Ariel University, Department of Physics

Non-renormalization of the fractional quantum Hall conductivity by interactions

Talk at the University of Minho in Braga (Portugal) at the conference

Weyl and Dirac Semimetals as a Laboratory for High-Energy Physics

Presented by

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1. Basics of the quantum Hall effect and Wigner-Weyl calculus
2. The quantum Hall fluid: Non-perturbative fractional topological phases
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4. The quantum Hall fluid in macroscopic motion
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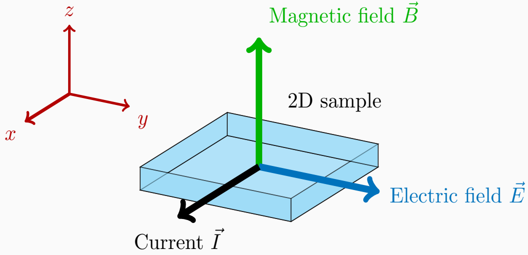
Basics of the quantum Hall effect and Wigner-Weyl calculus

The Hall conductivity

The quantum Hall effect in (2+1)D



non-dissipative, topological response function: Hall conductivity $I_x = \sigma_{xy} E_y$, $\sigma_{xy} = \frac{e^2}{h}\nu$

integer/fractional quantum Hall effect (IQHE/FQHE): ν is an integer/fraction visualization: topological quantum Hall plateaus ΔB in the $(\rho_{xy}$-$B)-$plane

$\rightarrow$ Landau level quantization of electrons in an external magnetic field

resistivity matrix $\rho = \begin{pmatrix} \rho_{xx} & \rho_{xy} \\ \rho_{yx} & \rho_{yy} \end{pmatrix}, \ \rho_{xx} = \rho_{yy}$ (rotationally invariant sample)

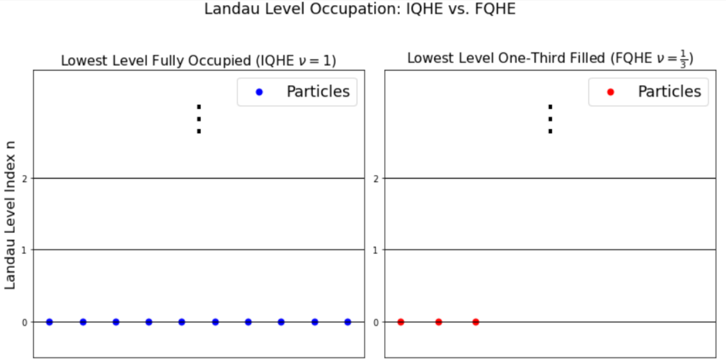
conductivity matrix $\sigma = \rho^{-1}, \ \sigma_{xx} = \sigma_{yy} = \dfrac{\rho_{xx}}{\rho_{xx}^2 + \rho_{xy}^2}, \quad \sigma_{xy} = \dfrac{-\rho_{xy}}{\rho_{xx}^2 + \rho_{xy}^2}$

on a Hall plateau: $\rho_{xy} = -\rho_{yx} = \frac{h}{e^2}\frac{1}{\nu}, \quad \rho_{xx} = \rho_{yy} = 0 \quad \Rightarrow \quad \sigma_{xx} = 0, \quad \sigma_{xy} = -\frac{1}{\rho_{xy}}$

$\frac{h}{e^2}$: von Klitzing constant, can be measured very precisely

IQHE vs. FQHE

Landau level quantization in an external magnetic field:



IQHE: gapped ground state due to Landau level energy gap ($\omega_c = \frac{eB}{m_e}$)

FQHE: gapped (!) due to strongly correlated electrons ($V(l_B) = \frac{e^2}{\epsilon l_B}$, $l_B \sim \frac{1}{\sqrt{B}}$)

