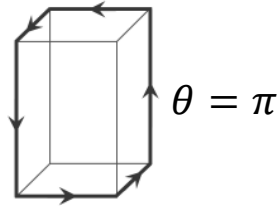
e.g. Inversion (I)



Varnava et al., Phys. Rev. B 98, 245117 (2018)

What is an Axion Insulator (AXI)?

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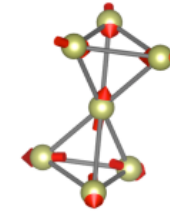
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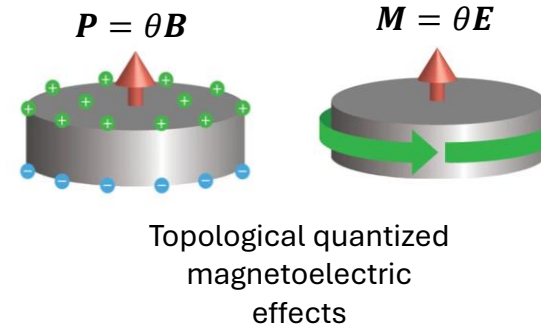
Varnava et al., Phys. Rev. B 98, 245117 (2018)

Axion-modified
electrodynamics

$$\mathcal{L} \propto \theta \mathbf{E} \cdot \mathbf{B}$$

$$\left\{ \begin{array}{l} \nabla \cdot (\varepsilon \mathbf{E}) = -\nabla \theta \cdot \mathbf{B} \\ \nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} \\ \nabla \cdot \mathbf{B} = 0 \\ \nabla \times \mu^{-1} \mathbf{B} = \frac{\partial (\varepsilon \mathbf{E})}{\partial t} + \nabla \theta \times \mathbf{E} + \frac{\partial \theta}{\partial t} \mathbf{B} \end{array} \right.$$

F. Wilczek, "Two applications of axion electrodynamics", PRL letters 58, 1799 (1987)



Outline

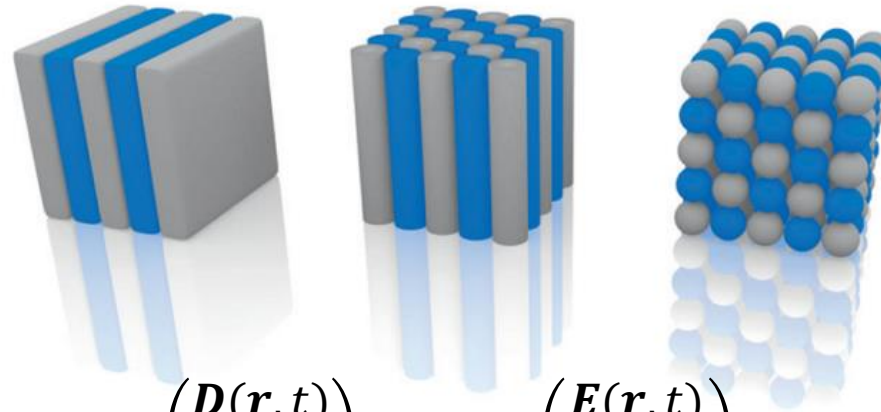
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1- From 3D Chern insulators to Relative AXI

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- Outlook and conclusions

What is a Photonic Crystal?



$$\begin{pmatrix} \mathbf{D}(\mathbf{r}, t) \\ \mathbf{B}(\mathbf{r}, t) \end{pmatrix} = \mathcal{K}(\mathbf{r}) \begin{pmatrix} \mathbf{E}(\mathbf{r}, t) \\ \mathbf{H}(\mathbf{r}, t) \end{pmatrix}$$

$$\mathcal{K}(\mathbf{r}) = \begin{pmatrix} \vec{\epsilon}(\mathbf{r}) & \vec{\xi}(\mathbf{r}) \\ \vec{\chi}(\mathbf{r}) & \vec{\mu}(\mathbf{r}) \end{pmatrix}$$

Constitutive dielectric tensor is periodic $\mathcal{K}(\mathbf{r}) = \mathcal{K}(\mathbf{r} + \mathbf{R})$

Periodic Maxwell equations: bandgaps for light

Outline

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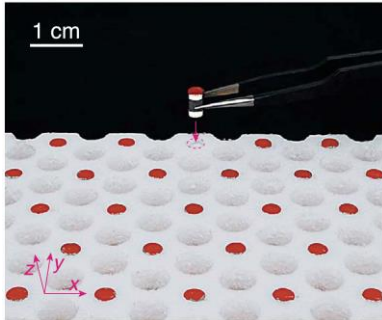
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Why do we seek a photonic AXI?

Why do we seek a photonic AXI?

Clean and flexible platform

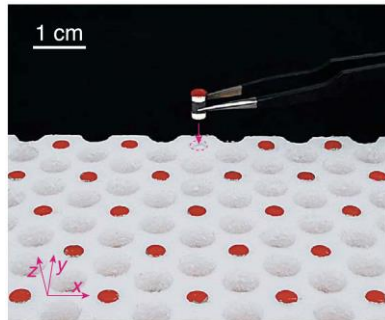
- Tunable symmetries (TRS, inversion)
- No stringent material constraints
- Compatible with inverse-design



Why do we seek a photonic AXI?

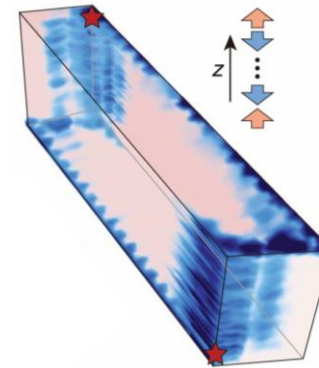
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Direct access to electromagnetic response

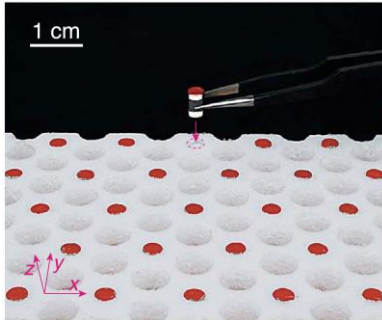
- Field-resolved experiments possible
- All **measurable via optical probes**



Why do we seek a photonic AXI?

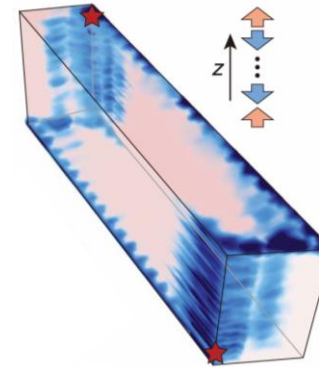
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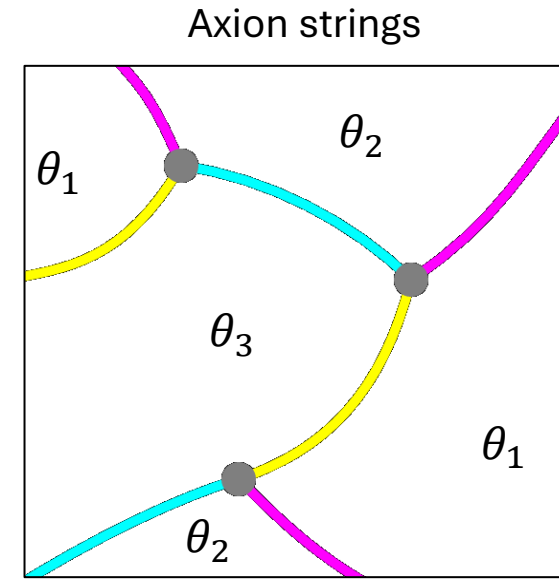
- Field-resolved experiments possible
- All **measurable via optical probes**



Can implement θ as a **spatially varying parameter $\theta(x)$** :
something **not feasible** in particle physics experiments

Simulating Axion Domain Walls

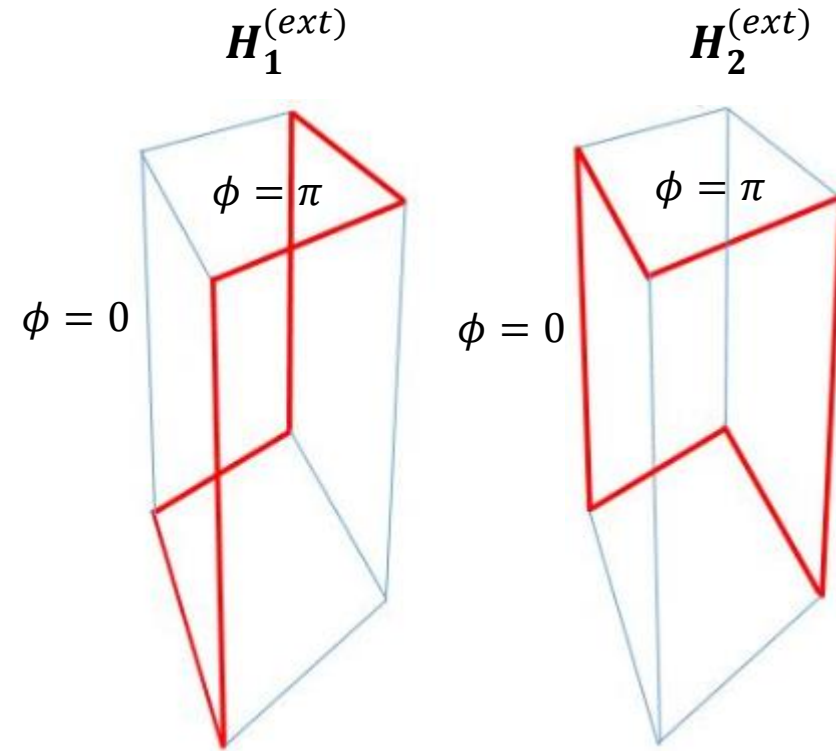
Can implement θ as a **spatially varying parameter** $\theta(x)$: enabling direct exploration of topological domain-wall physics



Armengaud, Journal of Cosmology and Astroparticle Physics, 2019(06), 047.

Manipulating Axion Domain Walls

Propose physically accessible ways
to control the axion string
connectivity



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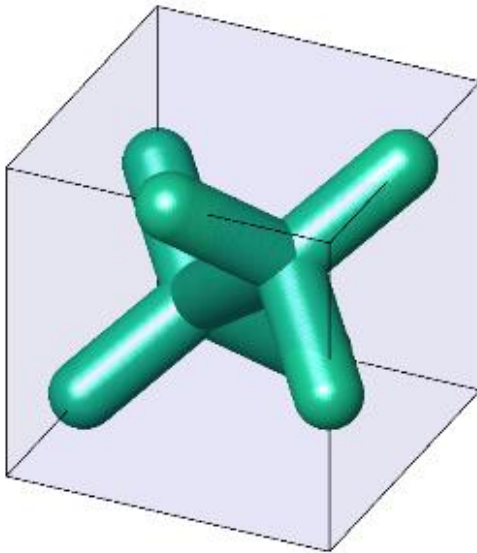
Design strategy

How to open an AXI gap in a PhC? I-symmetric and T-broken

Design strategy

How to open an AXI gap in a PhC? I-symmetric and T-broken

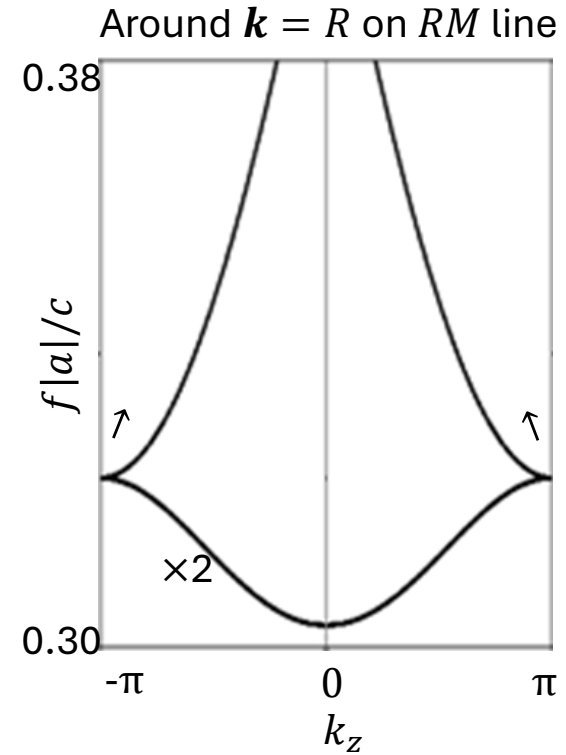
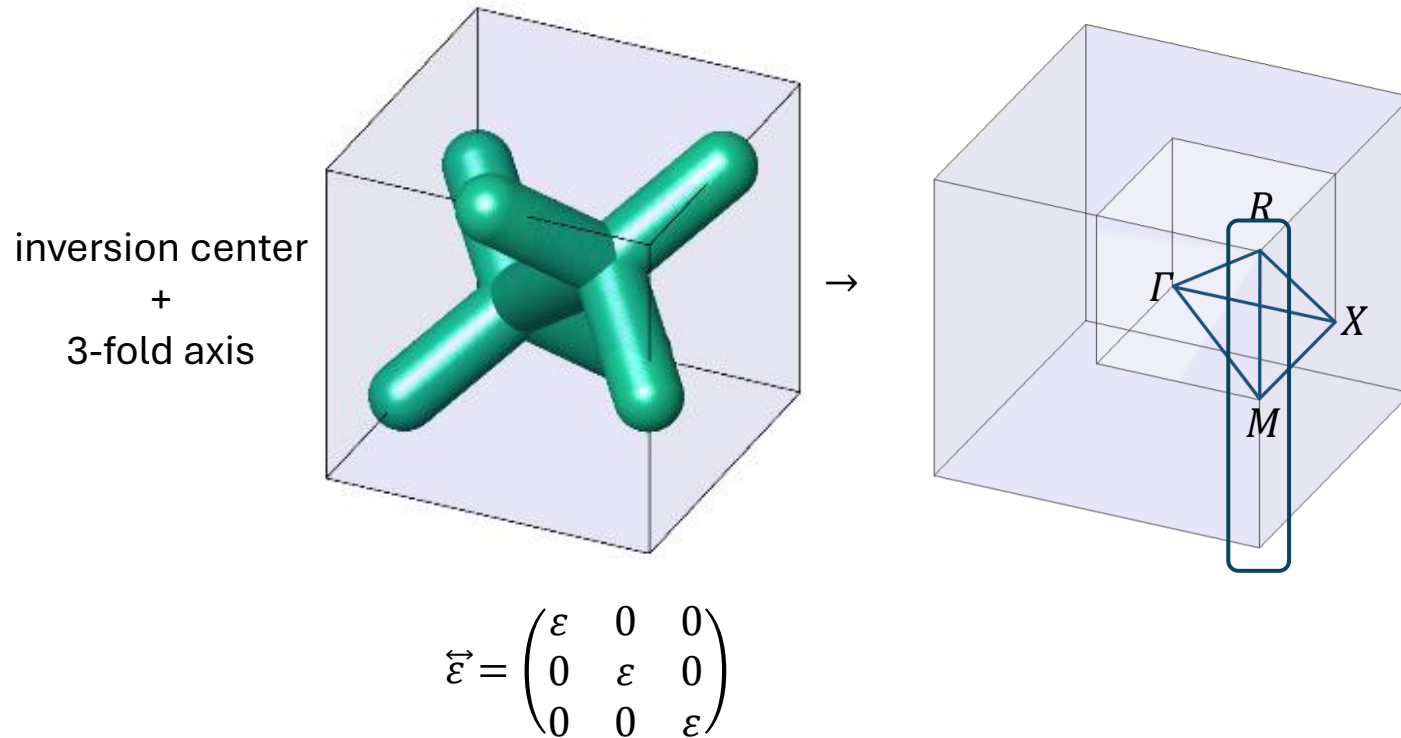
inversion center
+
3-fold axis



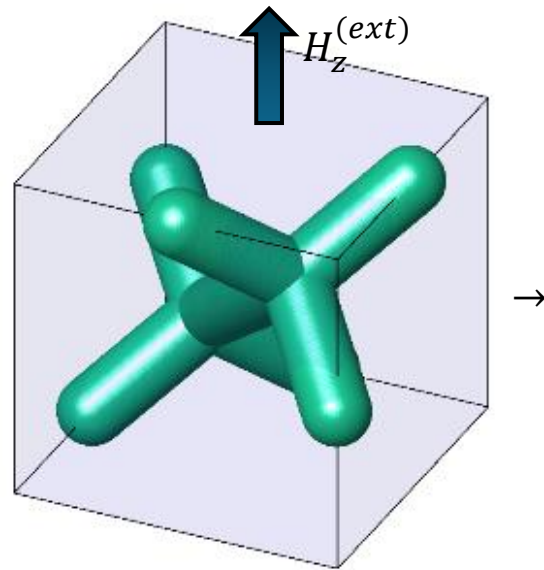
$$\hat{\epsilon} = \begin{pmatrix} \epsilon & 0 & 0 \\ 0 & \epsilon & 0 \\ 0 & 0 & \epsilon \end{pmatrix}$$

Design strategy

How to open an AXI gap in a PhC? I-symmetric and T-broken

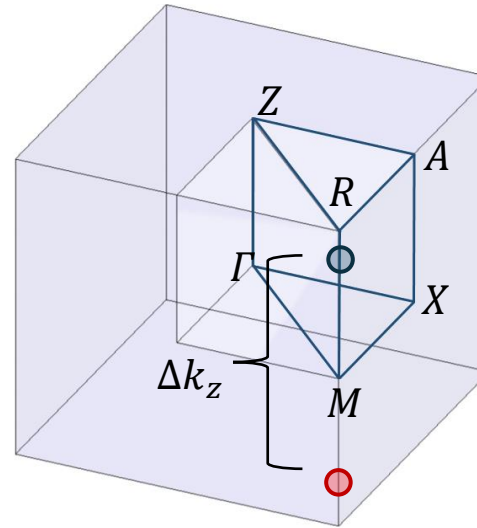


Step 1: Gyrotropy via Yttrium Iron Garnets (YIGs)



$$\vec{\varepsilon} = \begin{pmatrix} \varepsilon_{\perp} & i\eta_z & 0 \\ -i\eta_z & \varepsilon_{\perp} & 0 \\ 0 & 0 & \varepsilon \end{pmatrix}$$

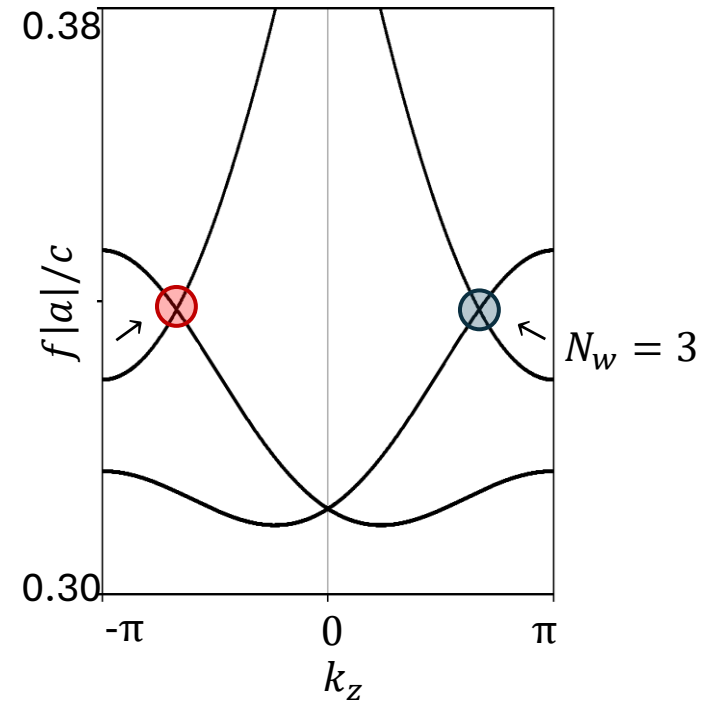
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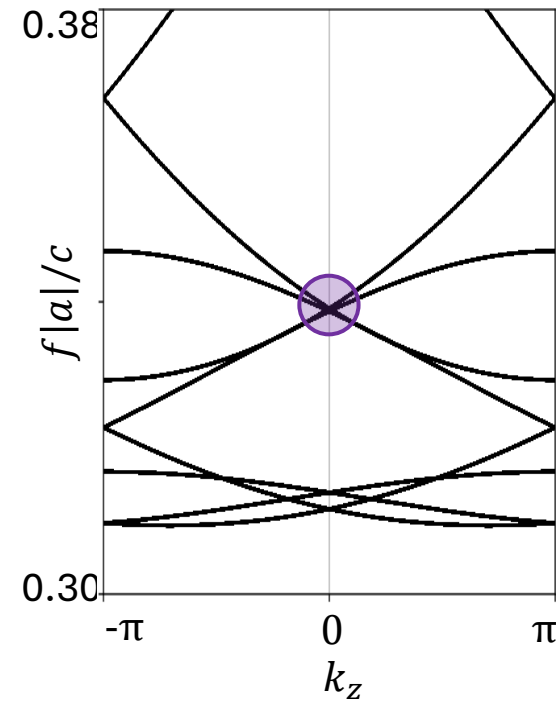
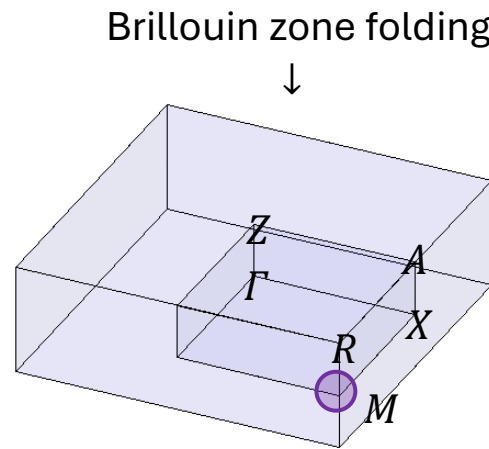
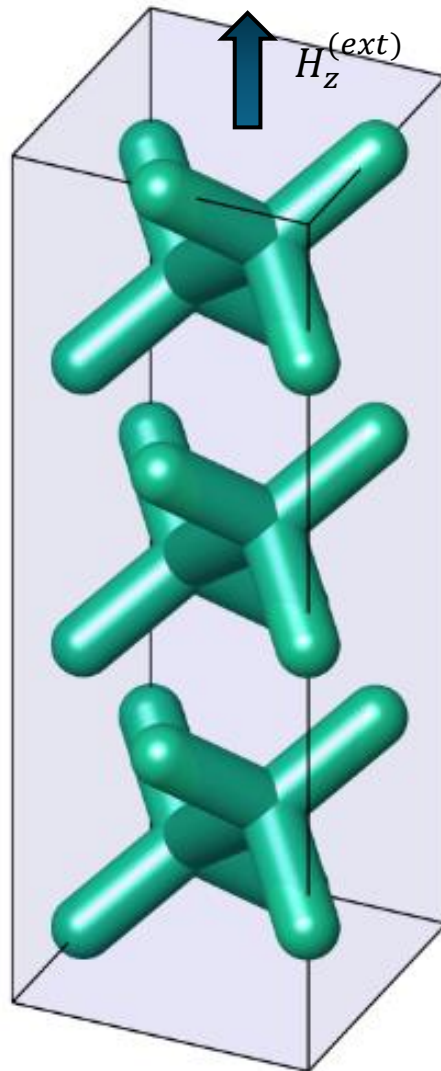
$$\Delta k_z = 2\pi/N_w$$



