Introduction to Supersymmetry

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LABORATÓRIO DE INSTRUMENTAÇÃO
E FÍSICA EXPERIMENTAL DE PARTÍCULAS

CMS
Compact Muon Solenoid

Outline

- Standard Model Refresher
- Supersymmetry Motivations:
  - Hierarchy Problem
  - Dark Matter
  - Coupling Constants
- The Supersymmetry Lagrangian

Standard Model Lagrangian

- The Standard Model construction:
  - Choice of symmetries respected by the model, i.e. specify the gauge group G: $U(1) \times SU(2) \times SU(3)$
  - Bosons are associated to vector fields of the gauge group
  - Matter fields to represent Fermions are chosen
  - Scalar fields are added → Give mass to some bosons
  - Write the most general Lagrangian invariant under G which couples all these fields: $\mathscr{L} = \mathscr{L}_{\text{free+interaction}} + \mathscr{L}_{\text{Gauge}} + \mathscr{L}_{\text{Higgs}} + \mathscr{L}_{\text{Yukawa}}$

Further Reading:

- Introduction to the Standard Model and Electroweak Physics - http://arxiv.org/abs/0901.0241
- Standard Model: An Introduction - http://arxiv.org/abs/hep-ph/0001283

Standard Model Lagrangian

$$\mathscr{L} = \mathscr{L}_{\text{free+interaction}} + \mathscr{L}_{\text{Gauge}} + \mathscr{L}_{\text{Higgs}} + \mathscr{L}_{\text{Yukawa}}$$

- The "free+interaction" term corresponds to the gauge invariant Dirac Lagrangian, it describes the free fermions and their interactions with the gauge fields:

$$\mathscr{L}_{\text{free+interaction}} = \sum_{i=1}^{3} \Big[\bar{\Psi}^i_L i\gamma^\mu D_\mu \Psi^i_L + \bar{R}_i i\gamma^\mu D_\mu R_i + \bar{Q}^i_L i\gamma^\mu D_\mu Q^i_L + \bar{U}^i_R i\gamma^\mu D_\mu U^i_R + \bar{D}^i_R i\gamma^\mu D_\mu D^i_R \Big]$$

- Covariant derivatives are determined from the transformation properties of the fields:

$$D_\mu \Psi^i_L = \left(\partial_\mu - i g_w \frac{\vec{\tau}}{2} \cdot \vec{W}_\mu + i \frac{g'}{2} B_\mu \right) \Psi^i_L$$

$$D_\mu R_i = \left(\partial_\mu + i g' B_\mu \right) R_i$$

$$D_\mu Q^i_L = \left(\partial_\mu - i g_s \vec{t} \cdot \vec{G}_\mu - i g_w \frac{\vec{\tau}}{2} \cdot \vec{W}_\mu - i \frac{g'}{6} B_\mu \right) Q^i_L$$

$$D_\mu U^i_R = \left(\partial_\mu - i g_s \vec{t} \cdot \vec{G}_\mu - i \frac{2}{3} g' B_\mu \right) U^i_R$$

$$D_\mu D^i_R = \left(\partial_\mu - i g_s \vec{t} \cdot \vec{G}_\mu + i \frac{g'}{3} B_\mu \right) D^i_R$$

g', g_w, g_s → weak hypercharge, weak isospin and strong couplings

$B_{\mu\nu}$, $\vec{W}_{\mu\nu}$ and $\vec{G}_{\mu\nu}$ → weak hypercharge, weak isospin and strong fields

Dirac spinors:

Ψ^i_L → Left-handed lepton and neutrino doublet of SU(2)

R_i → Right-handed lepton singlet of SU(2)

Q^i_L → Left-handed up and down quark doublet of SU(2)

U^i_R → Right-handed up quark singlet of SU(2)

D^i_R → Right-handed down quark singlet of SU(2)

Standard Model Lagrangian

$$\mathscr{L} = \mathscr{L}_{\text{free+interaction}} + \mathscr{L}_{\text{Gauge}} + \mathscr{L}_{\text{Higgs}} + \mathscr{L}_{\text{Yukawa}}$$

- The “Gauge” term corresponds to the kinetic energy for the vector fields, i.e. the gauge fields. It describes the free bosons and their self-interactions:

$$\mathscr{L}_{\text{Gauge}} = -\frac{1}{4}B_{\mu\nu}B^{\mu\nu} - \frac{1}{4}\vec{W}_{\mu\nu}\cdot\vec{W}^{\mu\nu} - \frac{1}{4}\vec{G}_{\mu\nu}\cdot\vec{G}^{\mu\nu}$$

Weak Hypercharge Field

Weak Isospin Field

Strong Field

- Non-abelian structure of SU(2) and SU(3) groups gives rise to the self interacting term (the third one in the equations below):

$$B_{\mu\nu}(x) = \partial_\mu B_\nu(x) - \partial_\nu B_\mu(x)$$

$$\vec{W}_{\mu\nu}(x) = \partial_\mu \vec{W}_\nu(x) - \partial_\nu \vec{W}_\mu(x) + ig_w\left(\frac{\vec{W}_\mu(x)\vec{W}_\nu(x) - \vec{W}_\nu(x)\vec{W}_\mu(x)}{2}\right)$$

$$\vec{G}_{\mu\nu}(x) = \partial_\mu \vec{G}_\nu(x) - \partial_\nu \vec{G}_\mu(x) + ig_s\left(\vec{G}_\mu(x)\vec{G}_\nu(x) - \vec{G}_\nu(x)\vec{G}_\mu(x)\right)$$