Feature	Integer Quantum Hall Effect (IQHE)	Fractional Quantum Hall Effect (FQHE)
Discovered	1980 (Klaus von Klitzing)	1982 (Tsui, Stormer, Gossard)
Hall Conductance	$\sigma_{xy} = \nu e^2/h,\ \nu \in \mathbb{Z}$	$\sigma_{xy} = \nu e^2/h,\ \nu = p/q,\ p, q \in \mathbb{Z}$
Theory	Non-interacting band theory	Strongly correlated field theory (e.g., composite fermions)
Origin	Non-interacting electrons in Landau levels, scale: $\omega_c = eB/m_e$	Strong electron-electron interactions, scale: $V(l_B) = e^2/l_B,\ l_B \sim 1/\sqrt{B}$
Topological Invariant	Chern number (integer) for electrons (fully filled Landau levels)	Chern number (integer) for composite fermions (fully filled effective Landau levels)
Quasiparticles	Electrons (fermions)	Anyons with fractional charge and statistics
Ground State Degeneracy	Unique (on torus)	Topologically degenerate
Wavefunction	Slater determinant of Landau level states	Laughlin wavefunction (e.g., for $\nu = \frac{1}{3}$),...
Edge States	Integer number of chiral modes	Chiral edge states with fractional excitations
Disorder Robustness	Very robust (mobility gap)	Robust but requires ultra-clean samples
Experimental Requirements	High magnetic field, low temperature	Higher field, cleaner samples, lower temperatures

Wigner-Weyl formalism (in $(2+1)$ dimensions, $e = \hbar = 1$)

Weyl symbol of an operator:

$$\hat{\mathcal{O}} = \int \frac{d^3k}{(2\pi)^3} \frac{d^3p}{(2\pi)^3} d^3y d^3x O_W(x,p) e^{i(k(x-\hat{x})+y(p-\hat{p}))}$$

configuration space representation:

$$\langle x|\hat{\mathcal{O}}|y\rangle = O(x,y) = \frac{1}{(2\pi)^3} \int d^3p O_W(\frac{x+y}{2}, p) e^{ip(x-y)}$$

$$\Leftrightarrow O_W(R,p) = \int d^3y O(R+\frac{y}{2}, R-\frac{y}{2}) e^{-ipy}$$

momentum space representation:

$$\langle p|\hat{O}|k\rangle = \tilde{O}(p,k) = \frac{1}{(2\pi)^3} \int d^3x O_W(x, \frac{p+k}{2}) e^{i(k-p)x}$$

$$\Leftrightarrow O_W(R,p) = \int d^3k \tilde{O}(p+\frac{k}{2}, p-\frac{k}{2}) e^{ikR}$$

on Hilbert space: operator product operation $\hat{\mathcal{O}} = \hat{\mathcal{A}}\hat{\mathcal{B}}$;   on phase space:

$$O_W(x,p) = A_W(x,p) \star B_W(x,p); \quad \star = \exp\Big(\frac{i}{2}\Big(\overleftarrow{(\partial_x)^\mu}\,\overrightarrow{(\partial_p)_\mu} - \overleftarrow{(\partial_p)_\mu}\,\overrightarrow{(\partial_x)^\mu}\Big)\Big)$$

Groenewold equation of the Weyl symbol of an operator $\hat{\mathcal{O}}$:

$$O_W(x,p) \star O_W^{-1}(x,p) = 1 \quad (\hat{\mathcal{O}} = \hat{Q} = \omega - \hat{H}, \hat{G} = \hat{Q}^{-1})$$

propagator matrix elements (time translation invariant):

$$G(\mathbf{x},\mathbf{y},\omega) = \langle \mathbf{x}|(\omega - \hat{H})^{-1}|\mathbf{y}\rangle, \quad \tilde{G}(\mathbf{p},\mathbf{q},\omega) = \langle \mathbf{p}|(\omega - \hat{H})|\mathbf{q}\rangle.$$

Feynman (or time ordered), Matsubara (or imaginary time ordered) Green functions

$$G_W^F(\mathbf{x},\mathbf{p},\omega) = G_W(\mathbf{x},\mathbf{p},\omega + i0^+ sign(\omega)) \Big/ \; G^F(x,y) = -i\langle \psi(x)\bar{\psi}(y)\rangle_{PI}$$

$$G_W^M(\mathbf{x},\mathbf{p},\omega_n) = -iG_W(\mathbf{x},\mathbf{p},i\omega_n) \Big/ \; G^M(x,y) = \langle \psi(x)\bar{\psi}(y)\rangle_{PI} \quad (\omega_n = (2n+1)\pi T \;\; n \in \mathbb{Z})$$