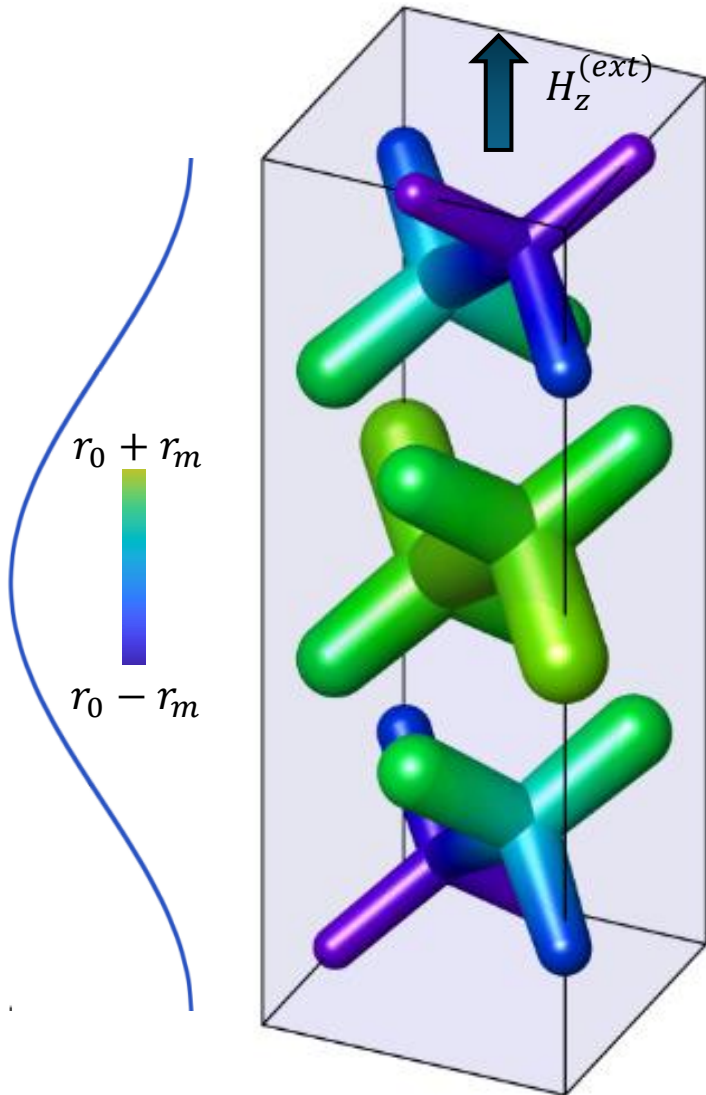
Weyl splitting at integer fractions of the BZ



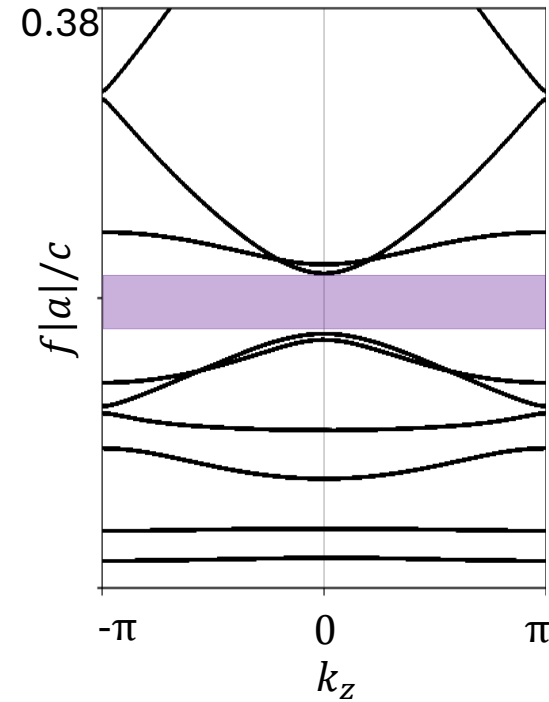
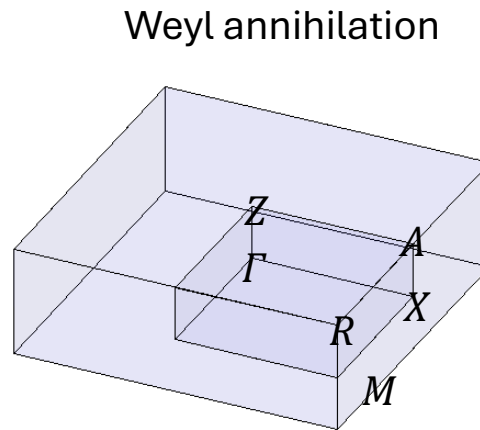
Step 2: Commensurate supercell



Step 3: Dielectric modulation



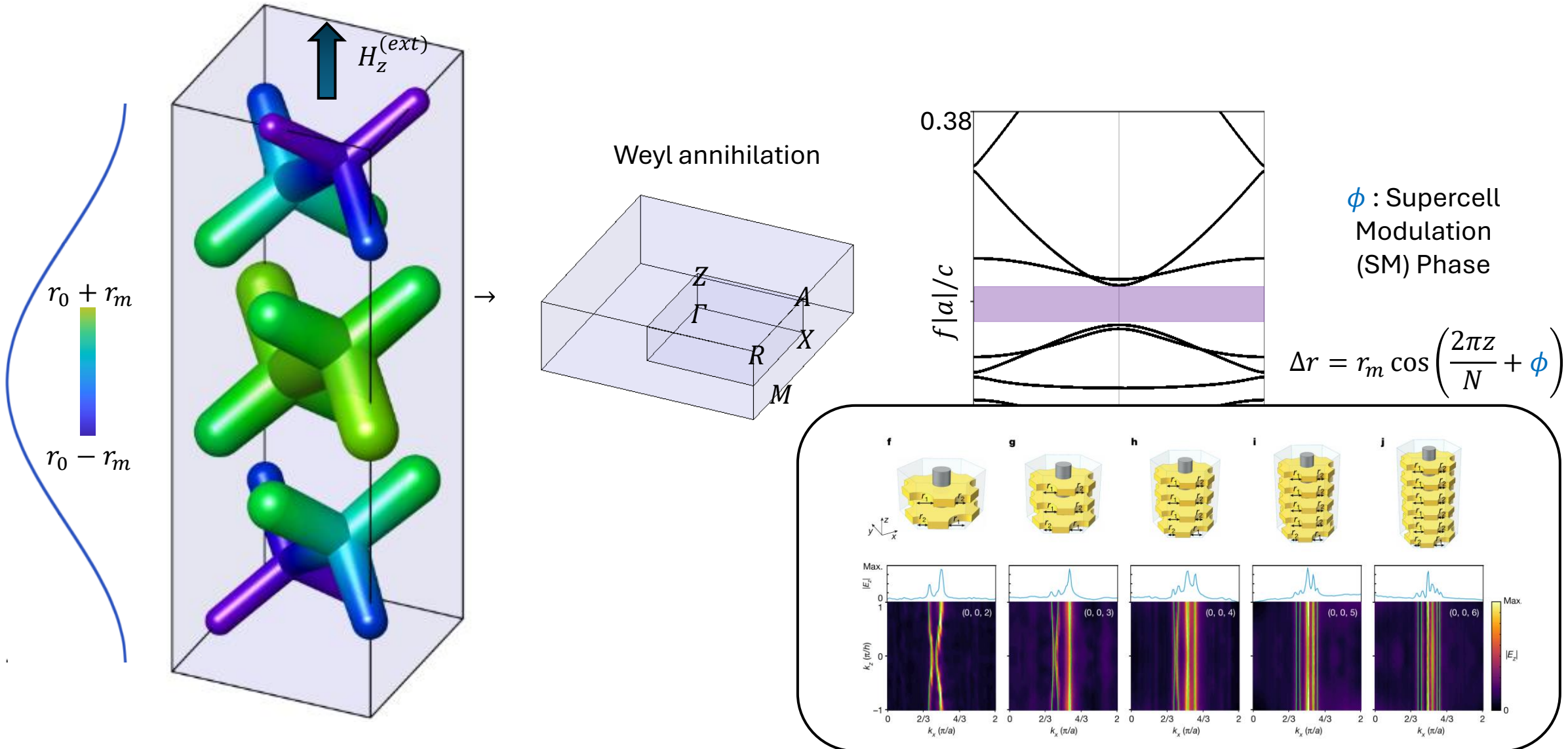
→



ϕ : Supercell Modulation (SM) Phase

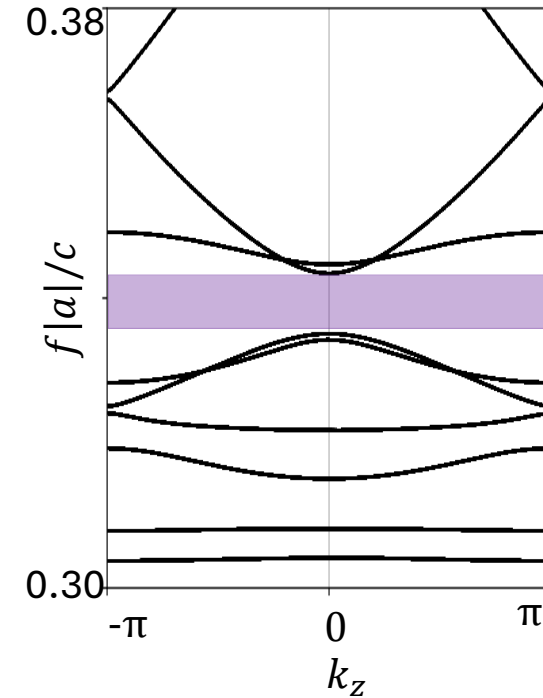
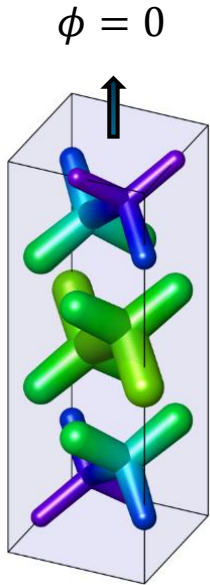
$$\Delta r = r_m \cos\left(\frac{2\pi z}{N} + \phi\right)$$

Step 3: Dielectric modulation



Final Step 4: Inversion symmetry pinning

Relative SM phase

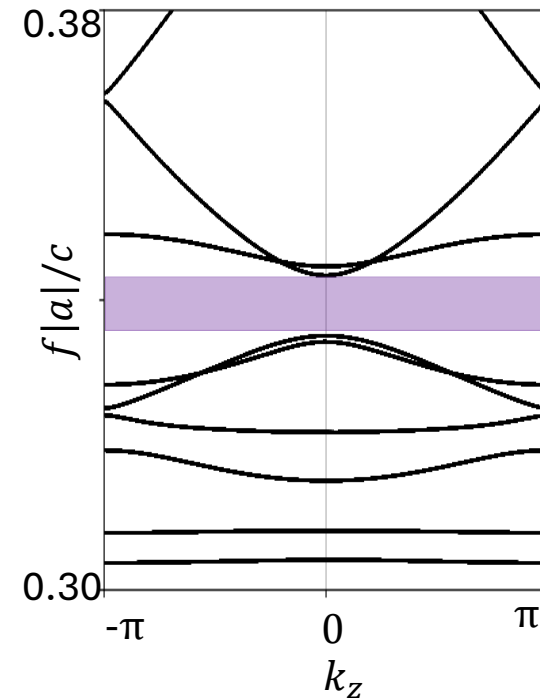
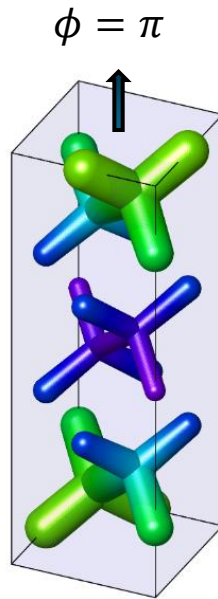
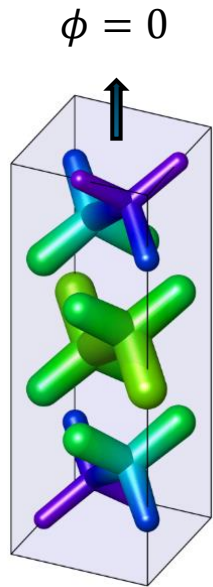


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Modulation
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Final Step 4: Inversion symmetry pinning

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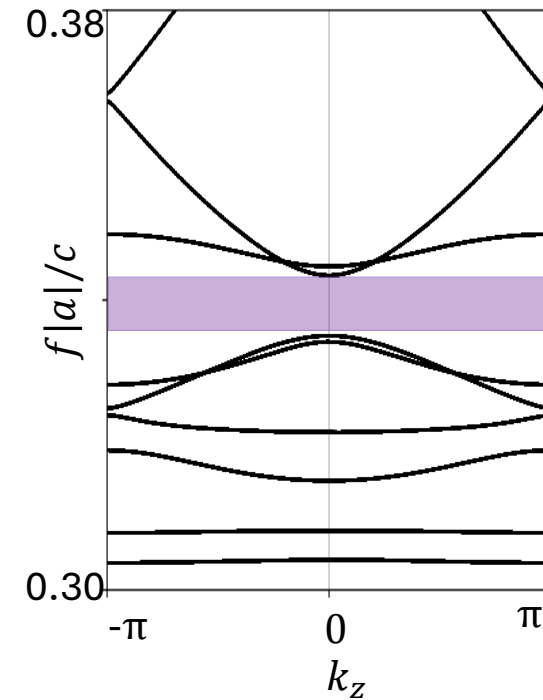
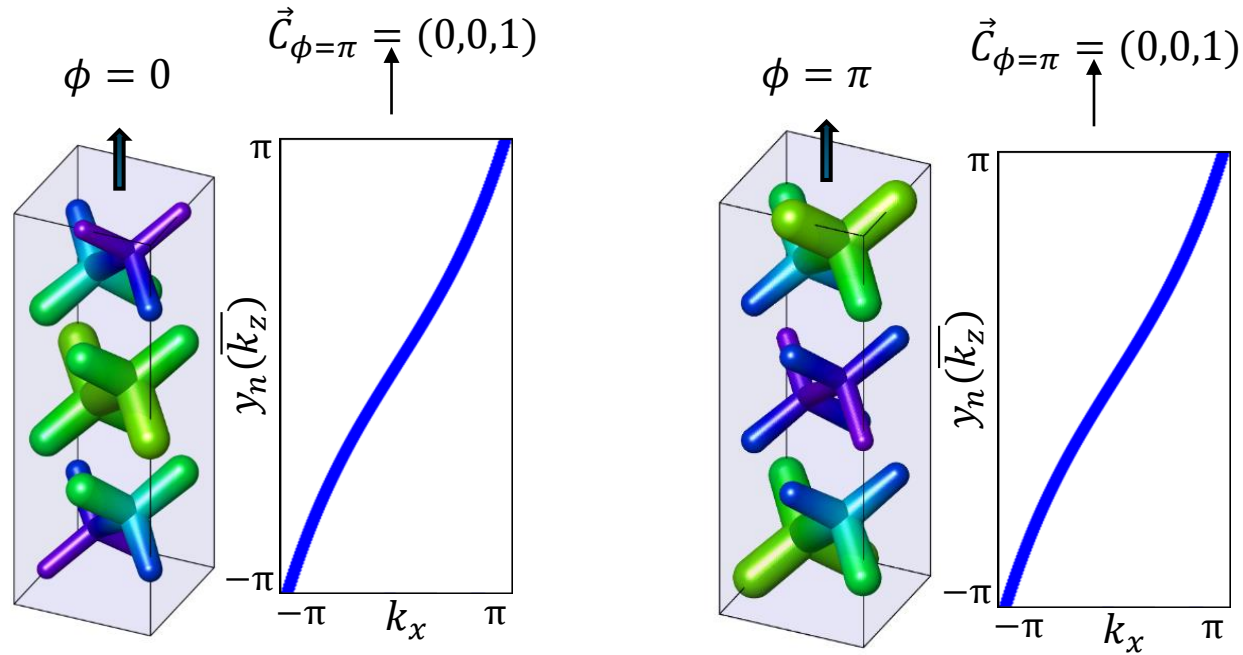


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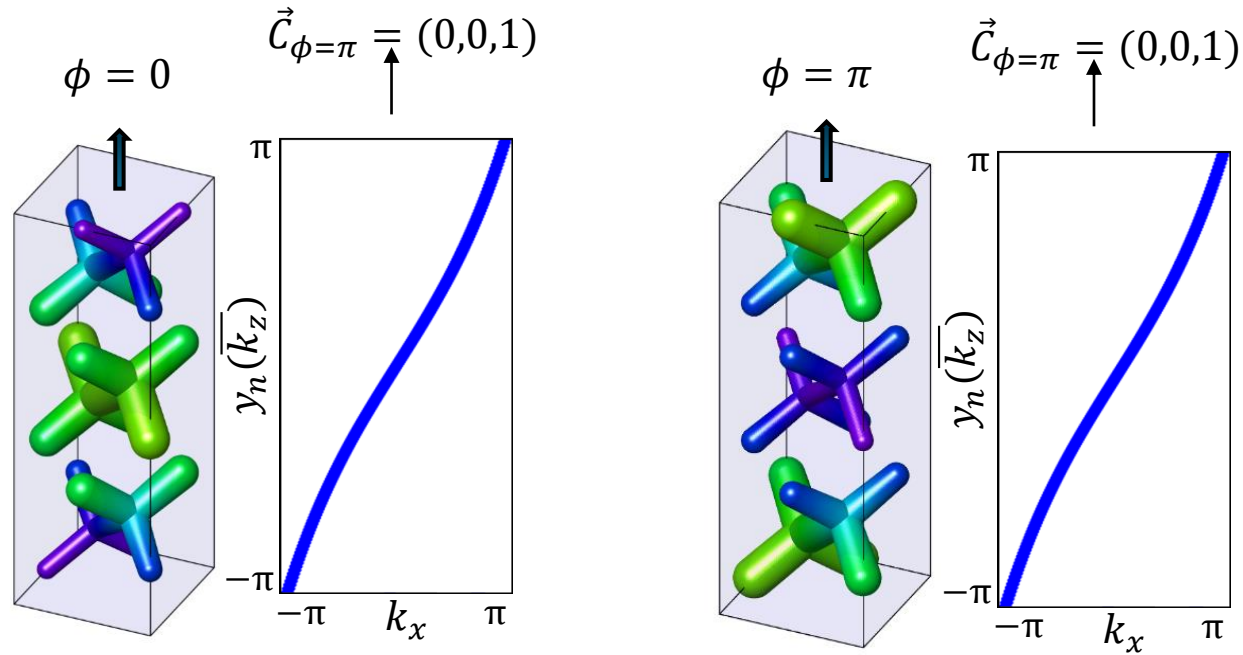


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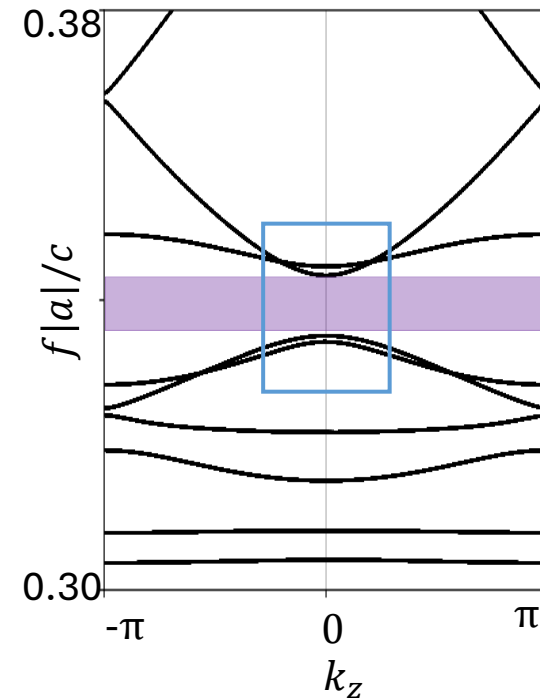
Relative SM phase



$$n_{\phi=0}^{k=V} = [2V_1^+ + 4V_1^-]$$

$$n_{\phi=\pi}^{k=V} = [4V_1^+ + 2V_1^-]$$

Double band-inversion at $V = (\pi, \pi, 0)$

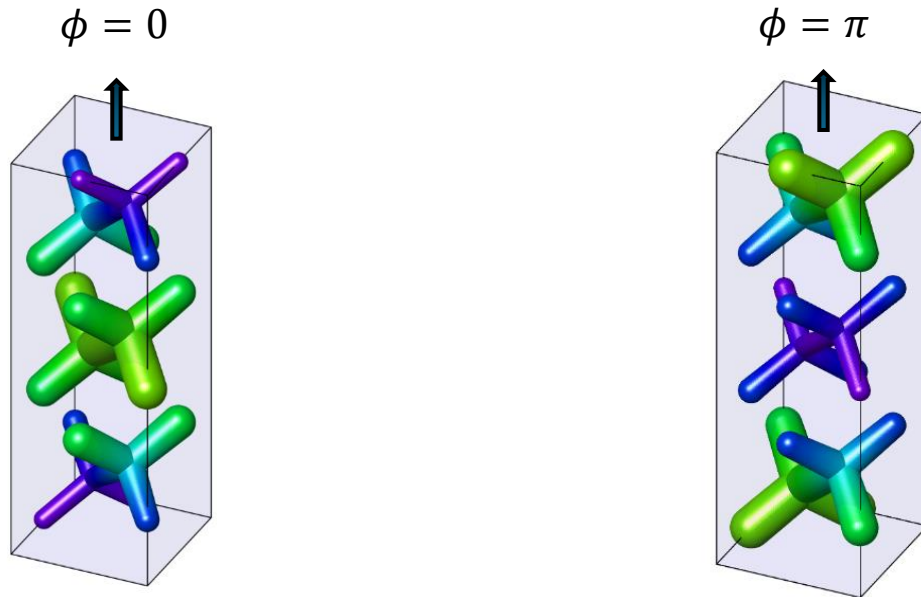


ϕ : Supercell Modulation (SM) Phase

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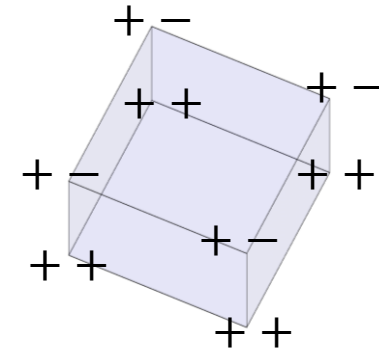
Obstructed magnetic symmetry indicators

Parity obstruction



Magnetic symmetry indicators

Phys. Rev. B 85, 165120 (2012)