Standard Model Lagrangian

$$\mathscr{L} = \mathscr{L}_{\text{free+interaction}} + \mathscr{L}_{\text{Gauge}} + \mathscr{L}_{\text{Higgs}} + \mathscr{L}_{\text{Yukawa}}$$

- The “Higgs” term introduces the Higgs potential to the model. It describes the dynamics of the Higgs field and interactions with the gauge bosons:

$$\mathscr{L}_{\text{Higgs}} = \left|D_\mu \Phi\right|^2 - V(\Phi)$$

$$V(\Phi) = \mu^2 \Phi^\dagger \Phi + \lambda \left(\Phi^\dagger \Phi\right)^2$$

- Covariant derivative is determined from the transformation properties of the field:

$$D_\mu \Phi = \left(\partial_\mu - i g_w \frac{\vec{\tau}}{2} \cdot \vec{W}_\mu - i \frac{g'}{2} B_\mu\right) \Phi$$

- Three of the four U(1)xSU(2) vector gauge bosons must acquire mass through electroweak symmetry breaking. At least 4 scalar fields are required and are placed in a complex doublet under SU(2):

$$\Phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix}$$

Standard Model Lagrangian

$$\mathscr{L} = \mathscr{L}_{\text{free+interaction}} + \mathscr{L}_{\text{Gauge}} + \mathscr{L}_{\text{Higgs}} + \mathscr{L}_{\text{Yukawa}}$$

- The “Yukawa” term describes the coupling between the scalar field and the fermion fields. Thus, it describes the interactions between the fermions and the scalar bosons:

$$-\mathscr{L}_{\text{Yukawa}} = \sum_{i=1}^{3} \left[G_i \left(\bar{\Psi}^i_L R_i \Phi + h.c. \right) \right] + \sum_{i=1}^{3} \left[G^i_u \left(\bar{Q}^i_L U^i_R \tilde{\Phi} + h.c. \right) \right] + \sum_{i,j=1}^{3} \left[\left(\bar{Q}^i_L G^{ij}_d D^j_R \Phi + h.c. \right) \right]$$

- Note that up and down type quarks can not be simultaneously diagonalised, by convention, the off diagonal terms are attributed to the down type quarks

- After Electroweak symmetry breaking, this term confers mass to the fermions

Standard Model Electroweak Symmetry Breaking

$$\mathscr{L} = \mathscr{L}_{\text{free+interaction}} + \mathscr{L}_{\text{Gauge}} + \mathscr{L}_{\text{Higgs}} + \mathscr{L}_{\text{Yukawa}}$$

- Note that all fields in the SM Lagrangian appear to be massless
- Choosing the μ^2parameter in the Higgs field to be negative triggers symmetry breaking
- The minimum of the Higgs potential is at a distance v from the origin, defined with: $v^2 = -\mu^2/\lambda$
- The field is translated and the Lagrangian is expanded around the minimum:

$$\boldsymbol{\Phi} \to \boldsymbol{\Phi} + \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v \end{pmatrix}$$

$$\boxed{\begin{aligned} \mathrm{Z}_\mu &= \cos\theta_W \boldsymbol{B}_\mu - \sin\theta_W {\boldsymbol{W}^3}_\mu \\ \mathrm{A}_\mu &= \cos\theta_W \boldsymbol{B}_\mu + \sin\theta_W {\boldsymbol{W}^3}_\mu \end{aligned} \quad \text{with } \tan\theta_W = \frac{g'}{g_w}}$$

- New terms appear in the Lagrangian, of particular interest are the mass terms:

$$m_\mathrm{H} = \sqrt{-2\mu^2} = \sqrt{2\lambda v^2}$$

$$m_\ell = \frac{1}{\sqrt{2}} G_\ell v \quad \text{with } \ell = \mathrm{e}, \mu,$$

$$m_q = G_\mathrm{u}^q v \quad \text{with } q = \mathrm{u}, \mathrm{c}, \mathrm{t}$$

Diagonalization of the down-type quark mass terms gives rise to the CKM matrix

$$m_q = G_\mathrm{d}^{ij} v$$

$$m_\mathrm{Z} = \frac{v\sqrt{{g_w}^2 + {g'}^2}}{2} = \frac{m_\mathrm{W}}{\cos\theta_W}$$