The quantum Hall fluid: Non-perturbative fractional topological phases

QFT for electrons with Coulomb interactions in Minkowski spacetime

$$\mathcal{Z} = \int D\psi D\bar{\psi} D\lambda e^{iS[\lambda]+i\int d^3x\bar{\psi}\hat{Q}[\lambda]\psi}; \quad S[\lambda] = \frac{1}{2}\int d^3z d^3\lambda(z) V^{-1}(z,z')\lambda(z'),$$

$$\hat{Q}[\lambda] = i\partial_0 - A_0(z) + \mu - \lambda(z) + \frac{1}{2m}\mathbf{D}^2, \quad \mathbf{D} = \boldsymbol{\partial} + i\mathbf{A}$$

the electric current:

$$j^k(x) = i\frac{\delta \log \mathcal{Z}}{\delta A_k} = i\frac{1}{\mathcal{Z}}\frac{\delta \mathcal{Z}}{\delta A_k} = -\frac{1}{\mathcal{Z}}\int D\psi D\bar{\psi} D\lambda e^{iS[\lambda]+i\int d^3x\bar{\psi}\hat{Q}[\lambda]\psi}\bar{\psi}(x)\frac{\delta}{\delta A_k}\hat{Q}[\lambda]\psi(x)$$

at zeroth order of perturbation theory ($\lambda = 0$):

$$j^k(x) = i\int \frac{d^3p}{(2\pi)^3} Tr\Big[G_W(x,p)\partial_{p_k} Q_W(x,p)\Big]$$

full inclusion of perturbative interactions ($\mathcal{Q}_W = Q_W - \Sigma_W$, $\mathcal{G}_W = \mathcal{Q}_W^{-1}$):

$$j^k(x) = i\int \frac{d^3p}{(2\pi)^3} Tr\Big[\mathcal{G}_W(x,p)\partial_{p_k} Q_W(x,p)\Big]$$

Topological vs. physical current for the interaction $a = 0$, *Yukawa*, *QED*

topological current (in Euclidean space):

$$J^k_{top,a}(x) = -\int \frac{d^3p}{(2\pi)^3} Tr\Big[(\mathcal{G}_a)_W(x,p) \cdot \frac{\partial}{\partial p_k}(\mathcal{Q}_a)_W(x,p)\Big]$$

$$\overline{J}^k_{top,a} = \frac{1}{\beta L^2} \int d^3x J^k_{aa}(x), \quad \cdot \to \star$$

physical current (in Euclidean space):

$$J^k_a(x) = -\int \frac{d^3p}{(2\pi)^3} Tr\Big[(\mathcal{G}_a)_W(x,p) \cdot \frac{\partial}{\partial p_k}(\mathcal{Q}_0)_W(x,p)\Big]$$

$$\overline{J}^k_a = \frac{1}{\beta L^2} \int d^3x J^k_a(x), \quad \cdot \to \star$$

inverse Dyson equation: $(\mathcal{Q}_a)_W = (\mathcal{Q}_0)_W - (\Sigma_a)_W$ with self-energy $(\Sigma_a)_W$

perturbative non-renormalization theorem: $\overline{J}^k_{top,a} = \overline{J}^k_a = \overline{J}^k_0$

The composite fermion QFT model of Lopez and Fradkin

$$\mathcal{Z} = \int D\psi D\overline{\psi} D\lambda D\mathcal{A}_\mu e^{iS_{CS}[\mathcal{A}]+iS[\lambda]+i\int d^3x \overline{\psi}\hat{Q}[\mathcal{A},\lambda]\psi}$$

$$S[\lambda] = \frac{1}{2}\int d^3x d^3x' \lambda(x) V^{-1}(x,x')\lambda(x'), \quad S_{CS}[\mathcal{A}] = \frac{\theta}{4}\int d^3x \epsilon^{\mu\nu\rho}\mathcal{A}_\mu \mathcal{F}_{\nu\rho},$$

$$\hat{Q}[\mathcal{A},\lambda] = i\partial_0 - A_0(x) - \mathcal{A}_0(x) + \mu - \lambda(x) + \frac{1}{2m}\mathbf{D}^2, \quad \mathbf{D} = \partial + i\mathbf{A}(x) + i\mathcal{A}(x)$$