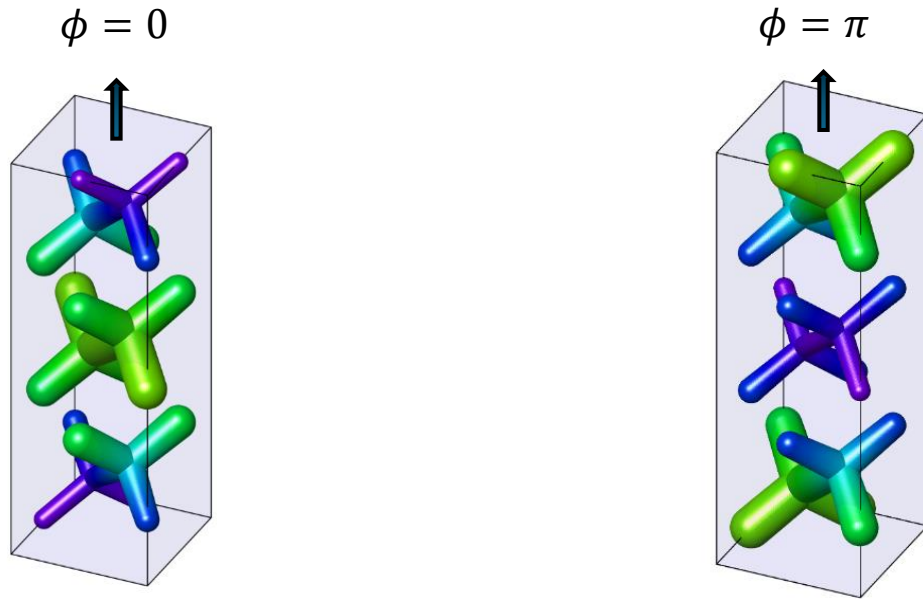


$$\nu = \{ \underbrace{\bar{z}_{2,x}, \bar{z}_{2,y}, \bar{z}_{2,z}}_{\text{Chern number parity}} \mid \underbrace{\bar{z}_4}_{\text{Weyl/Axion}} \}$$

Chern number parity Weyl/Axion

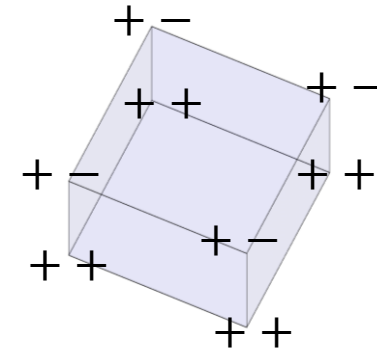
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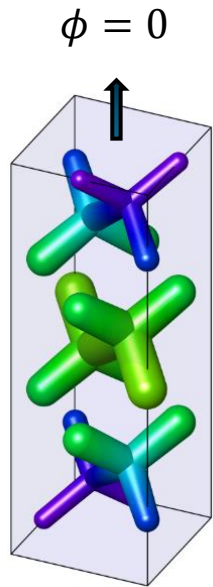
Chern number parity Weyl/Axion

Intrinsic axion insulator

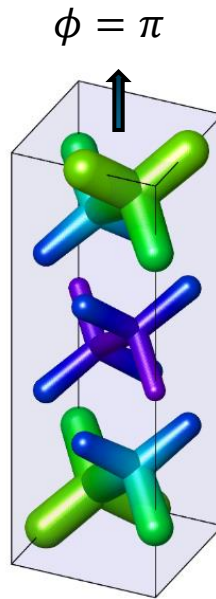
$$\nu_{AXI} = \{0,0,0|2\}$$

Obstructed magnetic symmetry indicators

Parity obstruction



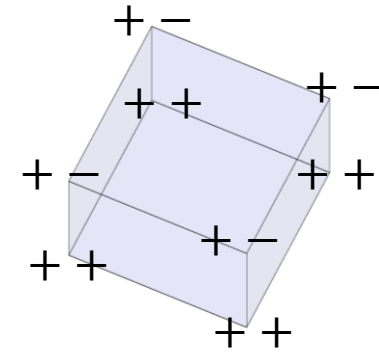
$$\nu_{\phi=0} = \{0, 0, 1 | 0\}$$



$$\nu_{\phi=\pi} = \{0, 0, 1 | 2\}$$

Magnetic symmetry indicators

Phys. Rev. B 85, 165120 (2012)



$$\nu = \{\bar{z}_{2,x}, \bar{z}_{2,y}, \bar{z}_{2,z} | \bar{z}_4\}$$

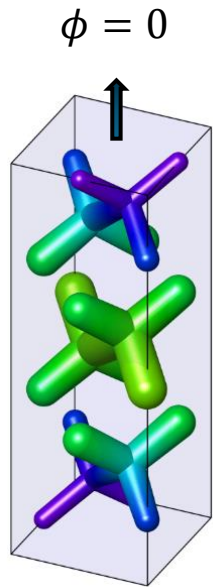
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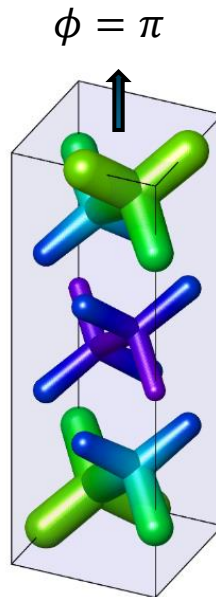
$$\nu_{AXI} = \{0, 0, 0 | 2\}$$

Obstructed magnetic symmetry indicators

Parity obstruction



$$\nu_{\phi=0} = \{0, 0, \textcolor{red}{1} | 0\}$$



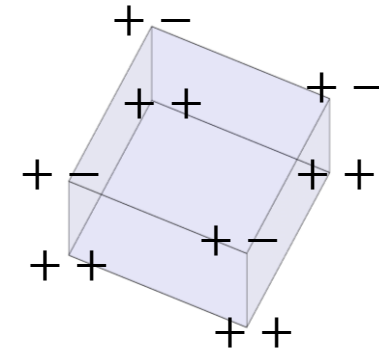
$$\nu_{\phi=\pi} = \{0, 0, \textcolor{red}{1} | 2\}$$

Relative axion topology

$$\nu_{\phi=\pi} - \nu_{\phi=0} = \{0, 0, 0 | 2\}$$

Magnetic symmetry indicators

Phys. Rev. B 85, 165120 (2012)



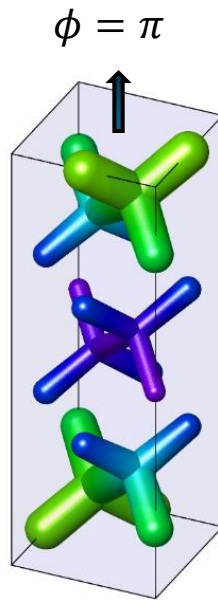
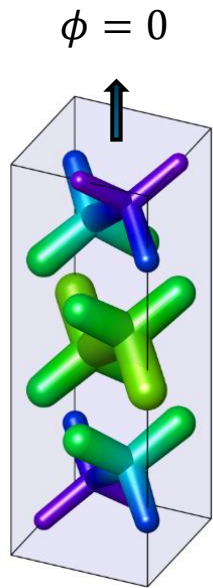
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$\underbrace{\qquad\qquad\qquad}_{\text{Chern number parity}} \quad \underbrace{\qquad}_{\text{Weyl/Axion}}$

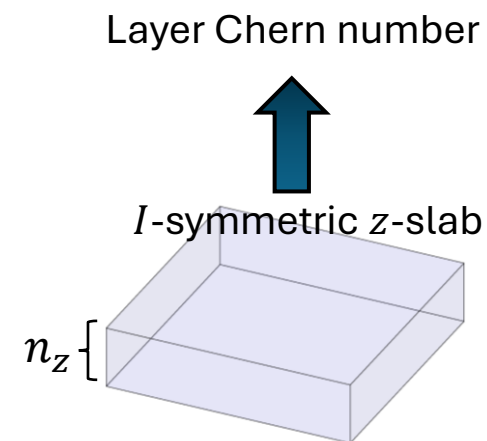
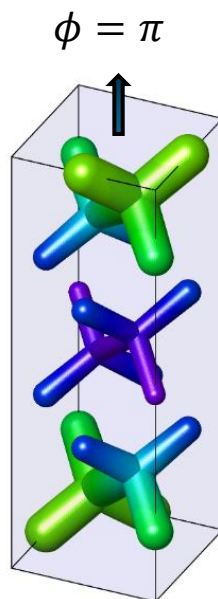
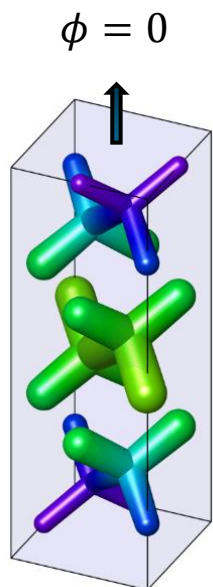
Intrinsic axion insulator

$$\nu_{AXI} = \{0, 0, 0 | 2\}$$

Relative axion angle



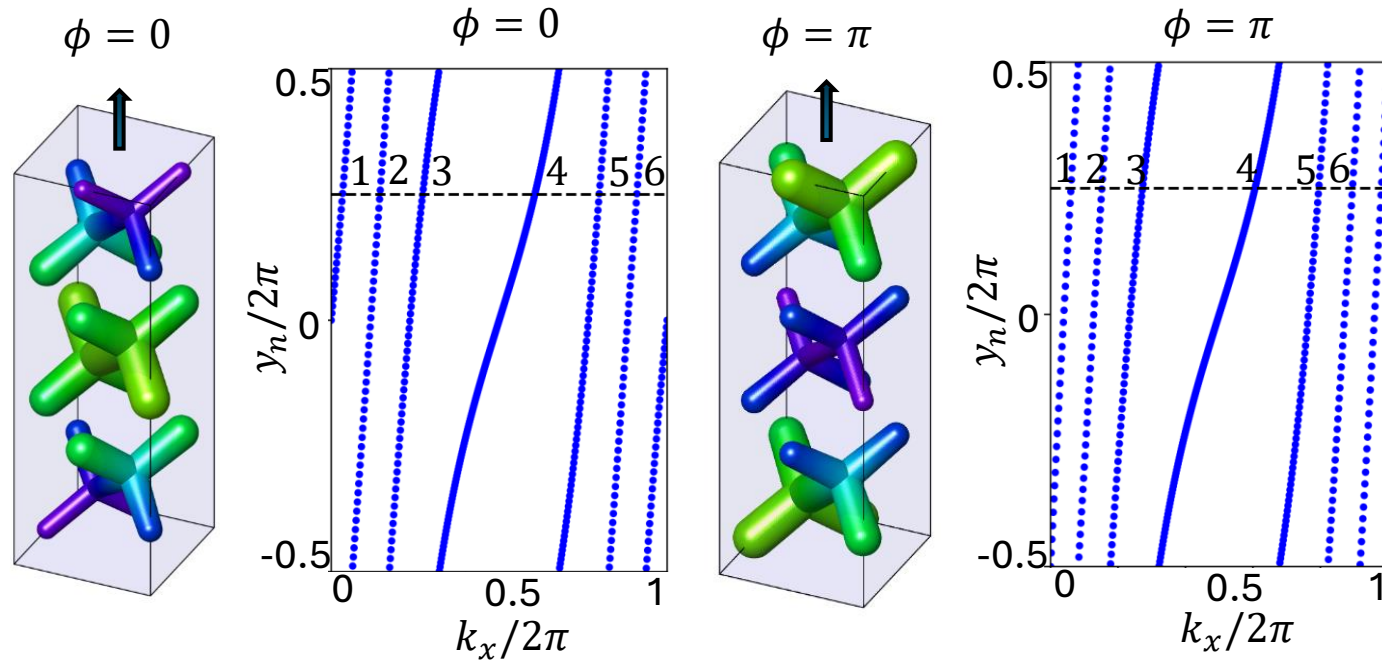
Relative axion angle



$$G_z = C_z n_z + \theta/\pi$$

Phys. Rev. B 98, 245117 (2018)
Phys. Rev. B 101, 155130 (2020)

Relative axion angle



Layer Chern number

$\uparrow$

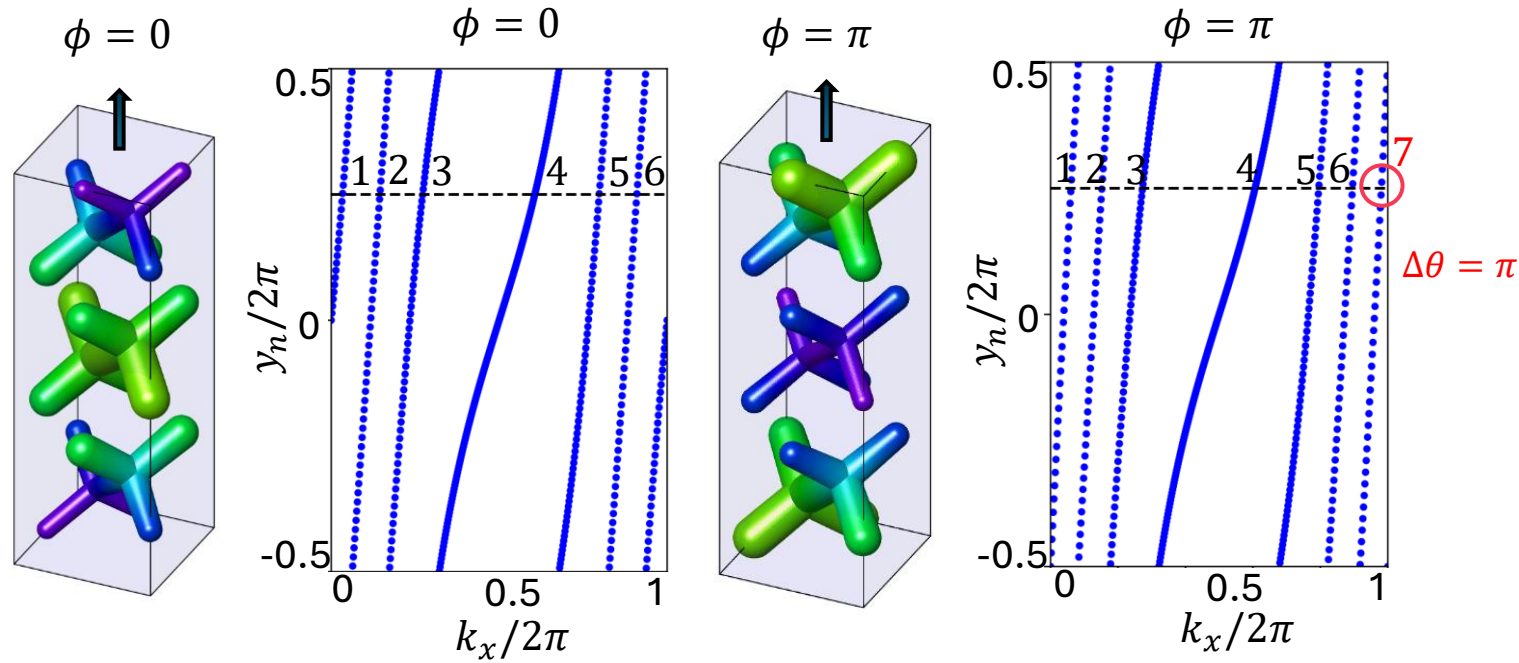
I -symmetric z -slab

$n_z \{$

$G_z = C_z n_z + \theta/\pi$

Phys. Rev. B 98, 245117 (2018)
Phys. Rev. B 101, 155130 (2020)

Relative axion angle



Layer Chern number

$\uparrow$

I -symmetric z -slab

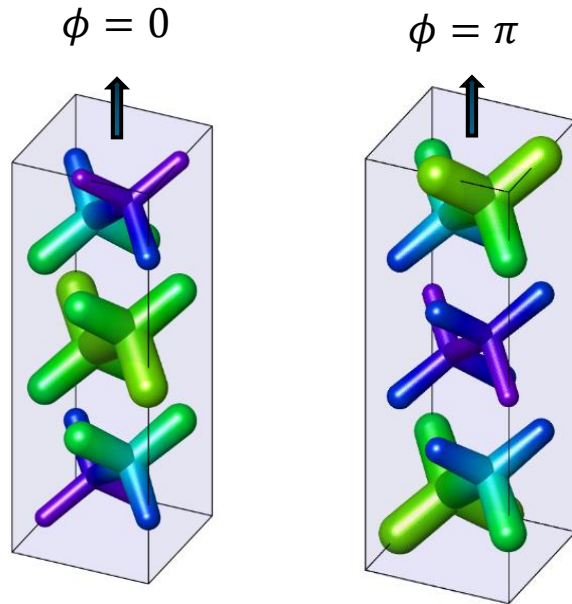
$n_z \{$

$G_z = C_z n_z + \theta/\pi$

Phys. Rev. B 98, 245117 (2018)
Phys. Rev. B 101, 155130 (2020)

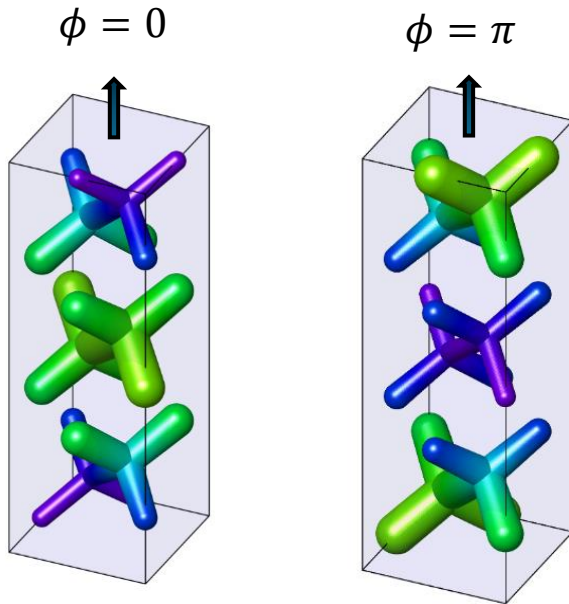
Making relative AXI topology manifest

Relative AXI

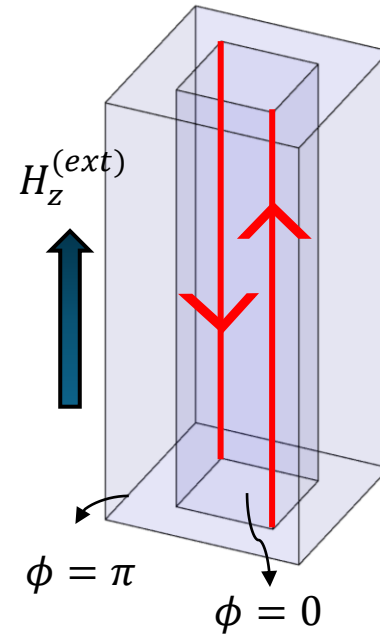


Making relative AXI topology manifest

Relative AXI



Phase-obstructed rod-like geometry

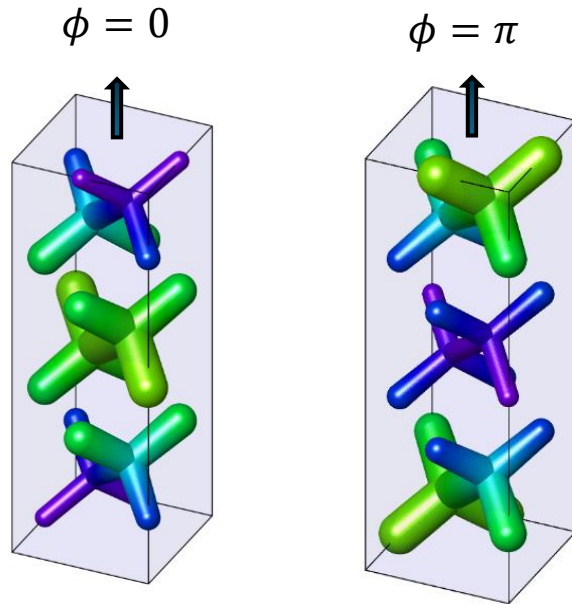


Chiral channels embedded in the bulk

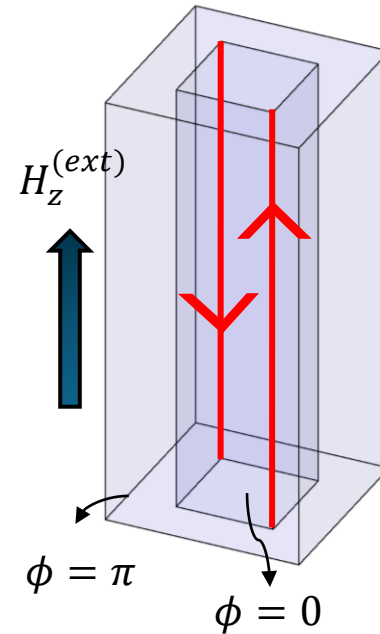
- Protected against radiation the EM continuum
 - Fully 3D connected PhC geometry

Making relative AXI topology manifest

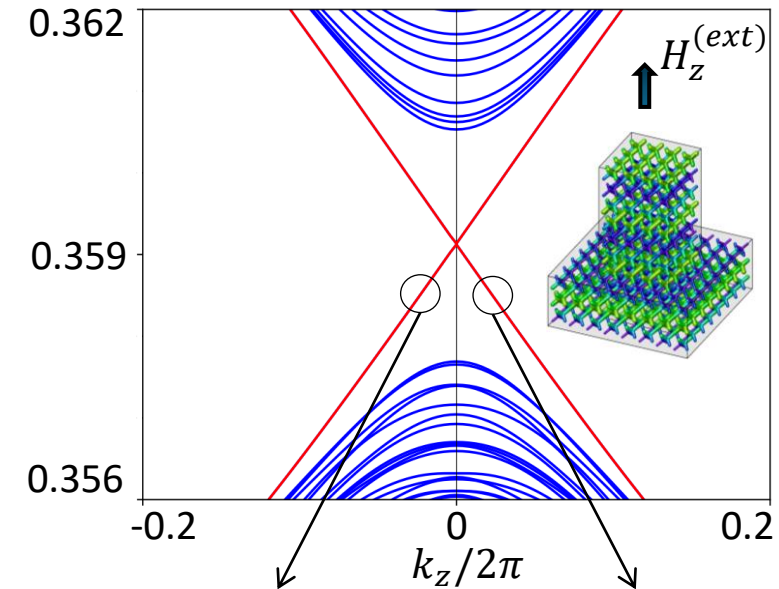
Relative AXI



Phase-obstructed rod-like geometry

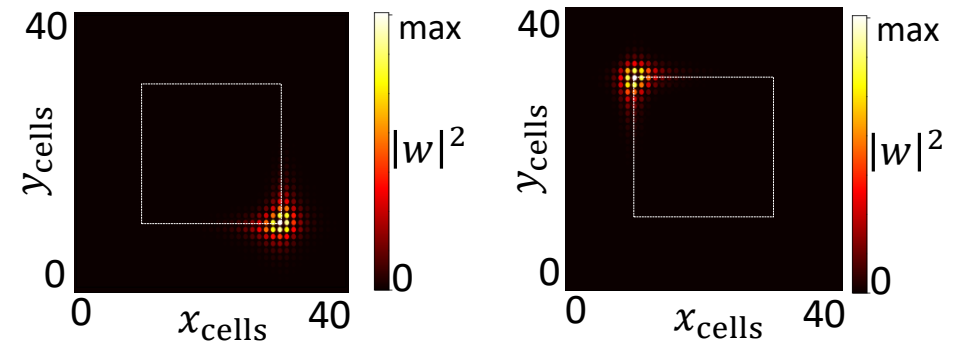


Phase-obstructed domain-walls



Chiral channels embedded in the bulk

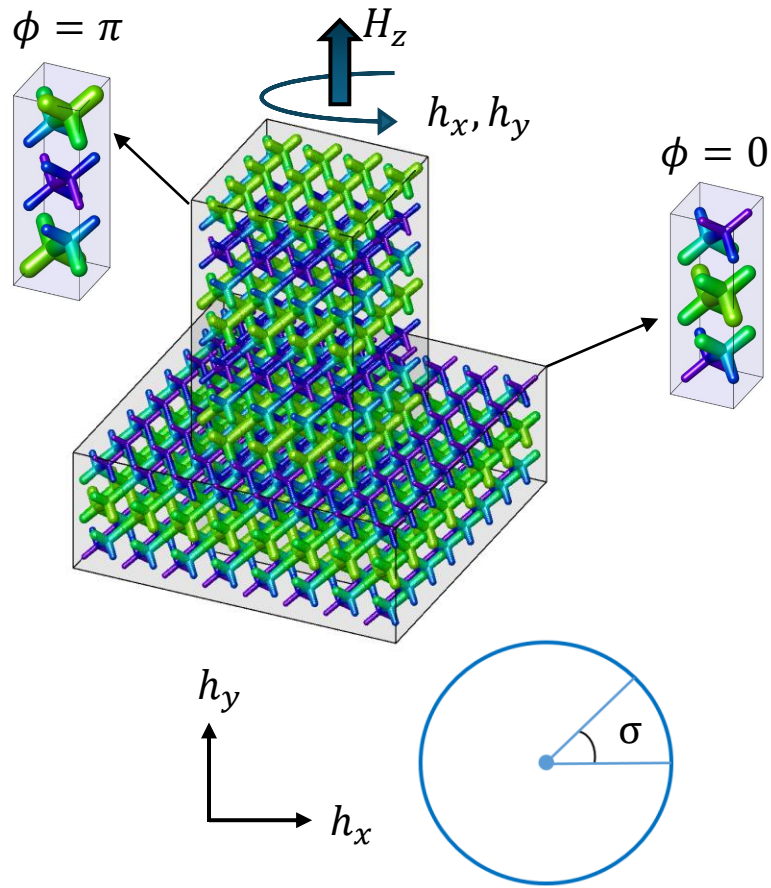
- Protected against radiation the EM continuum
 - Fully 3D connected PhC geometry