$$m_\mathrm{A} = 0$$

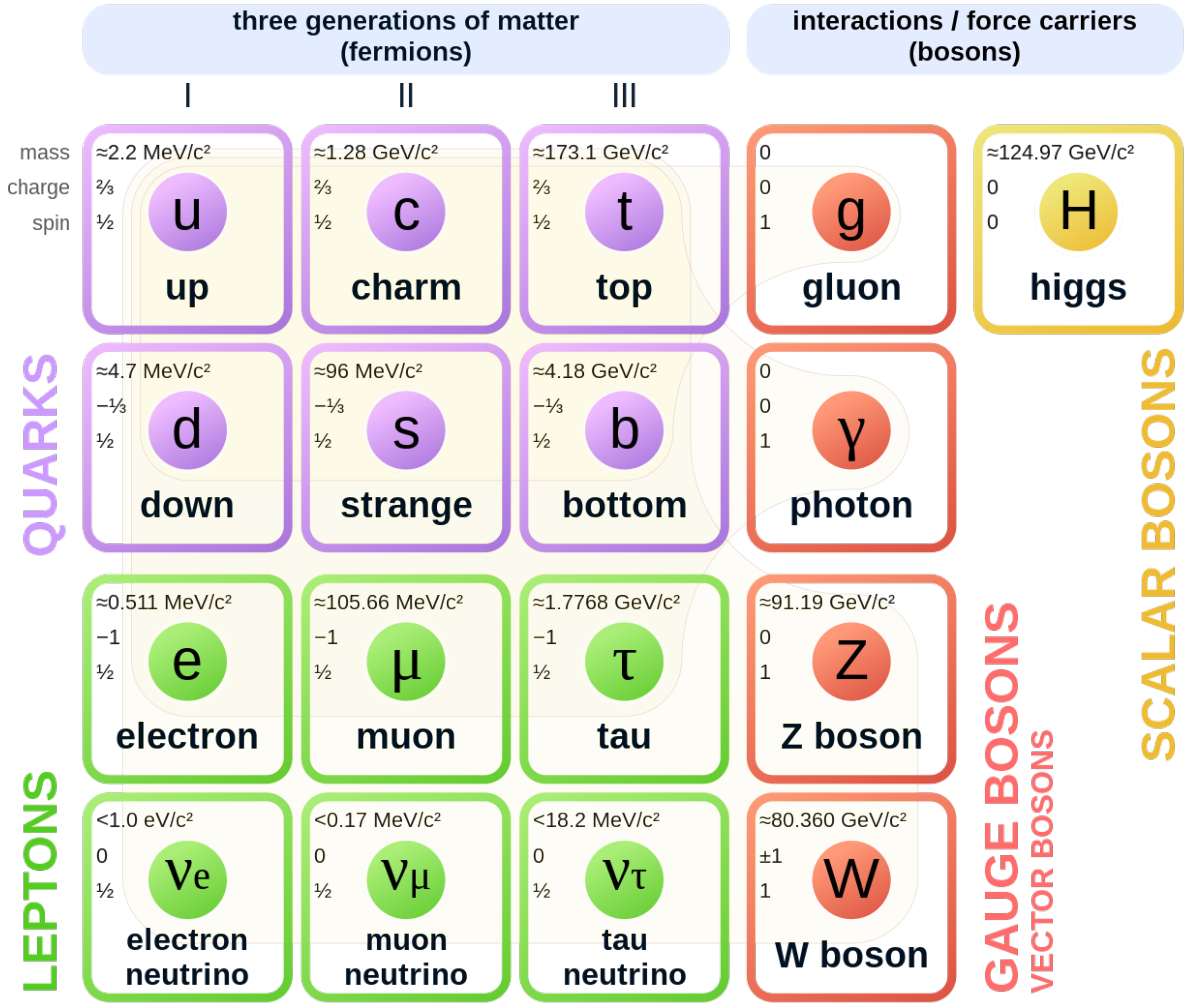
$$m_\mathrm{W} = \frac{v g_w}{2}$$

Standard Model Lagrangian

$$\mathscr{L} = \mathscr{L}_{\text{free+interaction}} + \mathscr{L}_{\text{Gauge}} + \mathscr{L}_{\text{Higgs}} + \mathscr{L}_{\text{Yukawa}}$$

- The Classical SM Lagrangian has 19 free parameters:
  - The three gauge coupling constants: g', g_w, g_s
  - The two parameters of the Higgs potential: λ and μ^2
  - Three Yukawa coupling constants for the three lepton families: G_ℓ with $\ell = \mathrm{e}, \mu, \tau$
  - Six Yukawa coupling constants for the three quark families: G_u^q with $q = \mathrm{u, c, t}$; G_d^q with $q = \mathrm{d, s, b}$
  - Four parameters of the CKM matrix, three angles and a phase
  - QCD theta angle

Particle content of Standard Model



Taken from Wikipedia

The discovery of the Higgs boson in 2012 completes the picture but…
(All SM parameters are now known)

Gauge Fields – Spin 1

Symbol	Associated Charge	Group	Coupling	Representation
B	Weak Hypercharge	$U(1)$	g'	$(\mathbf{1},\mathbf{1},0)$
W^i	Weak Isospin	$SU(2)$	g_w	$(\mathbf{1},\mathbf{3},0)$
G^i	Colour	$SU(3)$	g_s	$(\mathbf{8},\mathbf{1},0)$

Fermion Fields – Spin $\frac{1}{2}$

Symbol	Name	Representation
Q^i_L	Left-handed quark	$\left(\mathbf{3},\mathbf{2},\frac{1}{3}\right)$
$U^{i\,C}_R$	Left-handed antiquark (up)	$\left(\bar{\mathbf{3}},\mathbf{1},-\frac{4}{3}\right)$
$D^{i\,C}_R$	Left-handed antiquark (down)	$\left(\bar{\mathbf{3}},\mathbf{1},\frac{2}{3}\right)$
Ψ^i_L	Left-handed lepton	$(\mathbf{1},\mathbf{2},-1)$
R^C_i	Left-handed antilepton	$(\mathbf{1},\mathbf{1},2)$