$\theta = 1/(2\pi\, 2s)$, $s \in \mathbb{Z}$ to trivialize linking number contributions

mean field approximation: external electromagnetic fields are screened by the statistical electromagnetic field, maps FQHE to the IQHE

homogeneous mean field solution of the field equations

$$\theta\mathcal{B} = -\bar{\rho}, \quad \theta\mathcal{E}^i = \epsilon^{ij}\bar{j}^j, \quad \lambda = 0, \quad \bar{\rho} = \langle\bar{\psi}\psi\rangle, \quad \bar{j}^k = \left\langle \bar{\psi}\frac{(-i)D^k}{m}\psi \right\rangle$$

Screening and fractional quantization at the mean field level

assume p Landau levels are occupied for B_{eff} (principal Jain series)

$$\bar{\rho} = \frac{p}{2\pi} B_{eff}; \qquad B_{eff} = B + \mathcal{B} = B - \frac{\bar{\rho}}{\theta} = B - 2spB_{eff} \;\Leftrightarrow\; B_{eff} = \frac{B}{1+2sp}$$

$$\bar{j} = \frac{p}{2\pi} E_{eff}; \qquad E_{eff} = E + \mathcal{E} = E - \bar{j}/\theta = E - \; 2sp\, E_{eff} \;\Leftrightarrow\; E_{eff} = \frac{E}{1+2sp}$$

$$\bar{j} = \frac{p}{2\pi} E_{eff} = \frac{1}{2\pi}\frac{p}{1+2sp} E \;\Leftrightarrow\; \sigma_{xy} = \frac{1}{2\pi}\frac{p}{1+2sp} \quad (T \approx 0)$$

$$p = \mathcal{N} = \frac{T}{3!4\pi^2 A} \int d^3p\, d^3x\, \epsilon_{\mu\nu\rho}\, Tr\Big[(G_{eff})_W(x,p) \star \frac{\partial (Q_{eff})_W(x,p)}{\partial p_\mu} \star (G_{eff})_W(x,p) \star \frac{\partial (Q_{eff})_W(x,p)}{\partial p_\nu} \star (G_{eff})_W(x,p) \star \frac{\partial (Q_{eff})_W(x,p)}{\partial p_\rho}\Big]$$

minimal fractional charge of quasiparticles/holes: $e_{qp/h} = \pm e(1+2sp)^{-1}$

Relativistic field theory in macroscopic motion: The Zubarev statistical operator

Zubarev statistical operator method

relativistically covariant formulation of the statistical operator

- assumption 1: spacetime possesses foliation into a family of spacelike hypersurfaces Σ_σ parametrized by "time" σ with normal vector field n
- assumption 2: continuous medium (hydrodynamical) approximation is valid
- assumption 3: local thermalization timescale $\Delta\tau \ll \Delta t$ with Δt a characteristic time scale of interest (LTE)
- assumption 4: global thermalization of the physical system of interest (GTE)

the Zubarev statistical operator is constructed from conserved current densities which characterize the system macroscopically statistical operator from the maximum entropy principle with constraints:

$$n_\mu(x) Tr(\hat{\rho}\hat{T}^{\mu\nu}(x)) = n_\mu(x) T^{\mu\nu}_{cm}(x),\ n_\mu(x) Tr(\hat{\rho}\hat{j}^\mu(x)) = n_\mu(x) j^\mu_{cm}(x)$$

under Poincaré symmetry: $T^{\mu\nu} \equiv T^{\mu\nu}_{BR} \Big/ T^{\mu\nu}_g$ or $T^{\mu\nu} \equiv T^{\mu\nu}_{can}$ plus $M^{\mu\nu\rho}_{can}$

$$\hat{\rho}_{LTE} = \frac{1}{Z_{LTE}} exp\Big(- \int_{\Sigma_\sigma} d\Sigma_\sigma n_\mu (\hat{T}^{\mu\nu}(x)\beta_\nu(x) - \hat{j}^\mu(x)\zeta(x))\Big)$$

$$Tr(\hat{\rho}_{LTE}) = 1, \text{ timelike } \beta_\mu = \beta u_\mu,\ u_\mu u^\mu = 1,\ \zeta = \beta\mu$$

$$n_\mu(x) T^{\mu\nu}_{LTE}[\beta, \zeta, n](x) = n_\mu(x) T^{\mu\nu}_{cm}(x),\ n_\mu(x) j^\mu_{LTE}[\beta, \zeta, n](x) = n_\mu(x) j^\mu_{cm}(x)$$

$$T^{\mu\nu}_{LTE}[\beta, \zeta, n](x) = Tr(\hat{\rho}_{LTE}\hat{T}^{\mu\nu}(x)),\ \ j^\mu_{LTE}[\beta, \zeta, n](x) = Tr(\hat{\rho}_{LTE}\hat{j}^\mu(x))$$