External control of the hinge localization

External control of the hinge localization

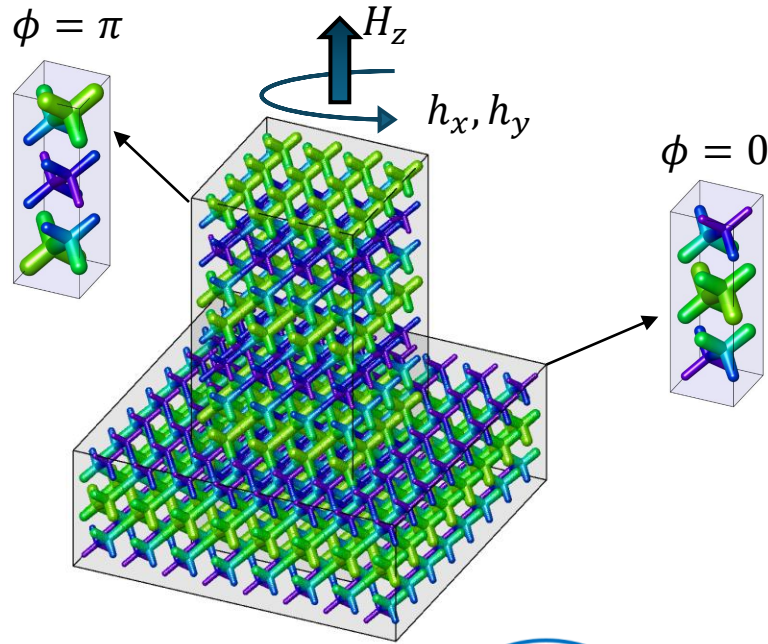
Manipulation of the axion chiral channels



$$\mathbf{H}^{(ext)} = (|h|\cos(\sigma), |h|\sin(\sigma), H_z)$$

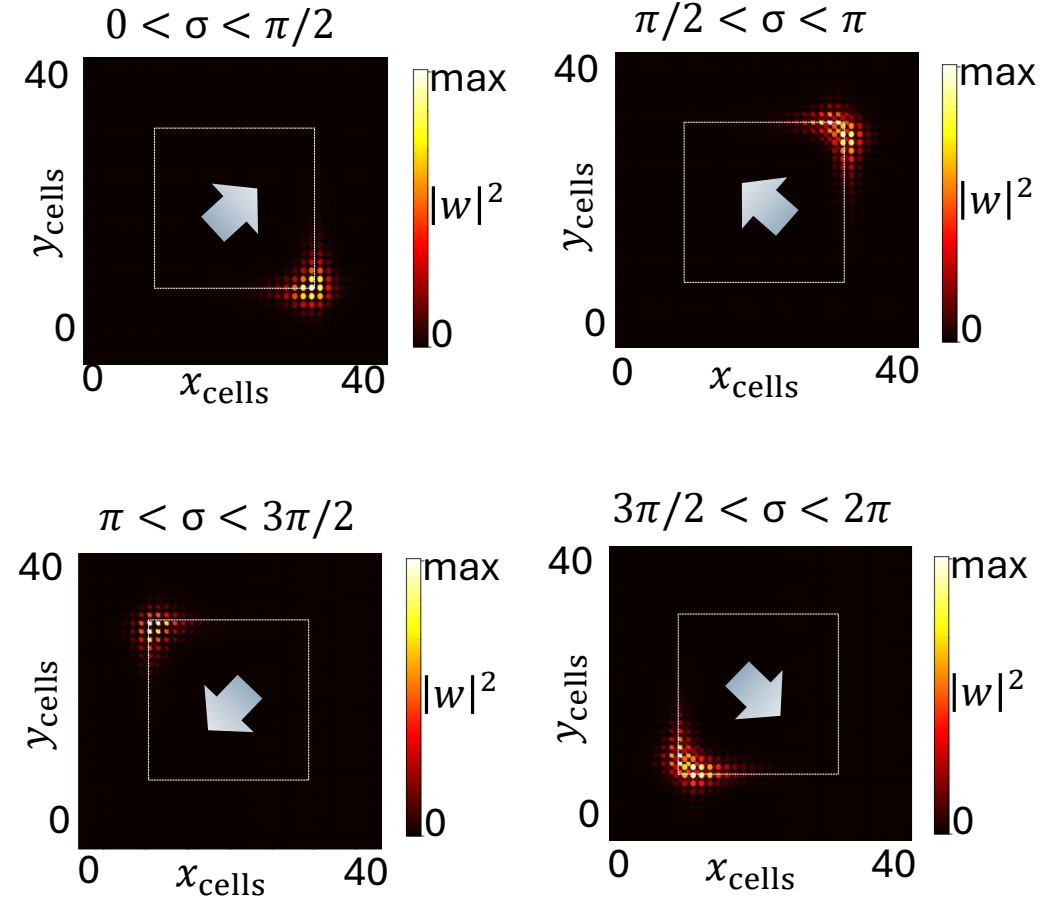
External control of the hinge localization

Manipulation of the axion chiral channels



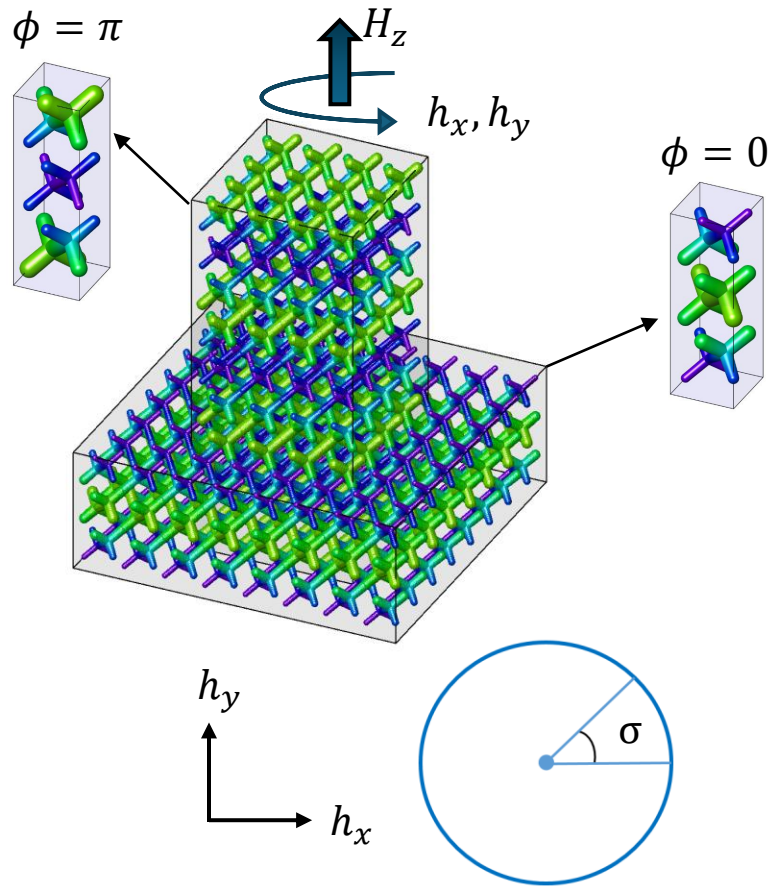
$$\mathbf{H}^{(ext)} = (|h|\cos(\sigma), |h|\sin(\sigma), H_z)$$

$v_z > 0$ upwards moving:

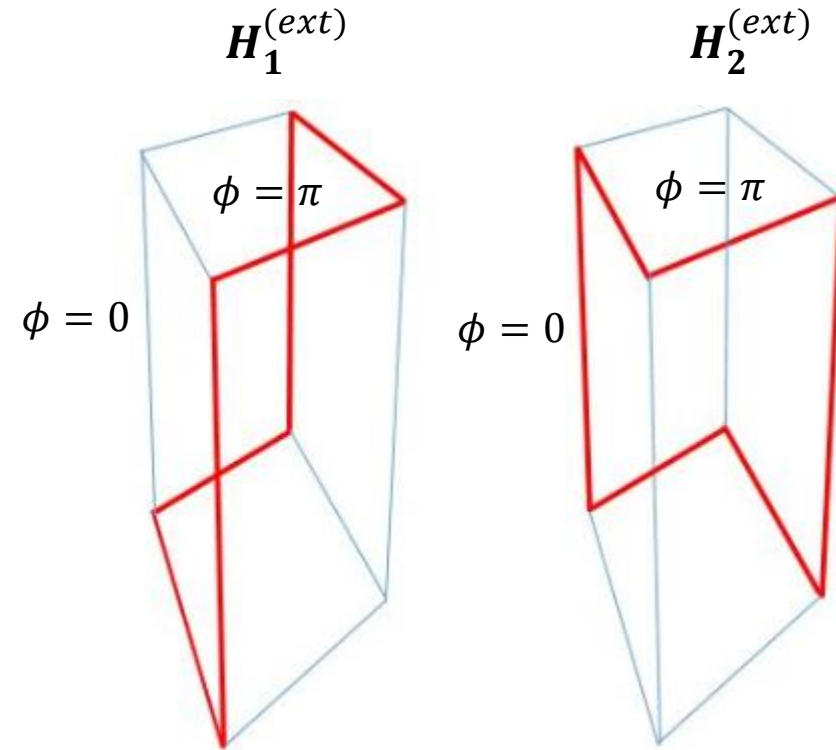


Gyrotropy-induced switching of AXI states

Manipulation of the axion chiral channels

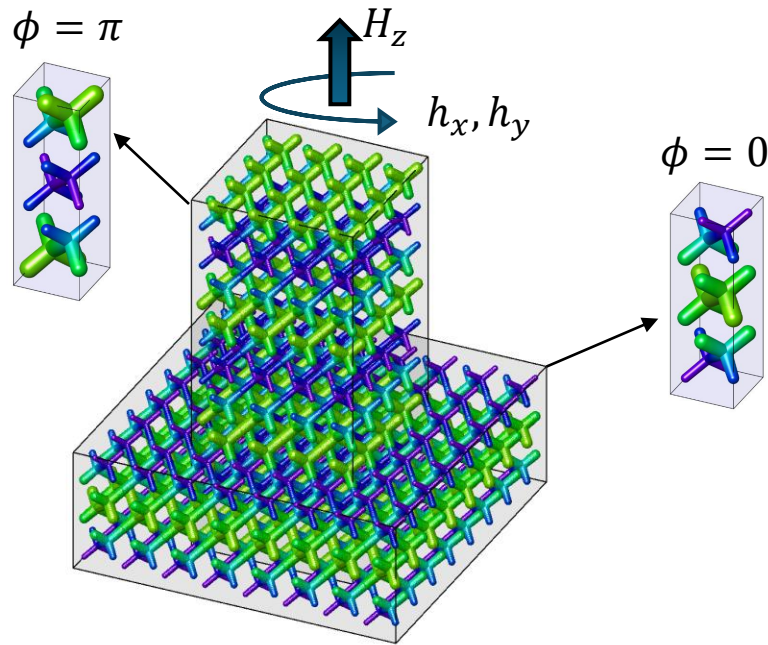


$$\mathbf{H}^{(ext)} = (|h|\cos(\sigma), |h|\sin(\sigma), H_z)$$

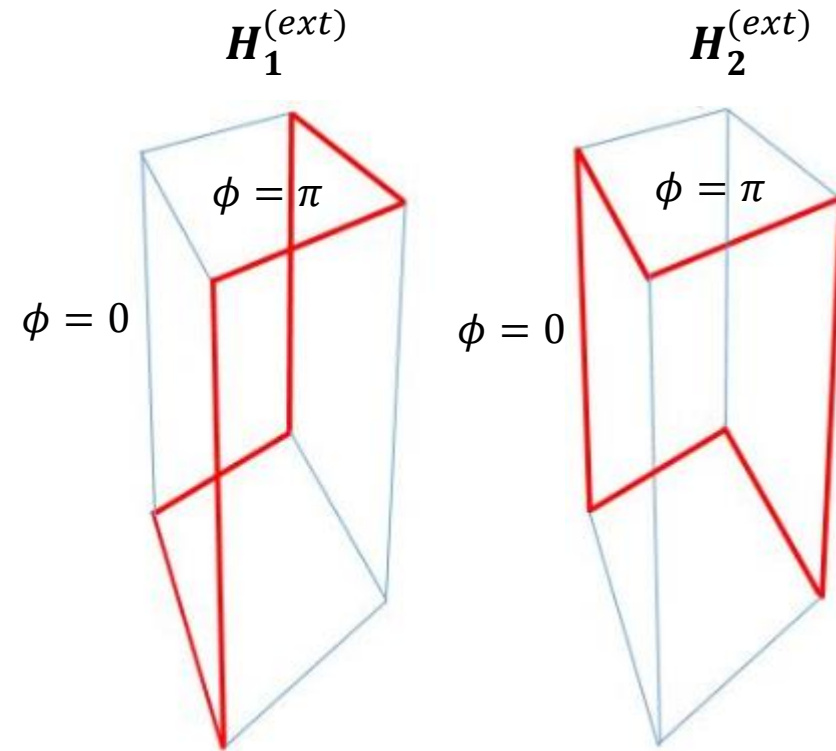


Gyrotropy-induced switching of AXI states

Manipulation of the axion chiral channels

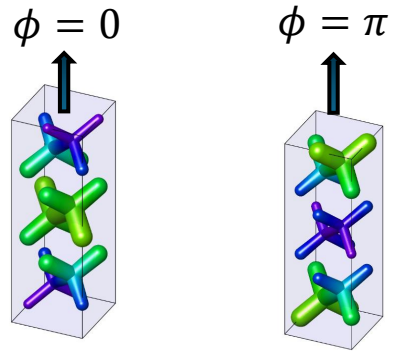


$$\vec{\epsilon} = \begin{pmatrix} \epsilon_{xx} & i\eta_z & -i\eta_y \\ -i\eta_z & \epsilon_{yy} & i\eta_x \\ i\eta_y & -i\eta_x & \epsilon_{zz} \end{pmatrix}$$



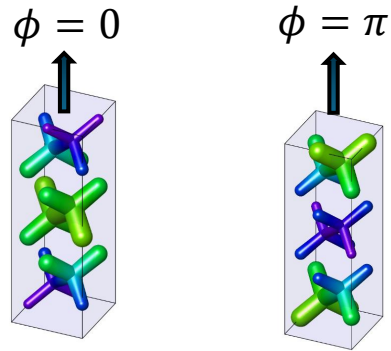
So far...

So far...

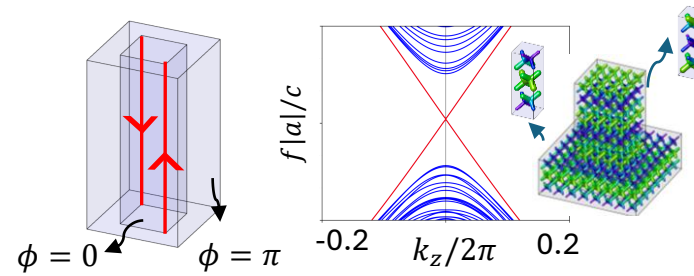


- Dielectric SM induces EM energy redistribution and opening of a relative axion gap

So far...

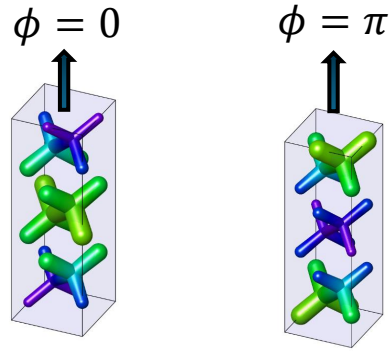


- Dielectric SM induces EM energy redistribution and opening of a relative axion gap

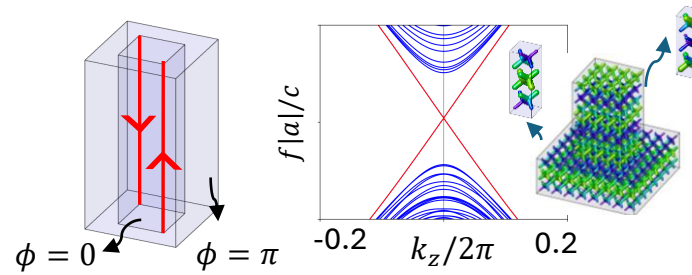


- Manifest axion topology at phase-obstructed domain walls

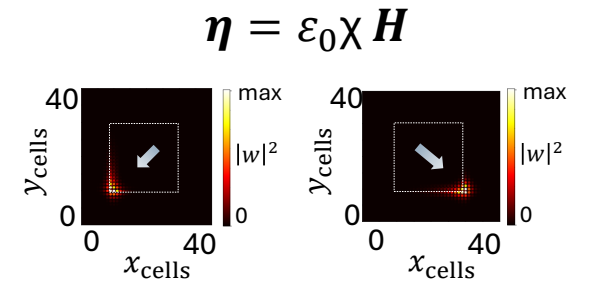
So far...



- Dielectric SM induces EM energy redistribution and opening of a relative axion gap



- Manifest axion topology at phase-obstructed domain walls



- Propose a physically accessible way to control of the chiral channels of light via gyromagnetism

Outline

- Context:
 - Axion Insulator (AXI)
 - Photonic crystals (PhC)
 - Why do we seek an AXI in a PhC

1- From 3D Chern insulators to Relative AXI

2- From Relative to Intrinsic AXI

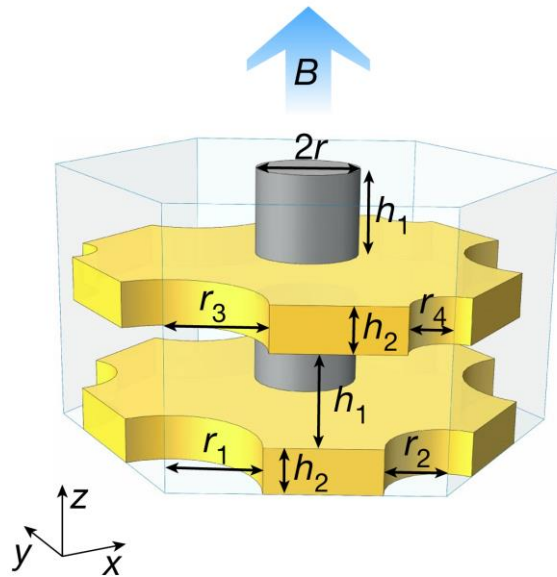
- Outlook and conclusions

From relative to intrinsic axion

From relative to intrinsic axion

External Magnetic field

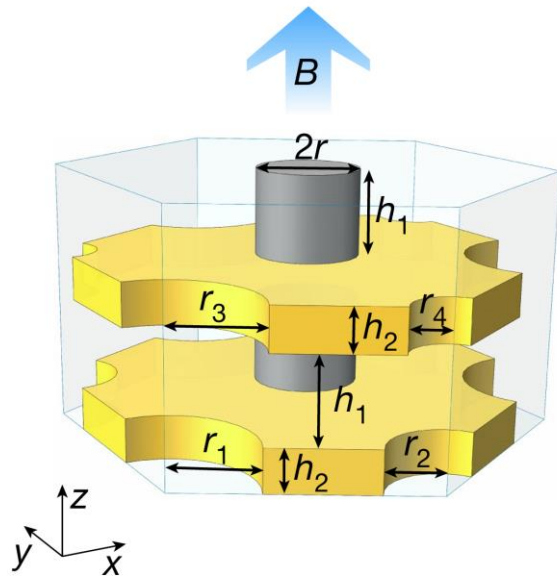
- YIG gyromagnetic cylinders
- Metallic plates



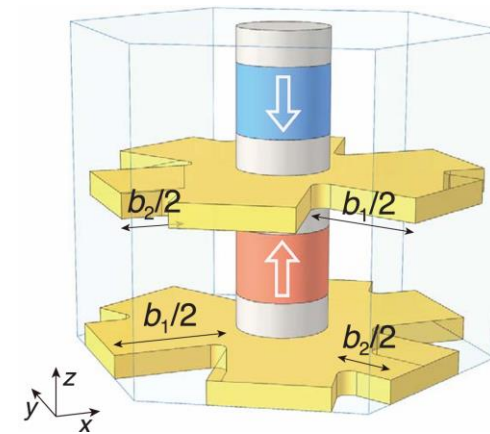
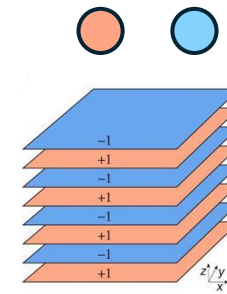
From relative to intrinsic axion