Higgs Fields – Spin 0

Symbol	Name	Representation
Φ	Higgs boson	$(\mathbf{1},\mathbf{2},1)$

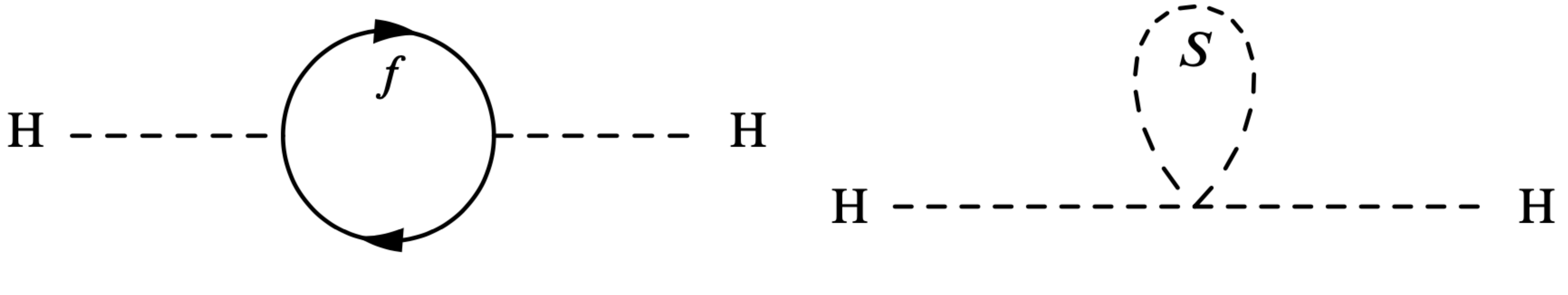
Shortcomings of the Standard Model

- Even though the SM accurately describes phenomena over several orders of magnitude, it is not a complete theory:
  - Neutrino Oscillations → Neutrinos have mass - Possible in the SM, but the exact mechanism is unknown
  - Matter/Anti-matter asymmetry observed in the universe - Complex CKM phase introduces CP violation, but not enough to explain observations
  - Observed matter only ~5% of the mass-energy content of the universe; Dark Matter ~26% and the rest is Dark Energy
  - Gravity not included

Hierarchy Problem

- The Electroweak scale (~250 GeV) is much smaller than the Planck scale. Why such a large difference?

- In an effective theory up to a scale Δ, the one loop corrections to the Higgs mass would be:



(a) Fermion loop **(b)** Scalar loop

$$M_h^2 \sim M_{h0}^2 + \frac{g_F^2}{4\pi^2}\left(\Delta^2 + m_F^2\right) - \frac{g_S^2}{4\pi^2}\left(\Delta^2 + m_S^2\right)$$

- The quadratic contributions diverge with the cutoff scale (Δ) of the effective theory
- At the Planck scale it is expected that gravity will become comparable to the other forces and a quantum theory of gravity would be needed, so the SM can be viewed as an effective theory at the electroweak scale.
- At the Planck scale an incredible fine tuning would be necessary to keep the Higgs mass at 125 GeV
- In other words, why is the Higgs mass unnaturally smaller than its natural theoretical value

Hierarchy Problem

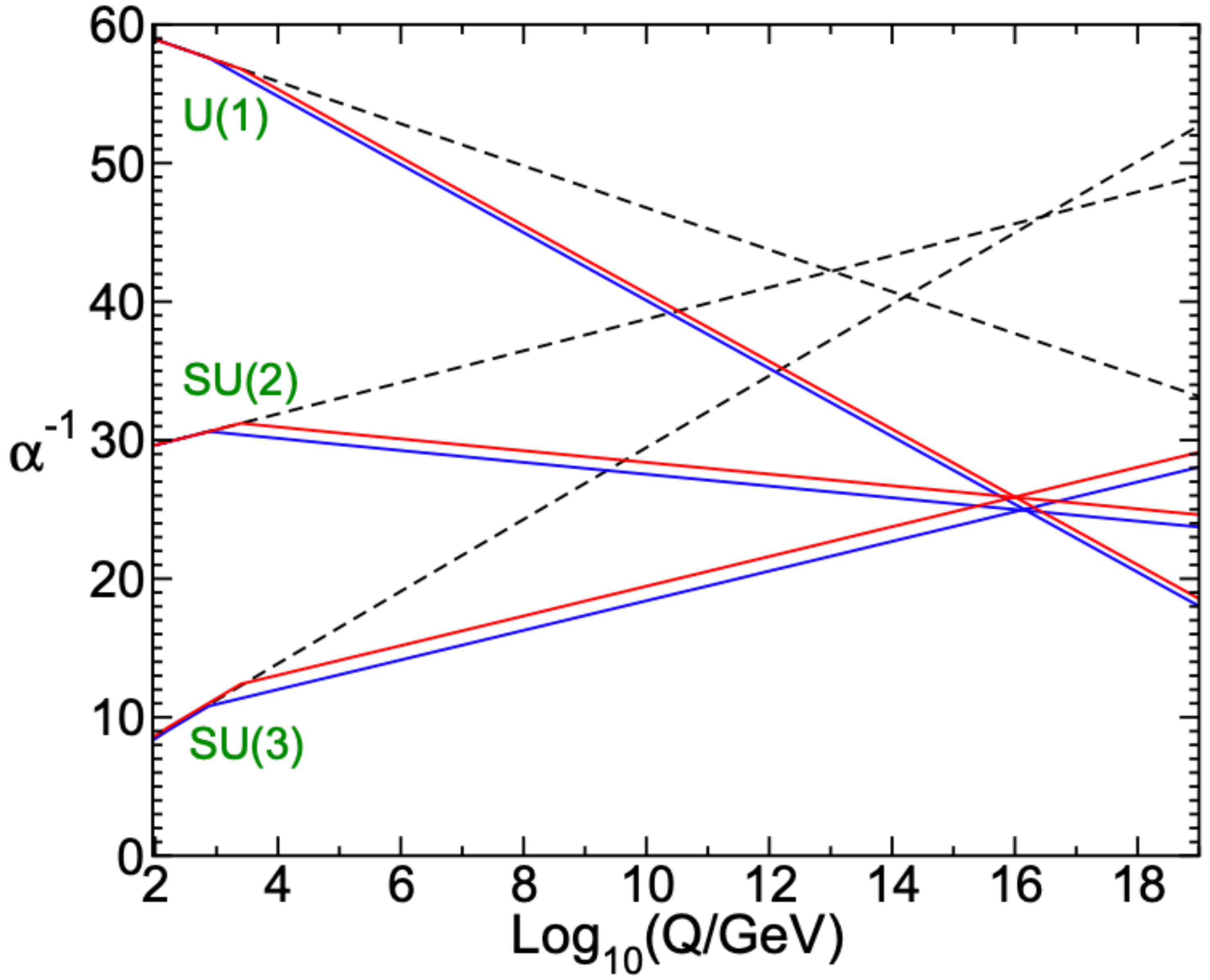
- Formula of the one loop corrections gives a hint to a possible solution:

$$M_h^2 \sim M_{h0}^2 + \frac{g_F^2}{4\pi^2}\left(\Delta^2 + m_F^2\right) - \frac{g_S^2}{4\pi^2}\left(\Delta^2 + m_S^2\right)$$

- If the fermion and scaler had the same coupling, the components that depend on the cutoff scale would cancel each other due to the opposite sign → One of the main motivations for Supersymmetry
- In supersymmetry, there should be a one-to-one correspondence between bosonic fields and fermionic fields
- Known particles are not good candidates for pairings since they do not share quantum numbers, thus do not have the same couplings

Supersymmetry: Other Motivations

- Phenomenology:
  - Dark Matter: In many Supersymmetric Theories, the lightest supersymmetric particle is stable, neutral and a weakly interacting massive particle, thus a good candidate for Dark Matter
  - Gauge coupling unification:
    - Coupling constants scale with energy
    - All 3 can meet at a single point in supersymmetry



Taken from "Supersymmetry Primer"

- Theory:
  - Supersymmetry is the most natural candidate to describe in a unified way not only all known interactions but also all matter and radiation together
  - String theory can only be consistent if it is supersymmetric

Supersymmetry

- Construction of the supersymmetric Lagrangian follows same recipe as the SM Lagrangian:
  - Choose the gauge group G of the symmetries respected by the model:
    $$U(1) \times SU(2) \times SU(3)$$
  - Group SM fields into superfields, 2 SM fields can not be grouped in a single superfield since they do not share quantum numbers:
    - Bosons are associated to vector superfields of the gauge group
    - Chiral superfields to represent fermions
    - Additional chiral superfields to add the scalars necessary for electroweak symmetry breaking and generating the masses of the required bosons
  - Write the most general Lagrangian invariant under G which couples all these fields:
    $$\mathscr{L}_{\text{MSSM}} = \mathscr{L}_{KE} + \mathscr{L}_{\text{interaction}} + \mathscr{L}_{W}$$

Further reading:

- Lectures on Supersymmetry : https://web.archive.org/web/20240202053316/https://people.sissa.it/~bertmat/susycourse.pdf
- Supersymmetry Primer: https://arxiv.org/abs/hep-ph/9709356
- Susy and Such: https://arxiv.org/abs/hep-ph/9612229

Supersymmetry Field Content

Chiral Superfields			
Superfield	Representation	Field Composition	**Particle Names**
$\widehat{Q}^i_L$	$\left(\mathbf{3}, \mathbf{2}, \frac{1}{3}\right)$	$Q^i_L, \widetilde{Q}^i_L$	
$\widehat{U}^{i\,C}_R$	$\left(\bar{\mathbf{3}}, \mathbf{1}, -\frac{4}{3}\right)$	$U^{i\,C}_R, \widetilde{U}^{i\,C}_R$	quark, squark
$\widehat{D}^{i\,C}_R$	$\left(\bar{\mathbf{3}}, \mathbf{1}, \frac{2}{3}\right)$	$D^{i\,C}_R, \widetilde{D}^{i\,C}_R$	
$\widehat{\Psi}^i_L$	$(\mathbf{1}, \mathbf{2}, -1)$	$\Psi^i_L, \widetilde{\Psi}^i_L$	lepton, slepton
$\widehat{R}^C_i$	$(\mathbf{1}, \mathbf{1}, 2)$	$R^C_i, \widetilde{R}^C_i$	
$\widehat{\Phi}_1$	$(\mathbf{1}, \mathbf{2}, 1)$	$\Phi_1, \widetilde{\Phi}_1$	higgs, higgsino
$\widehat{\Phi}_2$	$(\mathbf{1}, \mathbf{2}, -1)$	$\Phi_2, \widetilde{\Phi}_2$	
Vector Superfields			
Superfield	Representation	Field Composition	
$\widehat{B}$	$(\mathbf{1}, \mathbf{1}, 0)$	$B, \widetilde{B}$	B boson, bino
$\widehat{W}^i$	$(\mathbf{1}, \mathbf{3}, 0)$	$W^i, \widetilde{W}^i$	W boson, wino
$\widehat{G}^i$	$(\mathbf{8}, \mathbf{1}, 0)$	$G^i, \widetilde{G}^i$	gluon, gluino