Quantum electrodynamics under macroscopic motion in GTE

macroscopic motion Hamiltonian:

$$\hat{\rho}_{GTE} = \frac{1}{Z_{GTE}} exp\Big(- \int d\Sigma \beta \mathcal{H}_{mm} \Big), \quad \beta \mathcal{H}_{mm} = n_\mu(\vec{x}) \Big(\hat{T}^{\mu\nu}(\vec{x}) \beta_\nu(\vec{x}) - \hat{j}^\mu(\vec{x}) \zeta(\vec{x}) \Big)$$

$\rightarrow$ we converted this Hamiltonian into a Lagrangian via path integral methods

Dirac Lagrangian + gauge field Lagrangian:

$$\mathcal{L}(\overline{\psi}, \psi, \lambda_\mu) = \overline{\psi}(x) \Big(\frac{i}{2} \gamma^\mu \overleftrightarrow{D}_\mu - m \Big) \psi(x) - \frac{1}{4} F_{\mu\nu} F^{\mu\nu}$$

$$D_\mu = \partial_\mu - ie\lambda_\mu, \quad F_{\mu\nu} = \partial_\mu \lambda_\nu - \partial_\nu \lambda_\mu, \quad \overleftrightarrow{D}_\mu = \overrightarrow{D}_\mu - \overleftarrow{D}_\mu$$

Dirac field and gauge field BR energy momentum tensors:

$$T^{\mu\nu}_\psi(x) = \frac{i}{4} \overline{\psi}(x) \Big(\gamma^\mu \overleftrightarrow{D}^\nu + \gamma^\nu \overleftrightarrow{D}^\mu \Big) \psi(x)$$

$$T^{\mu\nu}_\lambda(x) = F^{\rho\mu}(x) F^\nu{}_\rho(x) + \frac{1}{4} g^{\mu\nu}(x) F_{\rho\sigma}(x) F^{\rho\sigma}(x)$$

GTE condition:

$$\frac{d\hat{\rho}}{d\sigma} = 0,\ \log\hat{\rho} = -log(Z) - \int d\Sigma_\sigma \beta n^\mu g_{\mu\nu}\Big((\hat{T}^{\nu\rho}_\psi + \hat{T}^{\nu\rho}_\lambda)u_\rho - \sum_i \mu_i \hat{j}^\nu_i\Big)$$

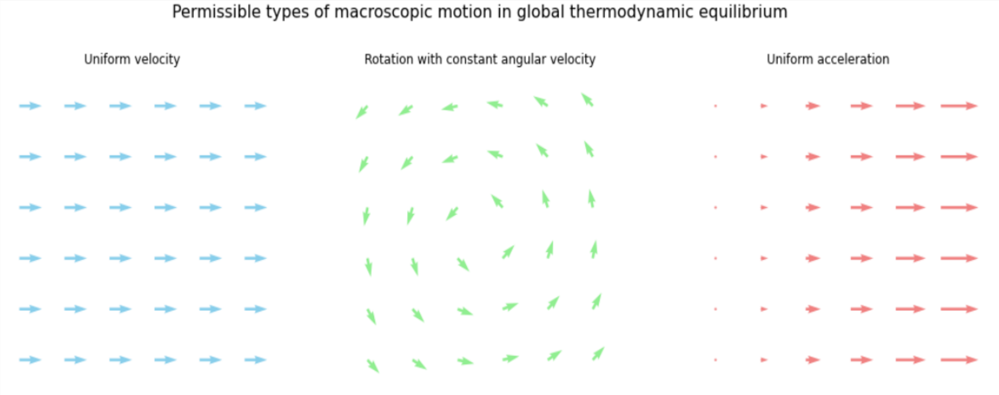
$$\Rightarrow 0 = \partial_\nu\Big(\hat{T}^{\nu\rho}\beta u_\rho - \sum_i \hat{j}^\nu_i \beta\mu_i\Big) = \hat{T}^{\nu\rho}\partial_\nu \beta u_\rho - \sum_i \hat{j}^\nu_i \partial_\nu \beta\mu_i$$