External Magnetic field

- YIG gyromagnetic cylinders
- Metallic plates



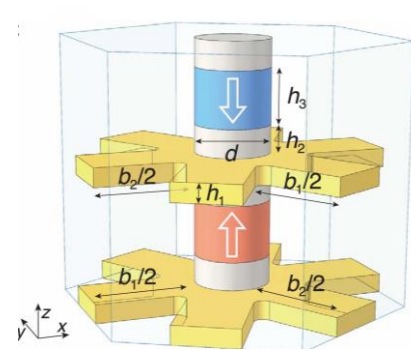
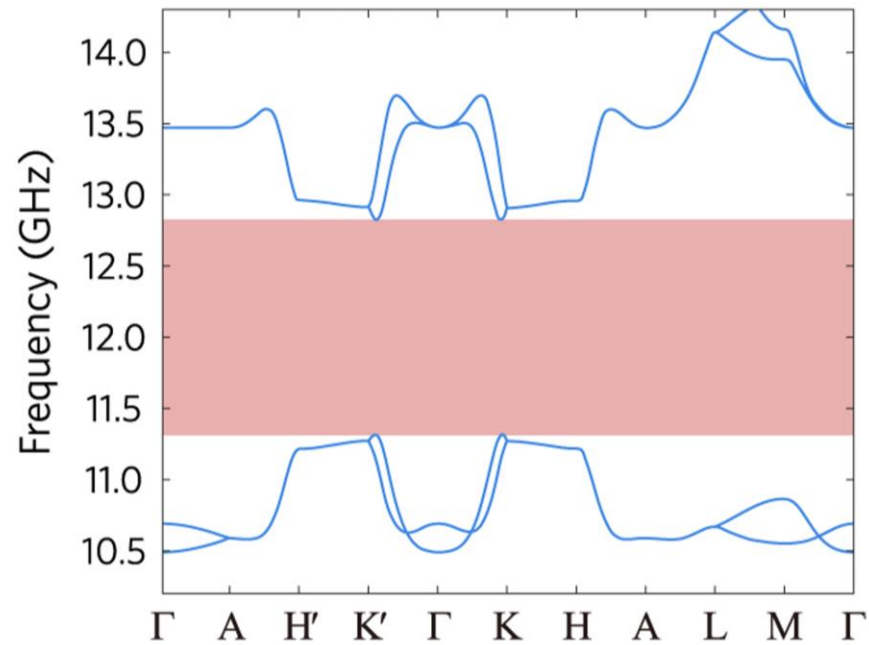
Alternating permanent magnets



Net zero Chern

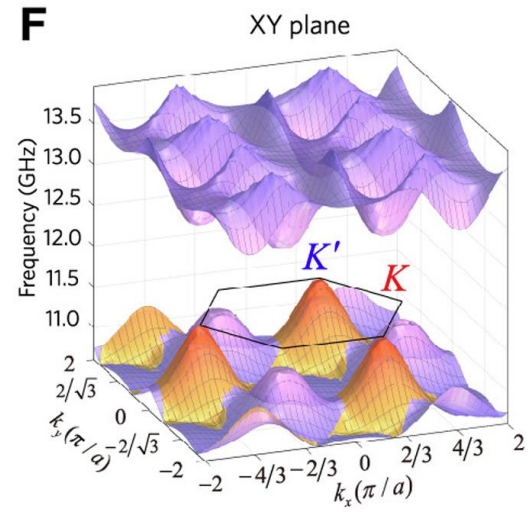
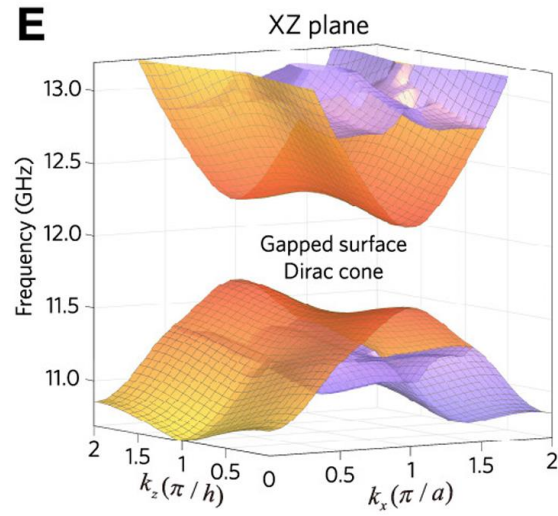
Gapped bulk

$b_1=10$ mm; $b_2=5$ mm

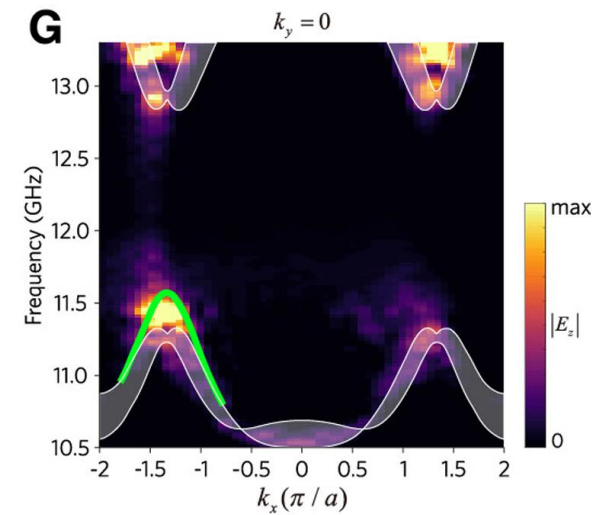
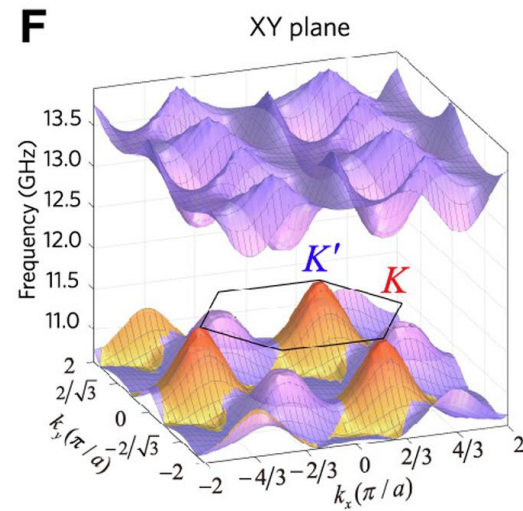
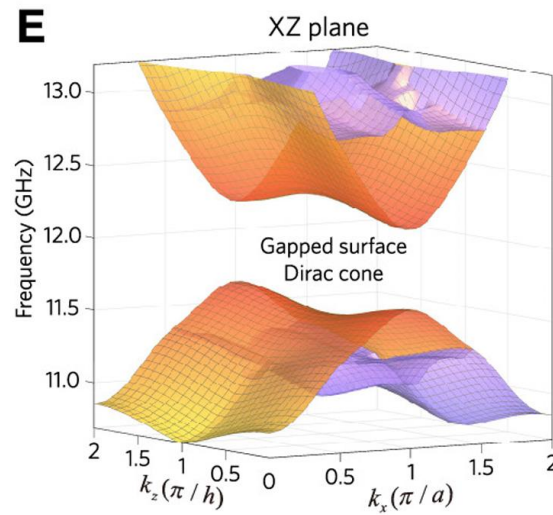


Gapped surface

Gapped surface

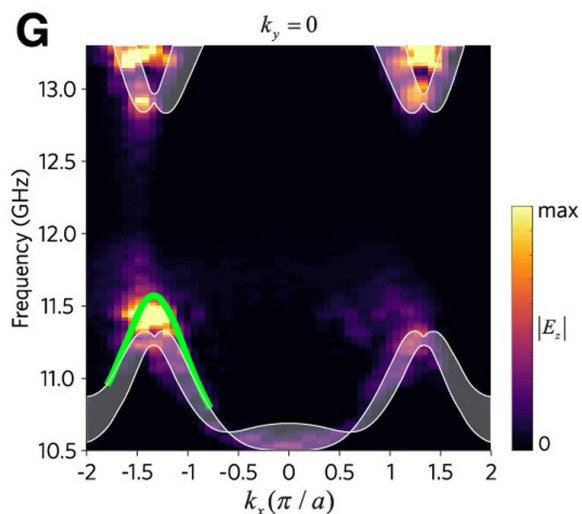
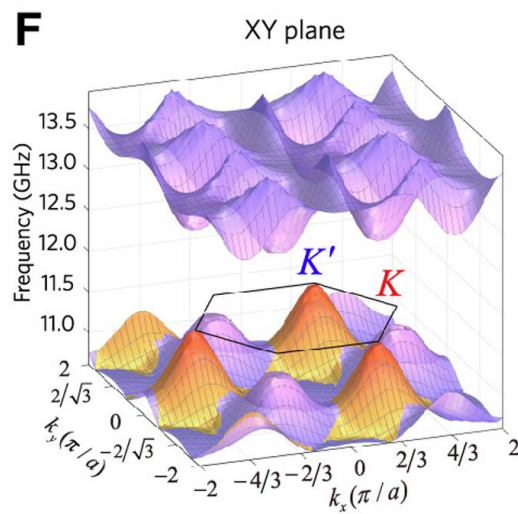
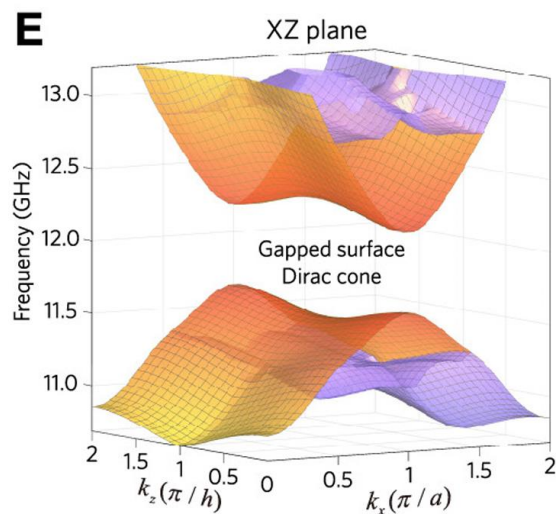


Gapped surface

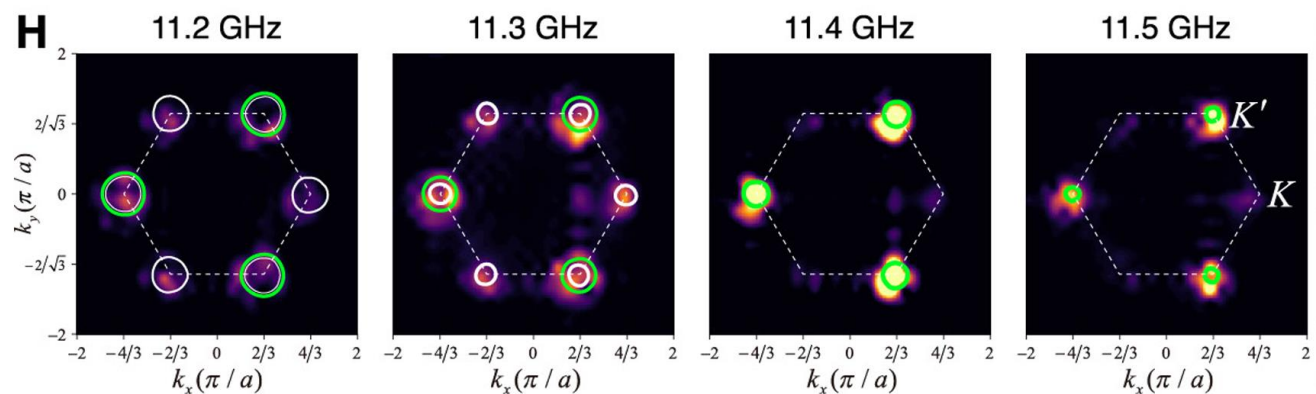


Top surface measurement

Gapped surface

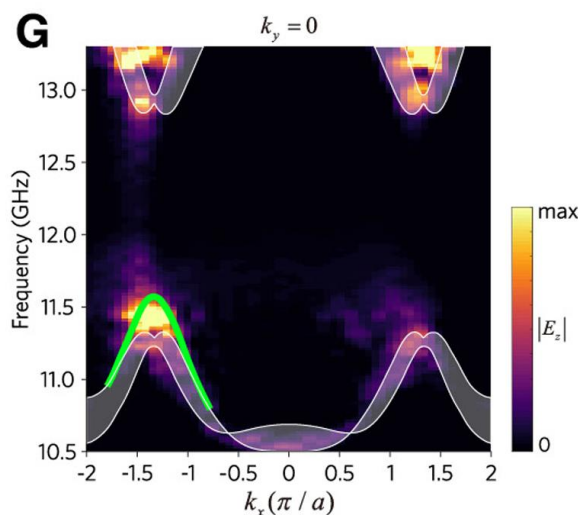
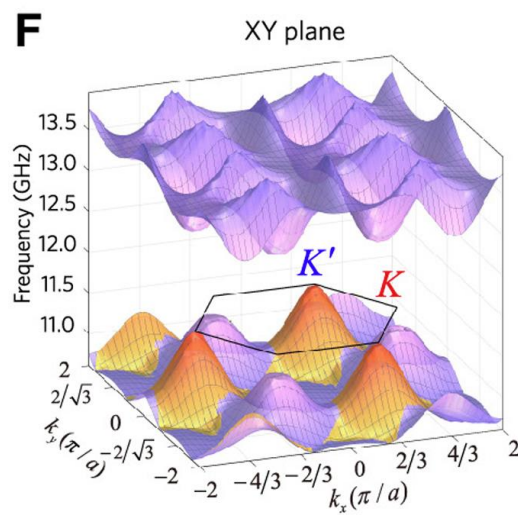
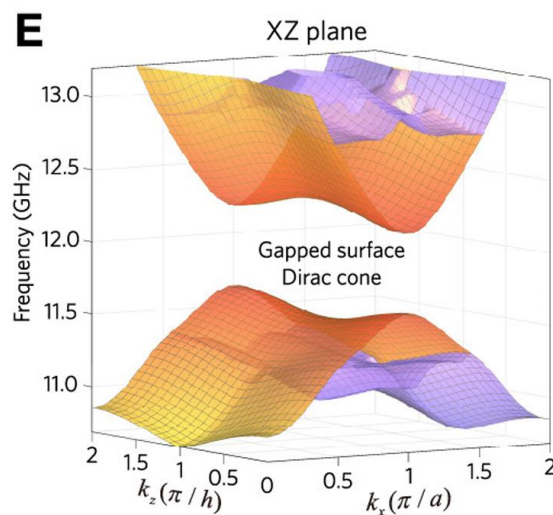


Top surface measurement

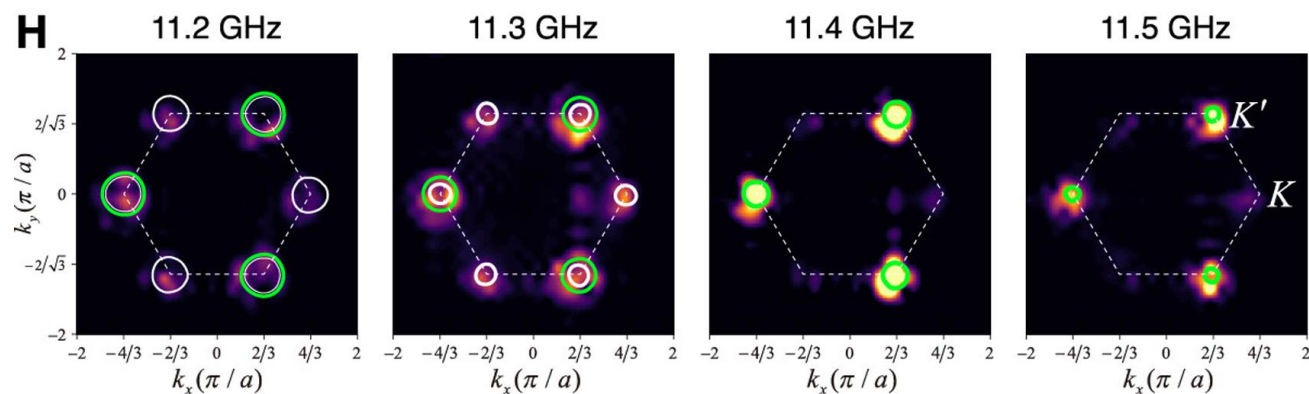


only one valley for surface state

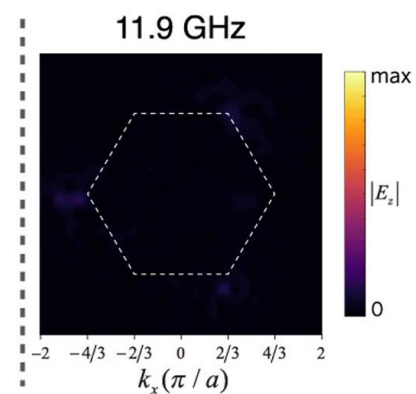
Gapped surface



Top surface measurement

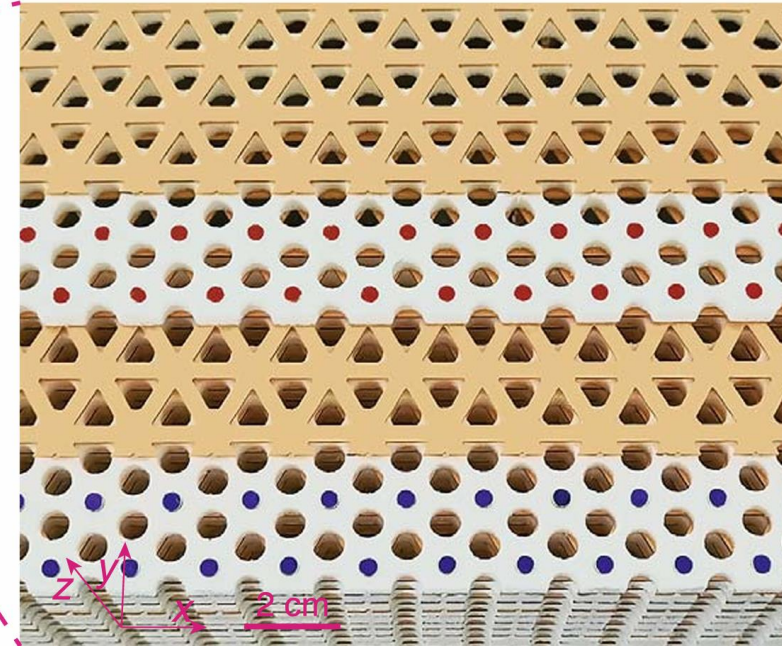
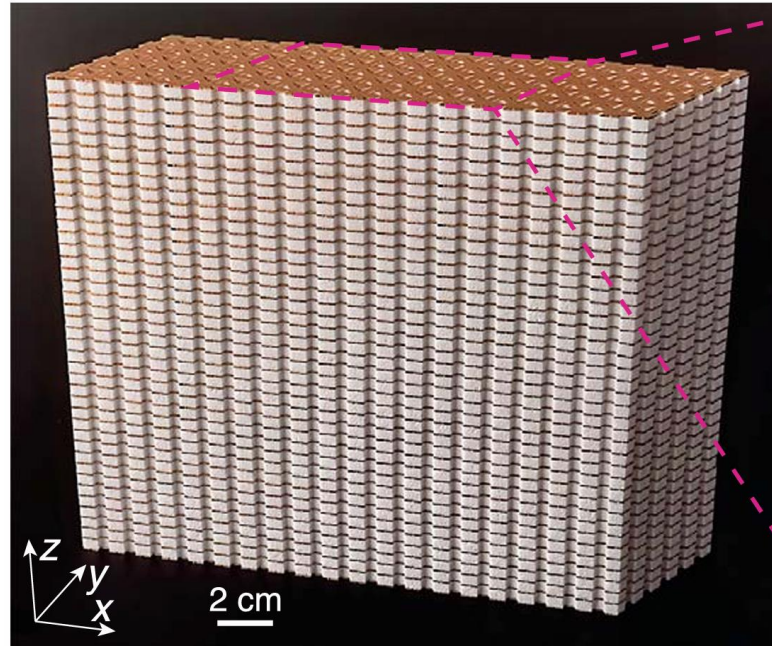