Higgs Superfields

- The Higgs field is the scalar part of a chiral superfield, however there is a fermion superpartner, an SU(2) doublet
- The fermion superpartner contributes to the triangle anomalies, which would be uncancelled
- Easiest way to remove the anomaly is by introducing a second higgs superfield with opposite weak hypercharge
- Gives supersymmetry 2 Higgs superfields and a very rich Higgs sector

Supersymmetry Lagrangian

$$\mathscr{L}_{\mathrm{MSSM}} = \mathscr{L}_{KE} + \mathscr{L}_{\mathrm{interaction}} + \mathscr{L}_{W}$$

- The “Kinetic Energy” term is summed over all the superfields (chiral and vector) and is analogous to the “free+interaction” term, “Gauge” term and the first part of the “Higgs” term from the SM Lagrangian

$$\mathscr{L}_{KE} = \sum_i \left\{ \left(D_\mu S_i^*\right)\left(D^\mu S_i\right) + i\bar{\psi}_i\gamma^\mu D_\mu \psi_i \right\} \quad \text{Chiral Superfields}$$

$$+ \sum_A \left\{ -\frac{1}{4} F_{\mu\nu}{}^A F^{\mu\nu A} + \frac{i}{2}\bar{\lambda}_A \gamma^\mu D_\mu \lambda_A \right\} \quad \text{Vector Superfields}$$

- Consequently, this term describes all free particles in supersymmetry as well as the interactions with the gauge bosons

- This term contains most of the SM, only missing electroweak symmetry breaking terms from Higgs and the Yukawa terms

Supersymmetry Lagrangian

$$\mathscr{L}_{\text{MSSM}} = \mathscr{L}_{KE} + \mathscr{L}_{\text{interaction}} + \mathscr{L}_{W}$$

- The "Interaction" term describes the interactions between the chiral superfields and the gauginos as well as the quartic interactions of the scalers:

$$\mathscr{L}_{\text{interaction}} = -\sqrt{2}\sum_{i,A} g_A \left[S_i^* T^A \bar{\psi}_i \lambda_A + h.c.\right] - \frac{1}{2}\sum_A \left(\sum_i g_A S_i^* T^A S_i\right)^2$$

New Interactions in Supersymmetry

Supersymmetry Lagrangian

$$\mathscr{L}_{\mathrm{MSSM}} = \mathscr{L}_{KE} + \mathscr{L}_{\mathrm{interaction}} + \mathscr{L}_{W}$$

- The "W" term results from the W superpotential. This superpotential is only a function of the chiral superfields and contains terms with 2 and 3 fields. This langrangian term contains the yukawa couplings and scalar field of the SM:

$$\mathscr{L}_W = -\sum_i \left|\frac{\partial W}{\partial z_i}\right|^2 - \frac{1}{2}\sum_{i,j}\left[\bar{\psi}_i \frac{\partial^2 W}{\partial z_i \partial z_j}\psi_j + h.c.\right]$$

- Most general superpotential, for a single family, is (in a more general approach, λ_i could be matrices):

$$W = \epsilon_{ij}\mu\widehat{\Phi_1^i}\widehat{\Phi_2^j} + \epsilon_{ij}\left[\lambda_L\widehat{\Phi_1^i}\widehat{\Psi_L^j}\widehat{R}^C + \lambda_D\widehat{\Phi_1^i}\widehat{Q_L^j}\widehat{D_R}^C + \lambda_U\widehat{\Phi_2^j}\widehat{Q_L^i}\widehat{U_R}^C\right]$$
$$+\epsilon_{ij}\left[\lambda_1\widehat{\Psi_L^i}\widehat{\Psi_L^j}\widehat{R}^C + \lambda_2\widehat{\Psi_L^i}\widehat{Q_L^j}\widehat{D_R}^C\right] + \lambda_3\widehat{U_R}^C\widehat{D_R}^C\widehat{D_R}^C$$

- The first term, $\mu\widehat{\Phi_1}\widehat{\Phi_2}$, gives rise to the Higgs mass term and thus electroweak symmetry breaking

- The terms proportional to $\lambda_L,\ \lambda_D$ and λ_U give rise to the Yukawa terms from the SM

- The terms proportional to $\lambda_1,\ \lambda_2$ and λ_3 are problematic and give rise to lepton and baryon number violation → One way to handle is by introduction of R-parity

R-Parity

- R-Parity is defined as a multiplicative quantum number where all particles of the SM have R=+1 and all their SUSY partners have R=-1; it can also be defined as:

$$R \equiv (-1)^{3(B-L)+s}$$

- Consequences:
  - Number of SUSY particles is conserved modulo 2
    - SUSY particles are always pair produced from SM particles (i.e. in colliders)
    - SUSY particle must decay to at least one other SUSY particle
  - There will be a stable lightest supersymmetric particle

Supersymmetry Lagrangian

- The Lagrangian presented so far has all the characteristics necessary for a supersymmetric gauge theory which exhibits spontaneous electroweak symmetry breaking
- However, in the current formulation, the supersymmetric particles exhibit the same mass as their partners → Supersymmetry would have already been discovered
- Observations show this is not the case, in fact no supersymmetric particle has yet been observed → Supersymmetry must be a broken symmetry
  - Breaking mechanism is unknown, but several candidates, such as mSUGRA
  - For now, will use the Minimal Supersymmetric Standard Model (MSSM) approach; use an effective Lagrangian with no assumption on the breaking mechanism itself