(employ Stokes' theorem + operators vanish at spacelike infinity, currents are conserved)

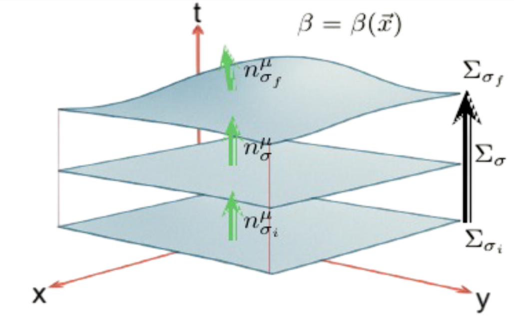
solution (uniform linear motion, rotation and accelerated motion at finite density):

$$\beta\mu_i = \zeta_i = \mathrm{const.},\quad \beta_\rho = \beta u_\rho = b_\rho + \omega_{\rho\sigma}x^\sigma,\quad b_\rho,\, \omega_{\rho\sigma} = \mathrm{const.},\quad \omega_{\rho\sigma} = -\omega_{\sigma\rho}$$

Macroscopic motion under full Poincaré symmetry (translations, rotations, boosts)



More on the path integral formulation



$$\mathcal{Z}[n(x), u(x), \beta(x), \mu_i(x), h] = \int D\psi D\bar{\psi} D\lambda_\mu \, e^{i \int d^4x \, \mathcal{L}(\overline{\psi}, \psi, \lambda_\mu)}$$

$$\mathfrak{U}(\vec{x}) = \frac{\beta(t_0, \vec{x})}{\mathfrak{B}(\vec{x})} u(t_0, \vec{x}), \quad \mathrm{\mu}(\vec{x}) = \frac{\beta(t_0, \vec{x})}{\mathfrak{B}(\vec{x})} \mu(t_0, \vec{x}), \; \Sigma_{\sigma_i} \to \Sigma_{\sigma_f}(h) = \{(h\mathfrak{B}(\vec{x}), \vec{x}) | \vec{x} \in \Sigma\}$$

Final effective Lagrangian comprising macroscopic motion

$$\begin{aligned}\mathcal{L}(\overline{\psi},\psi,\lambda_\mu) =&\overline{\psi}\Big(\gamma^\mu \frac{i}{2}\overset{\leftrightarrow}{D}_\mu - m\Big)\psi + \sum_i \upmu_i j_i^0 \\ &+\mathfrak{U}_k\Big(\overline{\psi}\gamma^0\Big(\frac{i}{8}[\gamma^j,\gamma^k](\overset{\leftarrow}{D}_j + D_j) - \frac{i}{2}\overset{\leftrightarrow}{D}^k\Big)\psi\Big) \\ &+\frac{1}{2}\Big(E^iE^i - B^iB^i\Big) \\ &+\frac{1}{2}\Big(\vec{\mathfrak{U}}^2 B^iB^i - (B_j\mathfrak{U}^j)(B_i\mathfrak{U}^i)\Big) - \epsilon_{ijk}E_iB_j\mathfrak{U}^k\end{aligned}$$

$$\mathfrak{B}(\vec{x}) = \beta(t_0,\vec{x})u^0(t_0,\vec{x}) \Leftrightarrow \mathfrak{U}^\mu(\vec{x}) = \frac{u^\mu(t_0,\vec{x})}{u^0(t_0,\vec{x})}, \quad \upmu(\vec{x}) = \frac{\mu(t_0,\vec{x})}{u^0(t_0,\vec{x})}$$

$$E_i = F_{\mu i}n^\mu, \quad B_i = -\frac{1}{2}\epsilon_{\mu ijk}F^{jk}n^\mu$$

The quantum Hall fluid in macroscopic motion

Relativistic extension of the model by Lopez and Fradkin

$$\mathcal{Z} = \int D\psi D\bar{\psi} D\lambda_\mu D\mathcal{A}_\mu e^{iS_{CS}[\mathcal{A}]+iS_g[\lambda]+i\int d^3x\bar{\psi}\hat{Q}[\mathcal{A},A,\lambda]\psi}$$

$$S_g = -\frac{1}{4e^2}\int d^4x(\partial_{[\mu}(A+\lambda)_{\nu]})^2 + \int d^4x\mathcal{L}_{gf}[A+\lambda], \quad S_{CS} = \frac{\theta}{4}\int d^3x\epsilon^{ijk}\mathcal{A}_i\mathcal{F}_{jk}$$