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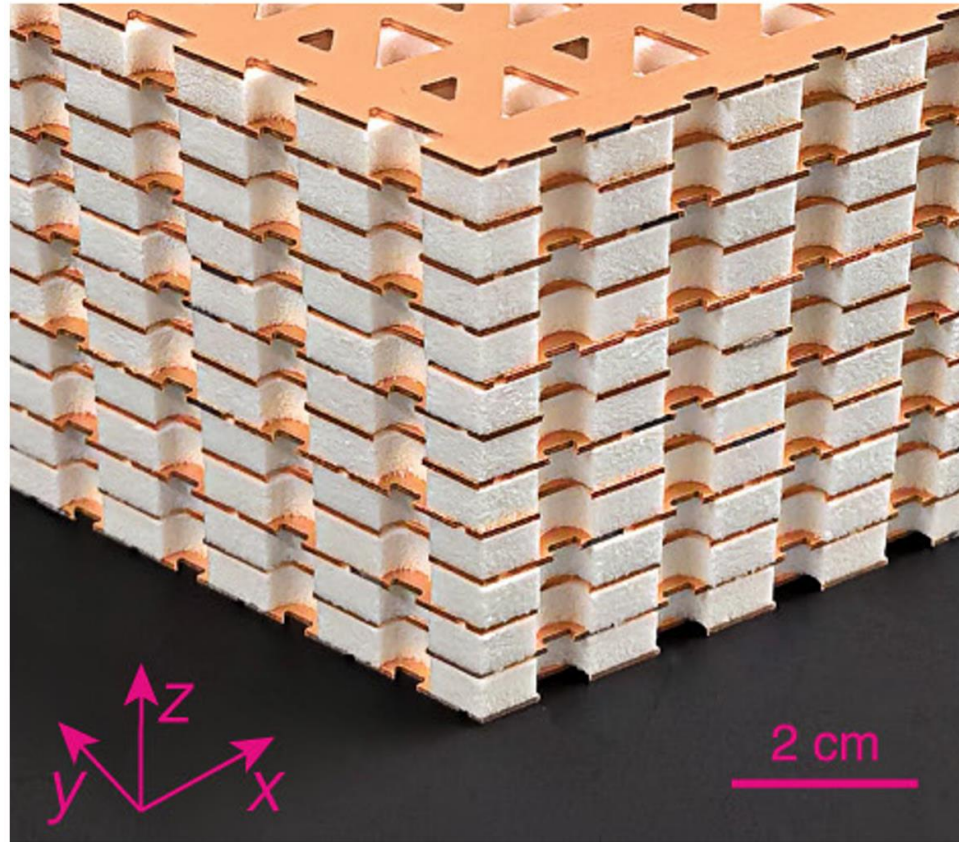


frequency used to observe hinge states

Finite 3D PhC

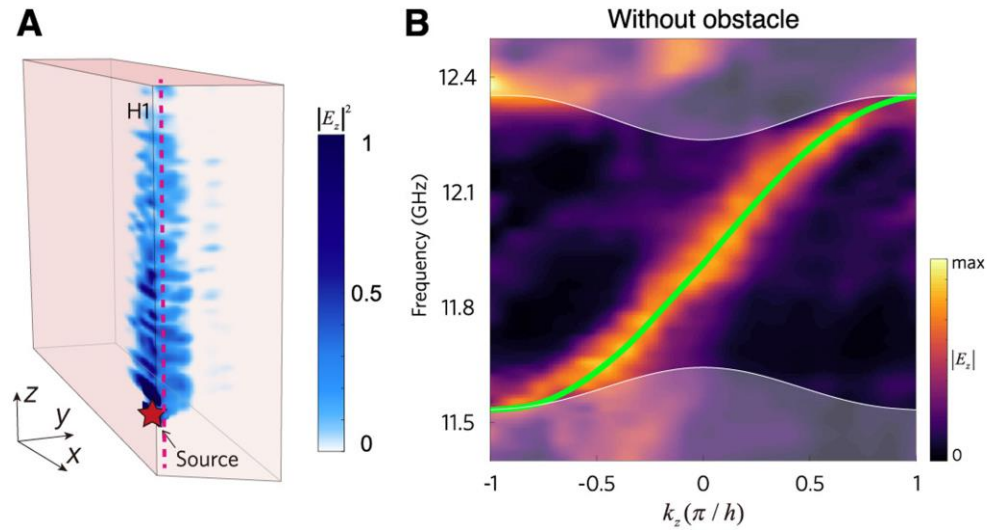


Hinge close up

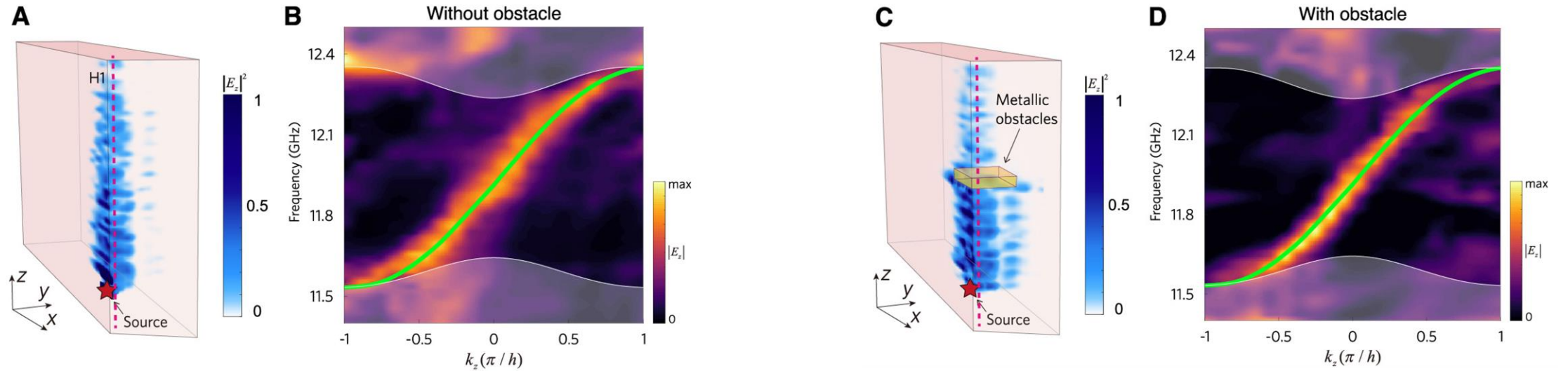


Hinge states robust to metallic obstacles

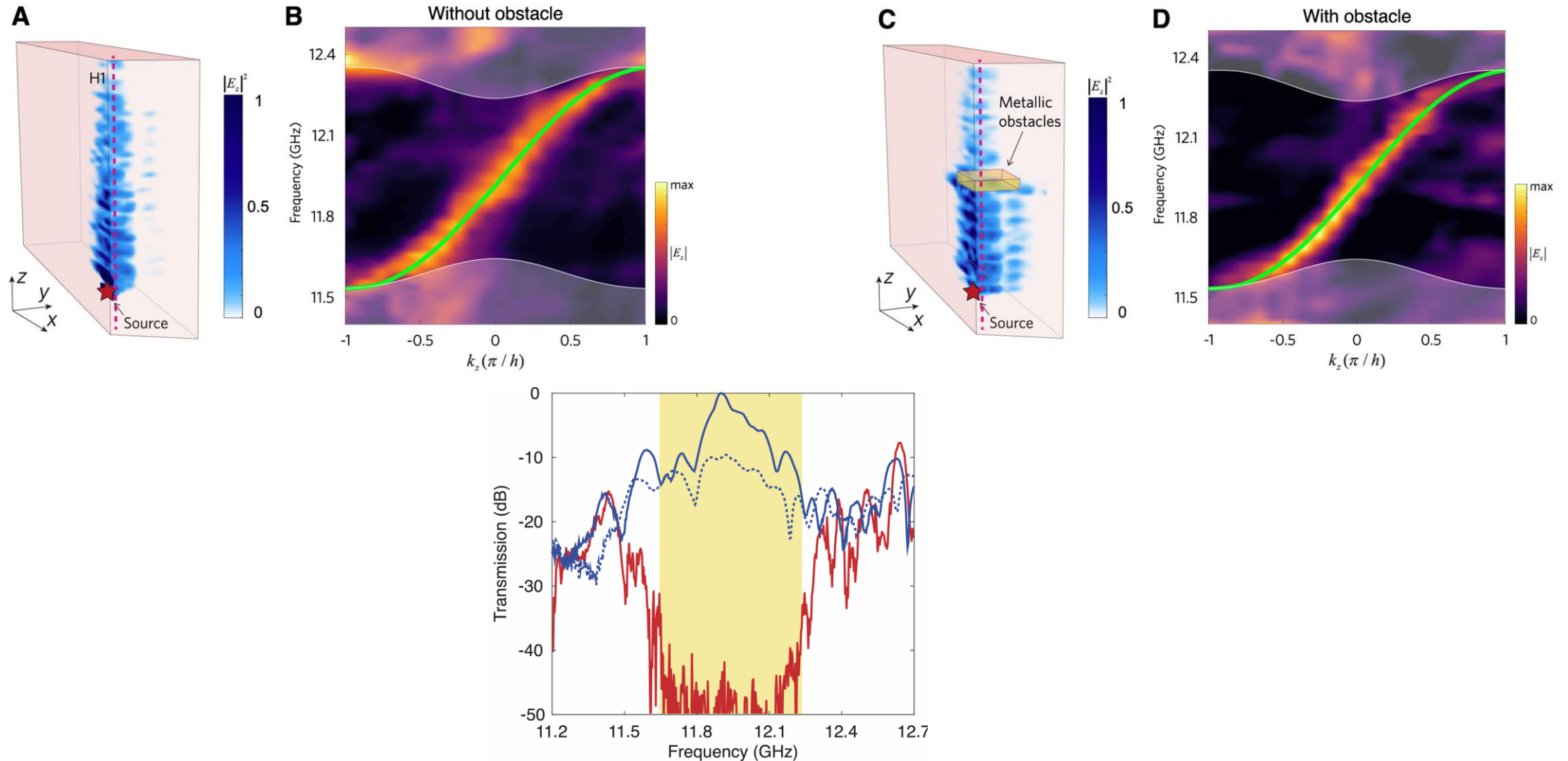
Hinge states robust to metallic obstacles



Hinge states robust to metallic obstacles

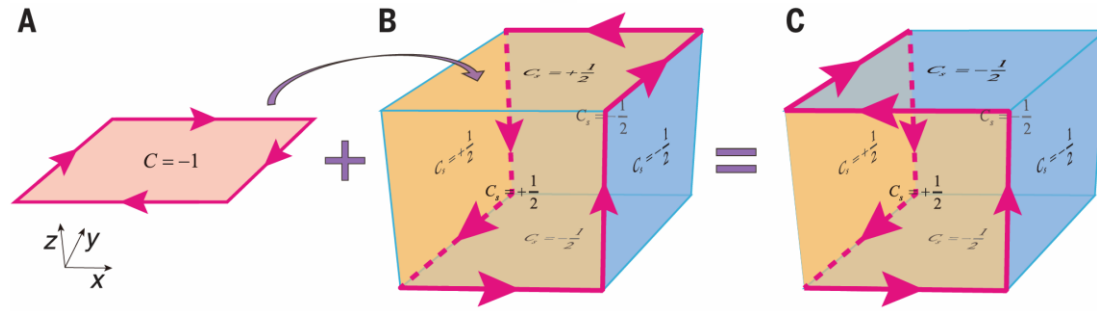


Hinge states robust to metallic obstacles

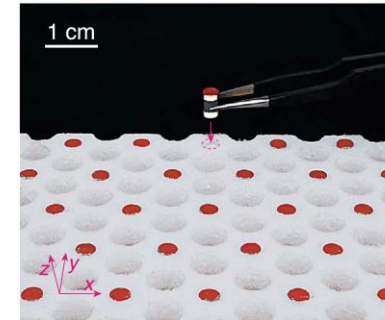
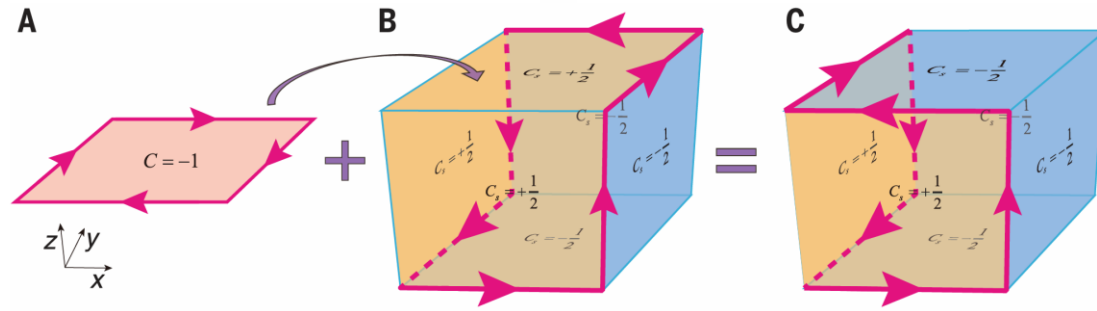


Even/odd layers: half-quantized Chern numbers

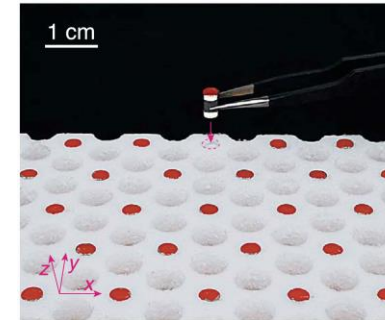
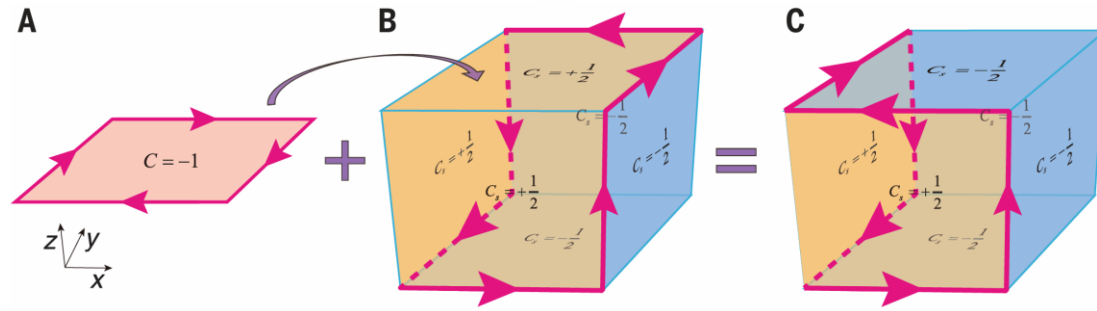
Even/odd layers: half-quantized Chern numbers



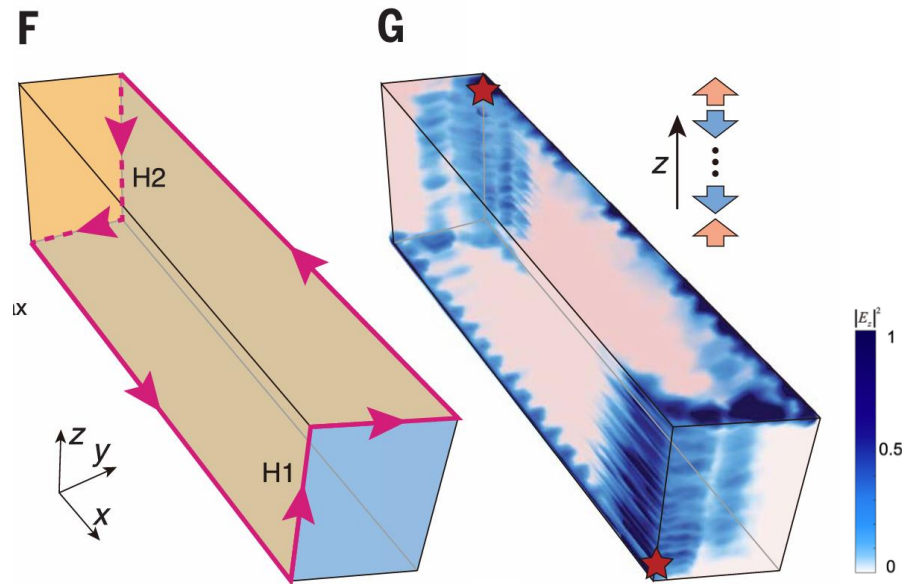
Even/odd layers: half-quantized Chern numbers



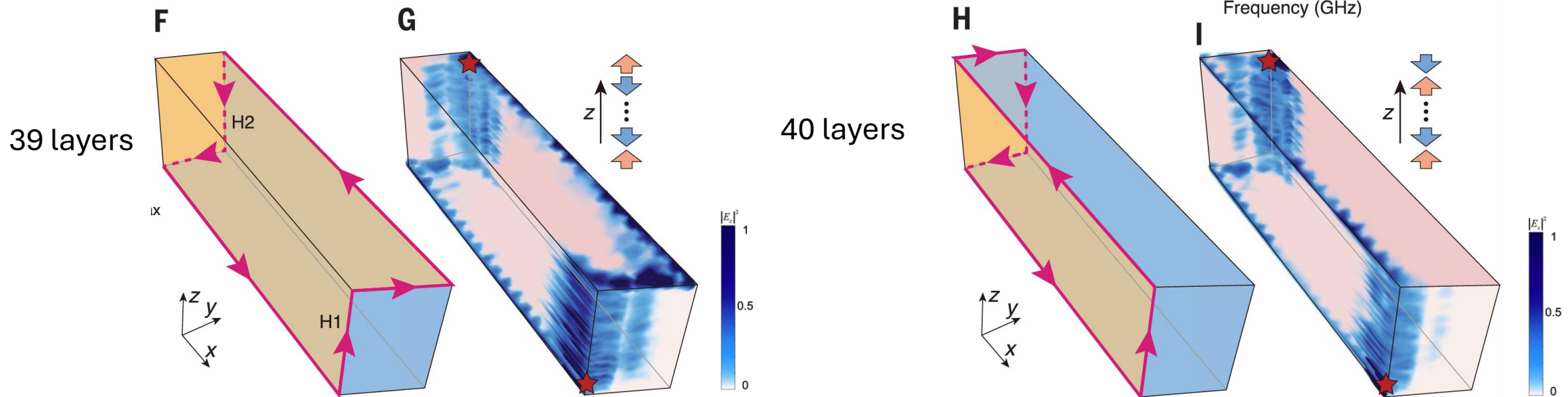
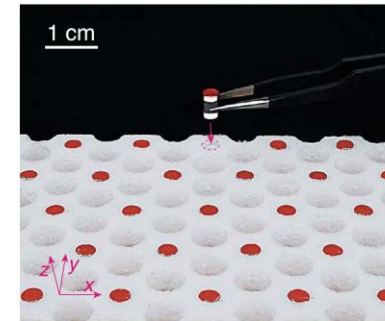
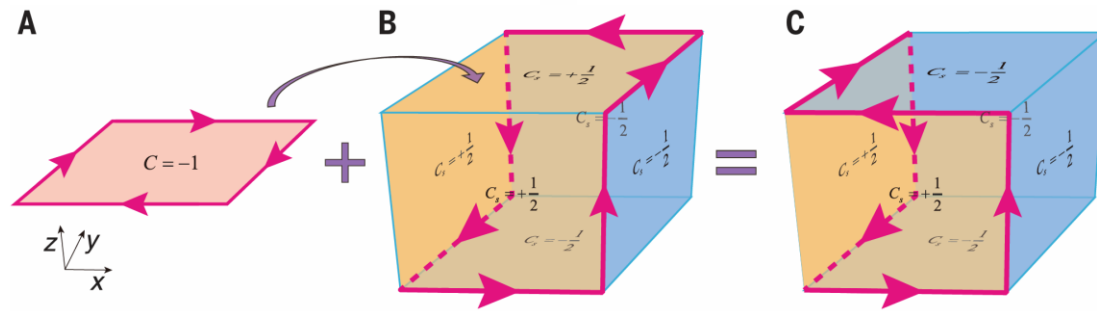
Even/odd layers: half-quantized Chern numbers



39 layers



Even/odd layers: half-quantized Chern numbers



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1- From 3D Chern insulators to Relative AXI

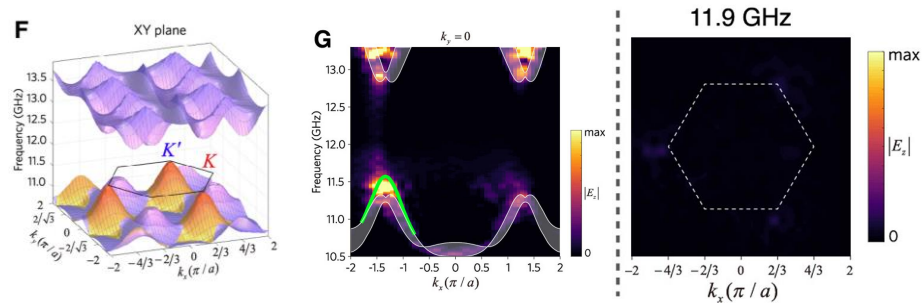
2- From Relative to Intrinsic AXI

- **Outlook and conclusions**

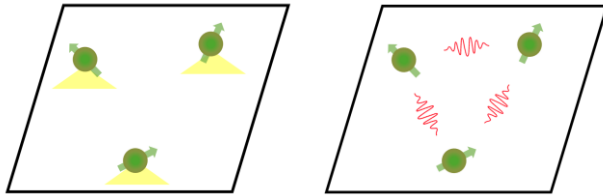
Surface topological photonic gaps

Surface topological photonic gaps

Surface photonic gap with non-trivial topological properties:

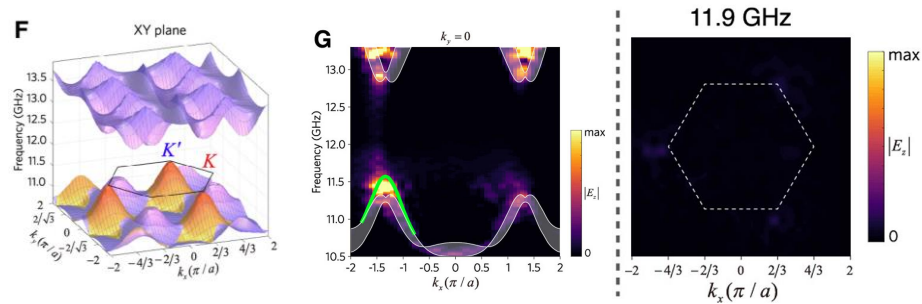


Coupling of emitters to the surface:



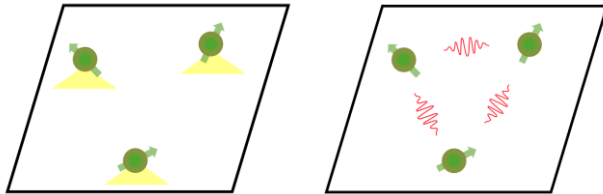
Surface topological photonic gaps

Surface photonic gap with non-trivial topological properties:



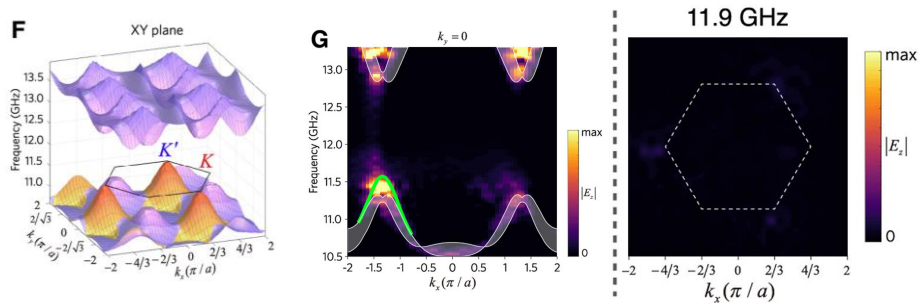
In contrast to coupling of QE in the bulk of....e.g.

Coupling of emitters to the surface:

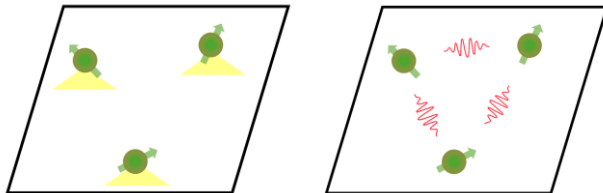


Surface topological photonic gaps

Surface photonic gap with non-trivial topological properties:

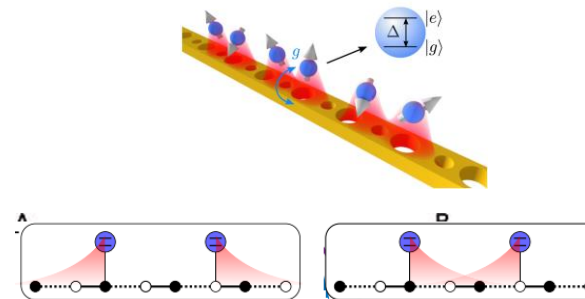


Coupling of emitters to the surface:



In contrast to coupling of QE in the bulk of....e.g.

a SSH Waveguide

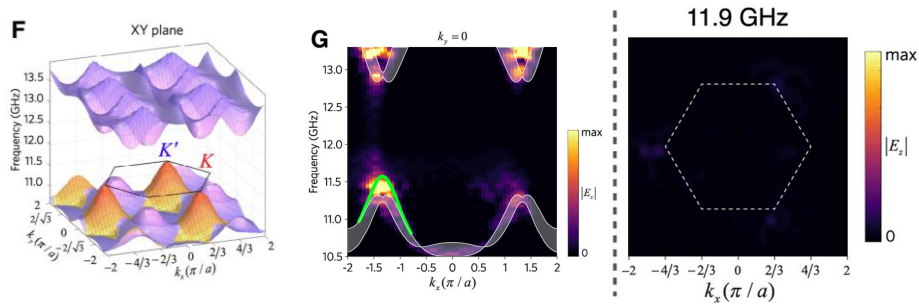


Single QE: chiral bound state
Many emitters: BS mediates
topological, tunable interactions

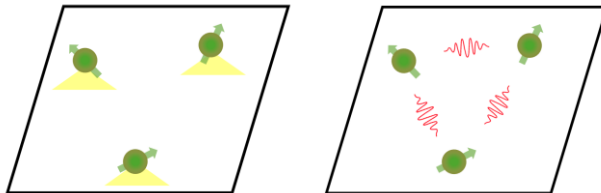
Bello M, Platero G, Cirac JJ, González-Tudela A.
Unconventional quantum optics in topological
waveguide QED. Science advances. 2019

Surface topological photonic gaps

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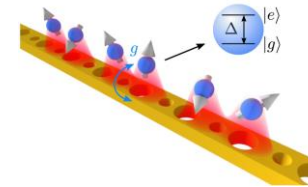


Coupling of emitters to the surface:



In contrast to coupling of QE in the bulk of....e.g.