MSSM Lagrangian

- Introduce a "Soft" supersymmetry breaking term into the Lagrangian such as with mass terms for the scalar members of the chiral superfields and for the gaugino members of the vector superfields

$$\mathscr{L}_{\text{MSSM}} = \mathscr{L}_{KE} + \mathscr{L}_{\text{interaction}} + \mathscr{L}_{W} + \mathscr{L}_{\text{soft}}$$

- Termed soft because they break the supersymmetry but not so much as to re-introduce the quadratic divergence which motivated Supersymmetry to start

$$\begin{aligned}
-\mathscr{L}_{\text{soft}} = {} & m_1^2 \left|H_1\right|^2 + m_2^2 \left|H_2\right|^2 - B\mu\epsilon_{ij}\left(H_1^i H_2^j + h.c.\right) + \tilde{M}_Q^2 \widetilde{Q_L}^* \widetilde{Q_L} \\
& + \tilde{M}_U^2 \widetilde{U_R}^* \widetilde{U_R} + \tilde{M}_D^2 \widetilde{D_R}^* \widetilde{D_R} + \tilde{M}_\Psi^2 \widetilde{\Psi_L}^* \widetilde{\Psi_L} + \tilde{M}_R^2 \widetilde{R}^* \widetilde{R} \\
& + \frac{1}{2}\left[M_3 \widetilde{G^i}^C \widetilde{G^i} + M_2 \widetilde{W^i}^C \widetilde{W^i} + M_1 \widetilde{B}^C \widetilde{B}\right] + \frac{g}{\sqrt{2}M_W}\epsilon_{ij}\left[\frac{M_D}{\cos\beta} A_D H_1^i \widetilde{Q_L^j} \widetilde{D_R}^* \right. \\
& \left. + \frac{M_U}{\sin\beta} A_U H_2^j \widetilde{Q_L^i} \widetilde{U_R}^* + \frac{M_R}{\cos\beta} A_E H_1^i \widetilde{\Psi_L^j} \widetilde{R}^* + h.c.\right]
\end{aligned}$$

- Mass terms for scalars, gauginos
- bi-linear and tri-linear mixing terms
- Each factor may be a matrix mixing families

MSSM Lagrangian

- Total of 124 free parameters:
    - 18 analogous to the SM
    - the rest, mostly introduced by the Soft supersymmetry breaking term

MSSM - Higgs Sector

- The second Higgs field in Supersymmetry lends the model a rich phenomenology in the Higgs sector, similar to a “Two Higgs Doublet Model”
- One of the Higgs couples to down-type quark fields and lepton fields and the other one couples to up-type quark fields
- Of the 8 degrees of freedom, 3 are absorbed to give mass to the W and Z bosons, resulting in 5 physical degrees of freedom which produce 5 bosons: 2 CP-even neutral Higgs bosons (h and H), one CP-odd Higgs boson (A) and 2 charged Higgs bosons
- Each Higgs field has its own VEV:

$$\begin{aligned} \langle H_1^0 \rangle &\equiv v_1 \\ \langle H_2^0 \rangle &\equiv v_2 \end{aligned} \xrightarrow[\text{Breaking}]{\text{EWK Symmetry}} M_W^2 = \frac{g^2}{2}(v_1^2 + v_2^2)$$

- Ratio of the two VEV is an important MSSM parameter: $\tan\beta \equiv \dfrac{v_2}{v_1}$

Additional reading:

- Susy and Such: https://arxiv.org/abs/hep-ph/9612229
- http://dx.doi.org/10.1016/0550-3213(79)90225-6

MSSM - Sfermion Sector

- There is a complex scalar superpartner field for each chiral state of SM field
- Tri-linear soft SUSY terms allow the complex scaler superpartners to mix when forming the mass eigenstates
- Mixing results in a 6x6 matrix (one for lepton SUSY partners, one for up-type SUSY partners and one for down-type SUSY partners)
- Focussing on the top squark sector, the left- and right-handed top squark mixing is given by:

$$M_{\tilde{t}}^2 = \begin{pmatrix} \tilde{M}_Q^2 + M_T^2 + M_Z^2(\frac{1}{2} - \frac{2}{3}\sin^2\theta_W)\cos 2\beta & M_T(A_T + \mu\cot\beta) \\ M_T(A_T + \mu\cot\beta) & \tilde{M}_U^2 + M_T^2 + \frac{2}{3}M_Z^2\sin^2\theta_W\cos 2\beta \end{pmatrix}$$

- The off-diagonal terms, the mixing effect, are proportional to the mass of the particle → lightest stop is often the lightest squark (similar for sbottom and stau)

MSSM - Chargino Sector

- There are 2 charge 1, spin 1/2 sfermions: wino (partner of charged W boson) and higgsino (partner of the charged Higgs boson)

- Physical mass states are formed from a linear combination of these states and are called charginos

$$M_{\tilde{\chi}^{\pm}} = \begin{pmatrix} M_2 & \sqrt{2}M_W \sin\beta \\ \sqrt{2}M_W \cos\beta & -\mu \end{pmatrix}$$

MSSM - Neutralino Sector

- There are 4 neutral sfermions: Bino (partner of the B boson), wino (partner of the neutral W boson), 2 higgsinos (partners of the neutral higgs bosons)