Dirac operator in the laboratory frame:

$$\hat{Q}^L_{eff} = i\gamma^0\partial_t - \gamma^0(\mathcal{A}_t - Ex) + \mu_{rel}\gamma^0 - \gamma^\mu\lambda_\mu + \gamma^k\Big(i\partial_k + Bx\hat{y}_k - \mathcal{A}_k\Big) - m_{rel}$$

boosted Dirac operator in the Hall fluid frame ($|v| \ll 1$):

$$\hat{Q}^F_{eff} = i\gamma^0\partial_t + \mu_{rel}\gamma^0 + \gamma^k\Big(i\partial_k + B_{eff}x\hat{y}_k\Big) - m_{rel}$$

The Zubarev statistical operator for the Hall fluid in GTE

$$\hat{\rho} = \frac{1}{Z} e^{-\int_\Sigma d^4x \beta n^\mu (T_{\mu\nu} u^\nu - \mu j_\mu)}, \quad \mathcal{H}_{mm} = n^\mu (T_{\mu\nu} u^\nu - \mu j_\mu)$$

$$\mathcal{Z}[\mathfrak{U}] = \int D\psi D\bar{\psi} D\lambda_\mu D\mathcal{A}_\mu e^{iS_{CS}[\mathcal{A}] + i\int d^3x \mathcal{L}_f(\overline{\psi},\psi,A,\lambda,\mathcal{A},\mathfrak{U}) + i\int d^4x \mathcal{L}_g(A,\lambda,\mathcal{A},\mathfrak{U})}$$

$$\mathcal{L}_f(\overline{\psi},\psi,A,\lambda,\mathcal{A},\mathfrak{U}) = \overline{\psi}\Big(\gamma^\mu \frac{i}{2} \overleftrightarrow{D}_\mu - m\Big)\psi + \mathfrak{U}_k \overline{\psi}\gamma^0 \Big(\frac{i}{8}[\gamma^j,\gamma^k](\overleftarrow{D}_j + D_j) - \frac{i}{2}\overleftrightarrow{D}^k\Big)\psi$$

$$\mathcal{L}_g(A,\lambda,\mathcal{A},\mathfrak{U}) = \frac{1}{2e^2}(\tilde{E}_i + \tilde{\Lambda}_i)^2 - \frac{1}{2e^2}(B+\Lambda)^2$$

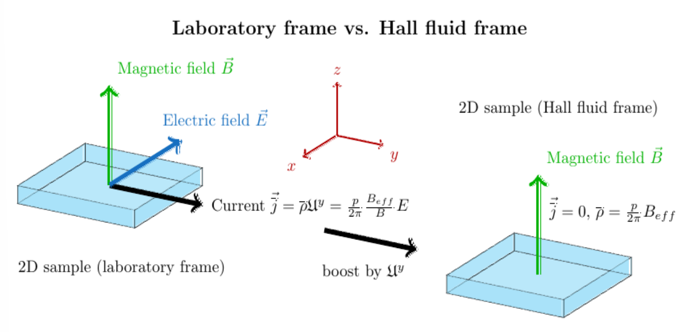
we assume constant external electric field E and magnetic field B ($\rightarrow$ no cross terms)

$$\tilde{E}_i = E_i - \epsilon_{ij} B \mathfrak{U}^j, \quad \tilde{\Lambda}_i = \Lambda_i - \epsilon_{ij} \Lambda \mathfrak{U}^j, \quad \Lambda_i = \partial_{[0}\lambda_{i]}, \quad \Lambda = -\epsilon^{0ij}\partial_{[i}\lambda_{j]}/2$$

gauge choice: $\mathfrak{U}^\mu = u^\mu / u^0$, non-relativistic limit $u^0 \approx 1 \Rightarrow \mathfrak{U}^\mu \approx u^\mu$

the state with nonzero electric field can not be stable due to pair production, unless

$$\mathfrak{U}^i = \epsilon^{ij} E_j / B = \epsilon^{ij} \mathcal{E}_j / \mathcal{B} \Rightarrow \text{boost into the Hall fluid frame where } E \text{ is zero}$$