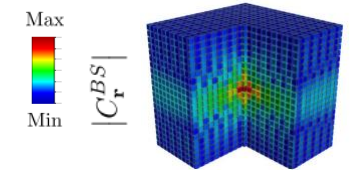
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Bello M, Platero G, Cirac JJ, González-Tudela A.
Unconventional quantum optics in topological
waveguide QED. Science advances. 2019

a Weyl Photonic Crystal



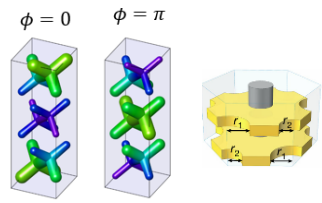
$$V_{eff} \propto \frac{1}{r^\alpha}$$

Single QE: Weyl bound state
Many emitters: BS mediates
coherent, long-range interactions

Iñaki García-Elcano, Alejandro González-Tudela,
and Jorge Bravo-Abad. Tunable and robust long-
range coherent interactions between quantum
emitters mediated by weyl bound states. Physical
Review Letters, 125(16):163602, 2020.

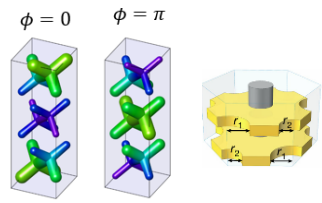
Summary

Summary

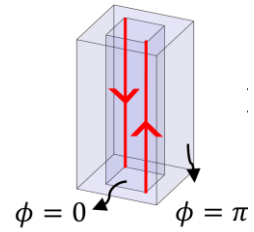


Supercell dielectric ~
inducing EM energy
redistribution

Summary

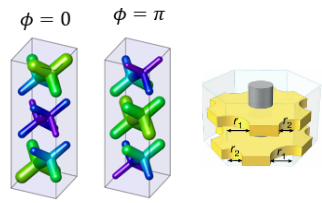


Supercell dielectric ~
inducing EM energy
redistribution

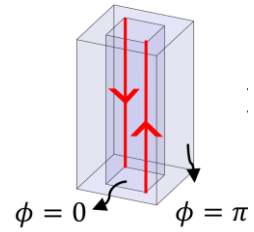


Axion hinge states
manifesting at the
domain wall

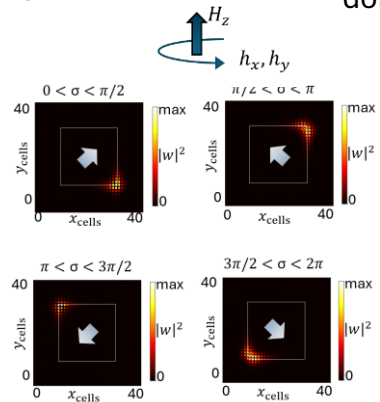
Summary



Supercell dielectric $\sim$
inducing EM energy
redistribution

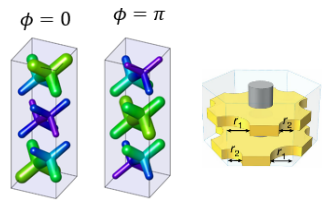


Axion hinge states
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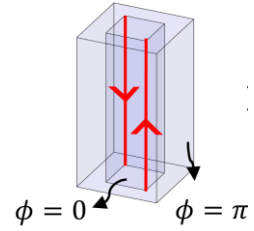


Small in-plane gyromagnetic bias to
control hinge string connectivity

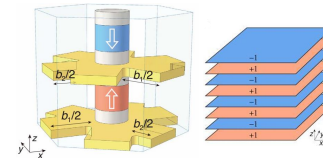
Summary



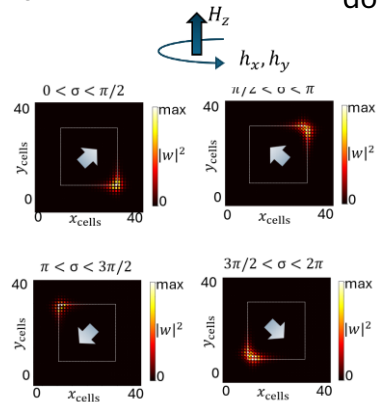
Supercell dielectric $\sim$
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Axion hinge states
manifesting at the
domain wall

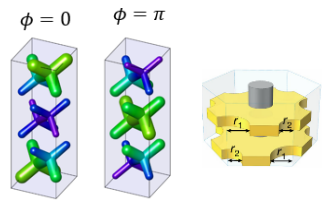


Intrinsic AXI can be realized via
an alternating stacking

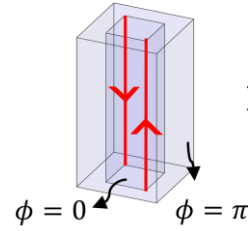


Small in-plane gyromagnetic bias to
control hinge string connectivity

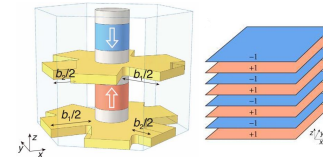
Summary



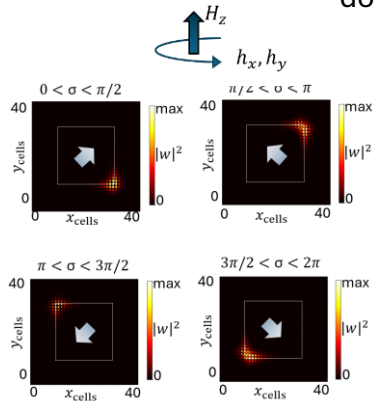
Supercell dielectric $\sim$
inducing EM energy
redistribution



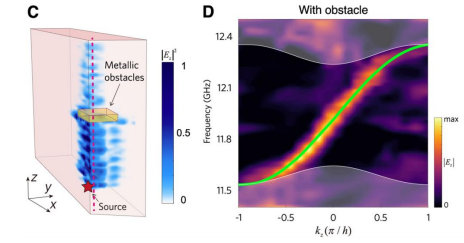
Axion hinge states
manifesting at the
domain wall



Intrinsic AXI can be realized via
an alternating stacking

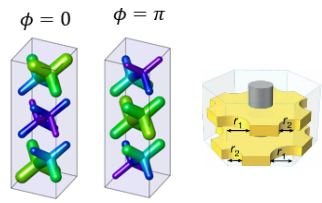


Small in-plane gyromagnetic bias to
control hinge string connectivity

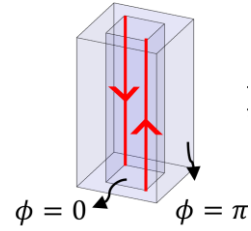


Unidirectional hinge states
robust to obstacles

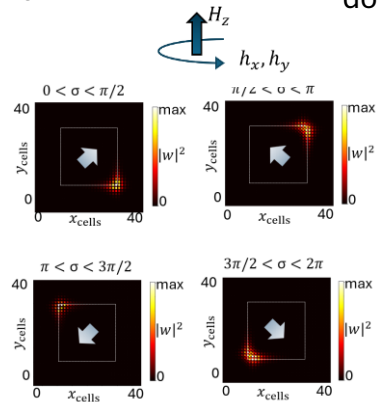
Summary



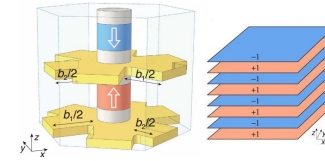
Supercell dielectric ~ inducing EM energy redistribution



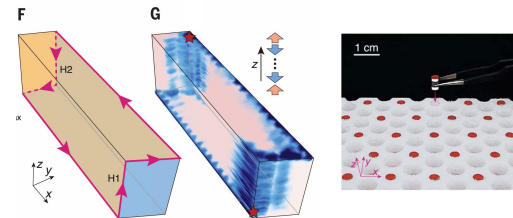
Axion hinge states manifesting at the domain wall



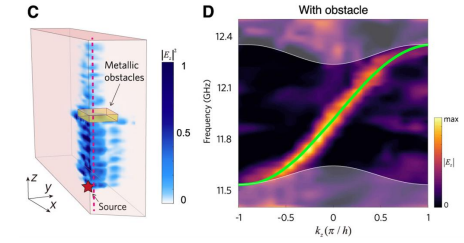
Small in-plane gyromagnetic bias to control hinge string connectivity



Intrinsic AXI can be realized via an alternating stacking

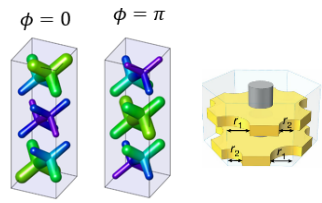


Half quantized anomalous Chern, whose sign can be probed by addition of physical layers

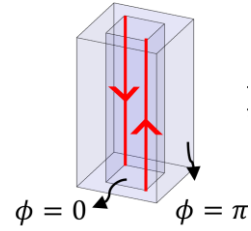


Unidirectional hinge states robust to obstacles

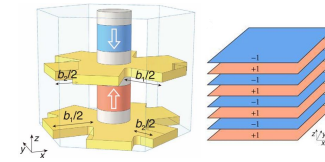
Summary



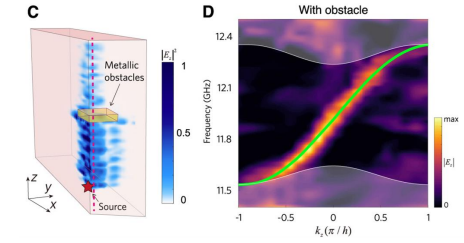
Supercell dielectric $\sim$
inducing EM energy
redistribution



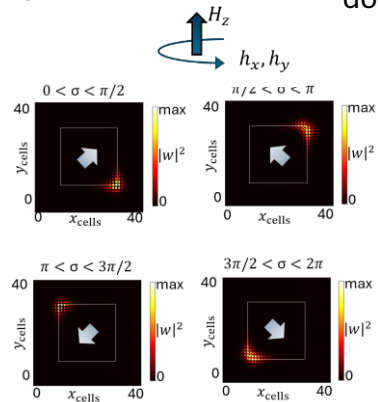
Axion hinge states
manifesting at the
domain wall



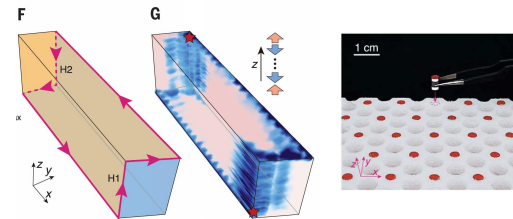
Intrinsic AXI can be realized via
an alternating stacking



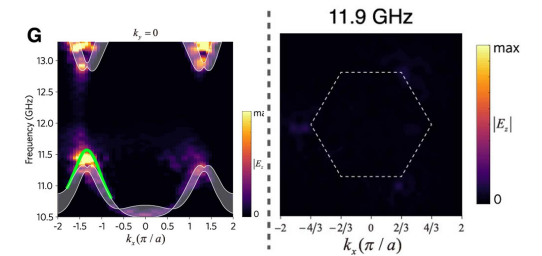
Unidirectional hinge states
robust to obstacles



Small in-plane gyromagnetic bias to
control hinge string connectivity

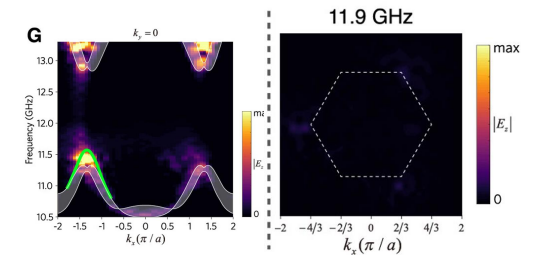
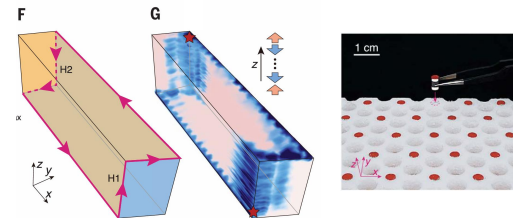
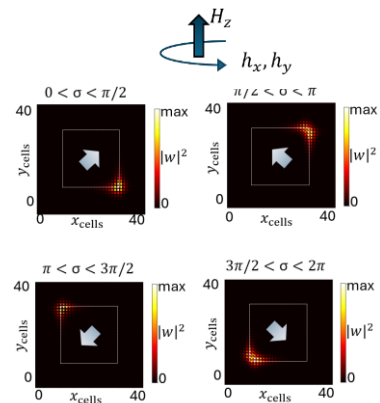
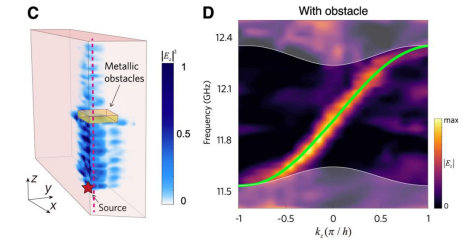
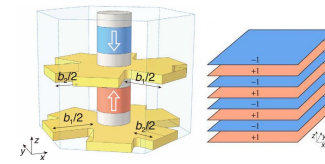


Half quantized anomalous Chern, whose
sign can be probed by addition of
physical layers



Potential platform to study surface
topological order

Thank you for listening!

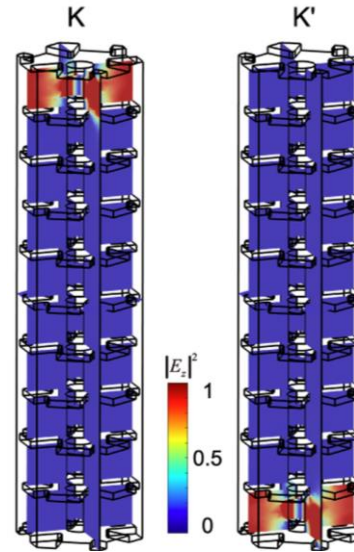
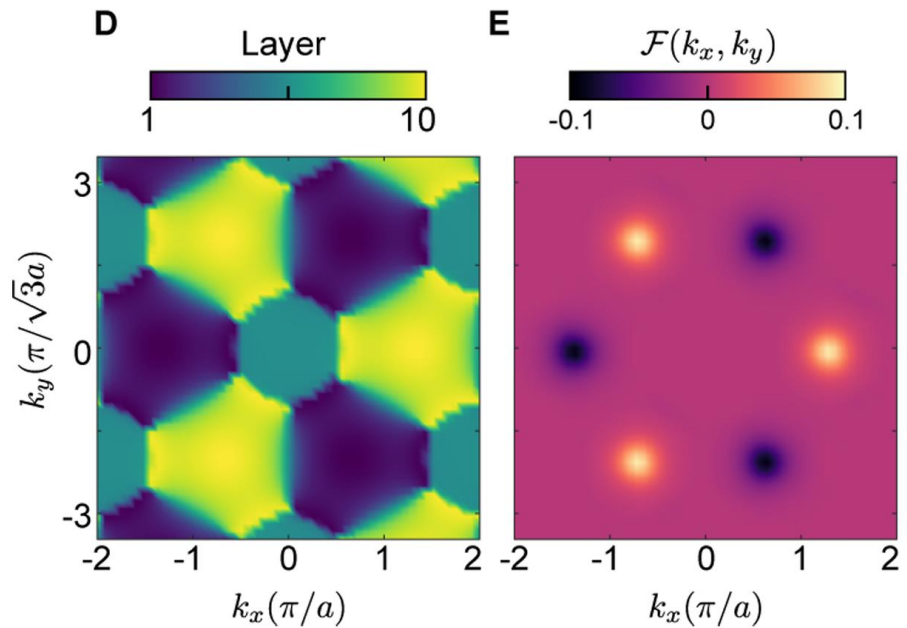


Axion topology in photonic crystals
Nat. Comm. 15, 6814 (2024)

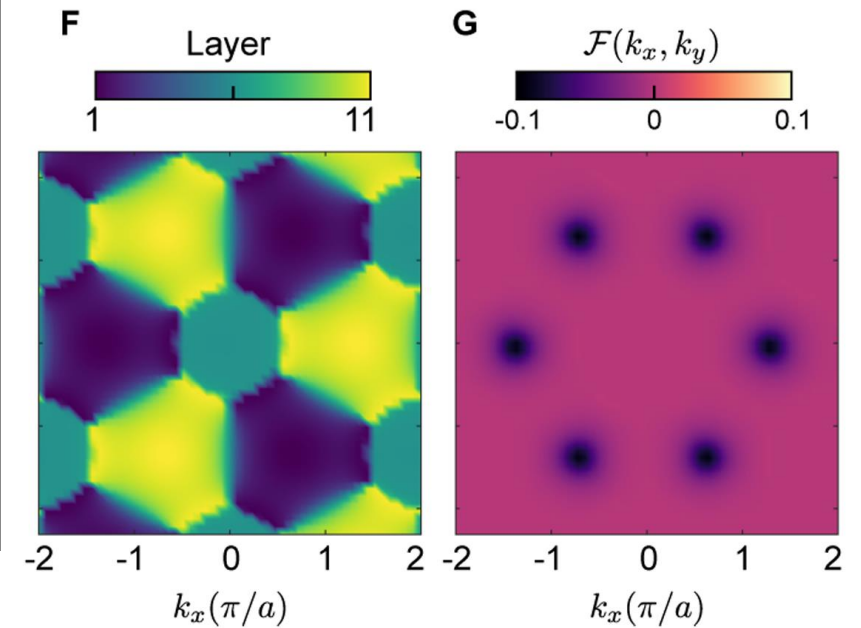
Photonic Axion Insulator
Science 387, 162–166 (2025)

Top/bottom valley contributions

Even number of layers

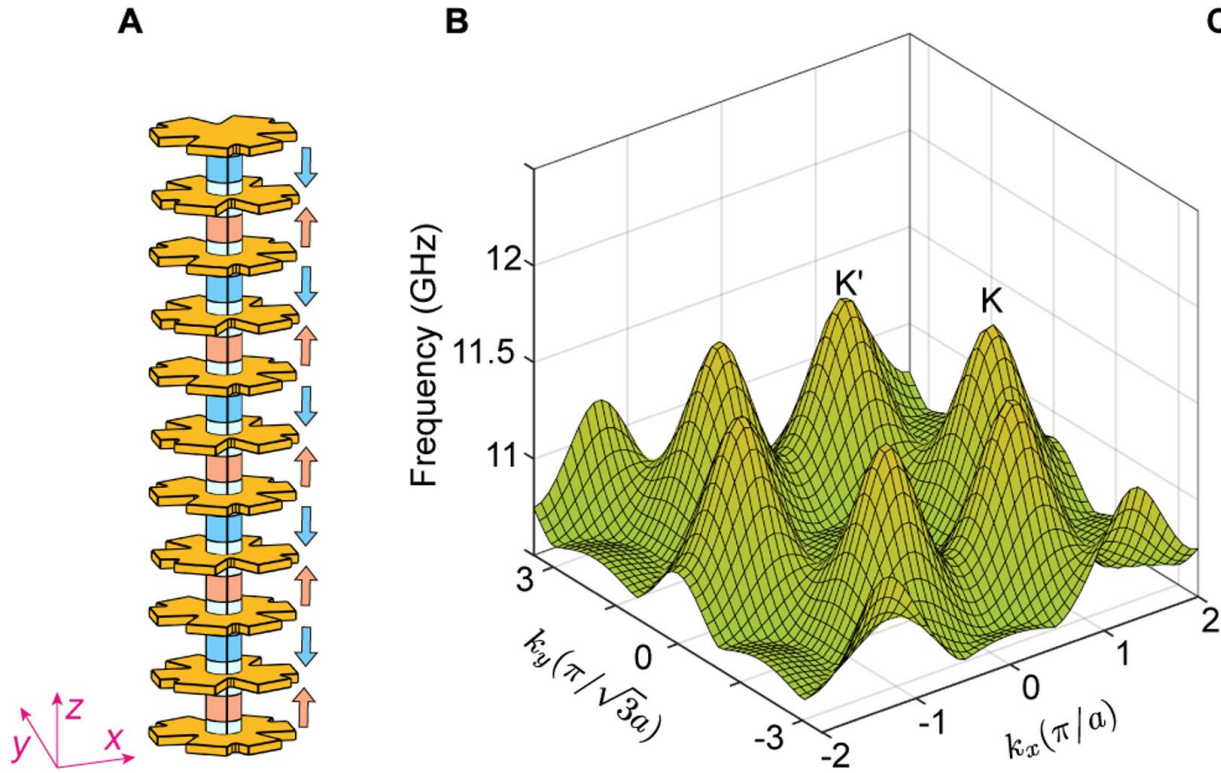


Odd number of layers

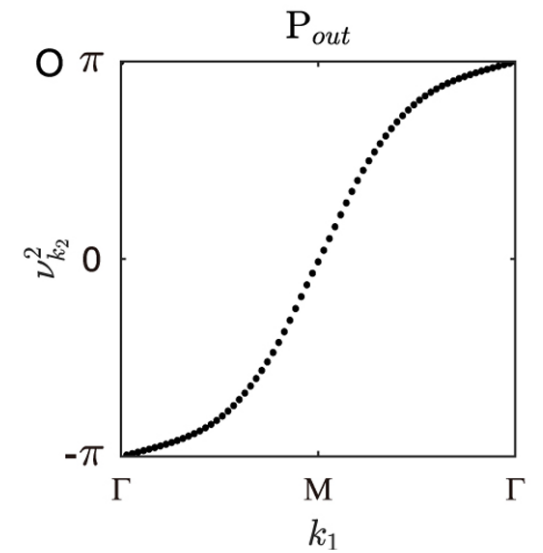
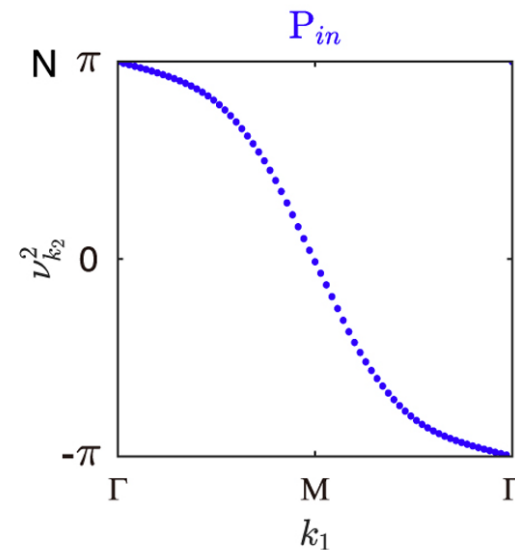
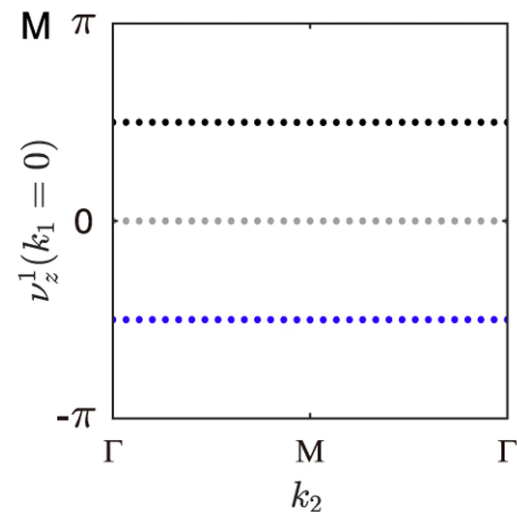
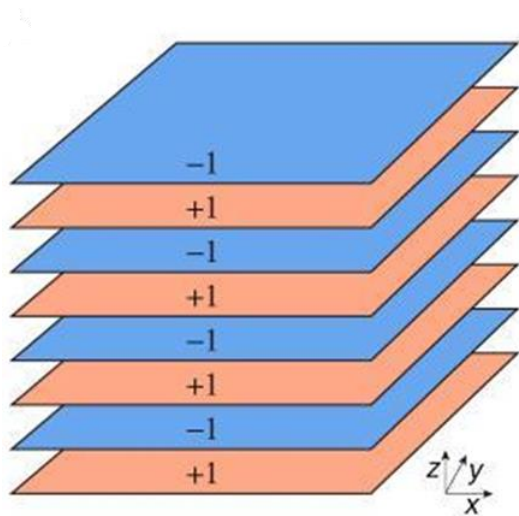