- Physical mass states are formed from a linear combination of these states and are called neutralinos

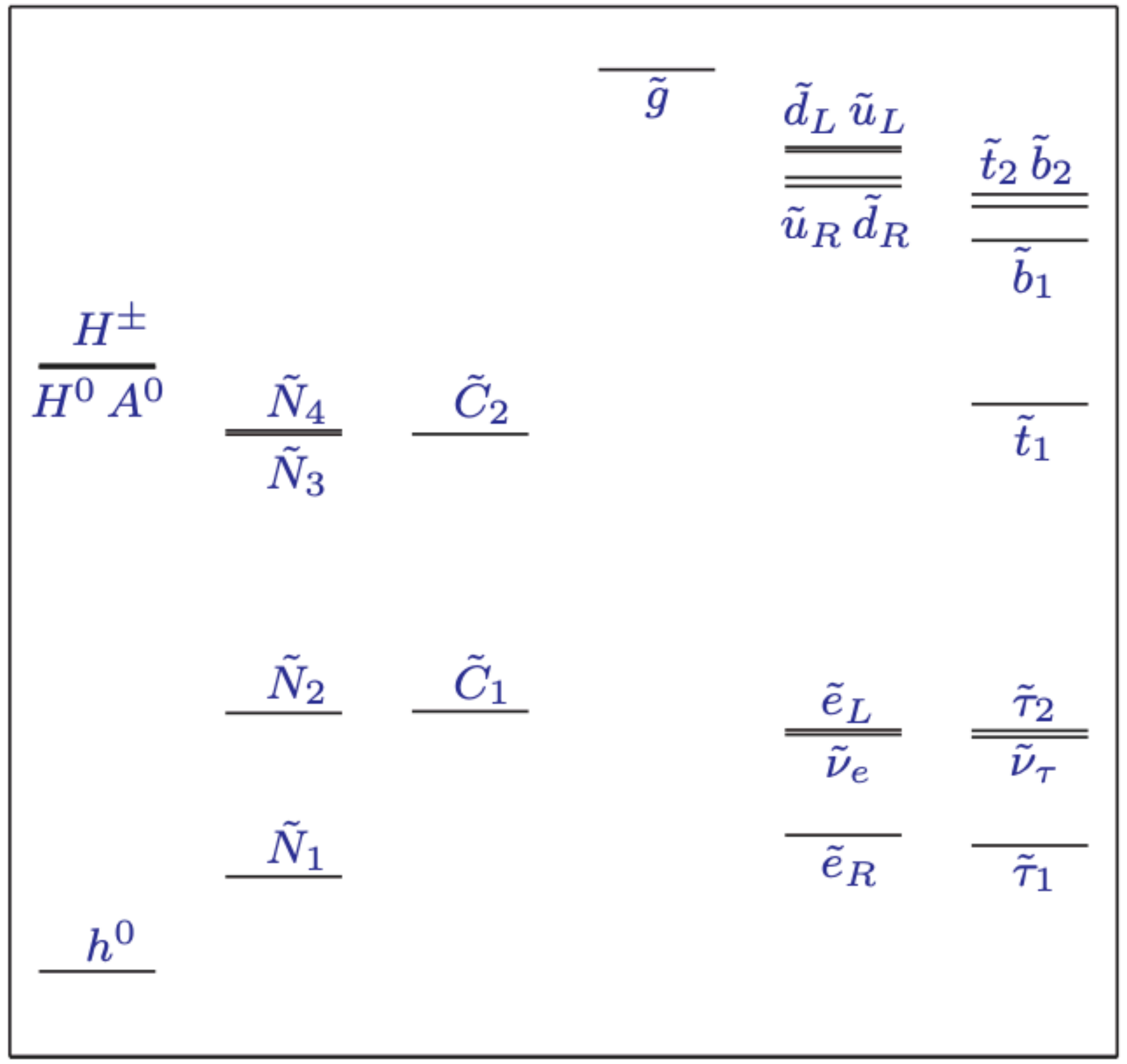
$$M_{\tilde{\chi}_i^0} = \begin{pmatrix} M_1 & 0 & -M_Z\cos\beta\sin\theta_W & M_Z\sin\beta\sin\theta_W \\ 0 & M_2 & M_Z\cos\beta\cos\theta_W & -M_Z\sin\beta\cos\theta_W \\ -M_Z\cos\beta\sin\theta_W & M_Z\cos\beta\sin\theta_W & 0 & \mu \\ M_Z\sin\beta\sin\theta_W & -M_Z\sin\beta\cos\theta_W & \mu & 0 \end{pmatrix}$$

- Neutralinos do not necessarily correspond to a photino (partner of the photon) or a zino (partner of the Z boson)

- The lightest neutralino is often assumed to be the lightest supersymmetric particle

Supersymmetry Particle Spectra

- The parameters of the model are chosen, then the supersymmetric particle masses can be computed and the spectra is drawn →
- SuSpect (http://suspect.in2p3.fr/) is a tool that can do this process for us
  - User writes an SLHA file describing the Supersymmetry model to use and relevant parameters and provides it to SuSpect
  - SuSpect outputs a more complete SLHA file containing all computed quantities, such as mass states of all the particles
  - SLHA files can be used with MC generators to simulate events, detector responses can be simulated as well. In this way MC samples for a given SUSY scenario can be created



Taken from "Supersymmetry Primer"