Dirac composite fermions in the lowest Landau level

comparison with the (particle-hole symmetric) Dirac composite fermion lowest Landau level projected theory (relates states with $\nu = \frac{1}{2} \pm \delta$):

Dirac fermions are necessarily massless to ensure particle-hole symmetry

in addition there is no Landau level mixing, as $\omega_c = \frac{eB}{m} \to \infty$ for $m \to 0$

$$\epsilon^{\mu\nu\lambda} \frac{F_{\nu\lambda}}{2B} \overline{\psi} \frac{i}{2} \overset{\leftrightarrow}{D}_\mu \psi = \overline{\psi} \gamma^0 \frac{i}{2} \overset{\leftrightarrow}{D}_0 \psi - \mathfrak{U}_k \overline{\psi} \gamma^0 \frac{i}{2} \overset{\leftrightarrow}{D}^k \psi \quad \text{(dipole term)}$$

$$\frac{1}{2} \frac{\nabla_i E^i}{2B} \overline{\psi} \psi \cong \mathfrak{U}_k \overline{\psi} \gamma^0 \frac{i}{8} [\gamma^j, \gamma^k] (\overset{\leftarrow}{D}_j + D_j) \psi \quad \text{(vorticity term)}$$

more generally: $\mathfrak{U}^i = \epsilon^{ij}(E_j - \partial_j \lambda_0)/B, \quad \lambda_j = \partial_j \phi$ fix gauge for λ_μ: $\quad \phi \to 0$

the fermionic actions are identical, but the topological couplings are different

Perturbative non-renormalization of the (fractional) Hall conductivity

the non-renormalization of the current for $E \neq 0$ (laboratory frame) is reduced to the non-renormalization of the density for $E = 0$ (Hall fluid frame)

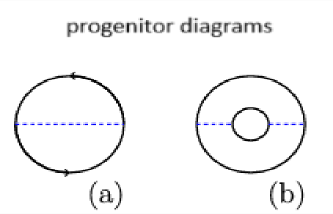
$$\rho = \frac{T}{(2\pi)^3 A} \int d^3p d^3x Tr\Big[\mathcal{G}_W(x,p) \star \frac{\partial (Q_{eff})_W(x,p)}{\partial p_3}\Big]$$

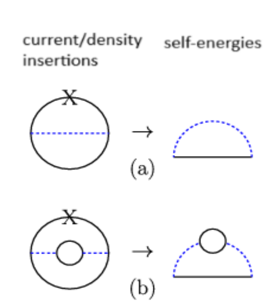
$$\rho' = \frac{T}{(2\pi)^3 A} \int d^3p d^3x Tr\Big[\mathcal{G}_W(x,p) \star \frac{\partial ((Q_{eff})_W - \Sigma_W)(x,p)}{\partial p_3}\Big]$$

$$\Delta\rho = \rho - \rho' \stackrel{!}{=} 0 \quad \text{under fluctuations of } \lambda \text{ or } \lambda_\mu$$

we show that $\mathcal{A}_\mu$ can not renormalize σ_{xy} via path integral methods for $\theta = 1/(2\pi\, 2s)$

the setup becomes spatially periodic and rotationally invariant in the Hall fluid frame





proof relates the perturbatively renormalized density contribution at each loop order to a sum of so-called progenitor diagrams (Feynman diagrams without external density insertion) supplemented by a total derivative operator inserted into the momentum integral of a fermion loop identical to a loop momentum component (which therefore vanishes identically)

symmetry factors present in the sum of progenitor diagrams are canceled exactly (symmetry factors are absent after density insertions into the progenitor diagrams)

Future research on the quantum Hall fluid

apply known techniques within the conductive response to viscoelastic responses:

- topological invariance of the Hall viscosity in theories with translational and rotational invariance (both in integer and fractional quantum Hall phases)
- how are the topological invariants for the quantum Hall conductivity and the Hall viscosity related to each other?
- how robust are the topological invariants relative to each other? (inhomogeneities, anisotropy, disorder, finite temperature, time dependence...)

Thank you for your attention and interest!

This talk is based on:

“Non-renormalization of the fractional quantum Hall conductivity by interactions.” `arXiv:2502.04047`