Sign change from positive to negative at the K valley
where top surface contributes

Surface band structure simulation

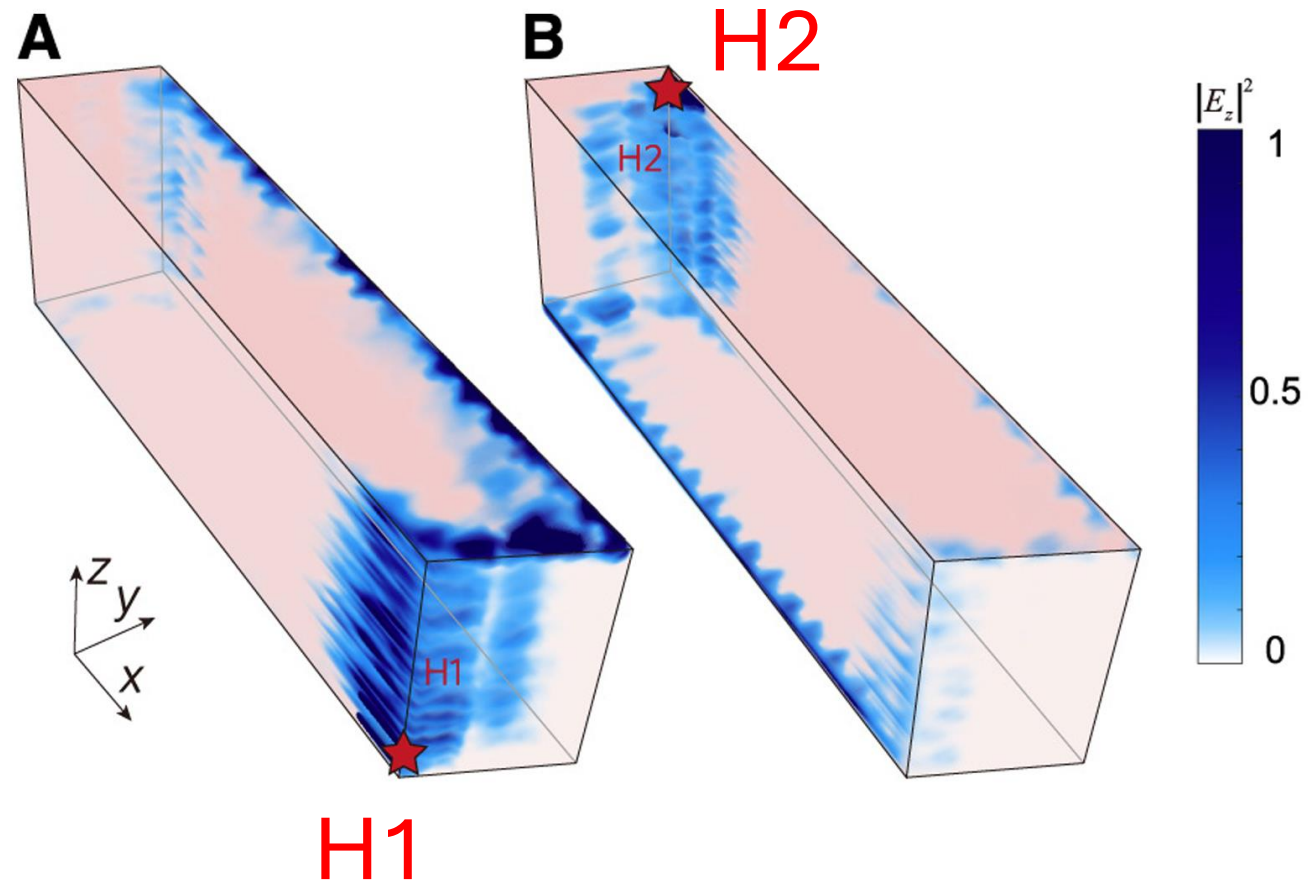


Nested Wilson loops



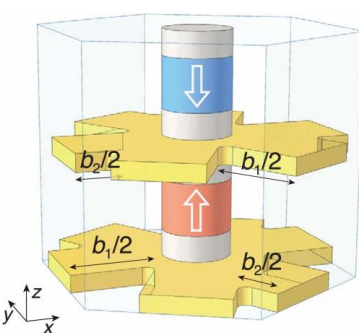
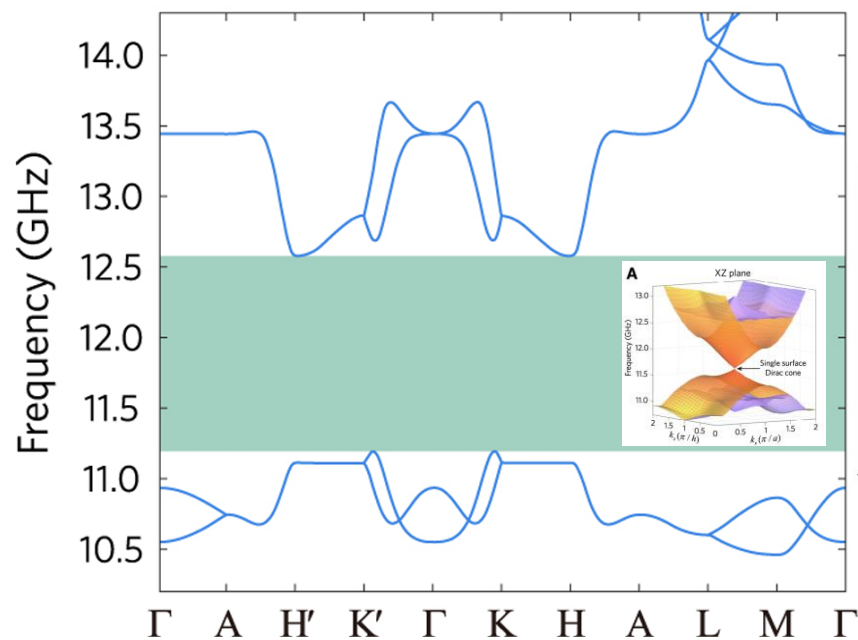
Hinge state profiles

Point source excitation: microwave dipole antennas

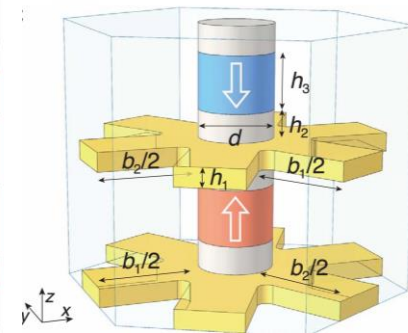
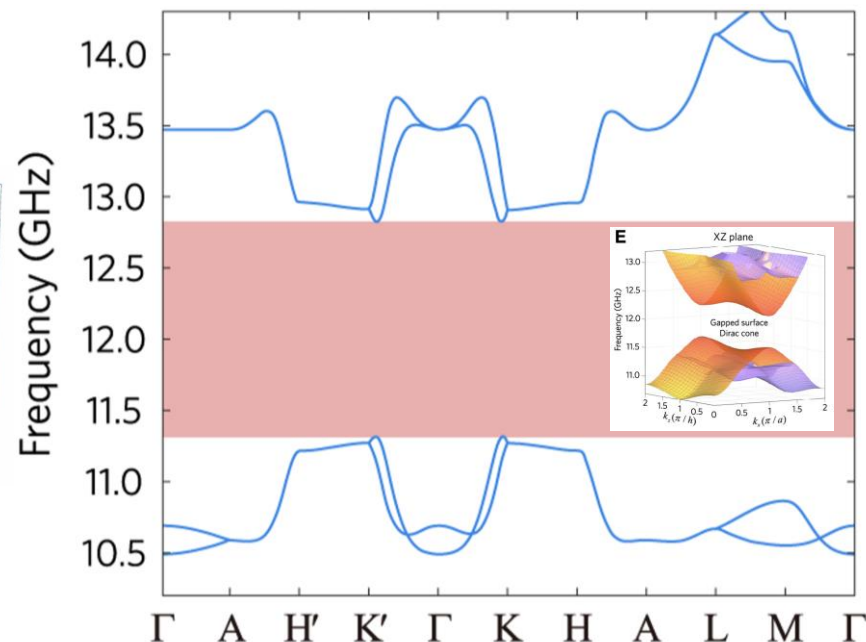


Gapped bulk

$b_1=10\text{ mm}; b_2=10\text{ mm}$

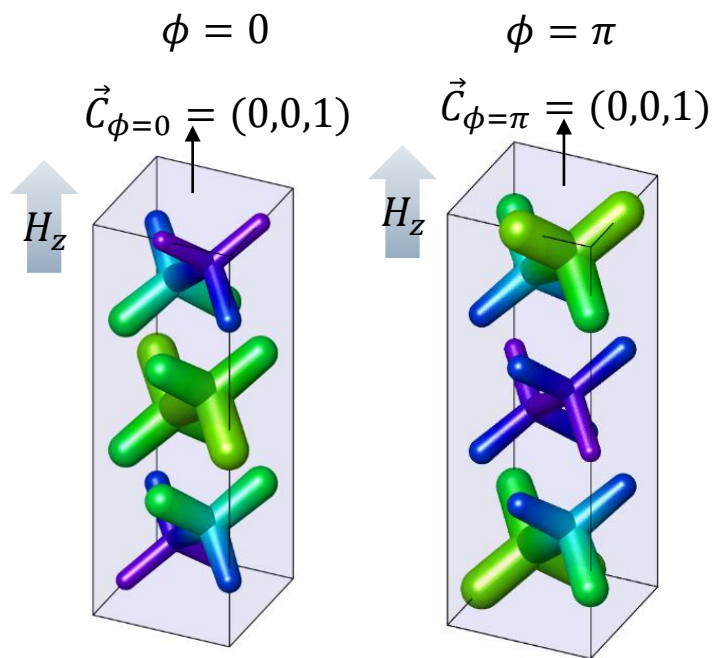


$b_1=10\text{ mm}; b_2=5\text{ mm}$



Magnetic space group

Tilted external magnetic field



$$Y = (\pi, \pi, 0)$$

$$Z = (0, 0, \pi)$$

$$C = (\pi, \pi, \pi)$$

$$\mathbb{Z}_2 \times \mathbb{Z}_2$$

$$\mathbf{n}_{\phi=0}^T$$

$$= [3A_1, 3B_1, C_1^- + 2C_2^+ + 3C_2^-, 3D_1, 3E_1, (\blacksquare)^{2T} + 2\Gamma_2^+ + 2\Gamma_2^-, Y_1^- + 2Y_2^+ + 3Y_2^-, 3Z_2^+ + 3Z_2^-]$$

$$\Rightarrow \mathbf{v}_{\phi=0}^T = (1, 0)$$

$$\mathbf{n}_{\phi=0}^T$$

$$= [3A_1, 3B_1, C_1^- + 2C_2^+ + 3C_2^-, 3D_1, 3E_1, (\blacksquare)^{2T} + 2\Gamma_2^+ + 2\Gamma_2^-, Y_1^+ + 3Y_2^+ + 2Y_2^-, 3Z_2^+ + 3Z_2^-]$$

$$\Rightarrow \mathbf{v}_{\phi=0}^T = (1, 1)$$

MSG 67.505

$$\begin{aligned} & \{\bar{1}|0\rangle \\ & \left\{C_{2z} \left| \frac{1}{2}, \frac{1}{2}, 0 \right. \right\} \\ & \left\{m_z \left| \frac{1}{2}, \frac{1}{2}, 0 \right. \right\} \end{aligned}$$

Unitary: P2/c (SG No. 13)

$$\begin{aligned} & \left\{C'_{2_{110}} \left| \frac{1}{2}, \frac{1}{2}, 0 \right. \right\}, \left\{m'_{110} \left| \frac{1}{2}, \frac{1}{2}, 0 \right. \right\} \\ & \left\{C'_{2_{1\bar{1}0}} \left| \frac{1}{2}, \frac{1}{2}, 0 \right. \right\}, \left\{m'_{1\bar{1}0} \left| \frac{1}{2}, \frac{1}{2}, 0 \right. \right\} \end{aligned}$$

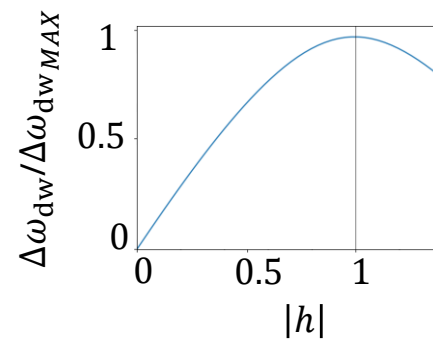
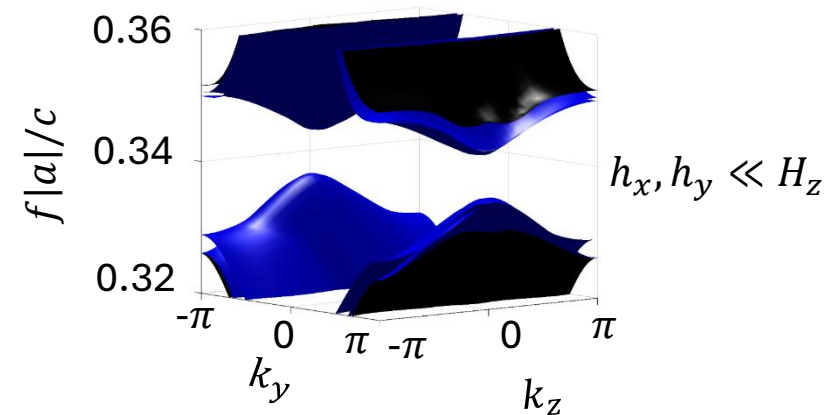
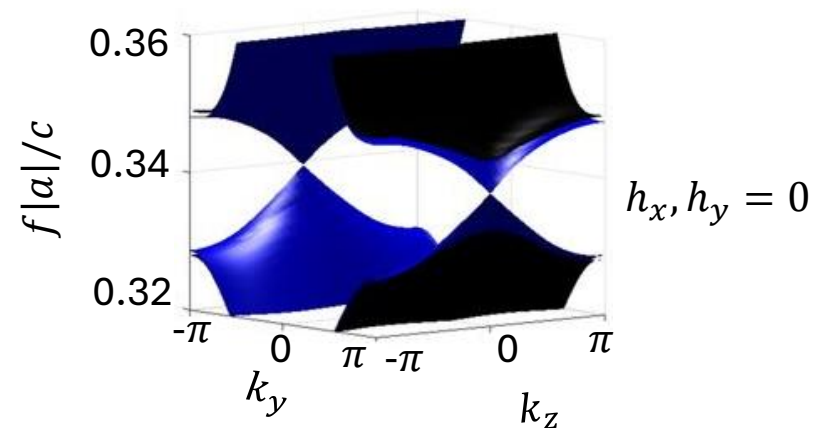
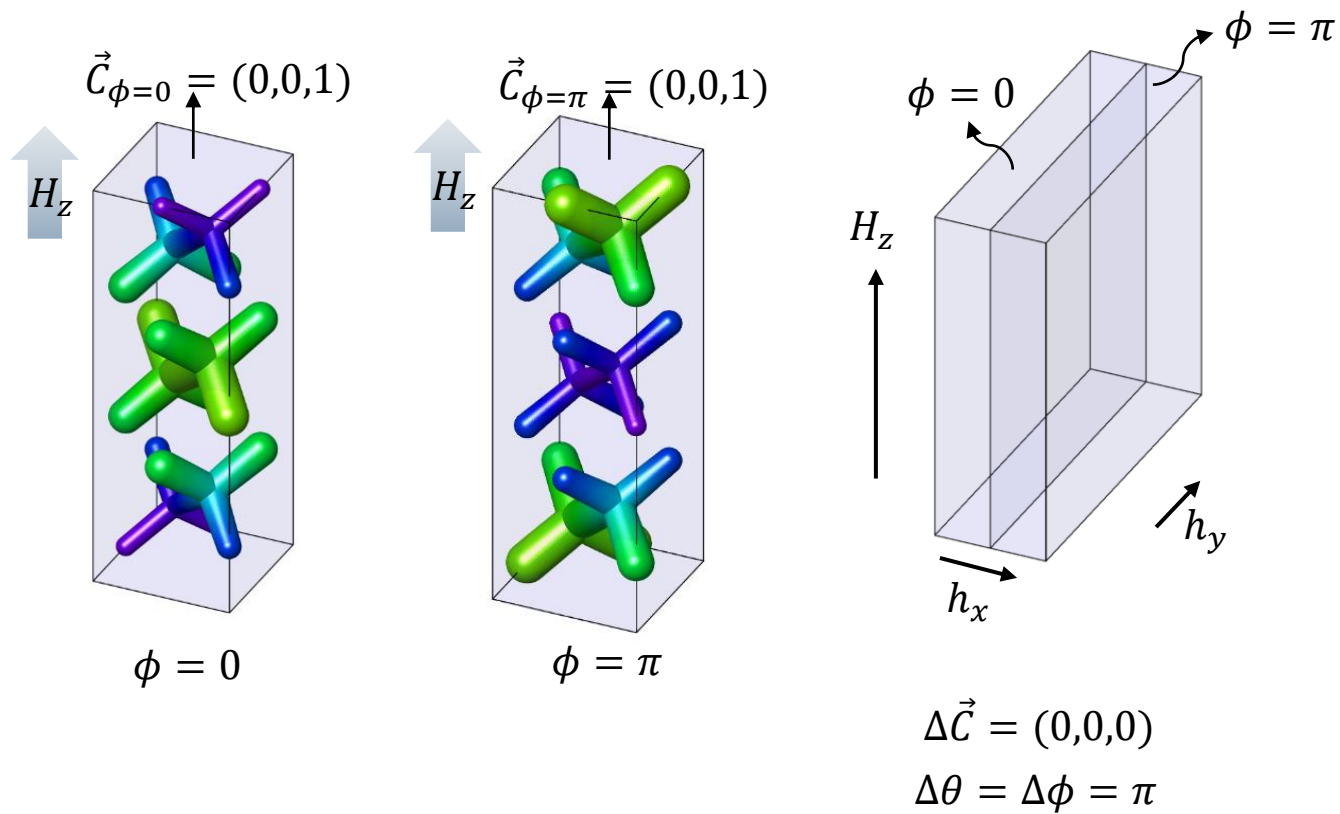
Anti-unitary

For $\mathbf{H} = (h_x, h_y, H_z)$, subduction to [MSG 2.4](#) without bulk-gap closing while $|h_x|, |h_y| \ll |H_z|$

$$\bar{z}_{2,x} = \bar{z}_{2,y} = 0 \quad \mathbf{v} = \left(\bar{z}_{2,z}, \frac{\bar{z}_{24}}{2} \right)$$

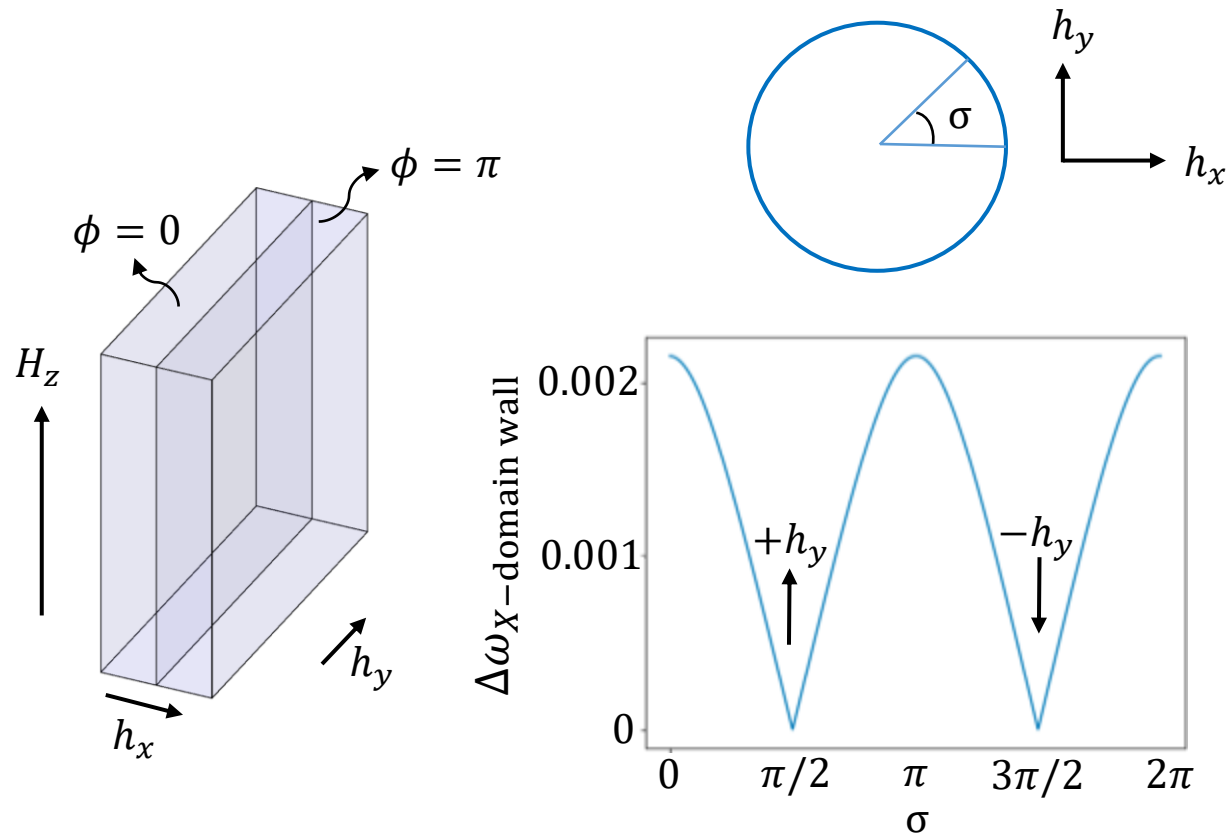
Higher-order response

Opening domain wall surface gap



Gyrotropy-induced switching of HOTI states

Boundary-obstructed transitions



$$|H_z| \gg |h_x|, |h_y|$$

